MHD turbulence in the solar corona and solar wind

Pablo Dmitruk

Departamento de Física, FCEN, Universidad de Buenos Aires

Motivations

- The role of MHD turbulence in several phenomena in space and solar physics (plasmas) $\rightarrow$ stars, Sun, solar corona, solar wind, interstellar space, magnetospheres, Earth interior

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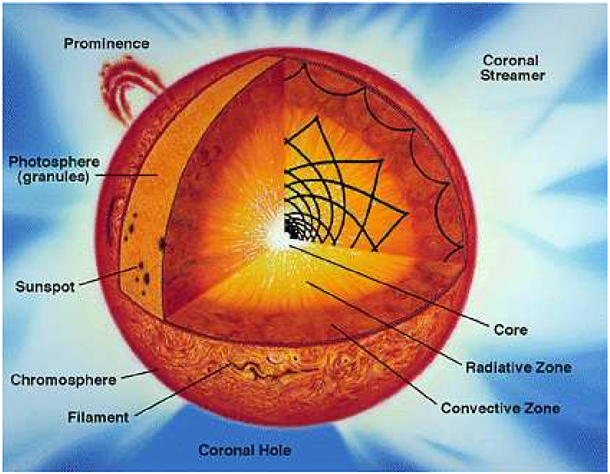
- Waves and turbulence: coexist ?
- Low-frequency fluctuations $\longrightarrow$ $1/f$ noise, magnetic field reversals

Basic structure of the Sun and the corona



$$T_{\mathrm{core}} \sim 14\ 10^6 K$$

$$T_{\mathrm{radiative}} \sim 10^5 K$$

$$T_{\mathrm{photosphere}} \sim 6\ 10^3 K$$

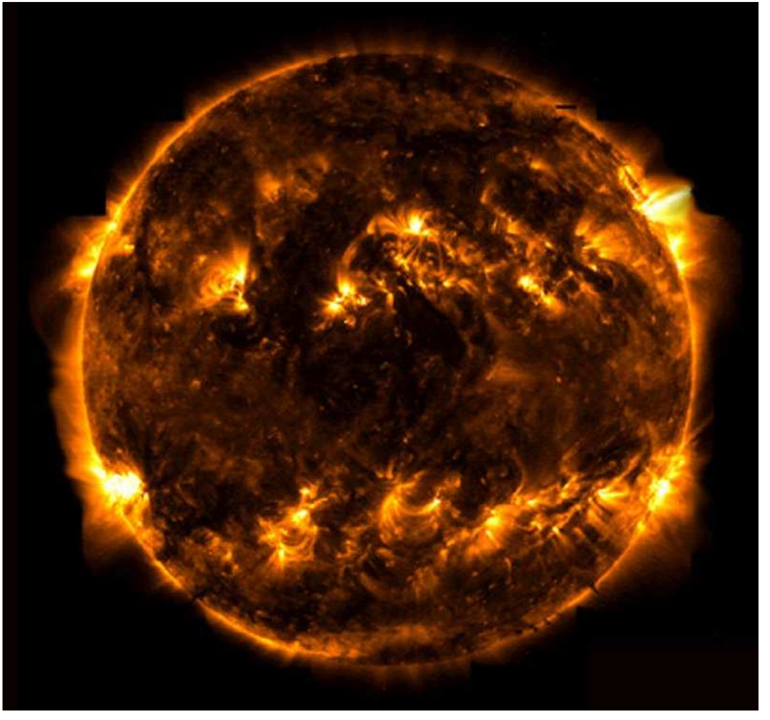
$$T_{\mathrm{corona}} \sim 2\ 10^6 K$$

Corona: external layer of Sun, atmosphere (heliosphere), from solar surface into interplanetary space. **Plasma**, electrons and ions gas, in motion, **electric currents** and **magnetic fields**.

Corona emits radiation, however its emision in the visible range is small as compared to the solar surface.

