Black Hole Thermodynamics: some old facts and questions and how one approaches them in LQG

Alejandro Perez
Centre de Physique Théorique, Marseille, France

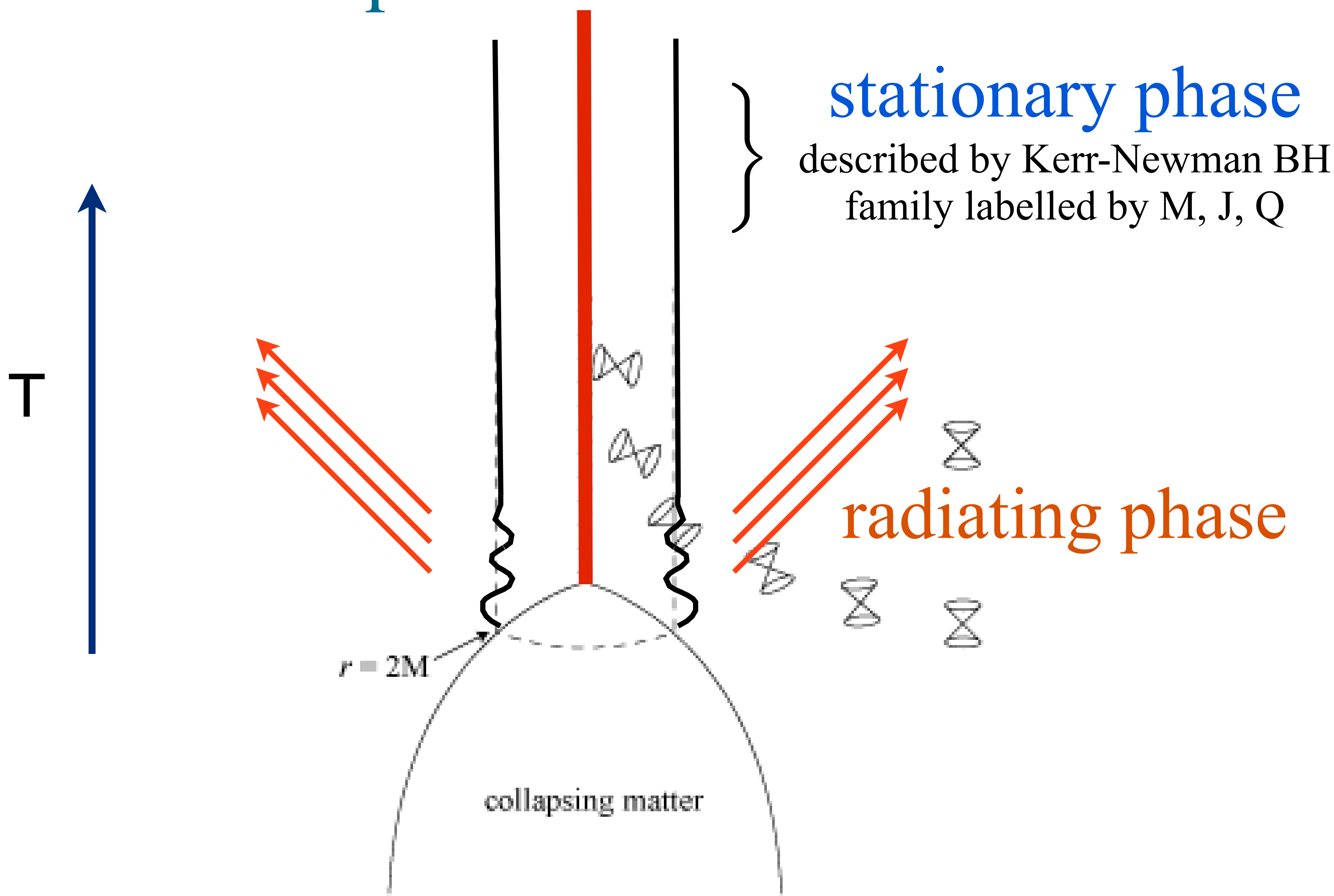
The new stuff is based on results obtained in collaboration with Amit Ghosh, Ernesto Frodden, and Karim Noui

Black Hole Mechanics

analogy with thermodynamics

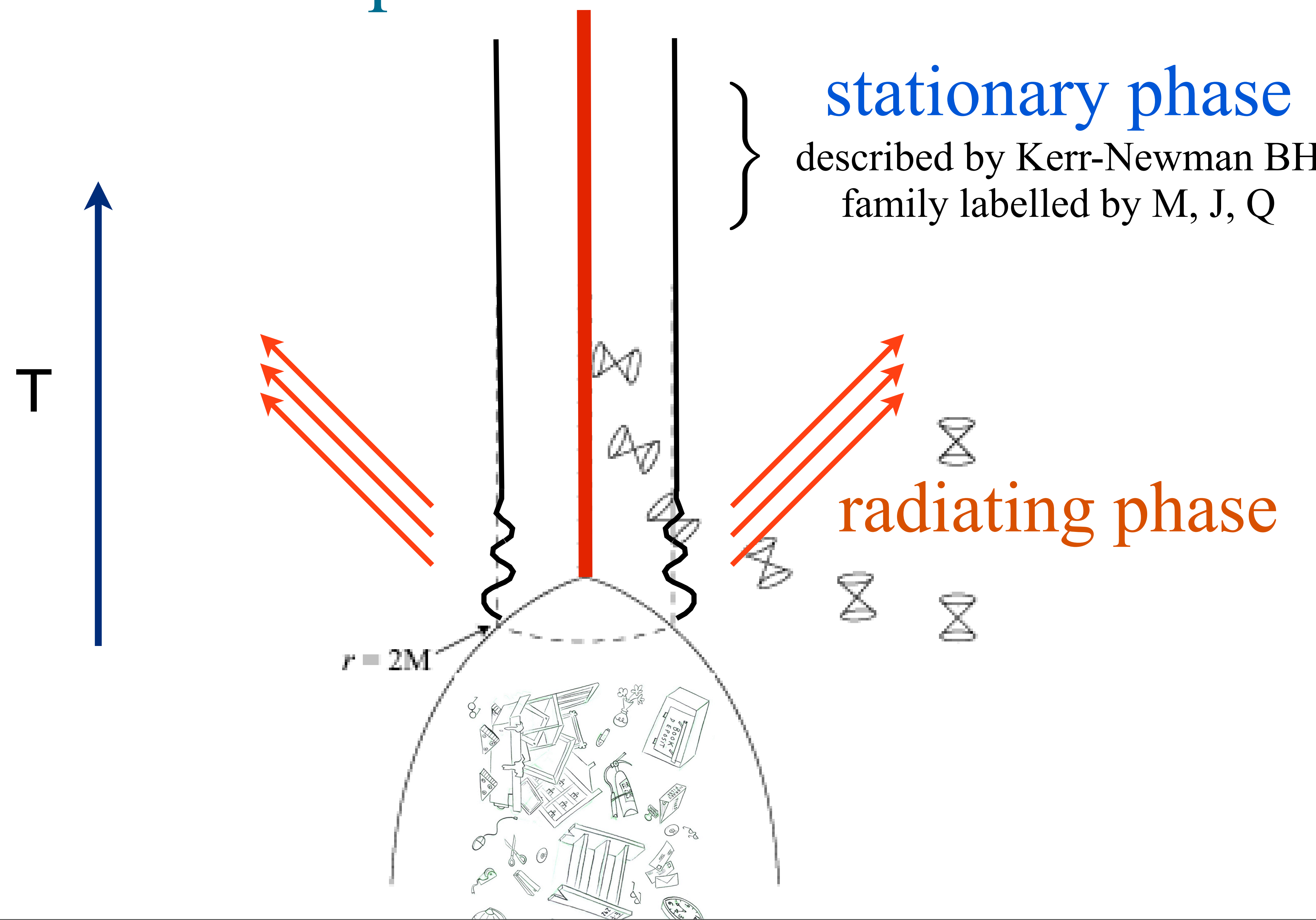
Black Hole Thermodynamics

The Robinson-Carter-Hawking-Israel uniqueness theorems



Black Hole Thermodynamics

The Robinson-Carter-Hawking-Israel uniqueness theorems

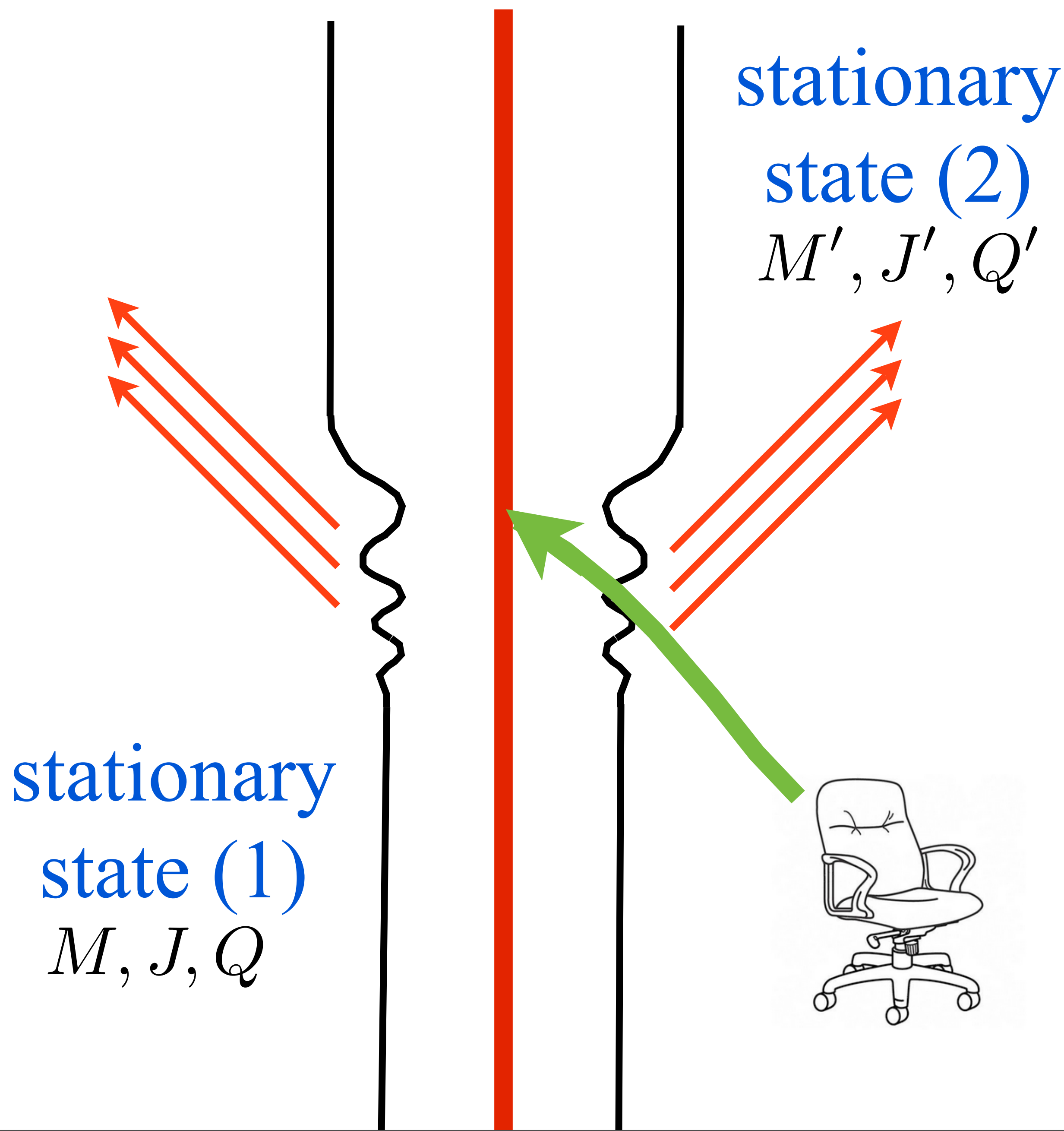


Black Hole Thermodynamics

The 0th, 1st, 2nd and 3rd laws of BH

Some definitions

- $\Omega \equiv$ horizon angular velocity
- $\kappa \equiv$surface gravity ('grav. force' at horizon)
  If ℓ^a =killing generator, then $\ell^a \nabla_a \ell^b = \kappa \ell^b$.
- $\Phi \equiv$electromagnetic potential.



0th law: the surface gravity κ is constant on the horizon.

1st law:

$$\delta M = \frac{\kappa}{8\pi}\delta A + \underbrace{\Omega \delta J + \Phi \delta Q}_{\text{work terms}}$$

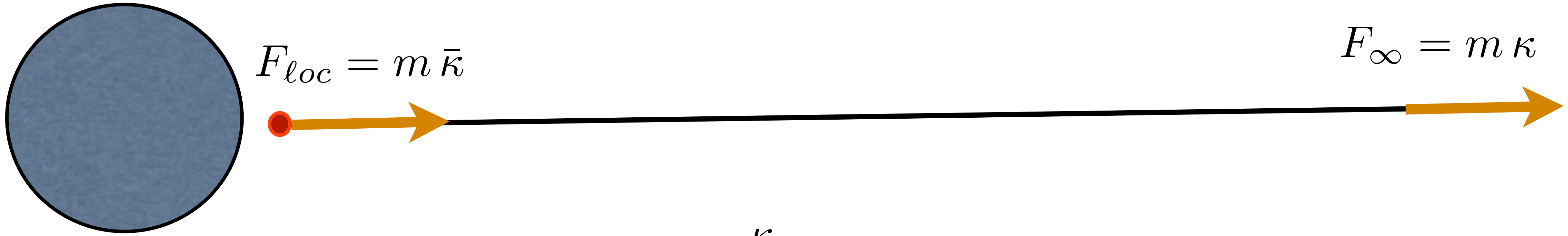
2nd law:

$\delta A \geq 0$

3rd law: the surface gravity value $\kappa = 0$ (extremal BH) cannot be reached by any physical process.

Back to the 1st law

The first law is a global relationship



$$\delta M = \frac{\kappa}{8\pi}\delta A + \Omega\,\delta J + \Phi\,\delta Q,$$

$\Omega =$ Angular velocity of non rotating observers as seen from infinity.

$J =$ Total angular momentum.

$\Phi =$ Potential difference from the horizon to infinity.

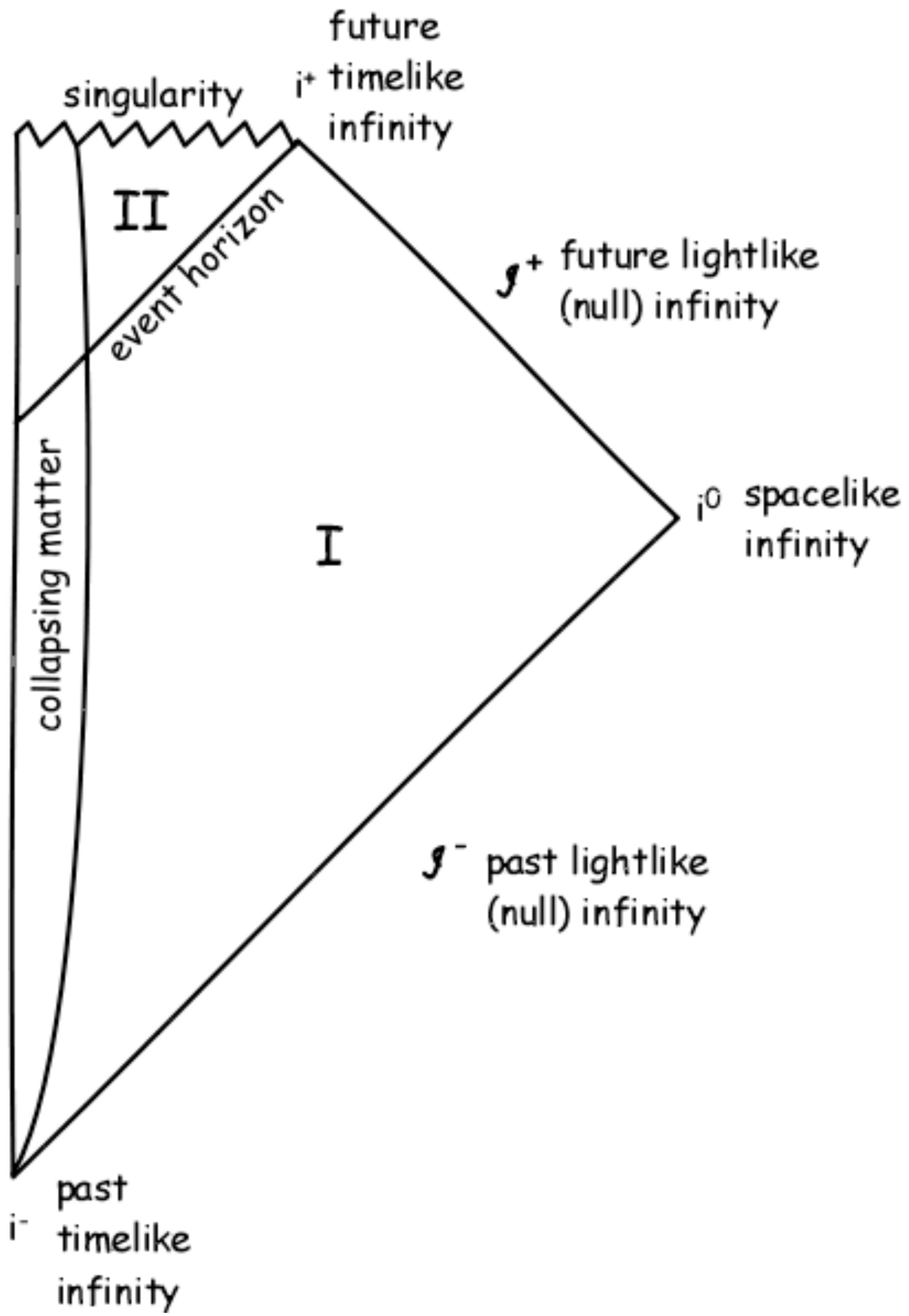
$Q =$ Total electric charge.