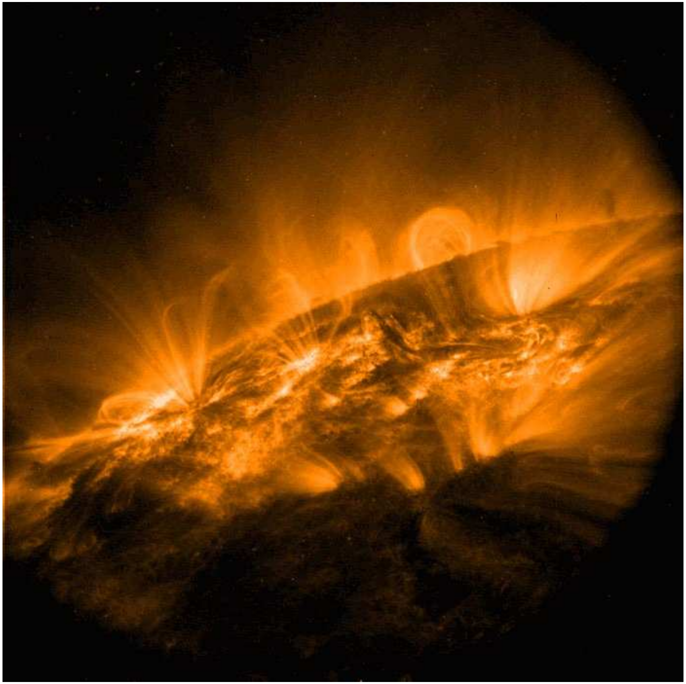
Corona in white light (in an eclipse)

Corona in UV (TRACE)



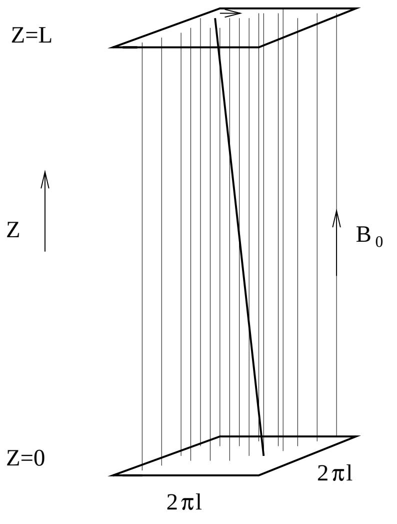
Magnetic loops

loops

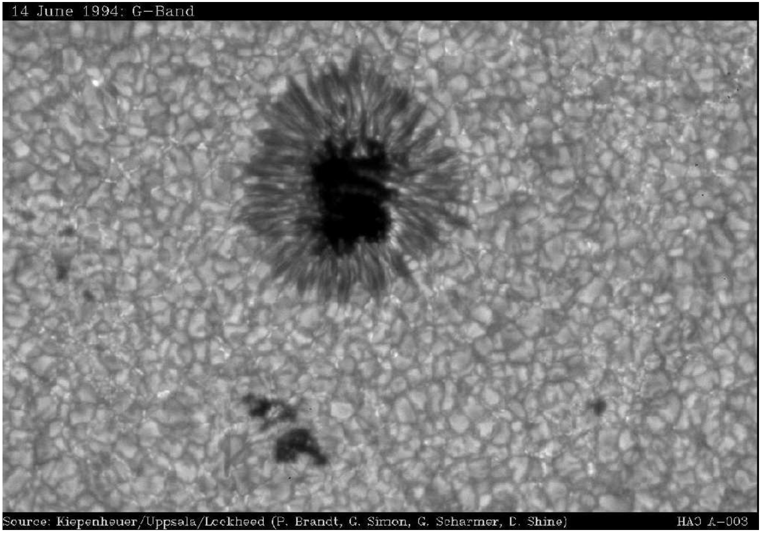


Very structured magnetic field, in a broad range of length scales.

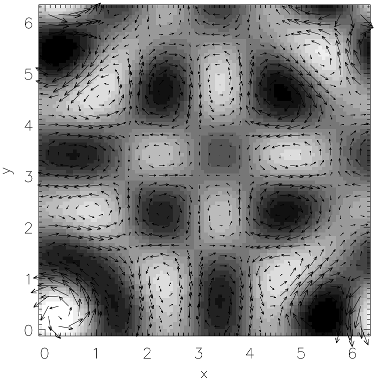
Magnetic loop: simple model



Photosphere granulation (movements in the loop footpoints)



Velocity at the loop footpoints



$l_p \sim 1000$ km

$t_p \sim 1000$ s

MagnetoHydroDynamics

Macroscopic description of a plasma: flows + electric currents

Fluid equations (Navier-Stokes) + Electrodynamics (Maxwell eqs, Ohm's law)

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$$\mathbf{u} = \mathbf{u}(x, y, z, t) = \text{plasma velocity}$$

$$\mathbf{B} = \mathbf{B}(x, y, z, t) = \text{magnetic field}$$

$$\rho(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}) = -\nabla p + \mathbf{J} \times \mathbf{B} + \rho\nu\nabla^2\mathbf{u}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta\nabla^2\mathbf{B}$$

$$\mathbf{J} = \nabla \times \mathbf{B} = \text{current density}, \quad \nabla \cdot \mathbf{B} = 0 \quad , \quad \nabla \cdot \mathbf{u} = 0$$

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Magnetic field tied to velocity field (frozen-in condition)

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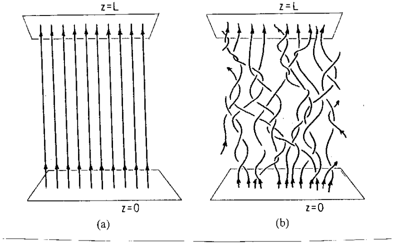

PROBLEM: $t_{\rm diss} \sim l^2/\eta$

but $\eta << 1$ in the corona and $l \sim 1000$ km in the loop footpoints producing the twist of the magnetic field

$$t_{\rm diss} \sim 10^4 \text{ years !!}$$

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$\rightarrow$ We need a mechanism to "accelerate" the energy dissipation

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Energy transfer from large to small scales, which enhances dissipative processes and mixing: **turbulent cascade**.

$$\mathbf{u} = \sum_{\mathbf{k}} \mathbf{u}_{\mathbf{k}} \mathrm{e}^{i\ \mathbf{k}\cdot\mathbf{r}}\ ,\ \ \mathbf{B} = \sum_{\mathbf{k}} \mathbf{B}_{\mathbf{k}} \mathrm{e}^{i\ \mathbf{k}\cdot\mathbf{r}}$$

$$\mathrm{non-linear\ terms:}\ \ \mathbf{u}\cdot\nabla\mathbf{u}\ ,\ \ \mathbf{J}\times\mathbf{B}\ ,\ \ \nabla\times(\mathbf{u}\times\mathbf{B}) \rightarrow \mathrm{mode-mode\ coupling}$$

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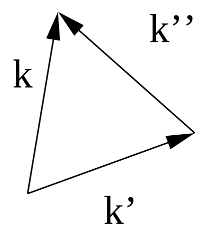
Involves convolution sums

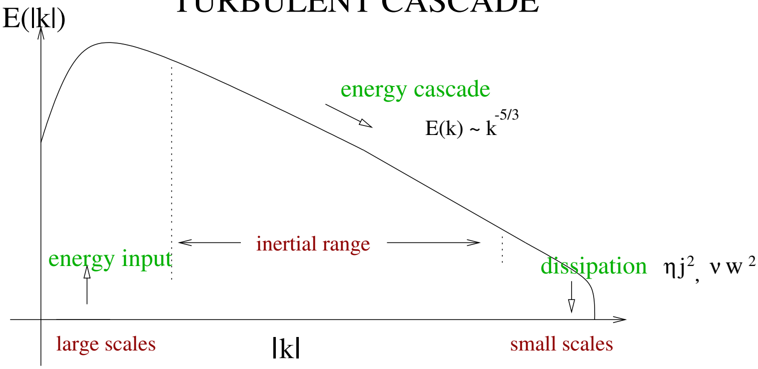
$$\frac{\partial \mathbf{u}_{\mathbf{k}}}{\partial t} \sim \sum_{\mathbf{k}'+\mathbf{k}''=\mathbf{k}} (\mathbf{u}_{\mathbf{k}'}\cdot i\mathbf{k}'')\mathbf{u}_{\mathbf{k}''}$$