Hawking Radiation

Black holes are thermal

Particle creation by collapsing spacetime

The Hawking effect



Particle creation by collapsing spacetime

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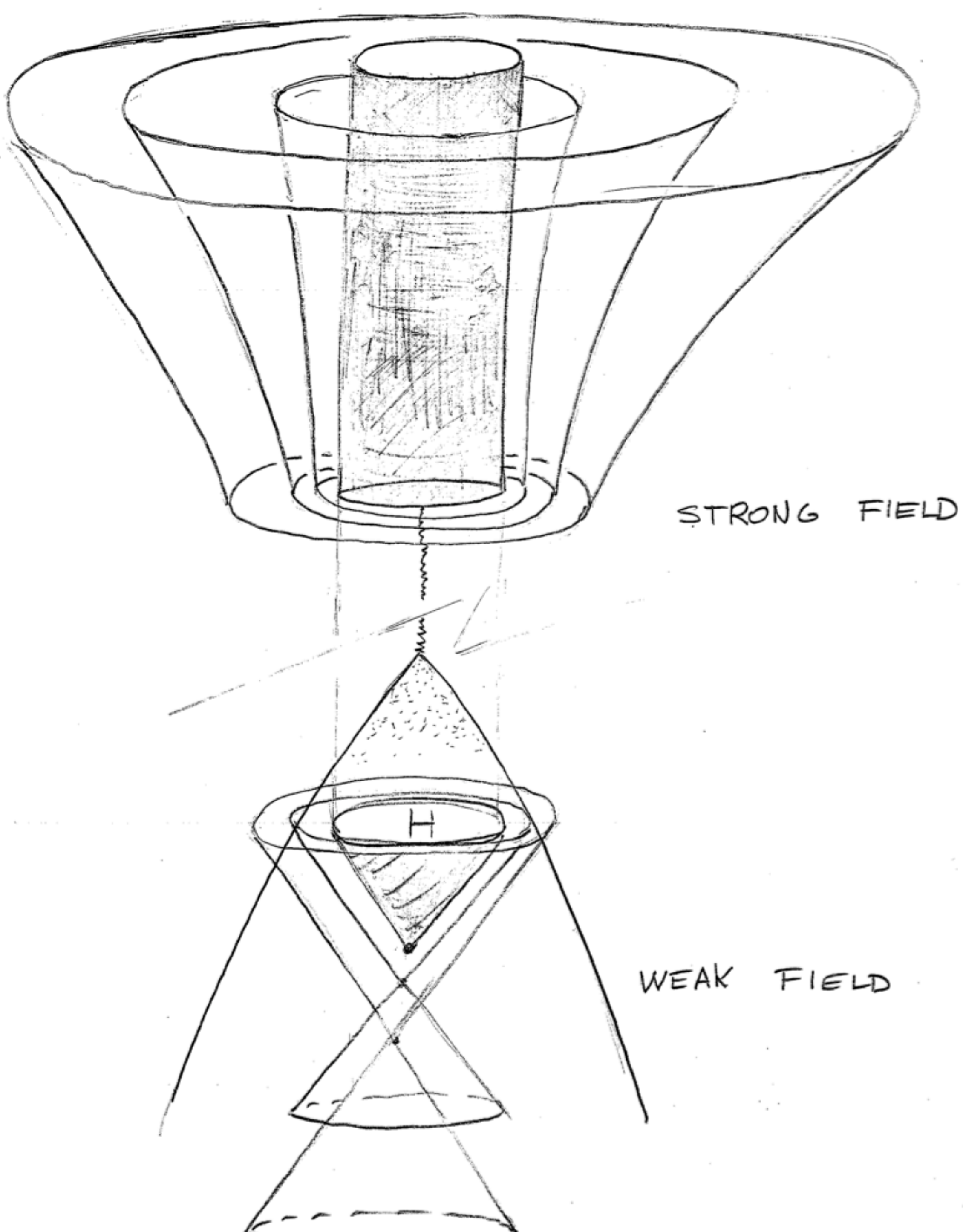
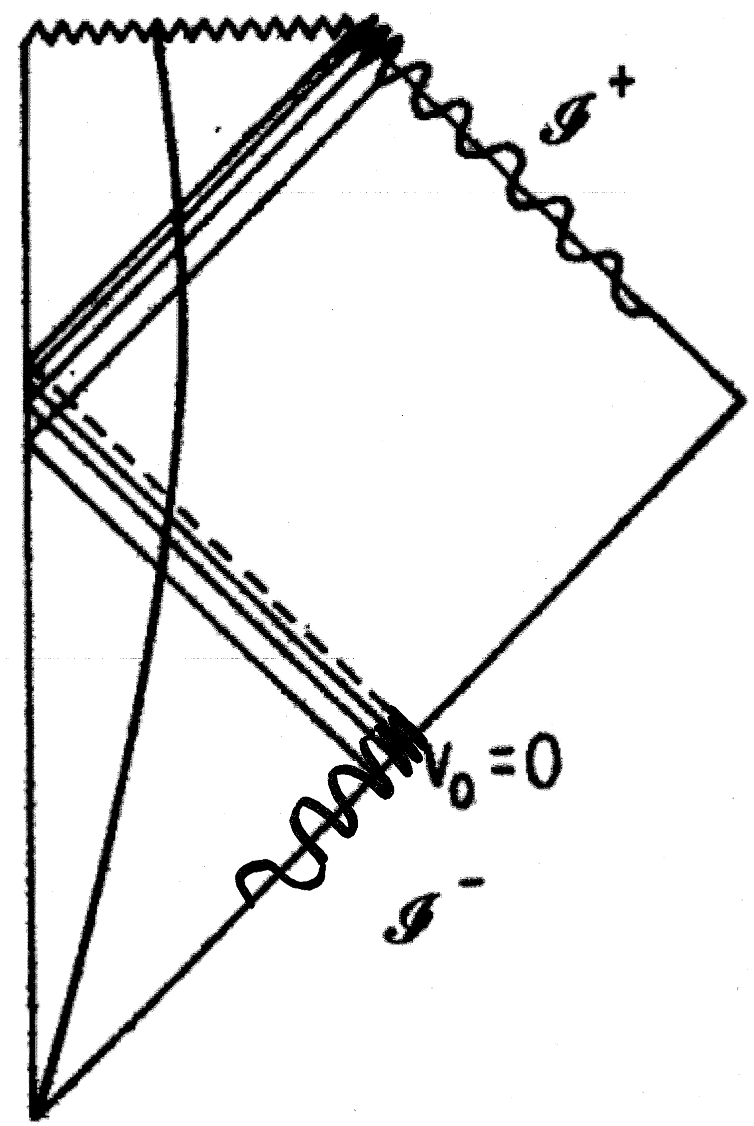
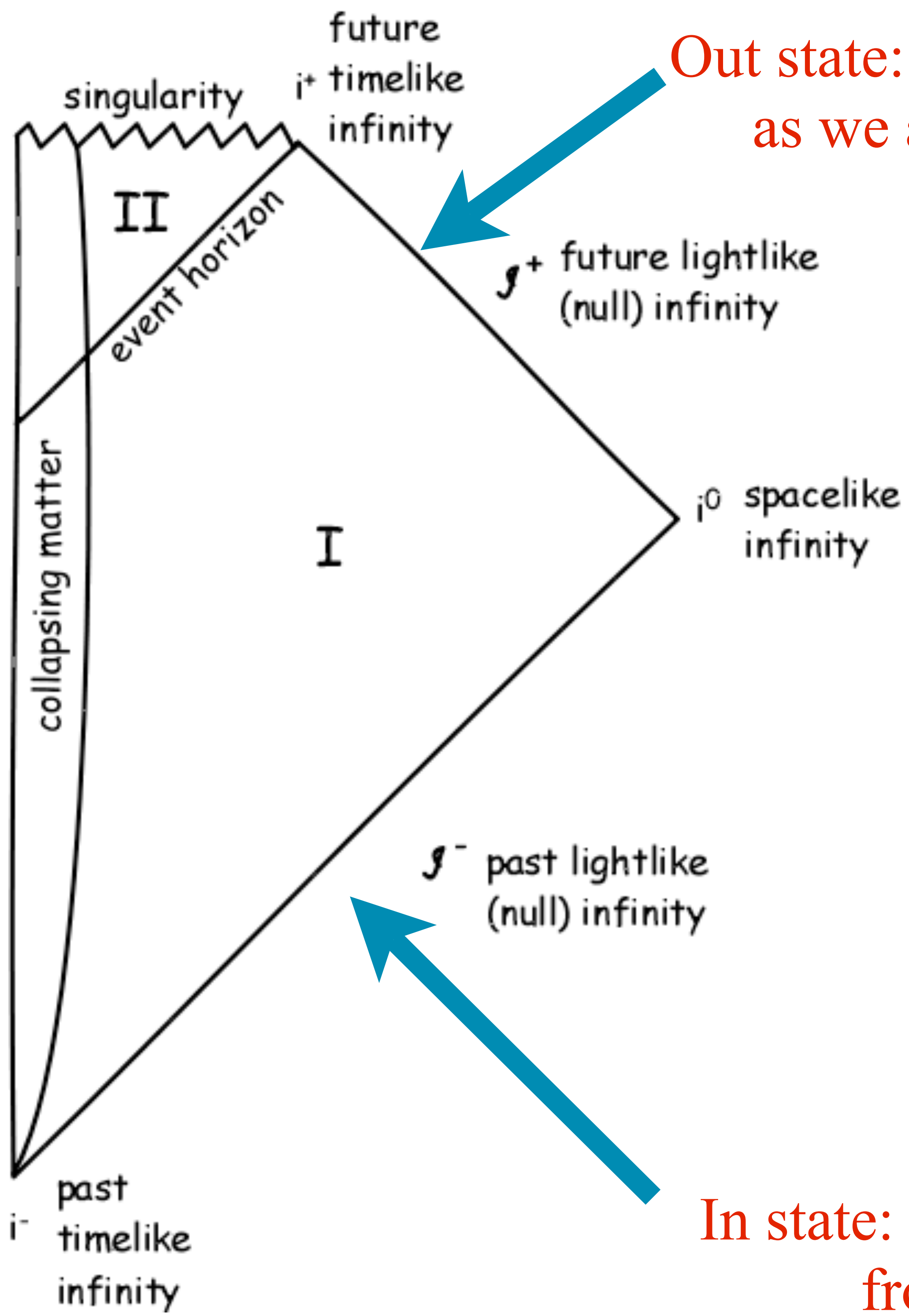




Black Hole Thermodynamics

Hawking Radiation: QFT on a BH background



Out state: thermal flux of particles as we approach the point **i**+

In state: vacuum far from **i**-

Temperature at infinity

$$T_\infty = \frac{\kappa}{2\pi}$$

From the first law

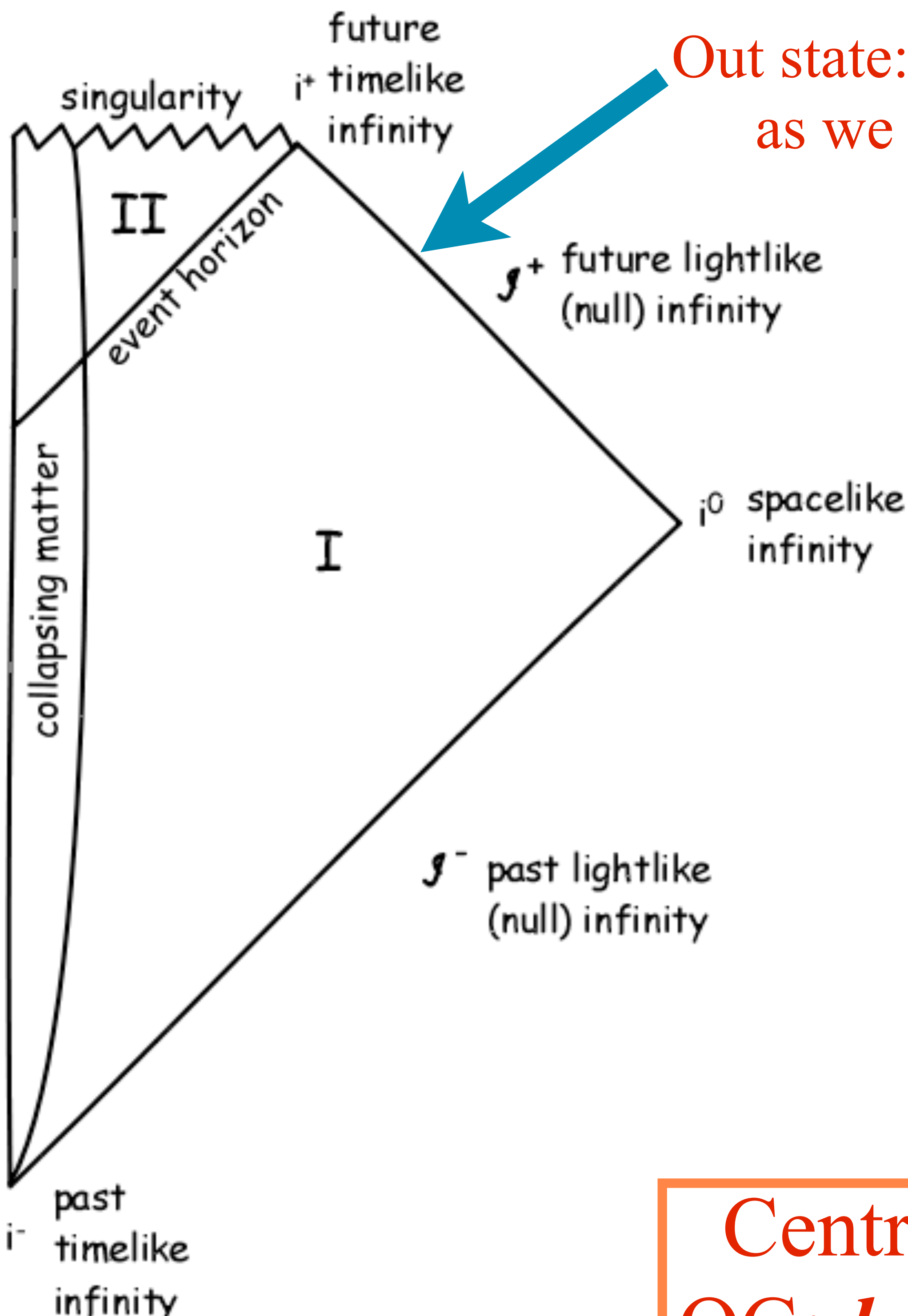
$$\delta M = \frac{\kappa}{8\pi}\delta A + \Omega\delta J + \Phi\delta Q$$

One infers the ENTROPY

$$S = \frac{A}{4\ell_p^2}$$

Black Hole Thermodynamics

Hawking Radiation: QFT on a BH background



Out state: thermal flux of particles as we approach the point **i**+

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$$T_\infty = \frac{\kappa}{2\pi}$$

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$$\delta M = \frac{\kappa}{8\pi}\delta A + \Omega\delta J + \Phi\delta Q$$

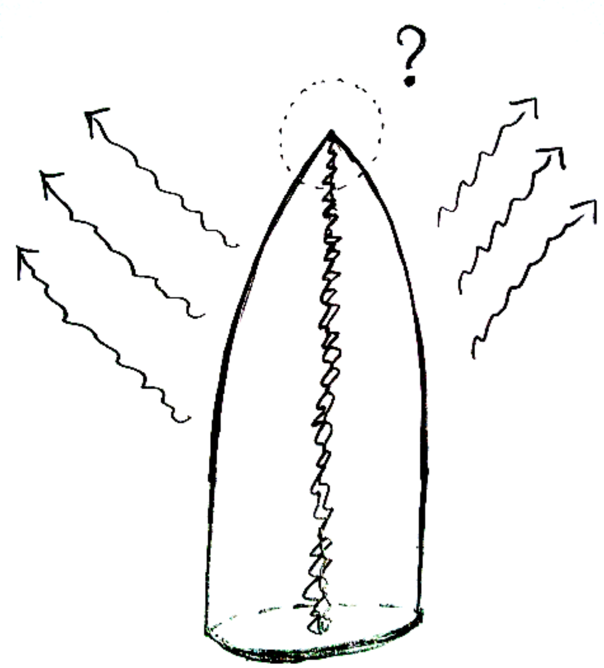
One infers the ENTROPY

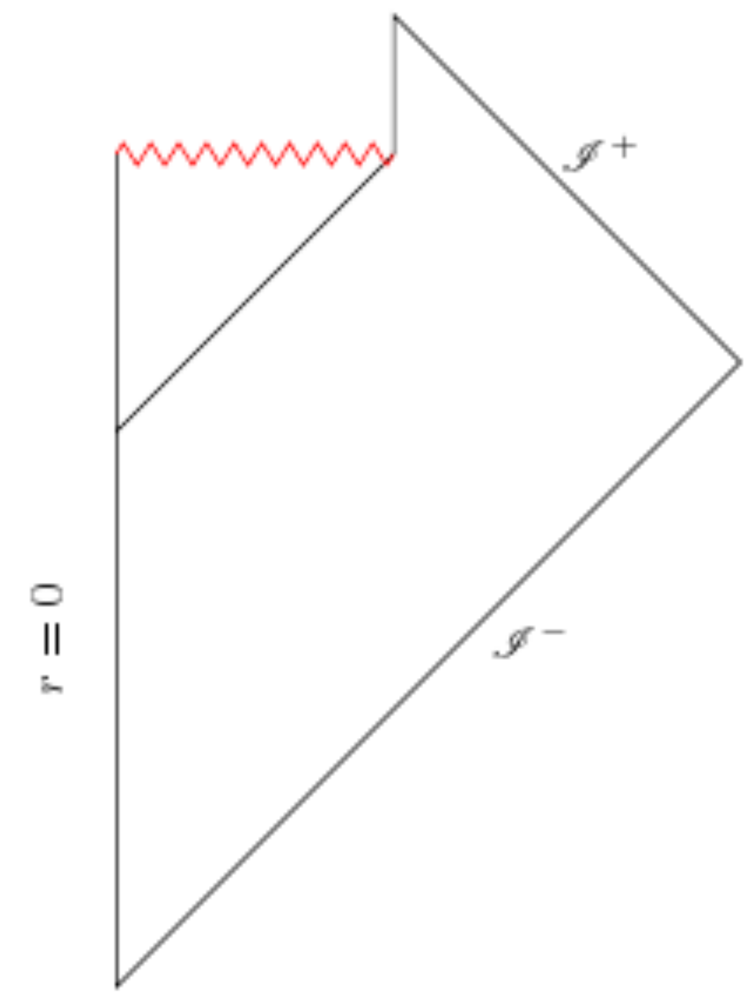
$$S = \frac{A}{4\ell_p^2}$$

Central Question for QG: ***how to get S from statistical mechanics***

Black holes evaporate

Hard Problem: fate of information, unitarity, singularity, etc...

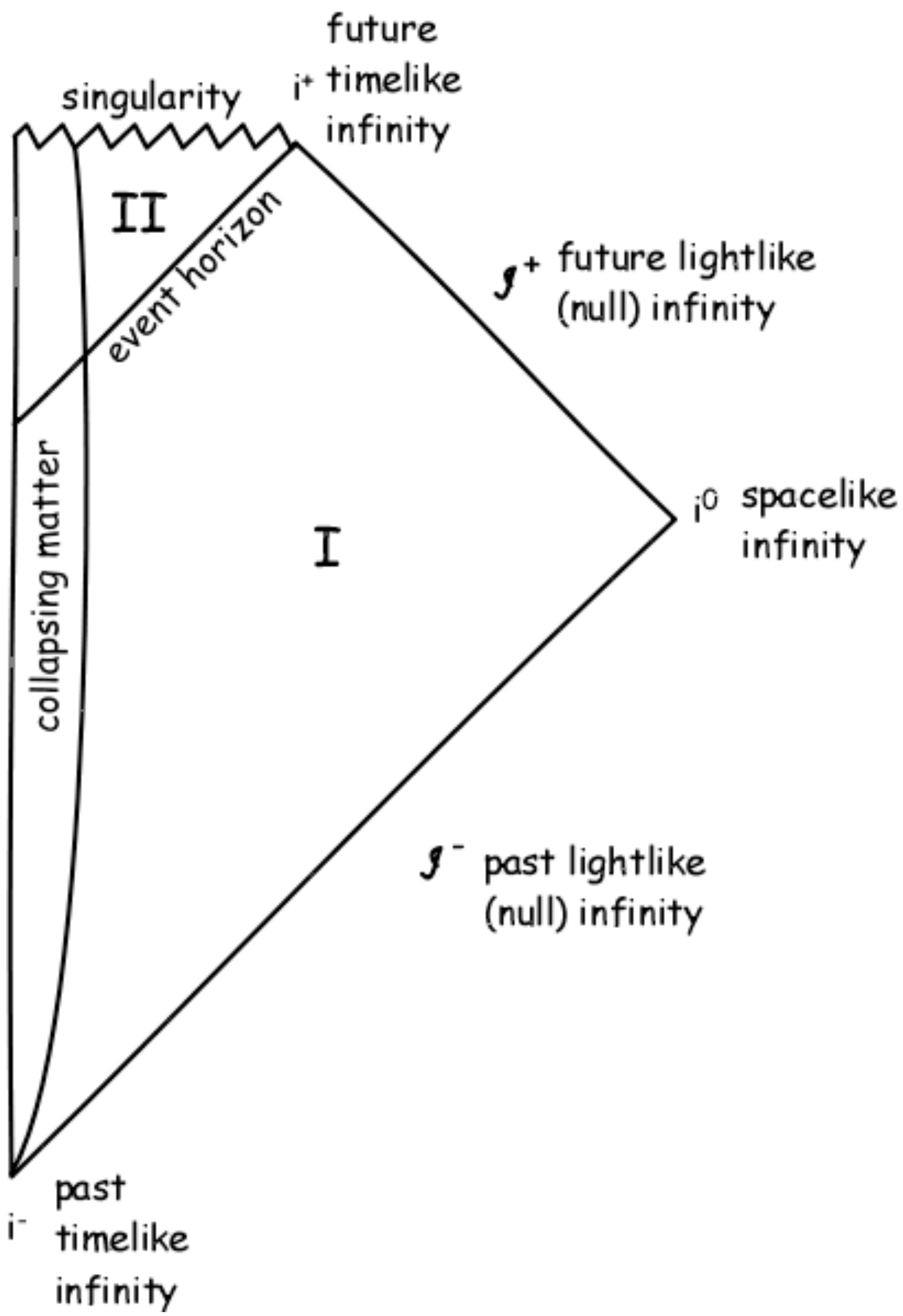


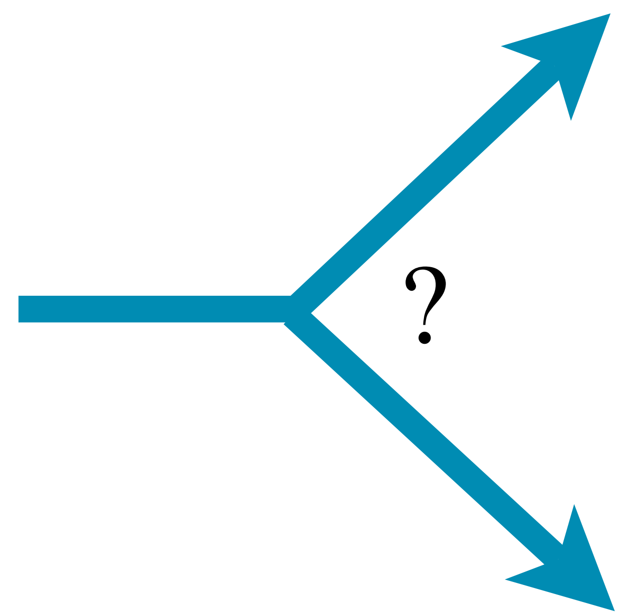


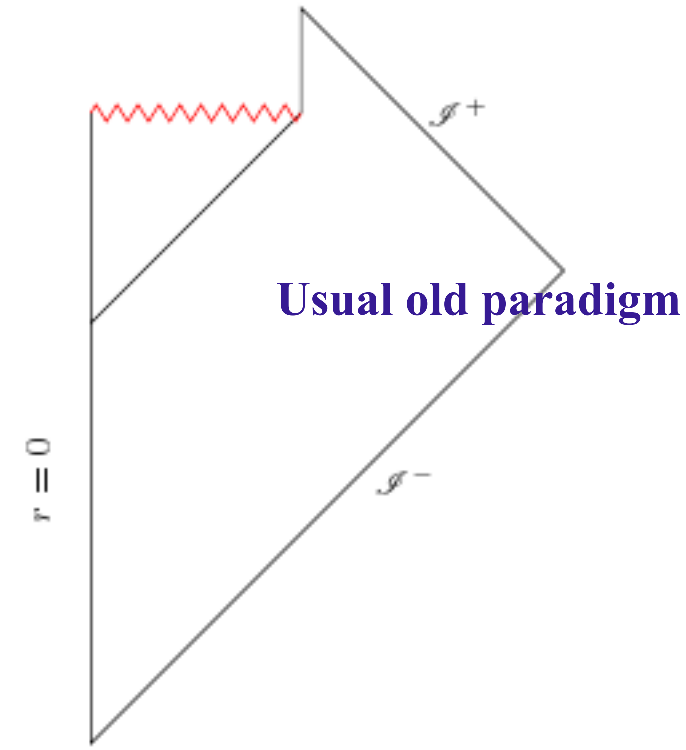
Simpler problem: The problem of BH thermodynamics (large BHs close to equilibrium)

Black Hole entropy in LQG

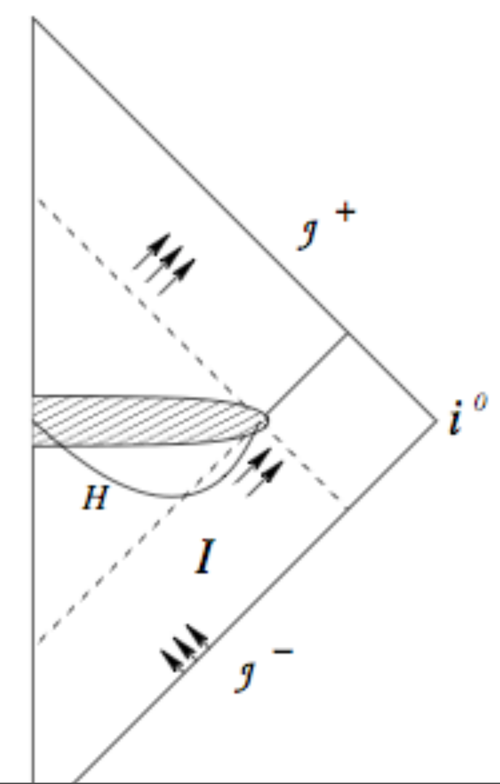
The standard definition of BH is GLOBAL
(need a quasi-local definition)







Usual old paradigm



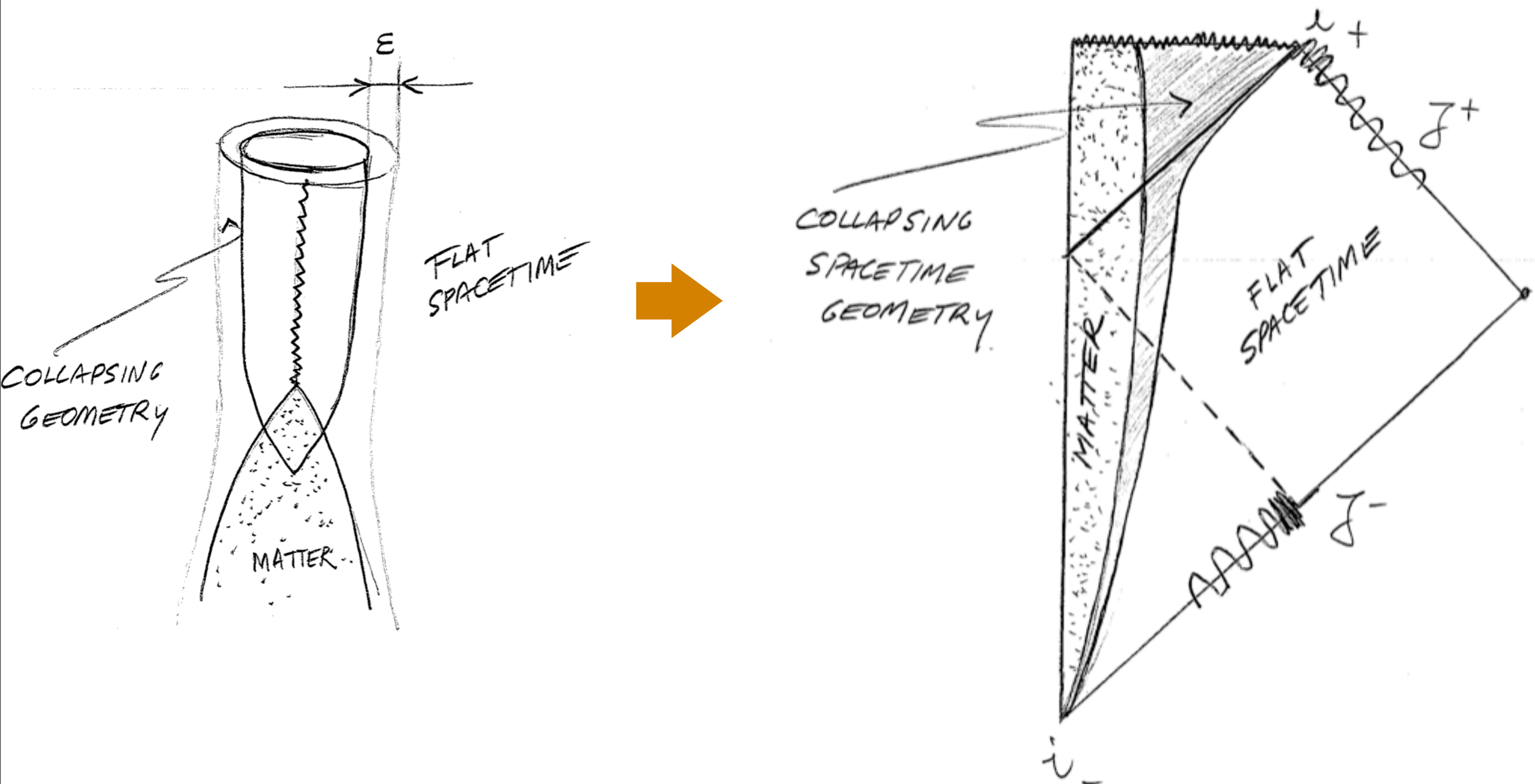
LQG Paradigm:
Ashtekar-Bojowald (2005),
Ashtekar-Taveras-Varadarajan (2008),
Ashtekar-Pretorius-Ramazanoglu (2011).

Let us consider the simpler problem: Understanding BH thermodynamics

An observation on Hawking computation

Particle creation by collapsing spacetime

The relevant physics is **Near-Horizon horizon physics**



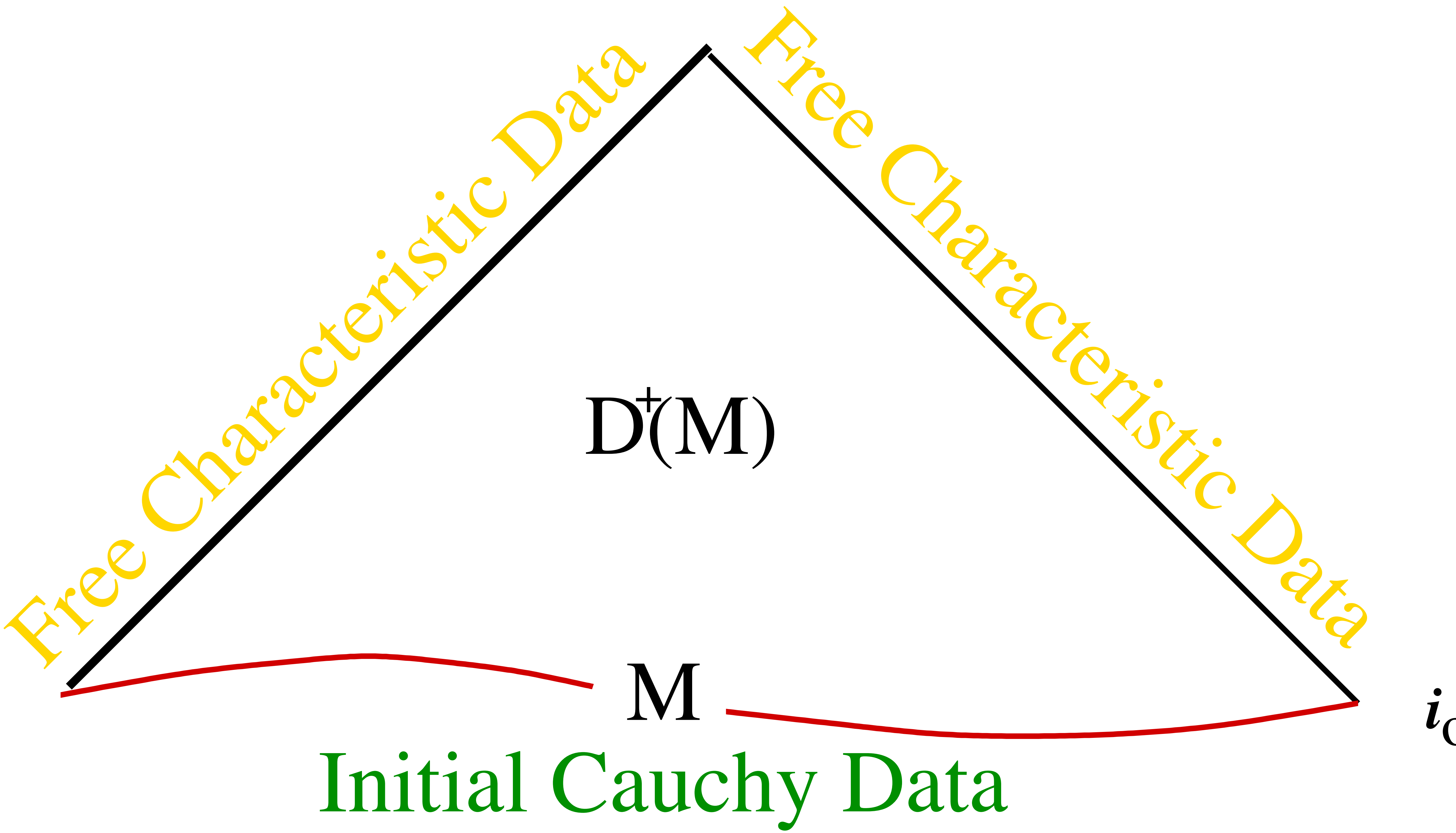
The horizon is the thermodynamical system; we need:

(**1**) A local definition of horizon in equilibrium

(**2**) A local version of BH thermodynamics

Black Hole Entropy from LQG

The Characteristic formulation of GR



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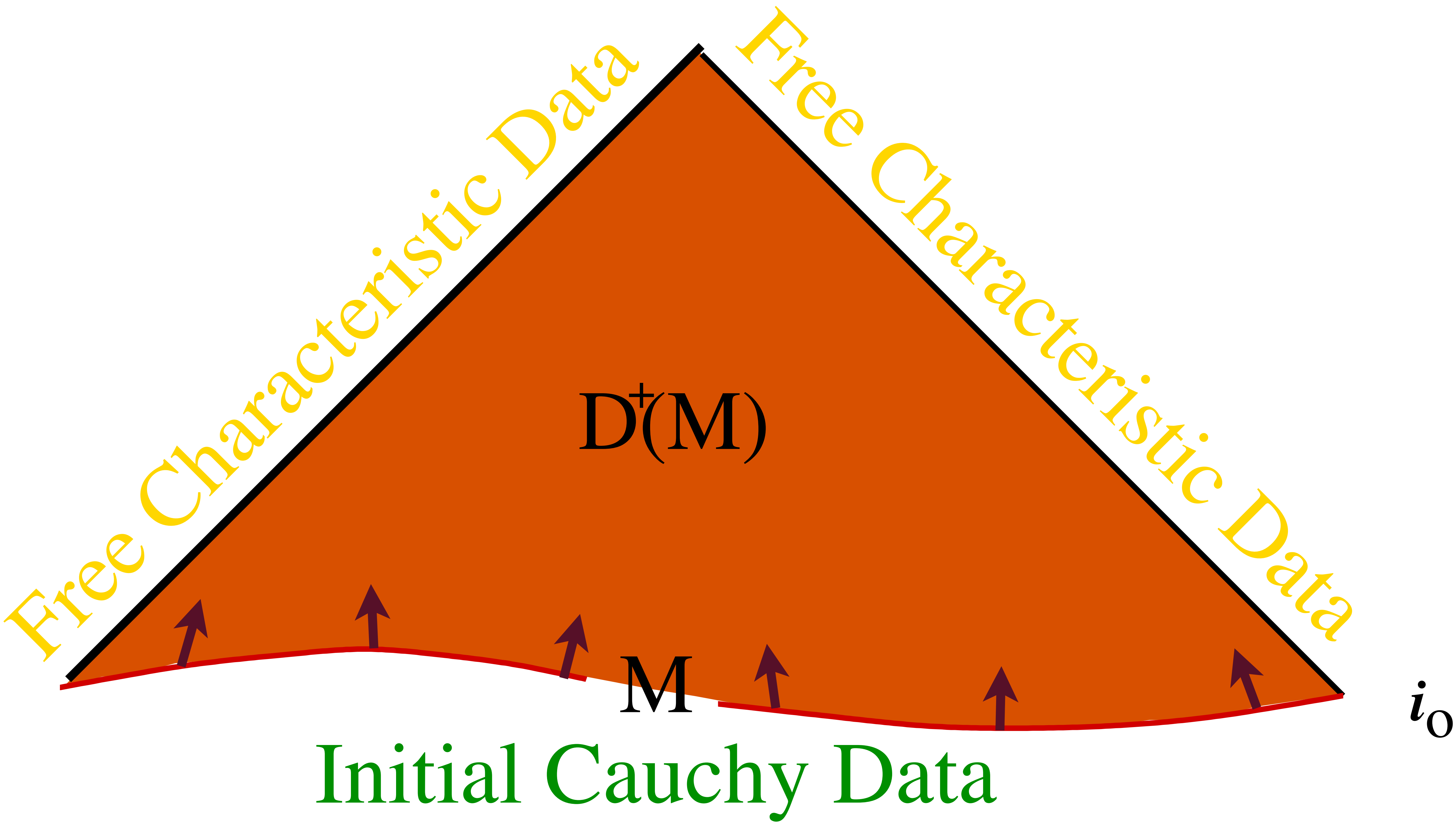

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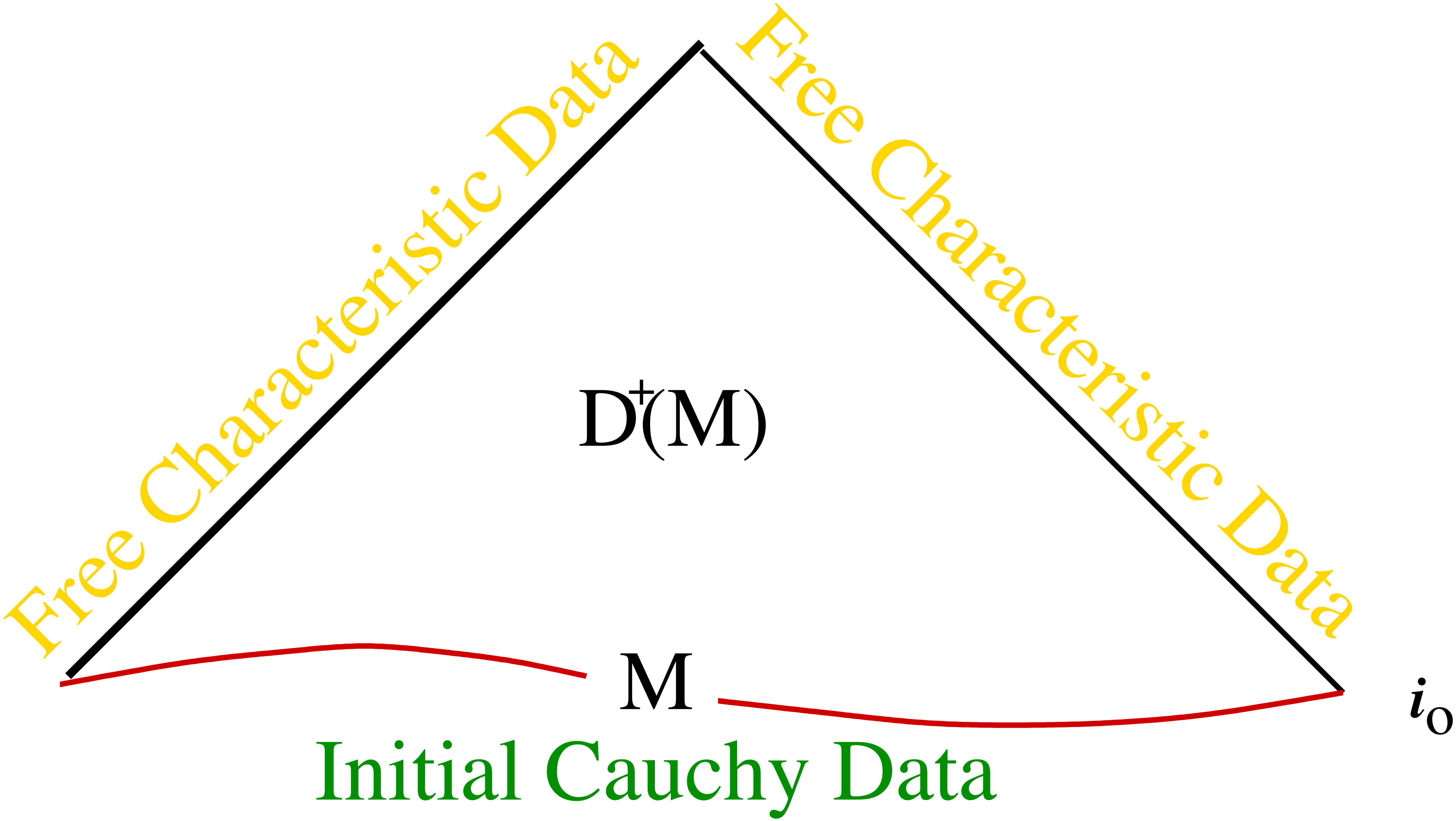

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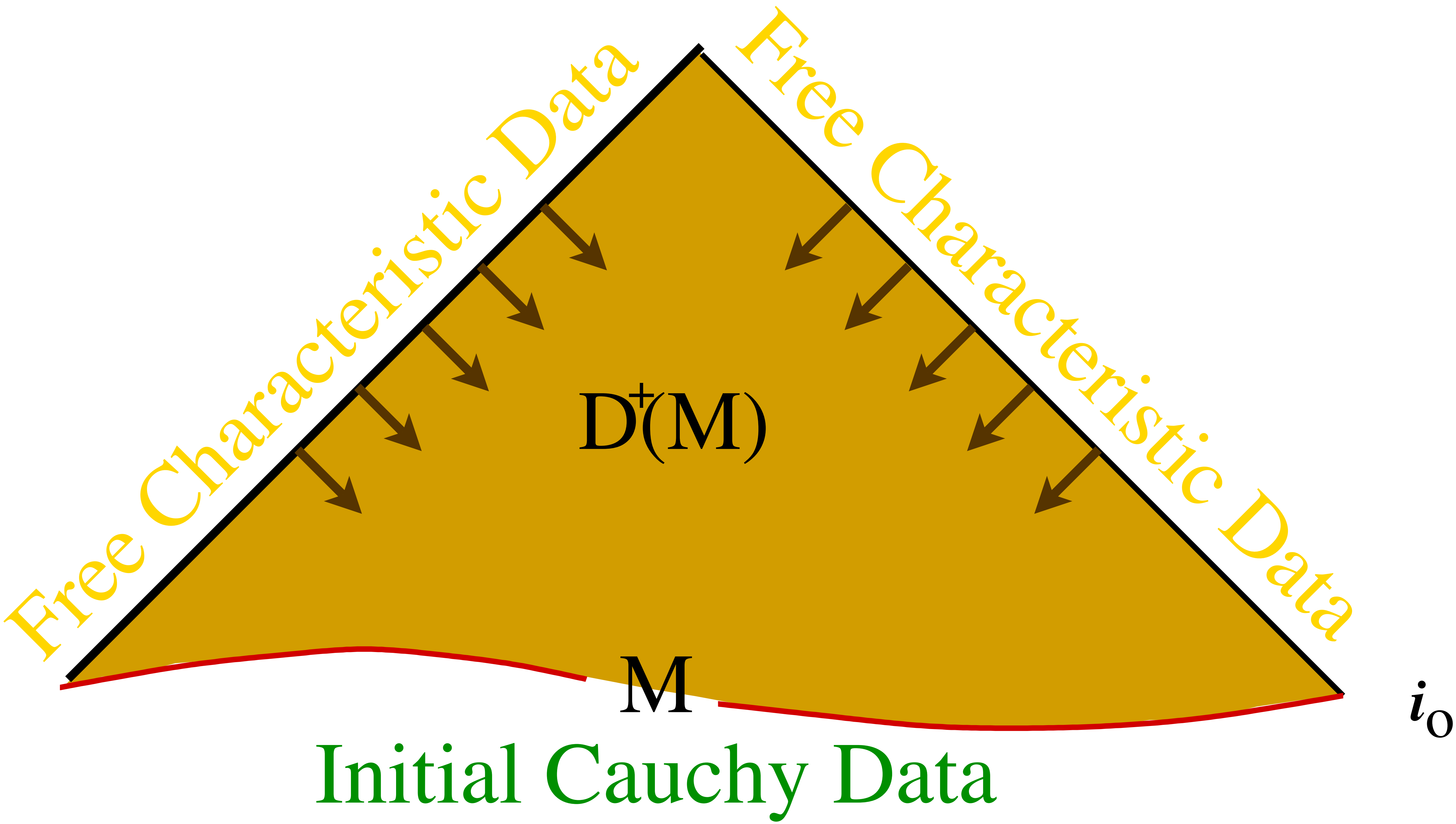
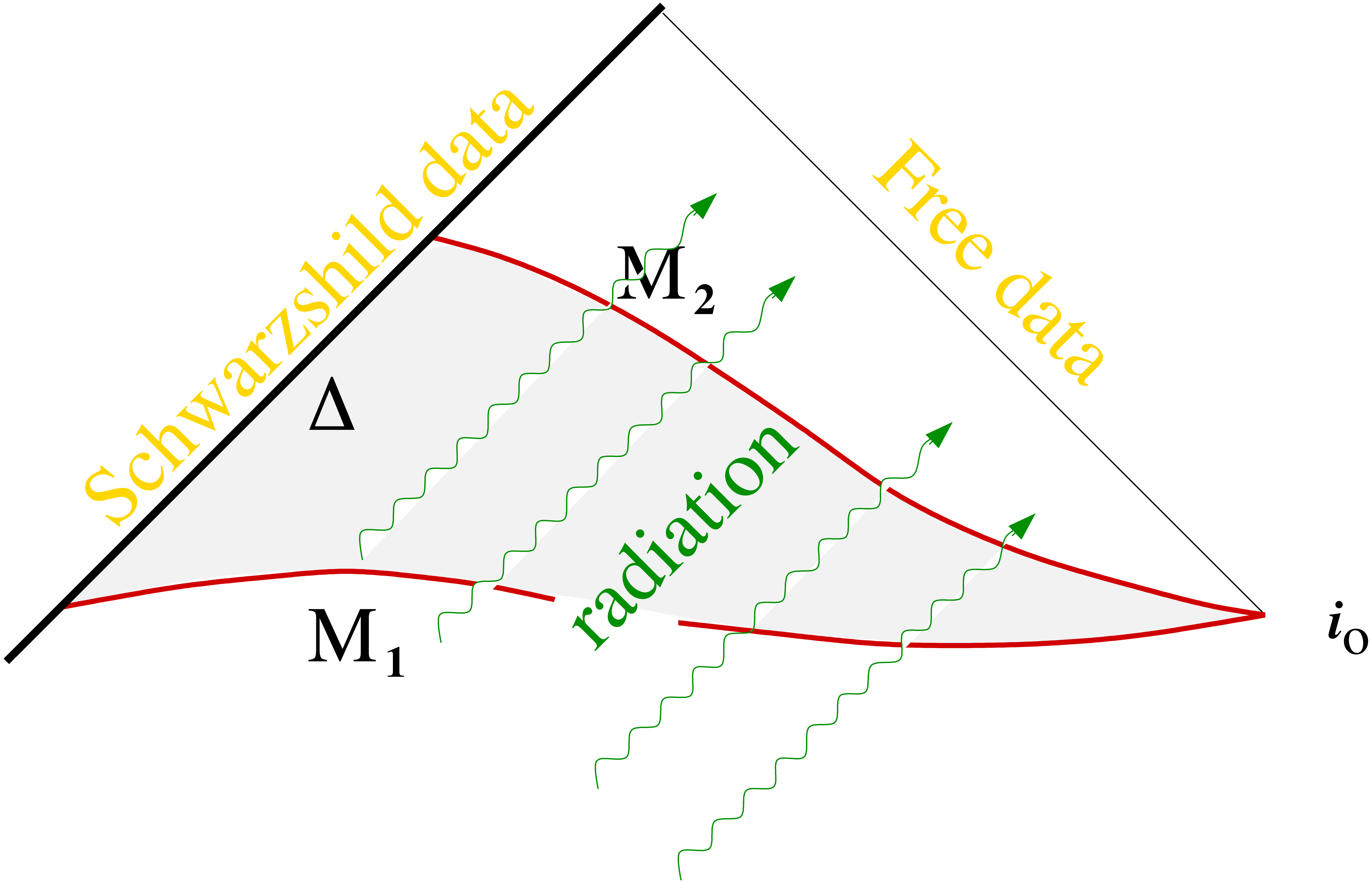