with coupling **triads**

$$\mathbf{k} = \mathbf{k}' + \mathbf{k}''$$



TURBULENT CASCADE



Turbulence transfers energy from large to small scales

$$k \sim \frac{1}{l}, \quad \text{large } k \rightarrow \text{small } l \quad \text{If } l \text{ small enough} \rightarrow t_{\text{diss}} \sim \text{hs}$$

Reynolds number: turbulence

$$R_l \sim \frac{\mathrm{non-linear}}{\mathrm{dissipative}} \sim \frac{(\mathbf{u}\cdot\nabla)\mathbf{u}}{\nu\nabla^2\mathbf{u}} \sim \frac{u_l\ l}{\nu}$$

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$l_d = l$ for which non-linear term $\sim$ dissipative term, that is $R_{l_d} \sim 1$

$$\rightarrow l_d \sim \frac{\nu}{u_{l_d}} \sim \frac{\nu^{3/4}}{\epsilon^{1/4}}$$

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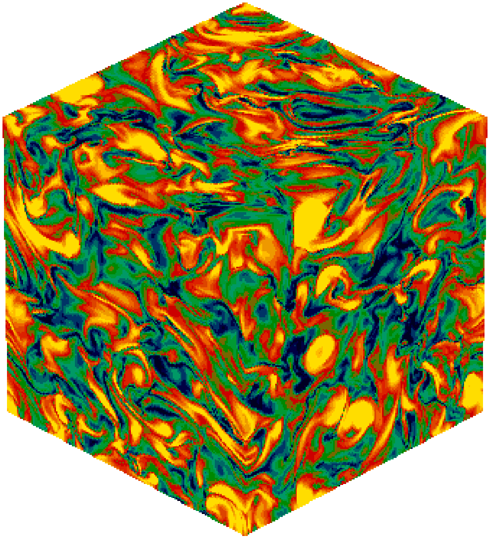
Number of modes $\sim R^{9/4} \sim 10^{23}$!!

We use **pseudospectral** method codes to accurately solve (through Direct Numerical Simulations) the MHD equations.

3D MHD turbulence Direct Numerical Simulation



B

But something happens if there is already a background magnetic field $\mathbf{B_0}$, large compared to magnetic field fluctuations $\mathbf{b}$, that is $\mathbf{B} = \mathbf{B_0} + \mathbf{b}$, $B_0 >> b$, $\rightarrow$ **anisotropy**.

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$$k_{\|}\ B_0\ b_k + k_{\perp}\ b_k\ b_k$$

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The first term is a “wave-like” term, the second term is a non-linear term. Non-linearities dominate for wave vectors for which

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For large $B_0 >> b_k$ this means $k_{\|} << k_{\perp}$, so $\nabla_{\|} << \nabla_{\perp}$.

Another way of looking at this is recalling mode-mode couplings imply wavenumber triad interactions

$$\mathbf{k} = \mathbf{k}' + \mathbf{k}''$$

But if we think on term of interacting Alfven waves (which we will see later have to be of opposite travelling direction), with e^{-iwt} time dependence, we also must have