Black Hole Entropy from LQG

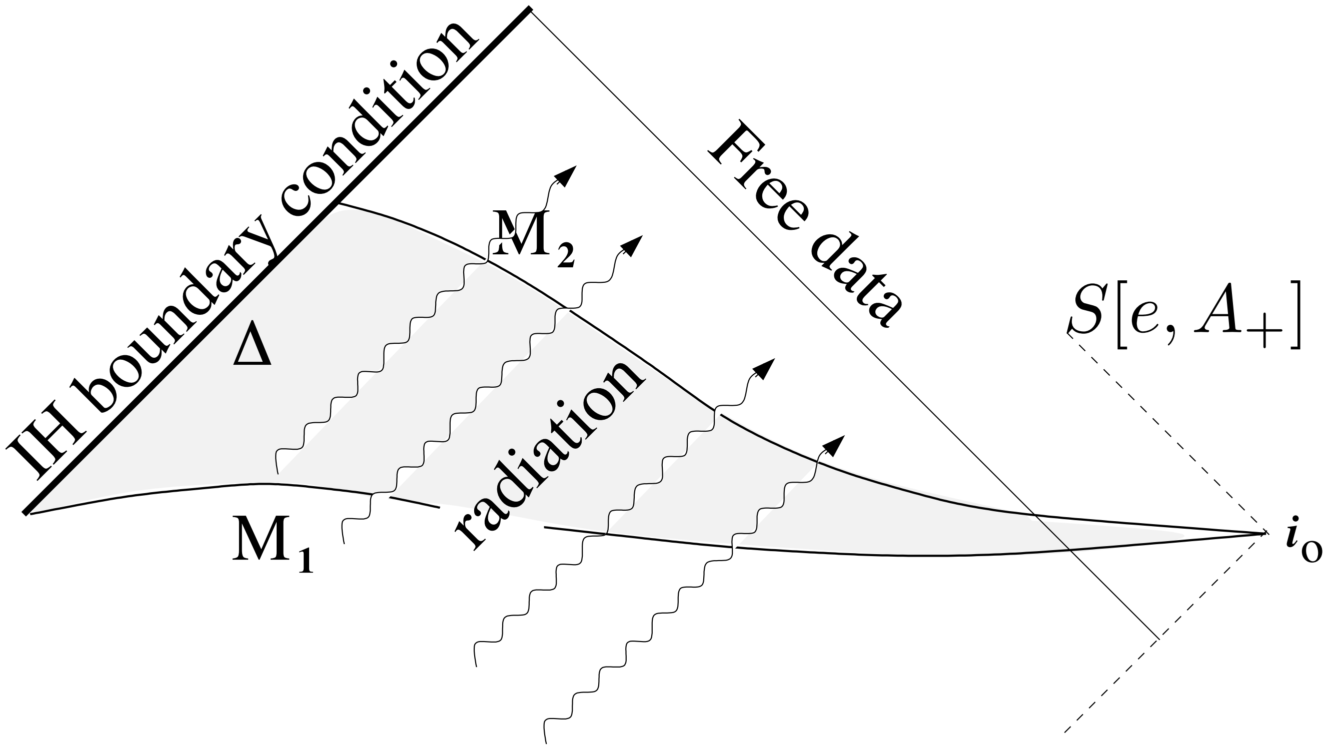
Definition of (spherical) Isolated Horizon**



** A. Ashtekar, J. Baez, A. Corichi and K. Krasnov, Phys. Rev. Lett. **80** (1998) 904. J. Lewandowski, Class. Quant. Grav. **17** (2000) L53. A. Ashtekar, S. Fairhurst and B. Krishnan, Phys. Rev. D**62** (2000) 104025. J. Engle, Th., Penn State (2006)

Generic (non-rotating) Isolated Horizons

Covariant phase space formulation



[Ashtekar, PRL, 1986]

$$S[e,A_+] = -\frac{i}{\kappa}\int_M \Sigma_i(e)\wedge F^i(A_+) + \frac{i}{\kappa}\int_{\tau_\infty}\Sigma_i(e)\wedge A^i_+$$

$$0 = \frac{i\kappa}{2}\int_{\partial B} J(\delta_1,\delta_2) = \underbrace{\int_\Delta \delta_{[1}\Sigma_i\wedge\delta_{2]}A^i_+}_{} + \int_{M_1}\delta_{[1}\Sigma_i\wedge\delta_{2]}A^i_+ - \int_{M_2}\delta_{[1}\Sigma_i\wedge\delta_{2]}A^i_+$$

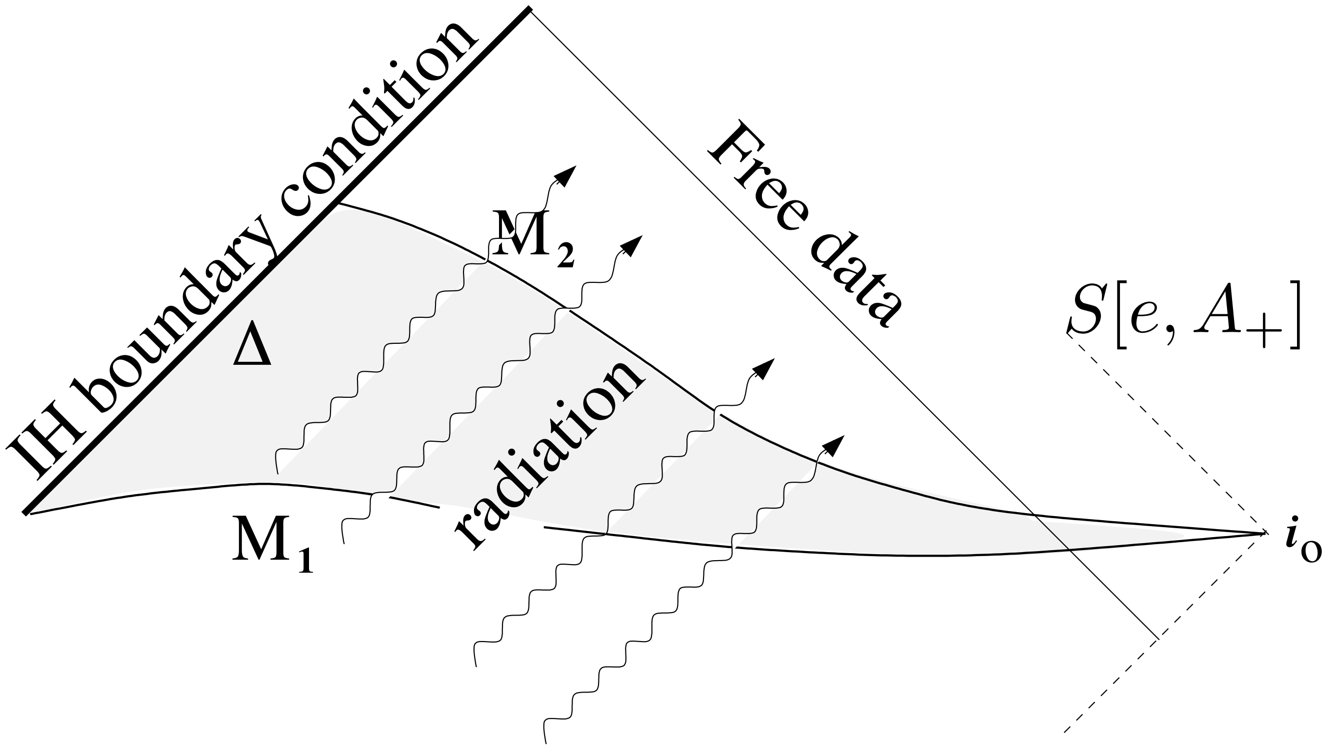
$$\overbrace{\frac{a}{2\pi}\int_{H_2}\delta_1 A_i\wedge\delta_2 A^i - \frac{a}{2\pi}\int_{H_1}\delta_1 A_i\wedge\delta_2 A^i}$$

Soldering Constraint: generating diffeomorphism and gauge transformations on the boundary

$$F_{ab}{}^i(A) = -\frac{2\pi}{a}\Sigma_{ab}{}^i$$

Generic (non-rotating) Isolated Horizons

Covariant phase space formulation



[Ashtekar, PRL, 1986]

$$S[e, A_+] = -\frac{i}{\kappa}\int_M \Sigma_i(e)\wedge F^i(A_+) + \frac{i}{\kappa}\int_{\tau_\infty}\Sigma_i(e)\wedge A_+^i$$

$$0 = \frac{i\kappa}{2}\int_{\partial B} J(\delta_1,\delta_2) = \underbrace{\int_\Delta \delta_{[1}\Sigma_i\wedge\delta_{2]}A_+^i}_{\frac{a}{2\pi}\int_{H_2}\delta_1 A_i\wedge\delta_2 A^i - \frac{a}{2\pi}\int_{H_1}\delta_1 A_i\wedge\delta_2 A^i} + \int_{M_1}\delta_{[1}\Sigma_i\wedge\delta_{2]}A_+^i - \int_{M_2}\delta_{[1}\Sigma_i\wedge\delta_{2]}A_+^i$$

$$\Omega(\delta_1,\delta_2) = \frac{1}{8\pi G\gamma}\int_M 2\delta_{[1}\Sigma^i\wedge\delta_{2]}A_i + \frac{a}{8\pi^2 G\gamma(1-\gamma^2)}\int_H \delta_1 A_i\wedge\delta_2 A^i$$

- Quantum geometry

  a Show that
$$\Sigma^i \delta\Gamma^i = d(e_i \wedge \delta e^i), \tag{1}$$
  where Γ^i is the spin connection satisfying first Cartan structure equation
$$de^i + \epsilon^{ijk}\Gamma^j \wedge e^k \tag{2}$$

  b From the differential equation defining holonomies (see notes of the second lecture) prove the following properties of holonomies.

    i) The holonomy associated to an oriented path is independent of its parametrization.

    ii) The holonomy along the product of two oriented paths that can be multiplied is the suitable product of holonomies.

    iii) Under a gauge transformation the holonomy h_e along the path e is mapped to $g_t h_e g_s^{-1}$ where g_t is the value of the gauge transformation at the target of the path and g_s is the value of the gauge transformation at the source.

    iv) Let $\phi : M \to M$ be a diffeomorphism then
$$h_{\phi(e)}[A] = h_e[\phi^* A] \tag{3}$$

We need:

(**1**) A local definition of horizon in equilibrium

(**2**) A local version of BH thermodynamics

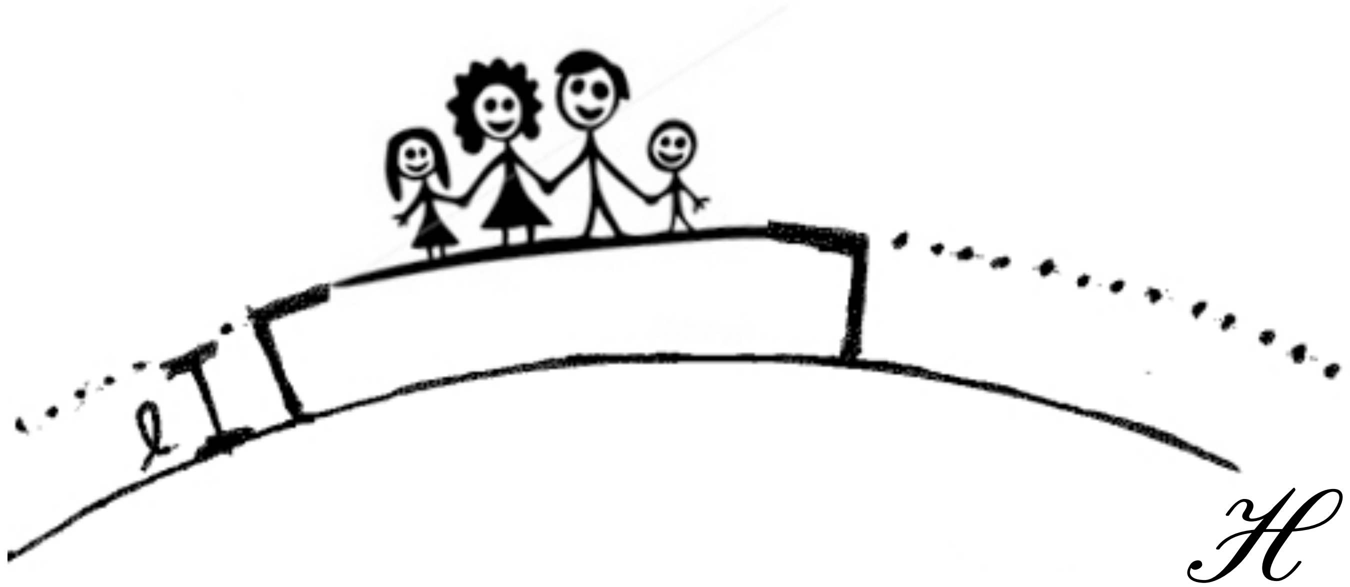
The local laws of BH mechanics

BH thermodynamics from a local perspective

[Frodden, Ghosh, Perez, 2012 PRD]

Black Hole Thermodynamics
A local perspective

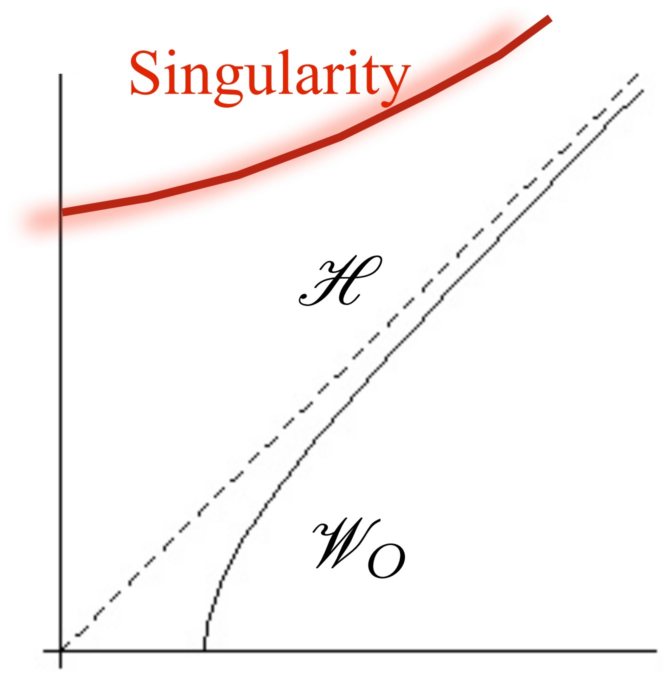
$\ell^2 << A$



Introduce a family of
local stationary observers
~ZAMOS

$$\chi = \xi + \Omega\,\psi = \partial_t + \Omega\,\partial_\phi$$

$$u^a = \frac{\chi^a}{\|\chi\|}$$



Black Hole Thermodynamics

A local perspective

$\ell^2 << A$

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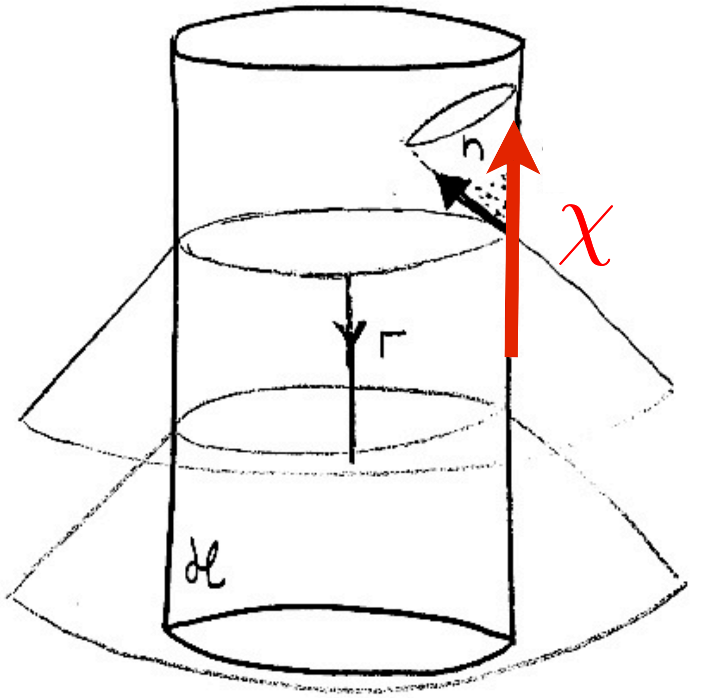
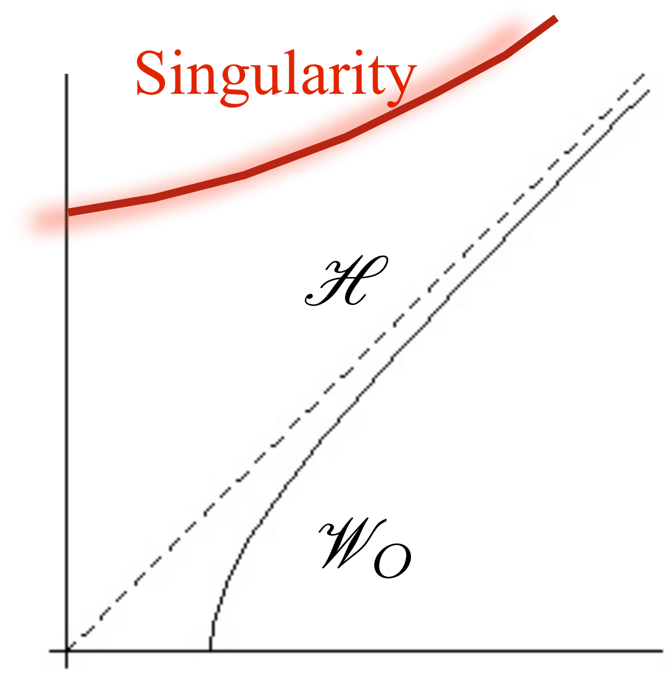


$$\chi = \xi + \Omega\,\psi = \partial_t + \Omega\,\partial_\phi$$

$$u^a = \frac{\chi^a}{\|\chi\|}$$

$$a = ||u^a \nabla_a u^b|| = \frac{1}{\ell}$$

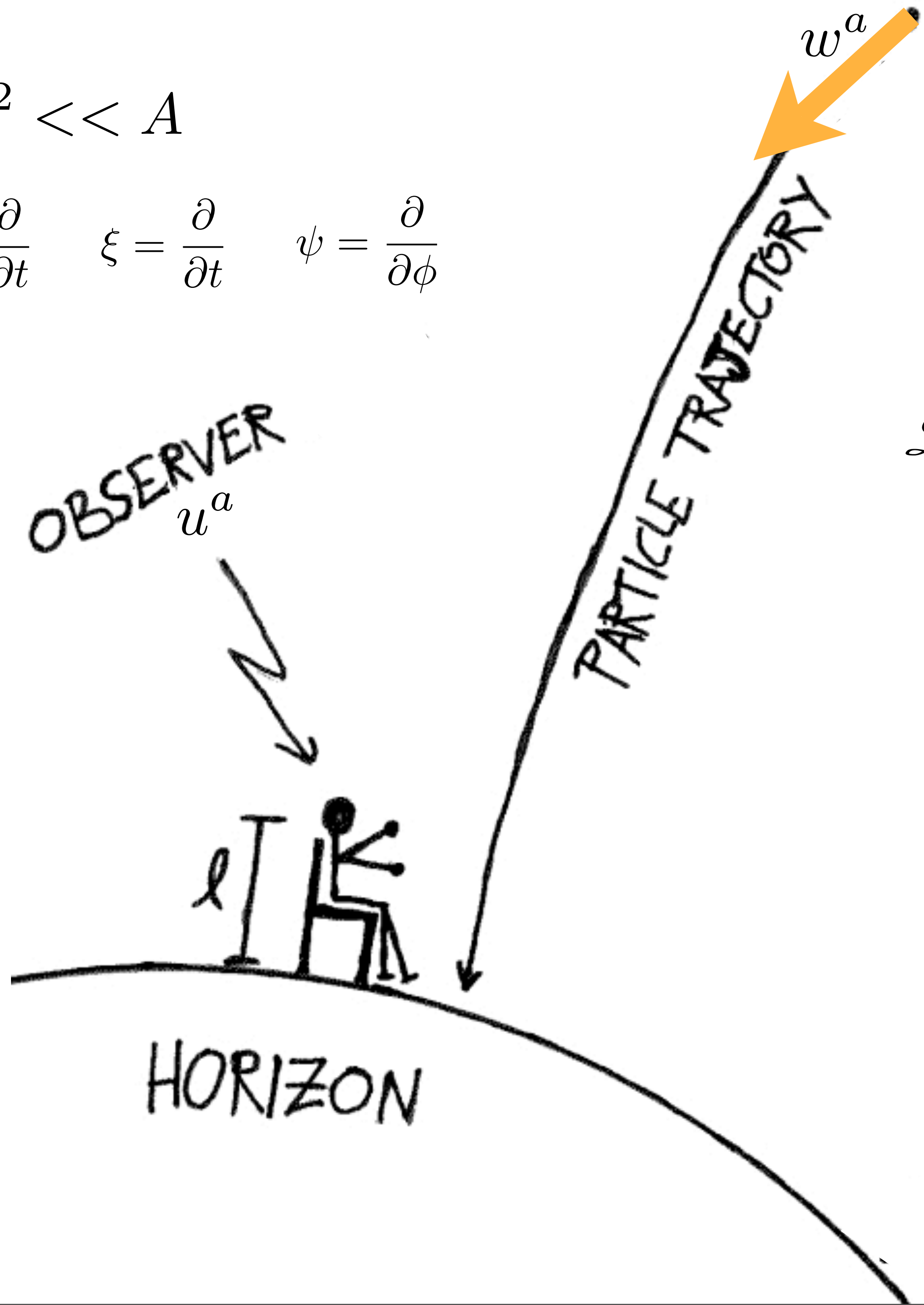


A thought experiment

throwing a test particle from infinity

$$\ell^2 << A$$

$$\chi = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial t} \qquad \xi = \frac{\partial}{\partial t} \qquad \psi = \frac{\partial}{\partial \phi}$$



Particle's equation of motion

$$w^a \nabla_a w_b = q\, F_{bc} w^c$$

Symmetries of the background

$$\mathscr{L}_\xi g_{ab} = \mathscr{L}_\psi g_{ab} = \mathscr{L}_\xi A_a = \mathscr{L}_\psi A_a = 0$$

Conserved quantities

$$\mathcal{E} \equiv -w^a \xi_a - q A^a \xi_a$$

$$L \equiv w^a \psi_a + q A^a \psi_a$$

A thought experiment
throwing a test particle from infinity

Conserved quantities

$$\mathcal{E} \equiv -w^a \xi_a - qA^a \xi_a$$

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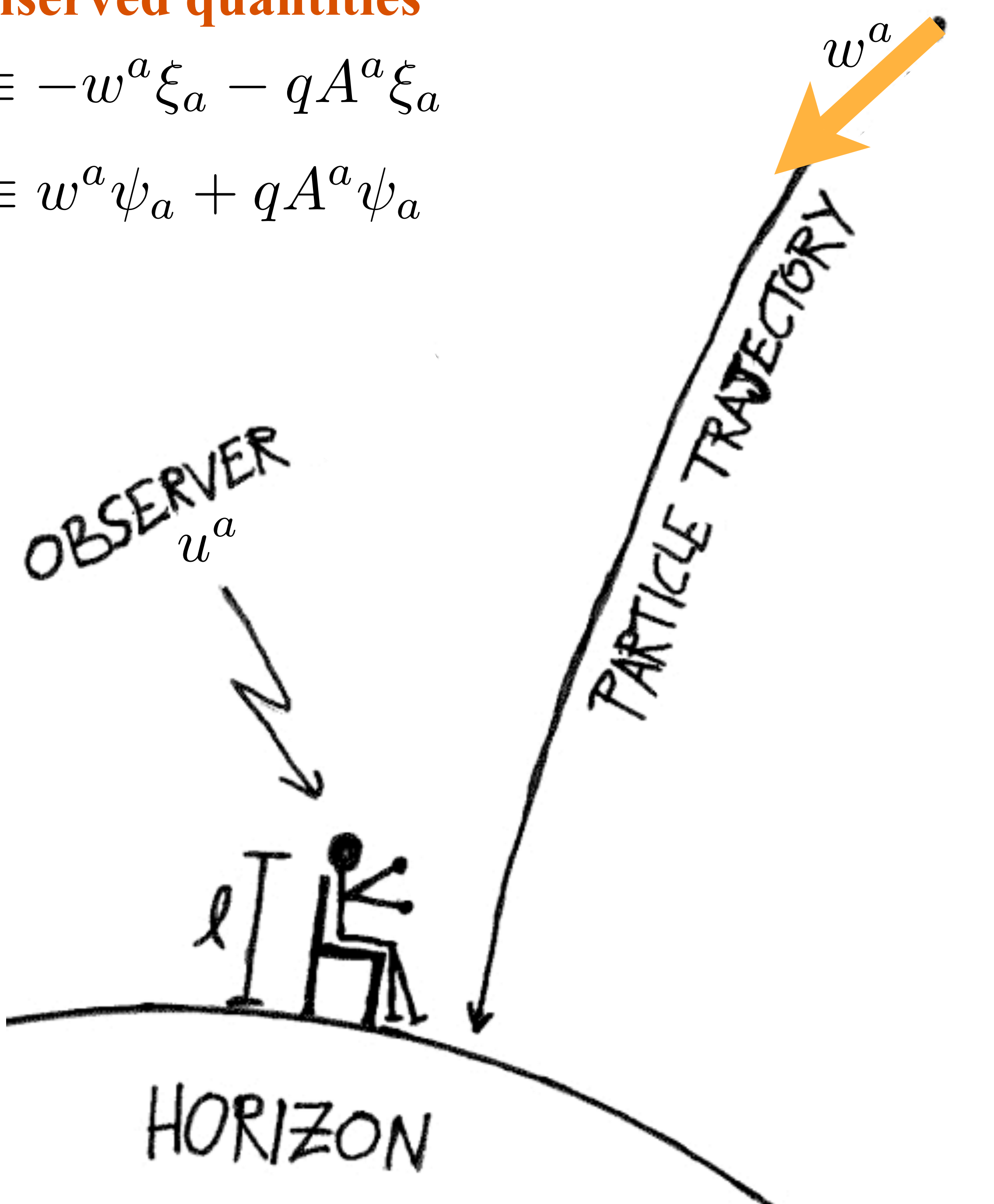


Particle at infinity

$$\mathcal{E} = -w^a \xi_a|_\infty \equiv \text{energy}$$

$$L = w^a \Psi_a|_\infty \equiv \text{angular momentum}$$

$$\chi = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial t} \qquad \xi = \frac{\partial}{\partial t} \qquad \psi = \frac{\partial}{\partial \phi}$$

A thought experiment

Throwing a test particle from infinity

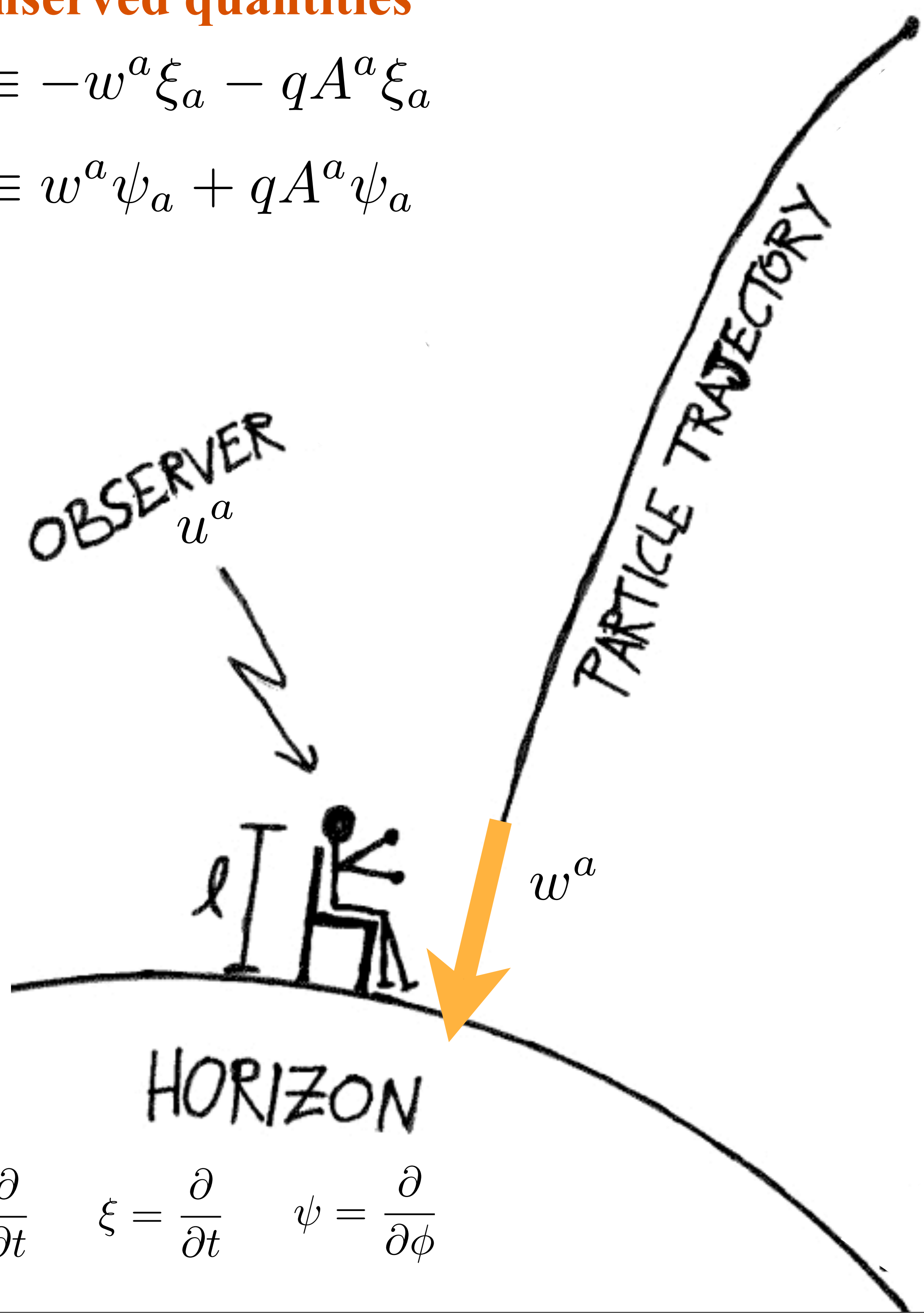
Conserved quantities

$$\mathcal{E} \equiv -w^a \xi_a - q A^a \xi_a$$

$$L \equiv w^a \psi_a + q A^a \psi_a$$

At the local observer

$$\mathcal{E}_{\ell oc} \equiv -w^a u_a \equiv \text{local energy}$$



$$\chi = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial t} \qquad \xi = \frac{\partial}{\partial t} \qquad \psi = \frac{\partial}{\partial \phi}$$

After absorption
seen from infinity

$$\chi = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial t} \qquad \xi = \frac{\partial}{\partial t} \qquad \psi = \frac{\partial}{\partial \phi}$$

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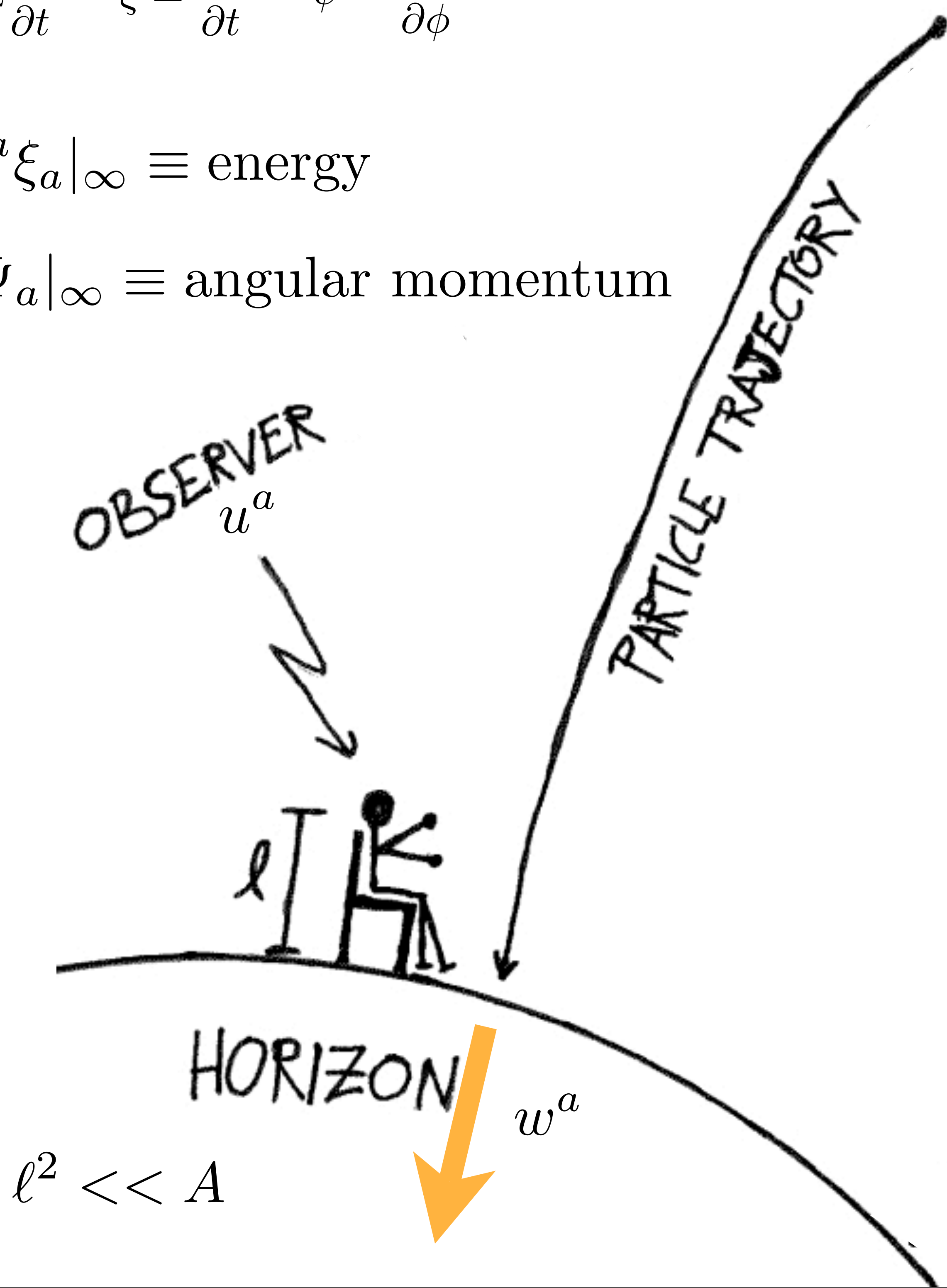


The BH readjusts parameters

$$\delta M = \mathcal{E} \qquad \delta J = L$$

$$\delta Q = q$$

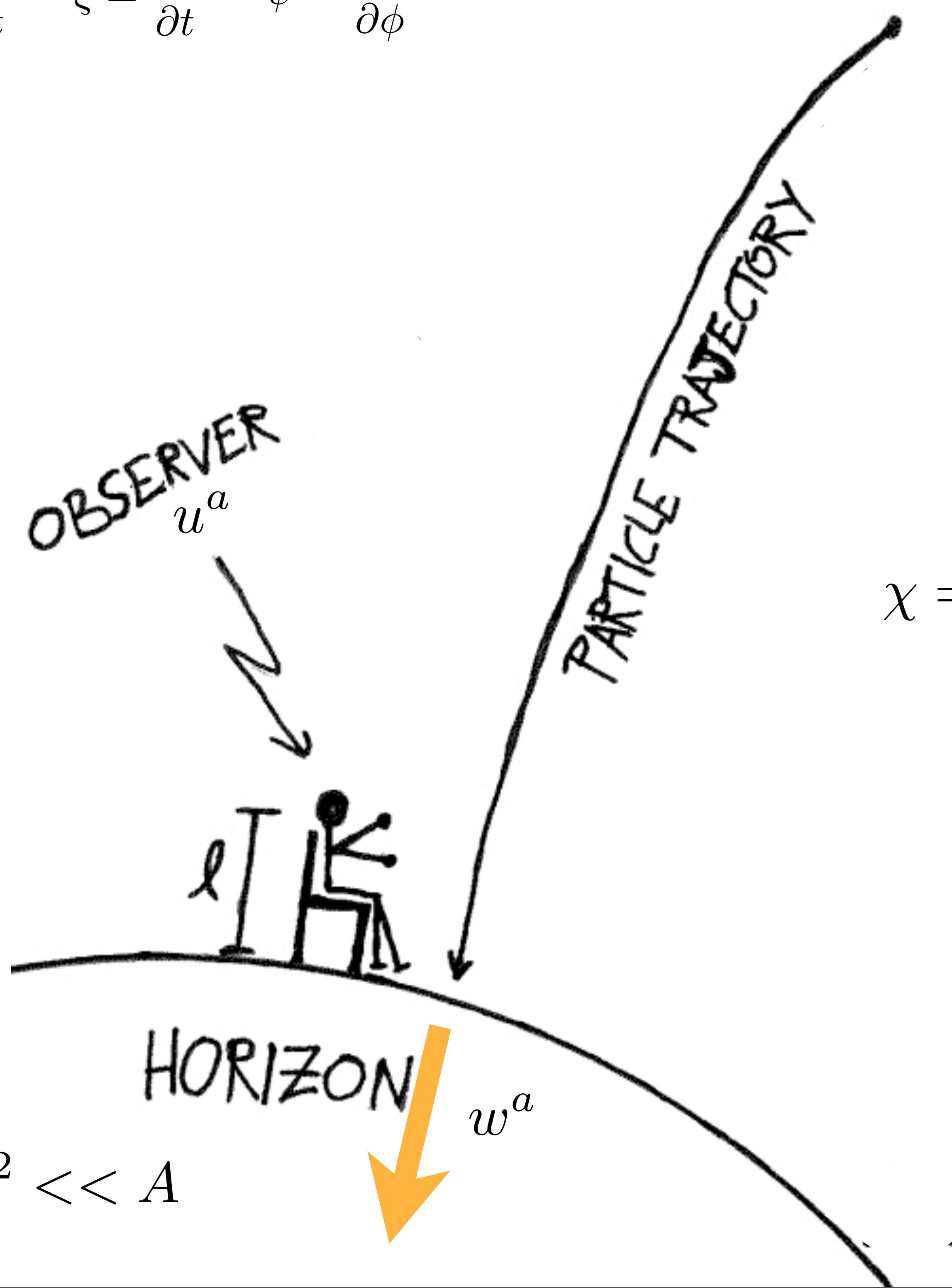
The area change from 1st law

$$\delta M = \frac{\kappa}{8\pi}\delta A + \Omega \delta J + \Phi \delta Q$$

$$\frac{\kappa}{8\pi}\delta A = \mathcal{E} - \Omega L - \Phi q$$

After absorption
seen by a local observer

$$\chi = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial t} \qquad \xi = \frac{\partial}{\partial t} \qquad \psi = \frac{\partial}{\partial \phi}$$



$\ell^2 << A$

$$\frac{\kappa}{8\pi}\delta A = \mathcal{E} - \Omega L - \Phi q$$

At the local observer

$$\mathcal{E}_{\ell oc} \equiv -w^a u_a \equiv \text{local energy}$$

$$\chi = \xi + \Omega\,\psi = \partial_t + \Omega\,\partial_\phi \qquad u^a = \frac{\chi^a}{\|\chi\|}$$

$$\mathcal{E}_{\ell oc} = -\frac{w^a \xi_a + \Omega w^a \psi_a}{\|\chi\|}$$

$$\boxed{\mathcal{E}_{\ell oc} = \frac{\mathcal{E} - \Omega L - q\Phi}{\|\chi\|}}$$

After absorption

seen by a local observer

$$\frac{\kappa}{8\pi}\delta A = \mathcal{E} - \Omega L - \Phi q$$

$$\mathcal{E}_{\ell oc} = \frac{\mathcal{E} - \Omega L - q\Phi}{\|\chi\|}$$

$$\mathcal{E}_{loc} = \frac{\kappa}{8\pi\|\chi\|}\delta A$$

$$\mathcal{E}_{loc} = \frac{\overline{\kappa}}{8\pi}\delta A$$

$$\overline{\kappa} \equiv \frac{\kappa}{\|\chi\|}$$

The appropriate local energy notion must be the one such that:

$$\delta E = \mathcal{E}_{\ell oc}$$

$$\boxed{\delta E = \frac{\overline{\kappa}}{8\pi}\delta A}$$

Local first law

Main classical result

$\ell^2 << A$

$$a = ||u^a \nabla_a u^b|| = \frac{1}{\ell}$$

$$\delta M = \frac{\kappa}{8\pi}\delta A + \Omega\,\delta J + \Phi\,\delta Q,$$

$$\delta E = \frac{\overline{\kappa}}{8\pi}\delta A \qquad \overline{\kappa} \equiv \frac{\kappa}{||\chi||} = \frac{1}{\ell} + o(\ell)$$

$$E = \frac{A}{8\pi\ell}$$

Local first law

Main classical result

$\ell^2 << A$

w^a

$$a = ||u^a \nabla_a u^b|| = \frac{1}{\ell}$$

$\mathscr{H}$

$$\delta M = \frac{\kappa}{8\pi} \delta A + \Omega\, \delta J + \Phi\, \delta Q,$$

$$\delta E = \frac{\overline{\kappa}}{8\pi} \delta A \qquad \overline{\kappa} \equiv \frac{\kappa}{||\chi||} = \frac{1}{\ell} + o(\ell)$$

$$\boxed{E = \frac{A}{8\pi\ell}}$$

- Quasilocal first law using test particles

  a Using that the spacetime geometry and maxwell fields representing the most general (physically relevant) stationary BH solution—the Kerr-Newman solution (see R.M. Wald GR, page 313) has killing fields ξ (stationarity) and ψ (axy-symmetry); therefore

$$\mathscr{L}_\xi g_{ab} = \mathscr{L}_\psi g_{ab} = 0 \tag{1}$$

$$\mathscr{L}_\xi A_a = \mathscr{L}_\psi A_a = 0 \tag{2}$$

  Show that the following two quantities are conserved along the trajectory of a unit mass test particle and charge q with four-velocity w^a

$$\mathcal{E} = -(\xi^a w_a + q\xi^a A_a) \tag{3}$$

$$L = -(\psi^a w_a + q\psi^a A_a) \tag{4}$$

  Show that these can be interpreted as total energy per unit mass and axial component of angular momentum for certain inertial observers placed at infinity (which ones?).

  b Show that the local surface gravity $\bar{\kappa} = \kappa/(||\chi||)$ defined for the special family of local observers previously introduced is universal in the leading order for proper distance $\ell \ll 1$ (i.e. independent of the BH parameters); more precisely show that

$$\bar{\kappa} = \frac{1}{\ell}(1 + curvature \ \ corrections) \tag{5}$$

  Interpret in terms of Rindler geometry. In what limit can one say that the near horizon geometry is Rindler?

  c With the above ingredients reproduce the argument leading to the local first law and the local energy formula, given in the previous pages, in all detail.