$$\pm w(\mathbf{k}) = w(\mathbf{k}') - w(\mathbf{k}'')$$

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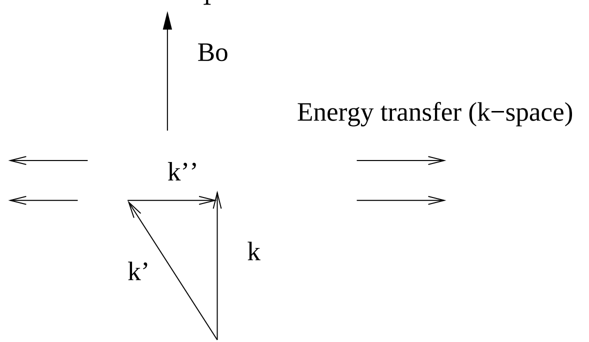
This imply that one of either $\mathbf{k}'$ or $\mathbf{k}''$ have zero parallel component along the background magnetic field direction and so the k-parallel component of $\mathbf{k}$ can not increase. In other words, the (energy) transfer in the k-parallel direction is "inhibited".



Reduced MHD (RMHD)

$$\mathbf{B} = B_0\ \hat{z} + \mathbf{b}(x, y, z, t), \qquad B_0 >> b$$

$$\mathbf{b} = \mathbf{b}_\perp(x, y, z, t), \qquad \mathbf{u} = \mathbf{u}_\perp(x, y, z, t) \quad \text{and} \quad \partial_z << \partial_{x,y}$$

and we also eliminate fast ∂_t variations (acoustic waves)

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We can put

$$\mathbf{b}_\perp = \nabla_\perp a(x, y, z, t) \times \hat{z} = \partial_y a\ \hat{x} - \partial_x a\ \hat{y}$$

$$\mathbf{u}_\perp = \nabla_\perp \psi(x, y, z, t) \times \hat{z} = \partial_y \psi\ \hat{x} - \partial_x \psi\ \hat{y}$$

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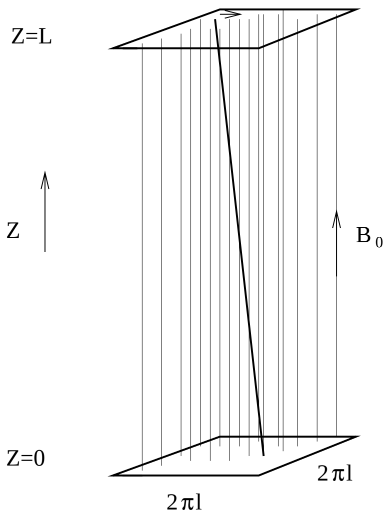
The 3D MHD equations become:

$$\partial_t a = B_0 \partial_z \psi + [\psi, a] + \eta \nabla_\perp^2 a$$

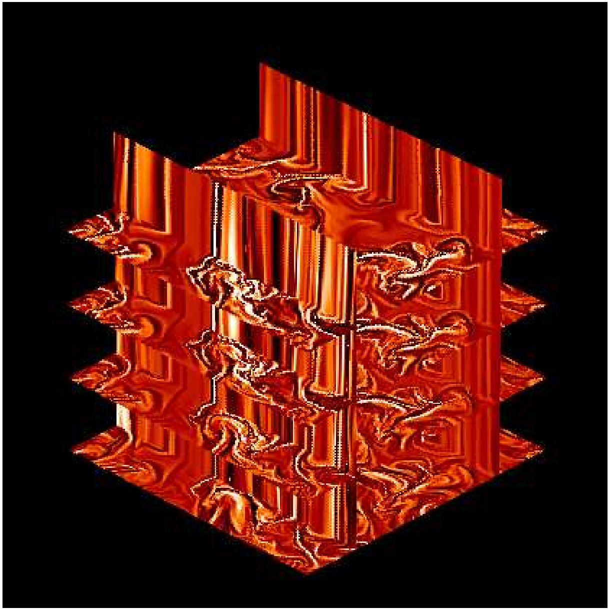
$$\partial_t \omega = B_0 \partial j + [\psi, w] - [a, j] + \nu \nabla_\perp^2 \omega$$

with $j = J_z$ the current density, $\omega = \omega_z$ the vorticity, and $[\psi, a] = \partial_x \psi \partial_y a - \partial_y \psi \partial_x a$.