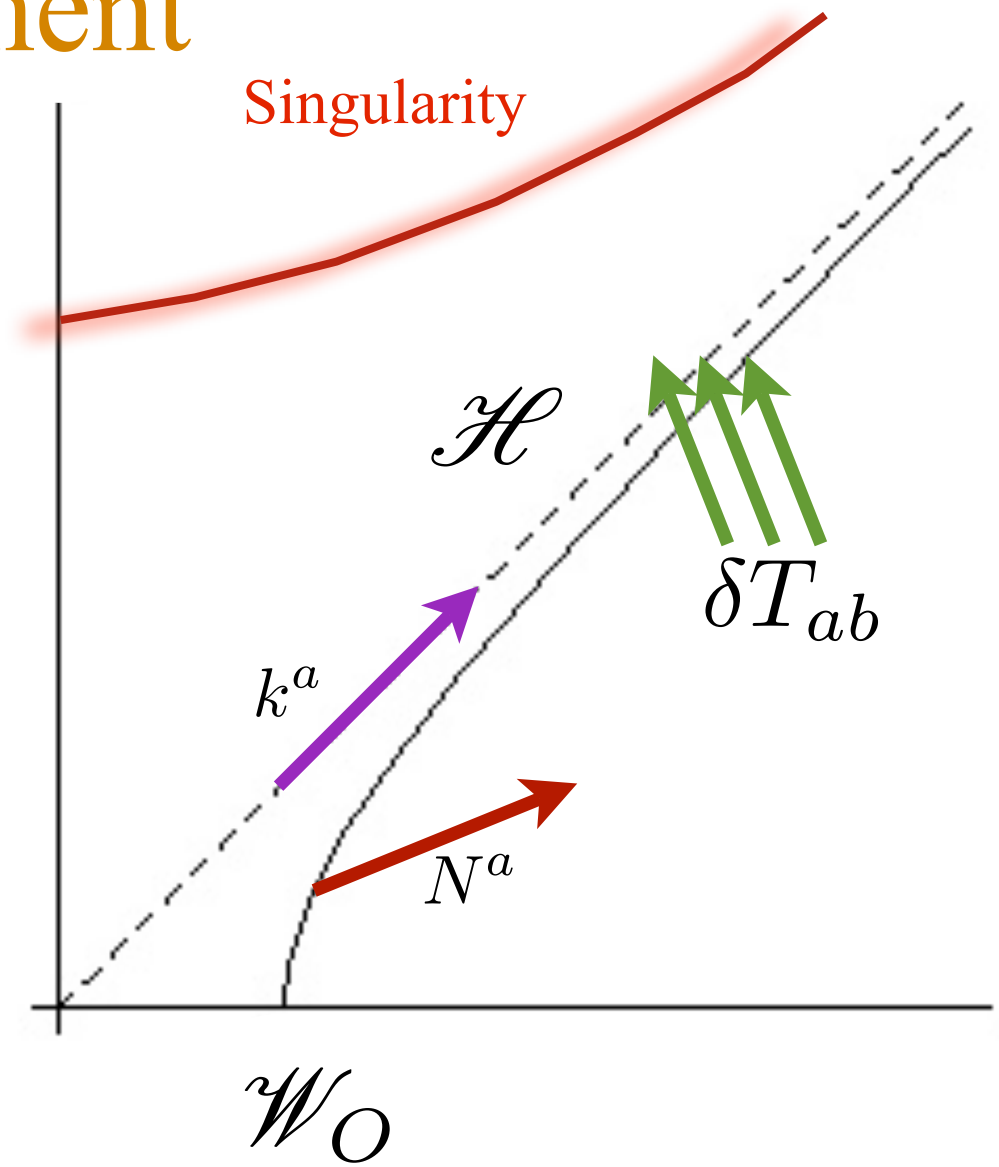
  d The quasi-local energy and the Komar-like energy formula. Show that the local energy can be written in terms of the Komar like integral

$$E = -\frac{1}{8\pi}\int_H \epsilon_{abcd}\nabla^c u^d \tag{6}$$

  where u^a is the four velocity of the local stationary observers defined in previous pages.

Local first law

A refined argument



$J^a = \delta T^a{}_b \chi^b$ is conserved thus

$$\int_{\mathscr{H}} dV dS\ \delta T_{ab}\chi^a k^b = \int_{W_{\mathscr{O}}} J_b N^b$$

$$\int_{\mathscr{H}} dV dS\ \delta T_{ab} \underbrace{\kappa V k^a}_{\chi^a} k^b = \int_{W_{\mathscr{O}}} \|\chi\|\, \delta T_{ab} u^a N^b$$

The Raychaudhuri equation

$$\frac{d\theta}{dV} = -8\pi \delta T_{ab} k^a k^b$$

$$\int_{\mathscr{H}} dV dS\ V \frac{d\theta}{dV} = -\frac{8\pi \|\chi\|}{\kappa} \delta E,$$

$$\delta E = \frac{\overline{\kappa}}{8\pi} \delta A$$

- Quasilocal first law using fields and Einsteins equations

  a Define the energy momentum current $J_a = T_{ab}\chi^b$ and show that it is conserved, where $T_{ab} = T^{(0)}_{ab} + \delta T_{ab}$ (i.e. a background term plus a perturbation representing a small amount of matter infalling into and otherwise stationary Kerr-Newman black hole).

  b Recalling that at the horizon the Killing generator χ satisfies $\chi^a \nabla_a \chi_b = \kappa \chi_b$ (where κ is the surface gravity), and that the Killing generator vanishes at the bifurcate horizon, show that there exist an affine generator k^a (i.e. $k^a \nabla_a k_b = 0$) such that

  $$\chi^a = \kappa V k^a, \tag{1}$$

  with V the affine parameter associated to k^a and singled out by the property that $V = 0$ at the bifrcate horizon.

  c Gauss law is subtle when applied to null surfaces (see Wald GR pages 432-434). Show that the flux of J^a across the horizon takes the form

  $$F_{horizon} = \int_H dV dS^2 T_{ab} \chi^a k^b \tag{2}$$

  where V is the affine parameter of point (b), and dS^2 is the area element of the spheres $V = constant$.

  d Use Gauss law in the region limited by the horizon and the worldsheet of local observers (see previous slide) combined with Raychaudhuri equation (Wald 9.2.32) and Einsteins equations to prove the quasilocal first law

  $$\delta E = \frac{\bar{\kappa}}{8\pi} \delta A. \tag{3}$$

  HINT: use the correct boundary condition for the expansion of k^a at $V = \infty$.

  e Notice that the derivation of the local first law does not need the normalization of the Killing field χ at infinity. Therefore, the local first law is valid in a more general context than asymptotically flat spacetimes. Indeed no asymptotic conditions are necessary for its validity due to its intrinsically quasilocal nature.

Local first law

A refined argument

$J^a = \delta T^a{}_b \chi^b$ is conserved thus

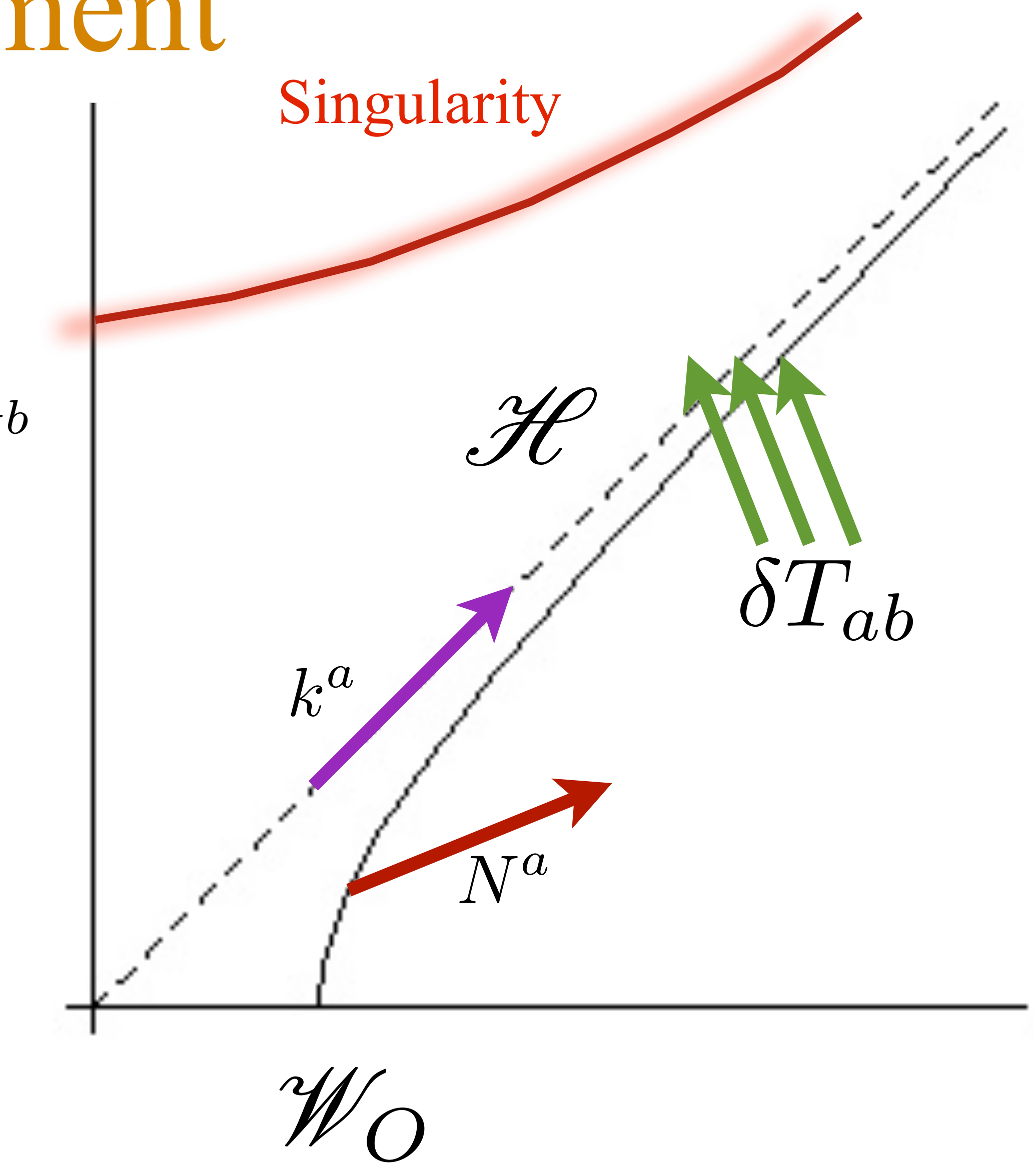
$$\int_{\mathscr{H}} dV dS\ \delta T_{ab} \chi^a k^b = \int_{W_{\mathscr{O}}} J_b N^b$$

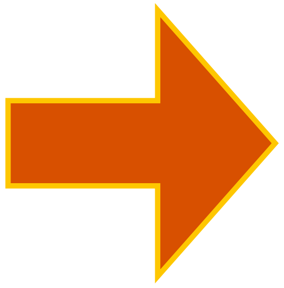
$$\int_{\mathscr{H}} dV dS\ \delta T_{ab} \underbrace{\kappa V k^a}_{\chi^a} k^b = \int_{W_{\mathscr{O}}} \| \chi \| \, \delta T_{ab} u^a N^b$$

The Raychaudhuri equation

$$\frac{d\theta}{dV} = -8\pi \delta T_{ab} k^a k^b$$

$$\int_{\mathscr{H}} dV dS\ V \frac{d\theta}{dV} = -\frac{8\pi \| \chi \|}{\kappa} \delta E,$$

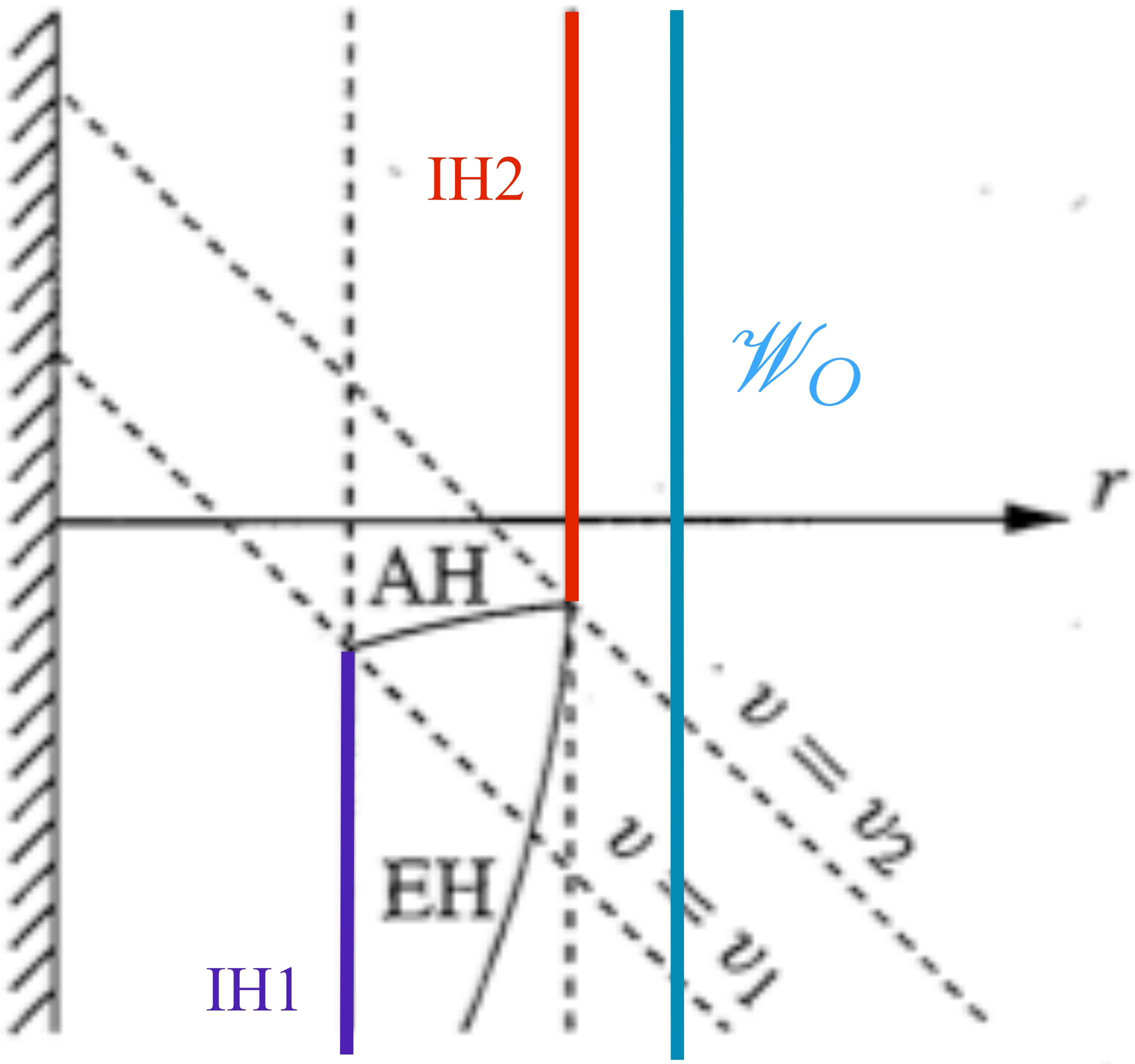


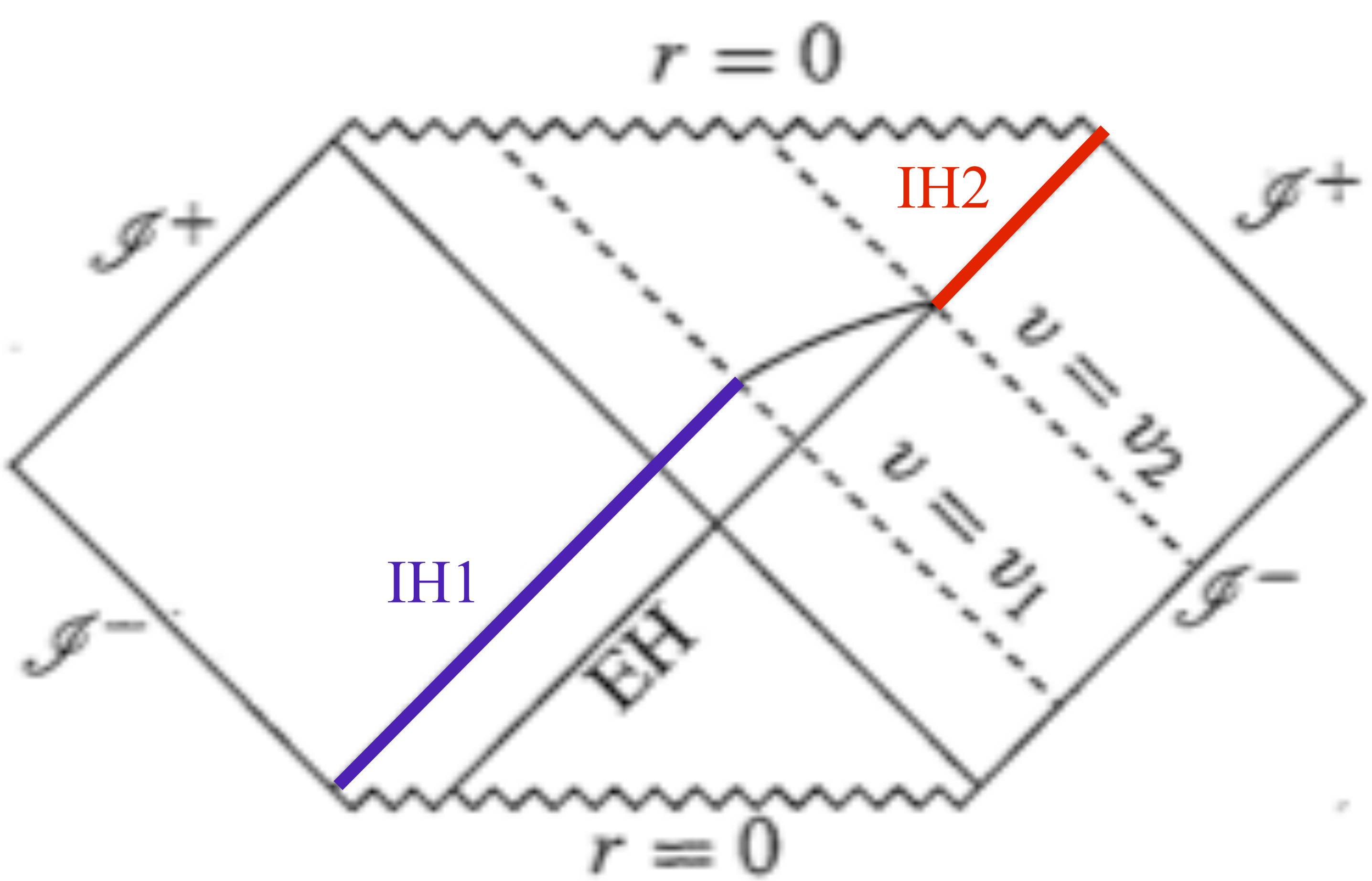


$$\delta E = \frac{\overline{\kappa}}{8\pi} \delta A$$

The Local first law is dynamical

Simple example: Vaidya spacetime





The same holds in non symmetric situations (detailed proof in progress AP, O. Moreschi, E. Gallo)

$$\delta E = \frac{\overline{\kappa}}{8\pi}\delta A$$

Implications for the quantum theory

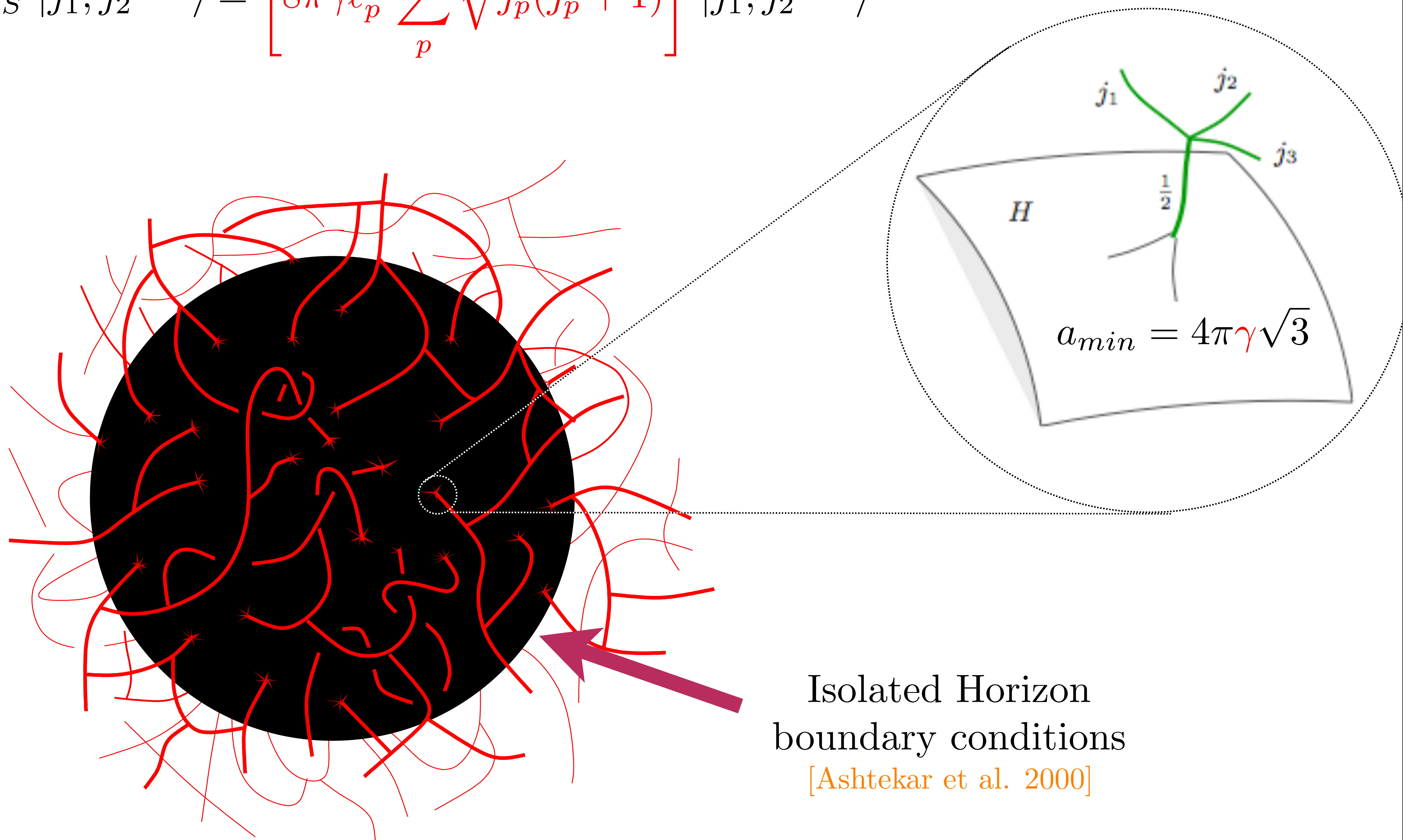
The quasilocal approach: insights into the statistical mechanical origin of BH entropy

[Ghosh, Perez, 2011 PRL]
[Frodden, Ghosh, Perez, to appear]