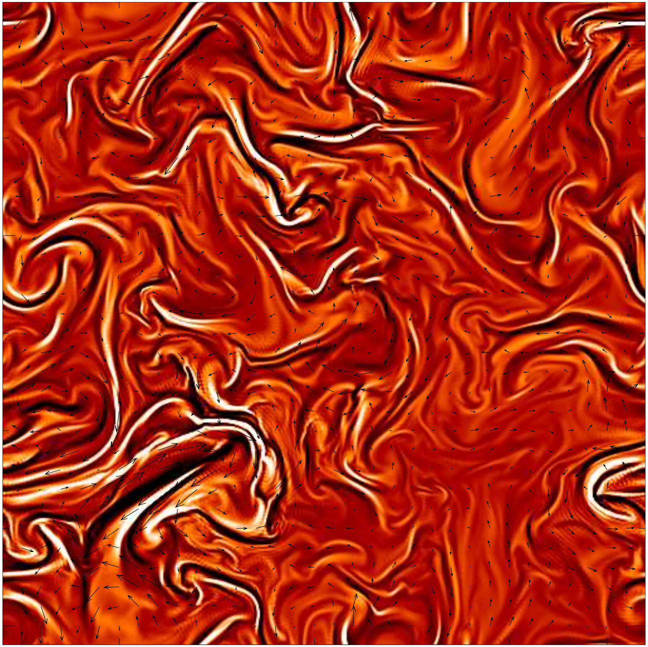
There is a clear numerical advantage on using RMHD equations vs the full 3D equations (for instance, we can increase the resolution in the perpendicular directions vs the parallel direction).



Current density inside a magnetic loop (numerical simulation)

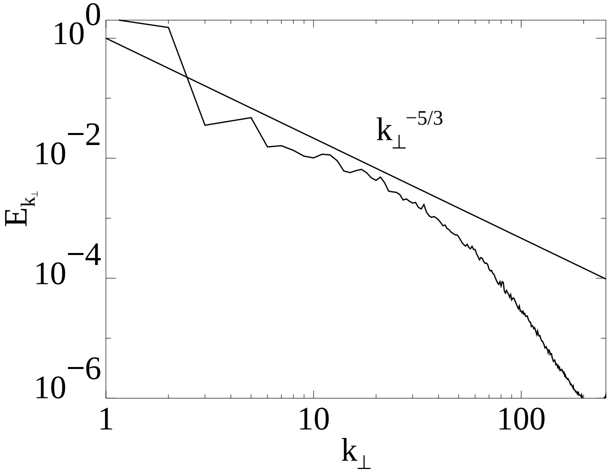


Current density and perpendicular magnetic field in a cross section of the magnetic loop



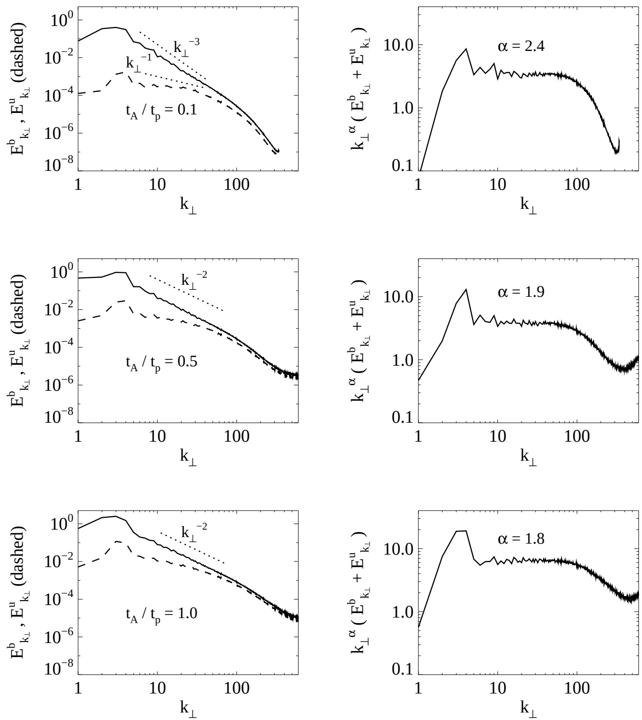
moviehd

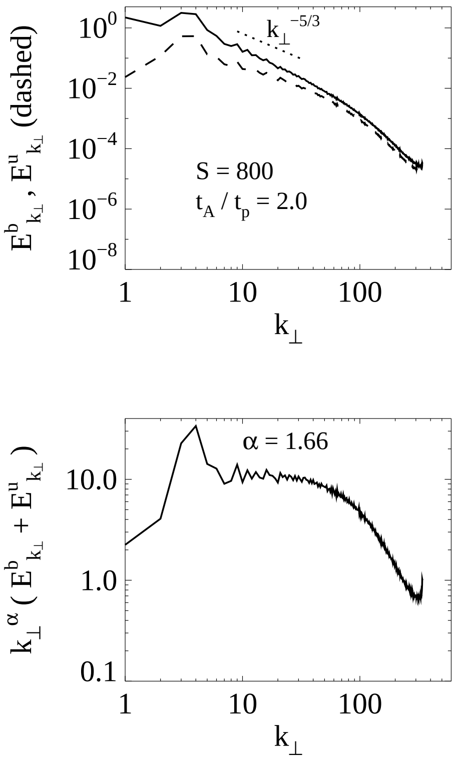
Energy spectrum



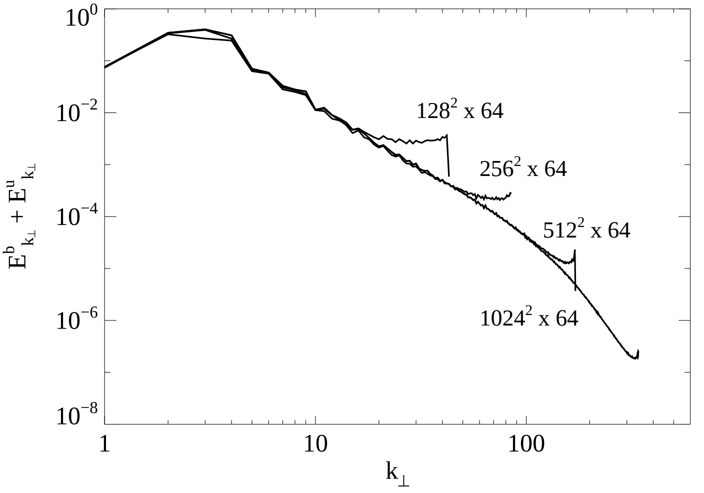
Consistent with Kolmogorov spectrum.

Energy spectrum depends on t_A/t_p (not universal)



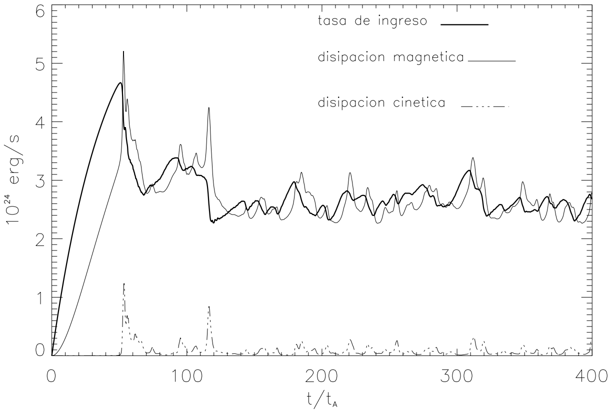


$k_\perp^{-5/3}$
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
$E^b_{k_\perp}, E^u_{k_\perp}$ (dashed)
S = 800
t_A / t_p = 2.0
1
10
100
$k_\perp$
α = 1.66
10.0
1.0
0.1
$k_\perp^\alpha (E^b_{k_\perp} + E^u_{k_\perp})$
1
10
100
$k_\perp$