The black hole area spectrum

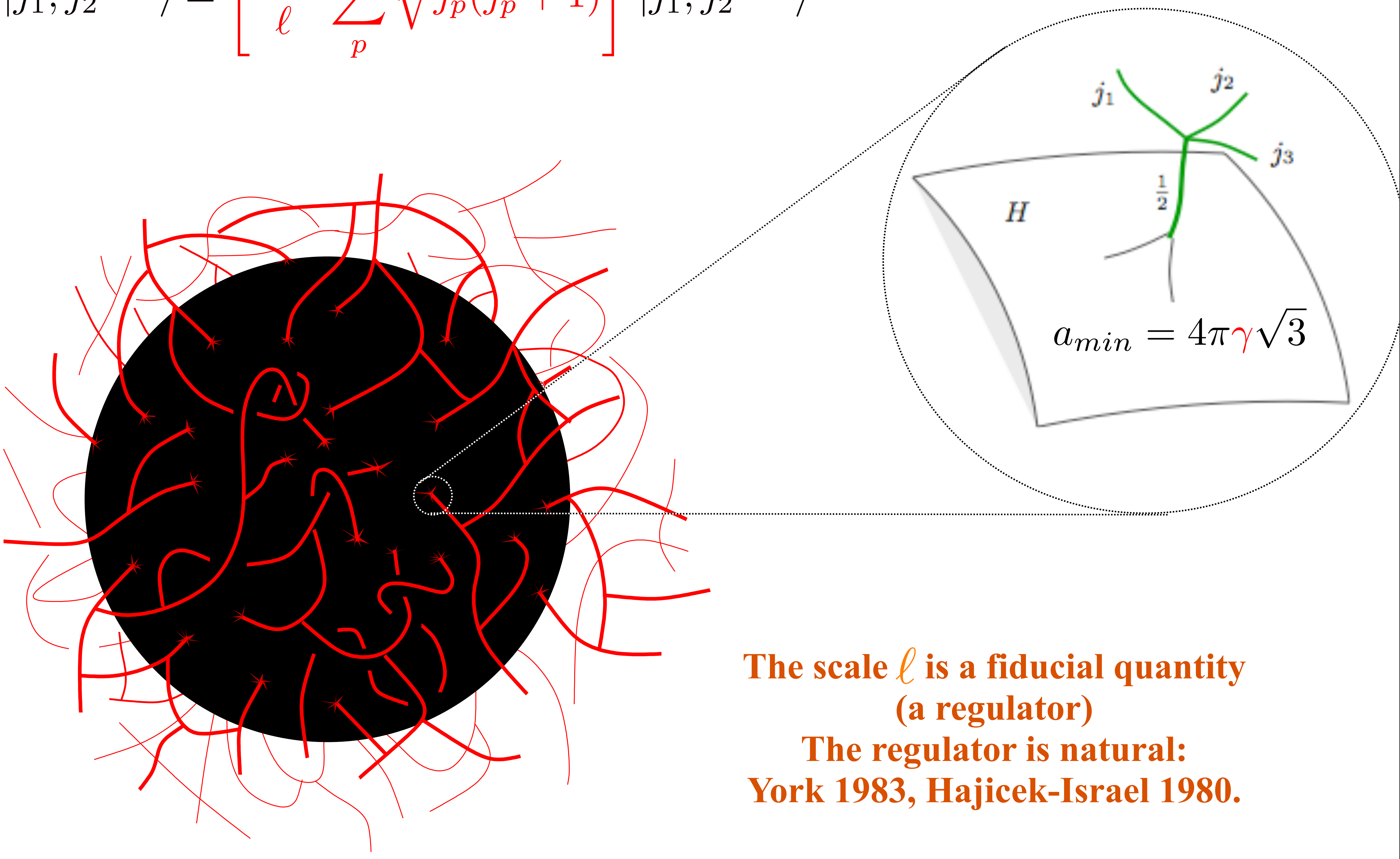
The area gap

$$\widehat{A}_S \, |j_1, j_2 \cdots\rangle = \left[8\pi\gamma\ell_p^2 \sum_p \sqrt{j_p(j_p+1)} \right] |j_1, j_2 \cdots\rangle$$



The energy spectrum
The area gap

$$\widehat{H}|j_1, j_2 \cdots\rangle = \left[\frac{\gamma \ell_p^2}{\ell} \sum_p \sqrt{j_p(j_p+1)}\right] |j_1, j_2 \cdots\rangle$$

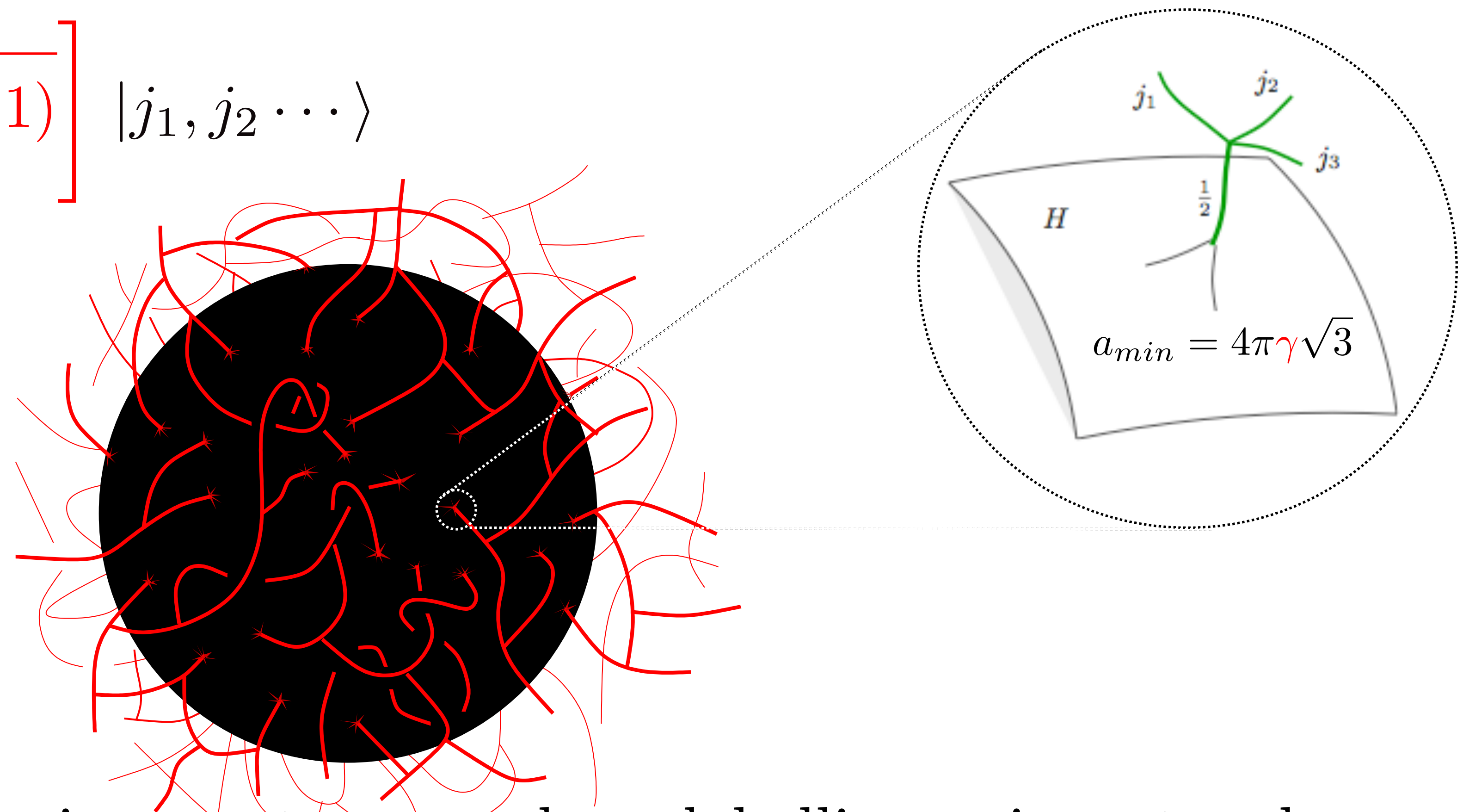


The scale ℓ is a fiducial quantity
(a regulator)
The regulator is natural:
York 1983, Hajicek-Israel 1980.

Is the number of punctures an important observable?

Energy Spectrum vs. Chemical Potential

$$\widehat{H}|j_1, j_2\cdots\rangle = \left[\frac{\gamma\ell_p^2}{\ell}\sum_p\sqrt{j_p(j_p+1)}\right]|j_1, j_2\cdots\rangle$$



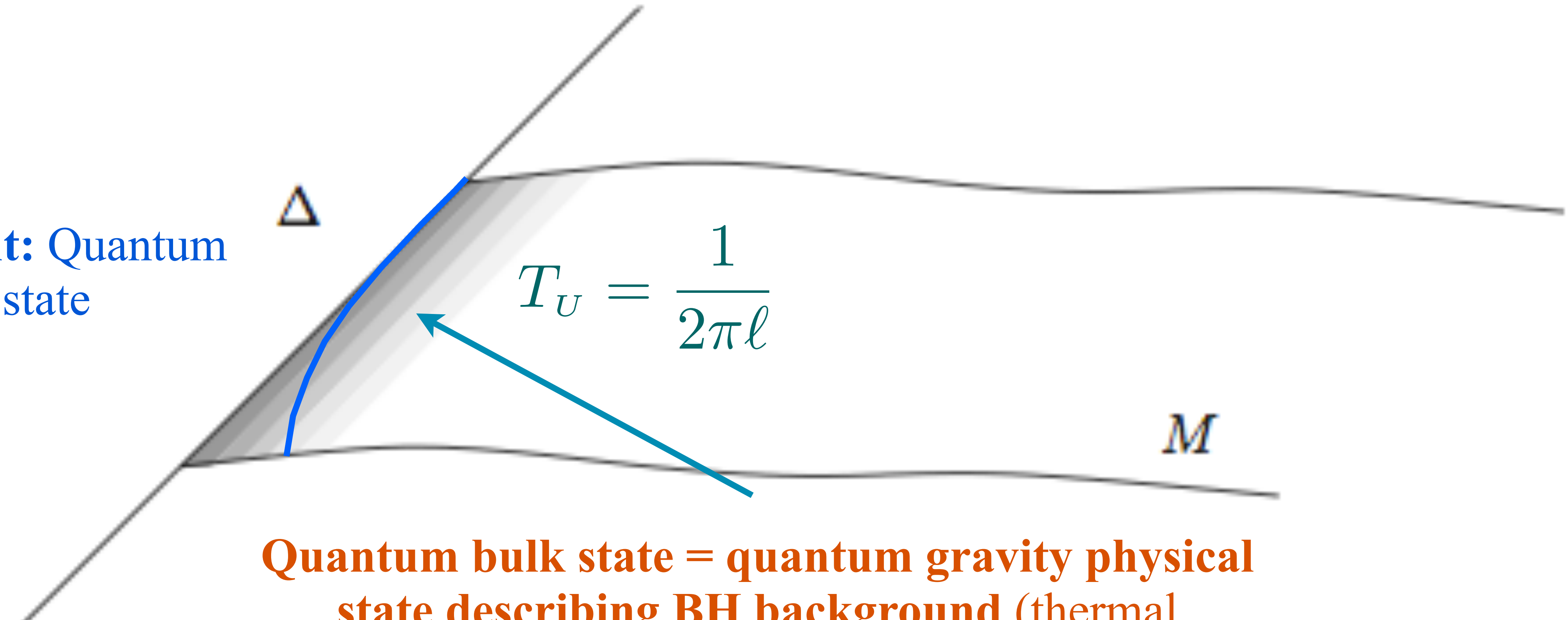
a) By a rearrangement of the spin quantum numbers labelling spin network links ending at punctures on the horizon without changing the number of punctures N (in the large area regime this kind of transitions allows for area jumps as small as one would like as the area spectrum becomes exponentially dense in $\mathbb{R}^+$ [Rovelli 96]

b) By the emission or absorption of punctures with arbitrary spin (such transitions remain discrete at all scales and are responsible for a modification of the first law: a chemical potential arises and encodes the mean value of the area change in the thermal mixture of possible values of spins j).

Computation of entropy in LQG

pure geometry calculation

Present ingredient: Quantum IH physical state



Quantum bulk state = quantum gravity physical state describing BH background (thermal equilibrium at Unruh temperature)

[for argument about how to perhaps avoid this assumption see Bianchi 2012 and Pranzetti 2013]

$\ell \equiv$ arbitrary fixed proper distance to the horizon

Black Hole Entropy from LQG

Pure gravity calculation
(neglecting matter contributions);
distinguishable punctures

- BH entropy

  a In this exercise we will compute BH entropy in the simplest LQG scenario. We assume punctures of the horizon (area quantum excitations) are distinguishable. In the microcanonical ensemble we must count how many states there are such that the following constraint is satisfied (according to the form of the area spectrum in LQG)

$$C_1 : \sum_j \sqrt{j(j+1)}\, s_j = \frac{A}{8\pi \ell_g^2}, \tag{1}$$

  Ignoring global constraints (due to Chern-Simons formulation) show that the number of states $d[\{s_j\}]$ associated with a configuration $\{s_j\}$ (where s_j denotes the number of punctures with spin j) is

$$d[\{s_j\}] = \Big(\sum_k s_k\Big)! \prod_j \frac{(2j+1)^{s_j}}{s_j!}. \tag{2}$$

  b Look for the configuration that maximizes the entropy $\log(d[\{s_j\}])$ subject to the above constraint.

  c Show that—using Stirling's approximation—the dominant configuration

$$\frac{s_j}{N} = (2j+1)e^{-\lambda\sqrt{j(j+1)}}, \tag{3}$$

  where is a solution of

$$1 = \sum_j (2j+1)e^{-\lambda\sqrt{j(j+1)}}. \tag{4}$$

  d Show that the entropy (defined as the value of $\log(d[\{s_j\}])$ on the dominant configuration) is

$$S = \frac{\gamma_0}{\gamma}\frac{A}{4\ell_p^2} \tag{5}$$

  where $\gamma_0 = \lambda/(2\pi)$.

  e Show that the previous result is in conflict with the local first law (and hence with the usual first law) unless $\gamma = \gamma_0$.

  f Redo the exercise by imposing an additional constraint

$$C_2 : \sum_j s_j = N.$$

  and show by computing $S(A, N)$ that the conflict with the first law disappears and all values of γ are allowed. Obtain an expression of the entropy as a function of the area alone using the equation of state.

Number of punctures contribute to S

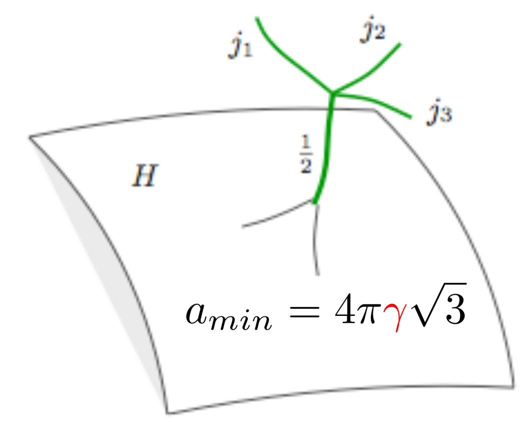
The canonical partition function is given by

K. Krasnov (1999), S. Major (2001), F. Barbero E. Villasenor (2011)

$$Z(N,\beta) = \sum_{\{s_j\}} \prod_j \frac{N!}{s_j!} [(2j+1)]^{s_j} e^{-\beta s_j E_j} \quad \Longrightarrow \quad \log Z = N \log[\sum_j [(2j+1)] e^{-\beta E_j}]$$

where $E_j = \ell_g^2 \sqrt{j(j+1)}/\ell$. A simple calculation gives

$$S = -\beta^2 \frac{\partial}{\partial \beta}(\frac{1}{\beta} \log Z)\Big|_{\beta = 2\pi\ell} = \frac{A}{4\ell_p^2} + \log Z$$



more precisely

$$S = \frac{A}{4\ell_p^2} + \sigma(\gamma) N$$

where

$$\sigma(\gamma) \equiv \log[\sum_j (2j+1) e^{-2\pi\gamma\sqrt{j(j+1)}}].$$

The (thermodynamical) local first law versus the (geometric) local first law

$$\delta M = \frac{\kappa}{2\pi}\,\delta S + \Omega\,\delta J + \Phi\,\delta Q + \mu\,\delta N \quad \Longleftrightarrow \quad \delta M = \frac{\kappa}{2\pi}\,\delta A + \Omega\,\delta J + \Phi\,\delta Q$$

$$\mu = -T \frac{\partial S}{\partial N}|_A = -\frac{\kappa}{2\pi}\sigma(\gamma)$$

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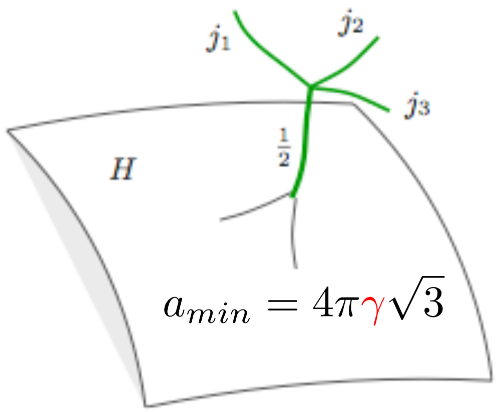

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Number of punctures contribute to S

Distinguishability

The canonical partition function is given by

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where $E_j = \ell_g^2 \sqrt{j(j+1)}/\ell$. A simple calculation gives

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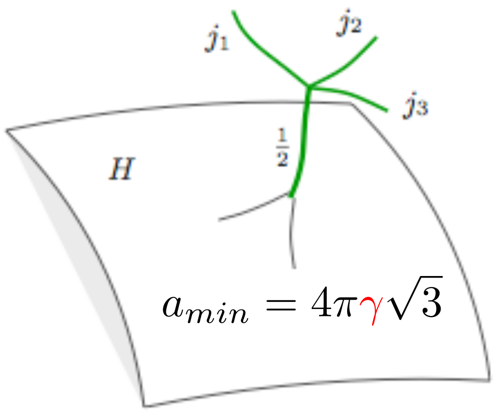

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Number of punctures contribute to S

Degeneracy

The canonical partition function is given by

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where $E_j = \ell_g^2 \sqrt{j(j+1)}/\ell$. A simple calculation gives

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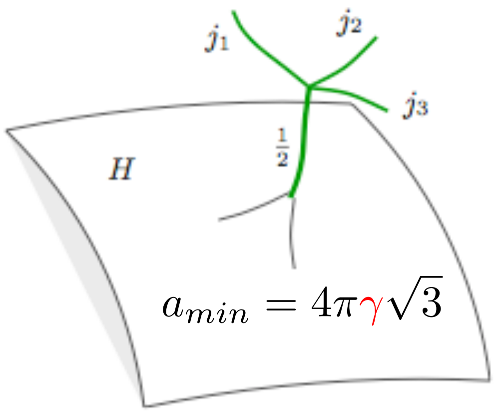

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Number of punctures: an important observable

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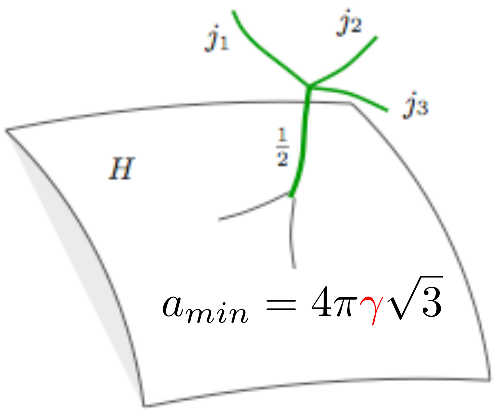

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Number of punctures: an important observable

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more precisely

$$S = \frac{A}{4\ell_p^2}\left(1 - \frac{\sigma(\gamma)}{\gamma \frac{d\sigma}{d\gamma}}\right) \qquad \text{from EOS} \qquad \langle E \rangle = -\frac{\partial}{\partial \beta} \log Z \Big|_{\beta=2\pi\ell} \quad \Longleftrightarrow \quad N = \frac{-A}{4\ell_p^2 \gamma \frac{d\sigma}{d\gamma}}$$

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where $E_j = \ell_g^2 \sqrt{j(j+1)}/\ell$. A simple calculation gives

$$S = -\, \beta^2 \frac{\partial}{\partial \beta} (\frac{1}{\beta} \log Z)\Big|_{\beta = 2\pi\ell} = \frac{A}{4\ell_p^2} + \log Z \neq \frac{A}{4\ell_p^2}$$

[Hawking-Gibbons, Carlip CFT, etc]

more precisely

$$S = \frac{A}{4\ell_p^2}\left(1 - \frac{\sigma(\gamma)}{\gamma \frac{d\sigma}{d\gamma}}\right) \quad \text{from EOS} \quad \langle E \rangle = -\frac{\partial}{\partial \beta} \log Z\Big|_{\beta=2\pi\ell} \quad \Longleftrightarrow \quad N = \frac{-A}{4\ell_p^2 \gamma \frac{d\sigma}{d\gamma}}$$

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$$\mu = -T \frac{\partial S}{\partial N}|_A = -\frac{\kappa}{2\pi} \sigma(\gamma)$$

Matter

Can we consistently compute BH entropy neglecting matter?