10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
1
10
100
$k_\perp$
$E^b_{k_\perp} + E^u_{k_\perp}$
128^2 x 64
256^2 x 64
512^2 x 64
1024^2 x 64

dissipation rate

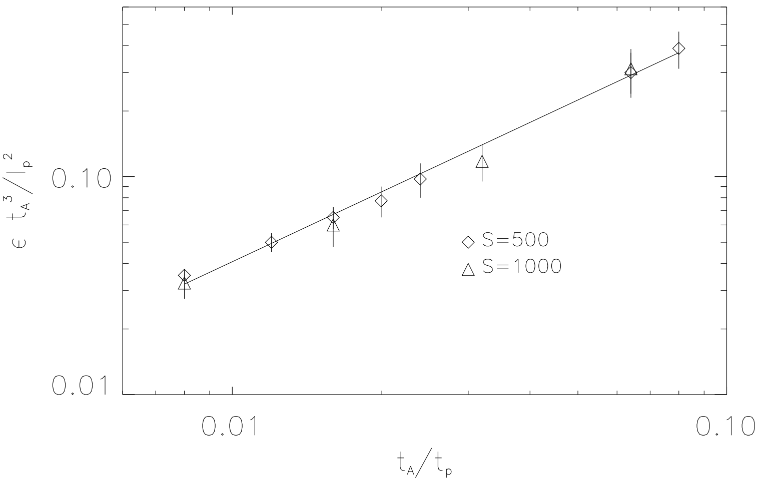


$t_A = L_{\|}/v_A =$ Alfven time, ≈ 20 s ($v_A = B_0/\sqrt{4\pi\rho}$=Alfven velocity)

dissipation rate consistent with observational value 10^7 erg/cm^2/s

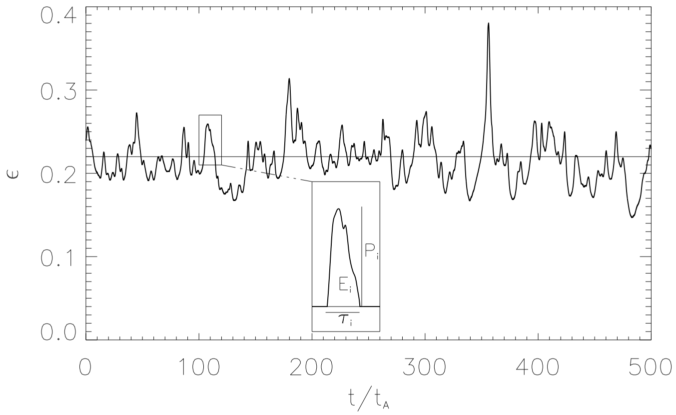
Initial transient 50 $t_A \approx 1000$ s
Fluctuations timescale 10 $t_A \approx 200$ s

Scaling law

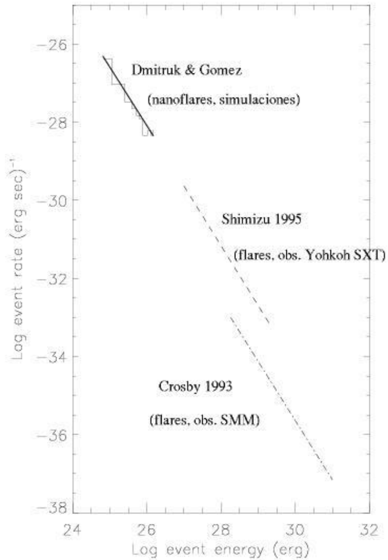


$$\epsilon \sim \frac{l_p^2}{t_A^3}\left(\frac{t_A}{t_p}\right)^q \quad , \quad q \approx \frac{3}{2}$$

Statistics of dissipation events (**nanoflares**)



$$\Delta E \sim 10^{24} \text{ erg}$$



$$N(E) \sim E^{-1.5}$$

Self-organized criticality (SOC)