[Frodden, Ghosh, Perez, to appear]

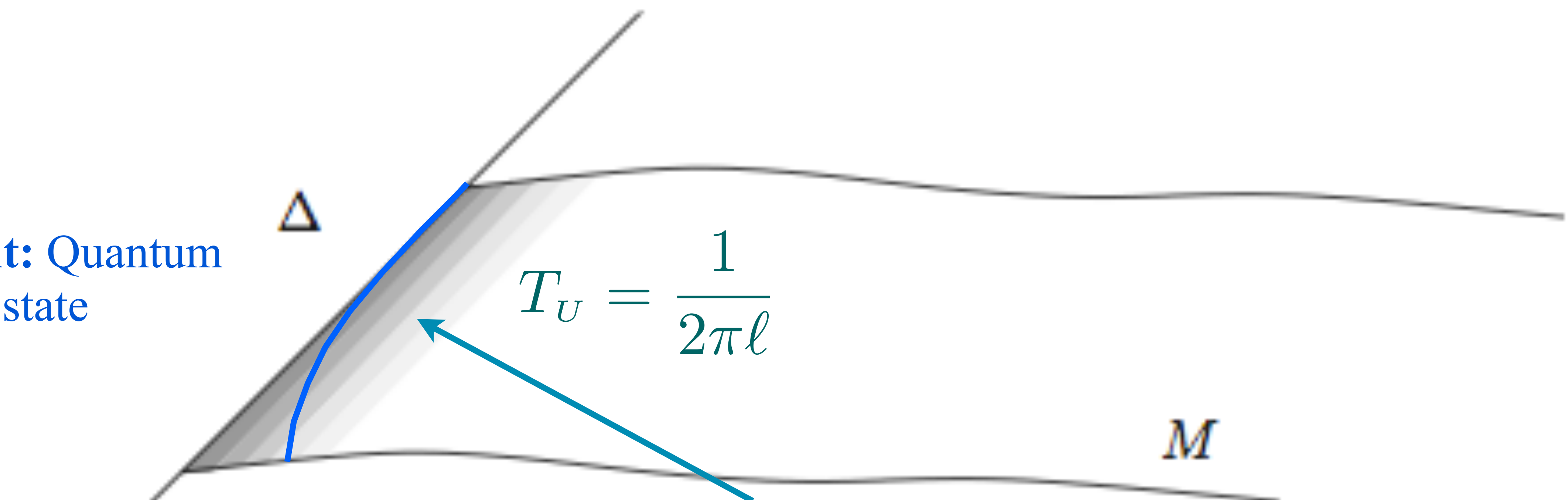
The vacuum in QFT is not an empty page...

The vacuum in QFT is not an empty page...

Computation of entropy in LQG

Including matter d.o.f.

Present ingredient: Quantum IH physical state

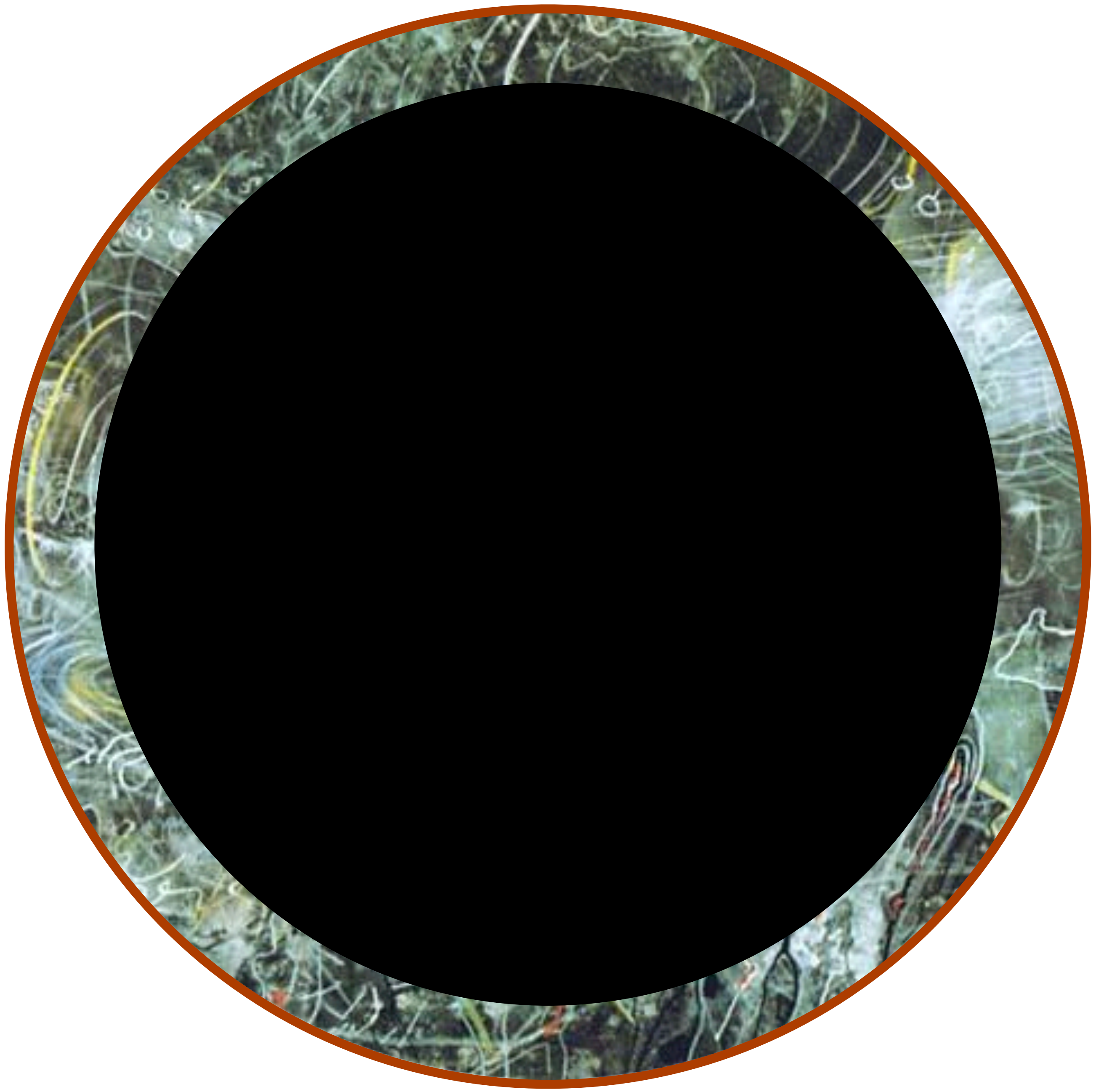


Quantum bulk state = quantum gravity physical state describing BH background (for matter fields in the semiclassical approximation = vacuum regular at horizon)

$\ell \equiv$ arbitrary fixed proper distance to the horizon

What about matter?

Matter entanglement, t'Hooft brick wall model, etc



$$S_{matter} = \lambda \frac{A}{\epsilon^2} + \text{corrections}$$

$\lambda =$ undertermined constant
(UV regularization dependent
species problem)

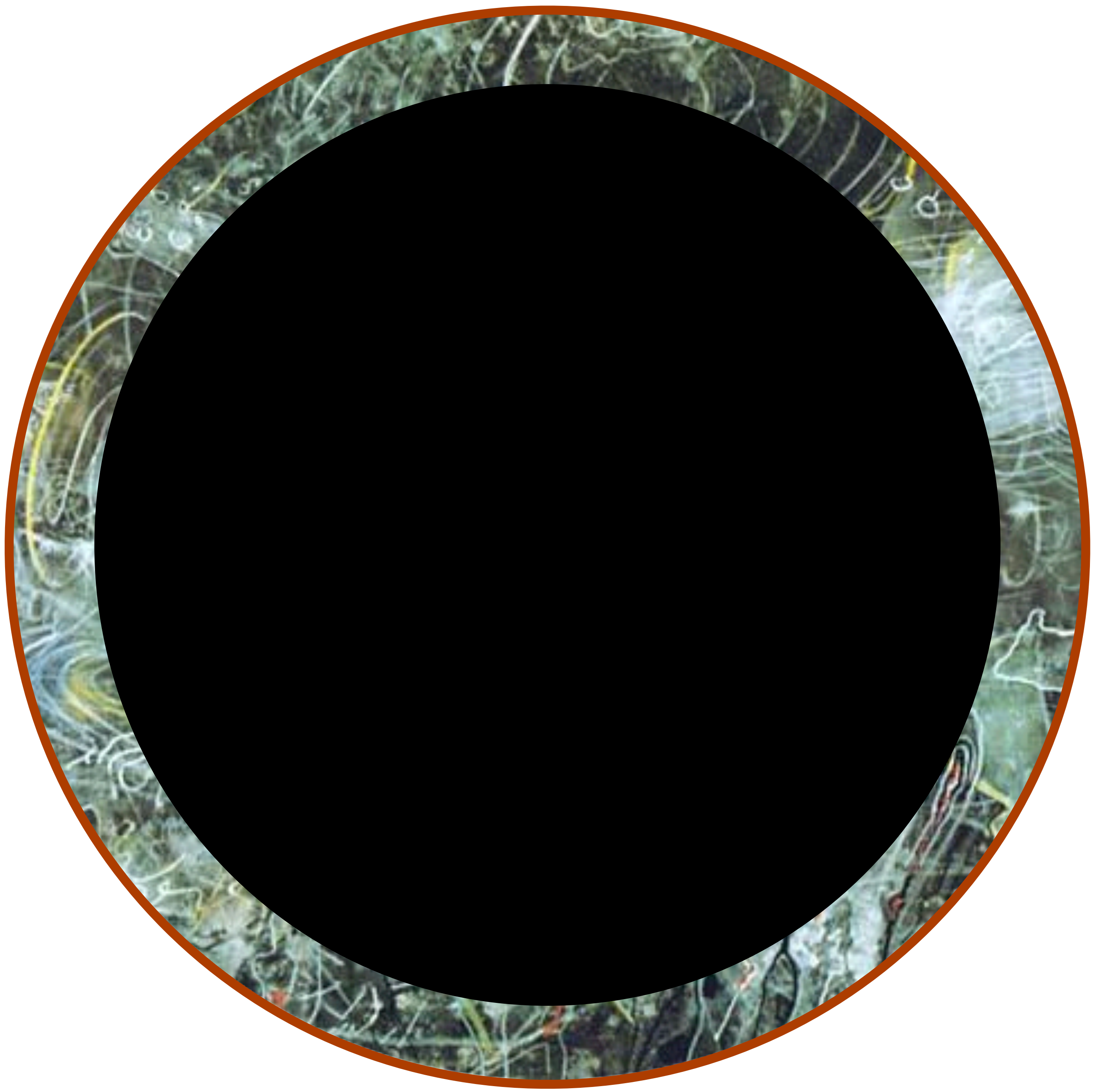
$\epsilon =$ UV cut-off

Number of d.o.f. dominated by boundary contribution

$$D \approx \exp(\lambda A/(4\ell_p^2))$$

What about matter?

Matter entanglement, t'Hooft brick wall model, etc



$$S_{matter} = \lambda \frac{A}{\epsilon^2} + \text{corrections}$$

In LQG: Energy=Area

Matter d.o.f. = degeneracy of area spectrum

$$\epsilon = \ell_p$$

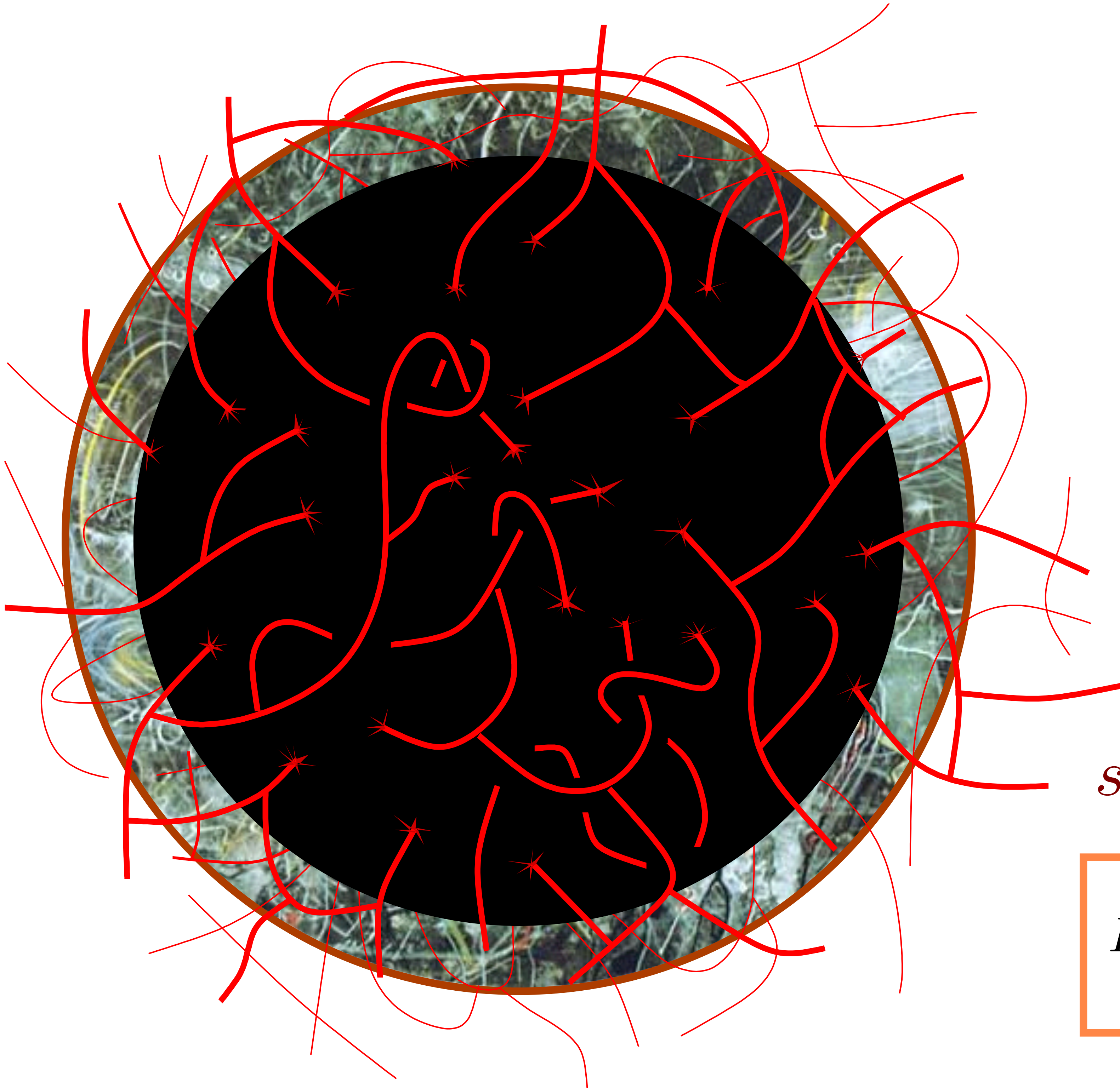
Just a new notation $\lambda = \frac{1-\delta}{4}$

Degeneracy grows exponentially with A

$$D \approx \exp\left[\lambda \frac{A}{\epsilon^2}\right]$$

What about matter?

Matter entanglement, t'Hooft brick wall model, etc



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In LQG: Energy=Area

Matter d.o.f. = degeneracy of area spectrum

$$\epsilon = \ell_p$$

Just a new notation $\lambda = \frac{1-\delta}{4}$

s_j $\equiv$ **number of punctures with spin j**

$$D[\{s_j\}] \approx \prod_j \exp \frac{(1-\delta) a_j s_j}{4\ell_p^2}$$

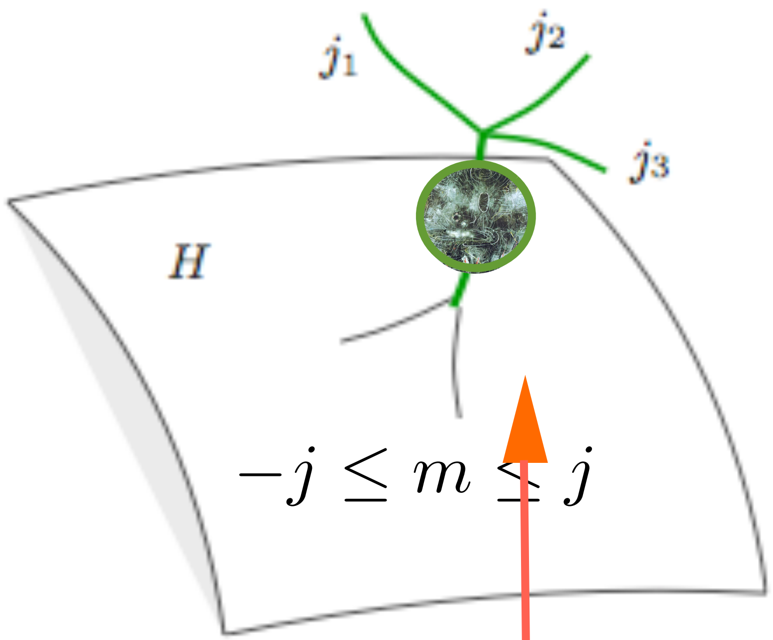
Black Hole Entropy from LQG

Gravity+Matter; indistinguishable punctures

$$Z[\beta] = \sum_{\{s_j\}} \prod_j e^{-(\beta - \beta_U + \delta\beta_U) s_j E_j}$$

$$Z[\beta_U] = \prod_j [1 \pm \exp(-2\pi\gamma\delta j)]^{\pm 1}$$

$$\beta_U = 2\pi\ell$$



The puncture is *dressed by matter d.o.f.*

$$U = \frac{A}{8\pi\ell} = -\partial_\beta \log Z$$

$$= \frac{\gamma \ell_p^2}{\ell} \sum_{j=1/2}^{\infty} \frac{j}{\exp(2\pi\gamma\delta j) \pm 1}$$

$$A \approx \frac{4\ell_p^2}{\pi\gamma\delta^2} \int_0^\infty \frac{x\, dx}{e^x \pm 1} = \frac{\epsilon_\pm \pi \ell_p^2}{3\gamma\delta^2}$$

$$\delta = \sqrt{\frac{\pi \epsilon_\pm \ell_p^2}{3\gamma A}} \ll 1$$

Black Hole Entropy from LQG

Gravity+Matter; indistinguishable punctures

INPUTS:

- Quasilocal Hamiltonian + LQG quantum geometry (Area spectrum)
- LQG UV finiteness + QFT
  $$D \approx \exp\left[\frac{1-\delta}{4}\frac{A}{\ell_p^2}\right]$$
- Indistinguishability of punctures

RESULTS:

- Matter saturates Holographic bound: $\delta = \sqrt{\frac{\pi\epsilon_\pm \ell_p^2}{3\gamma A}} \ll 1$
- $$S = \beta U + \log Z$$
  $$S = \frac{A}{4\ell_p^2}\left[1 + \sqrt{\frac{\pi\epsilon_\pm \ell_p^2}{3\gamma A}} + o\left(\frac{\gamma\ell_p^2}{A}\right)\right]$$
- Area fluctuations are small
  $$\frac{\Delta U}{U} = \frac{\Delta A}{A} = \sqrt{\pi\gamma\delta}$$
- $$\langle N\rangle \approx \sqrt{A/\ell_p^2}, \quad \langle j\rangle \approx \sqrt{A/\ell_p^2},$$
  $$\Delta j \approx \sqrt{A/\ell_p^2}$$
- $$C = -\beta^2\partial_\beta U \approx \frac{2}{\pi\gamma\delta^3}$$
- The chemical potential vanishes $\mu = 0$

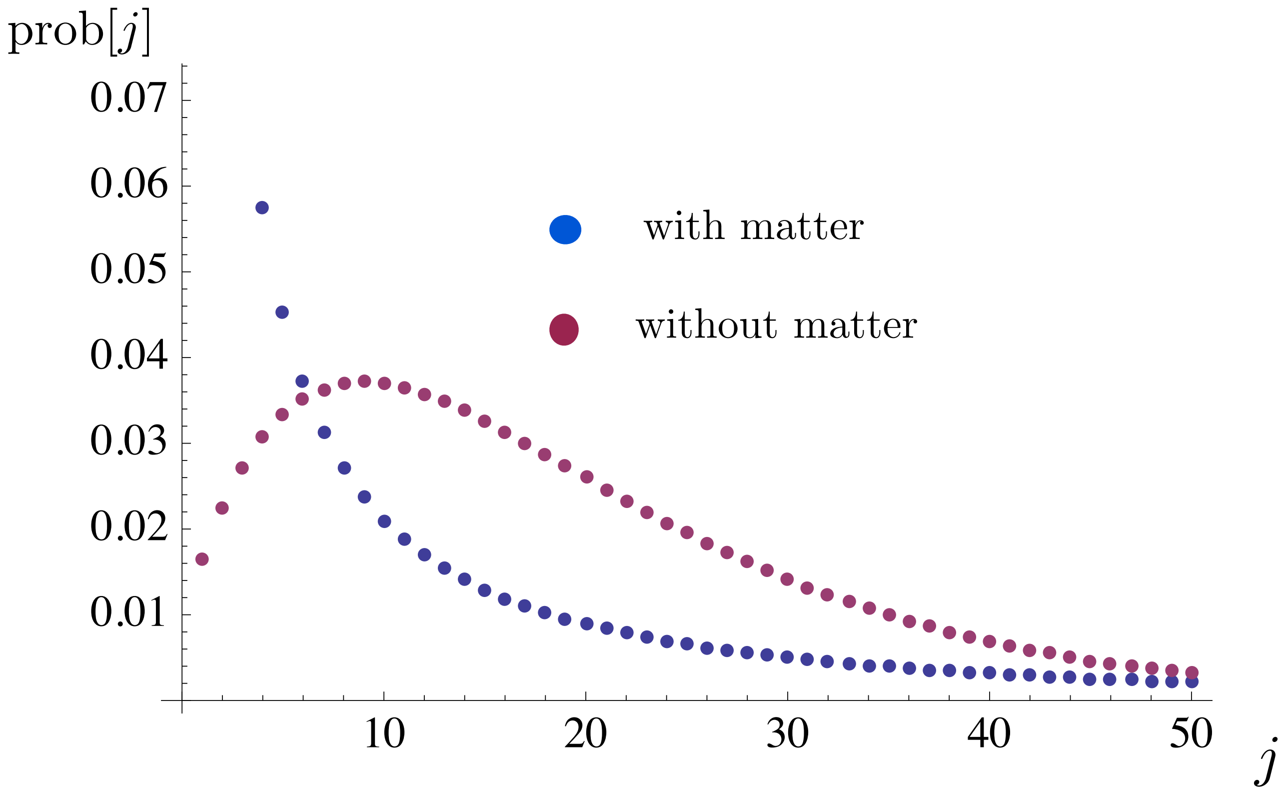
Black Hole Entropy from LQG

Thermal state is a semiclassical low energy state

$$Z[\beta] = \sum_{\{s_j\}} \prod_j e^{-(\beta - \beta_U + \delta\beta_U) s_j E_j}$$

$$\langle N \rangle \approx \sqrt{A/\ell_p^2}, \quad \langle j \rangle \approx \sqrt{A/\ell_p^2},$$

$$\Delta j \approx \sqrt{A/\ell_p^2}$$



Semiclassical and low energy regime

$$\ell_p^2 \ll \bar{a} = \ell_p^2 \langle j \rangle \ll A$$

Thiemann, Sahlmann, Winkler (2001)
Ashtekar et al. (2001)

Han et al. (2012) see spin foam talk

Q: can we get a more explicit manifestation that we are indeed in the semiclassical regime?

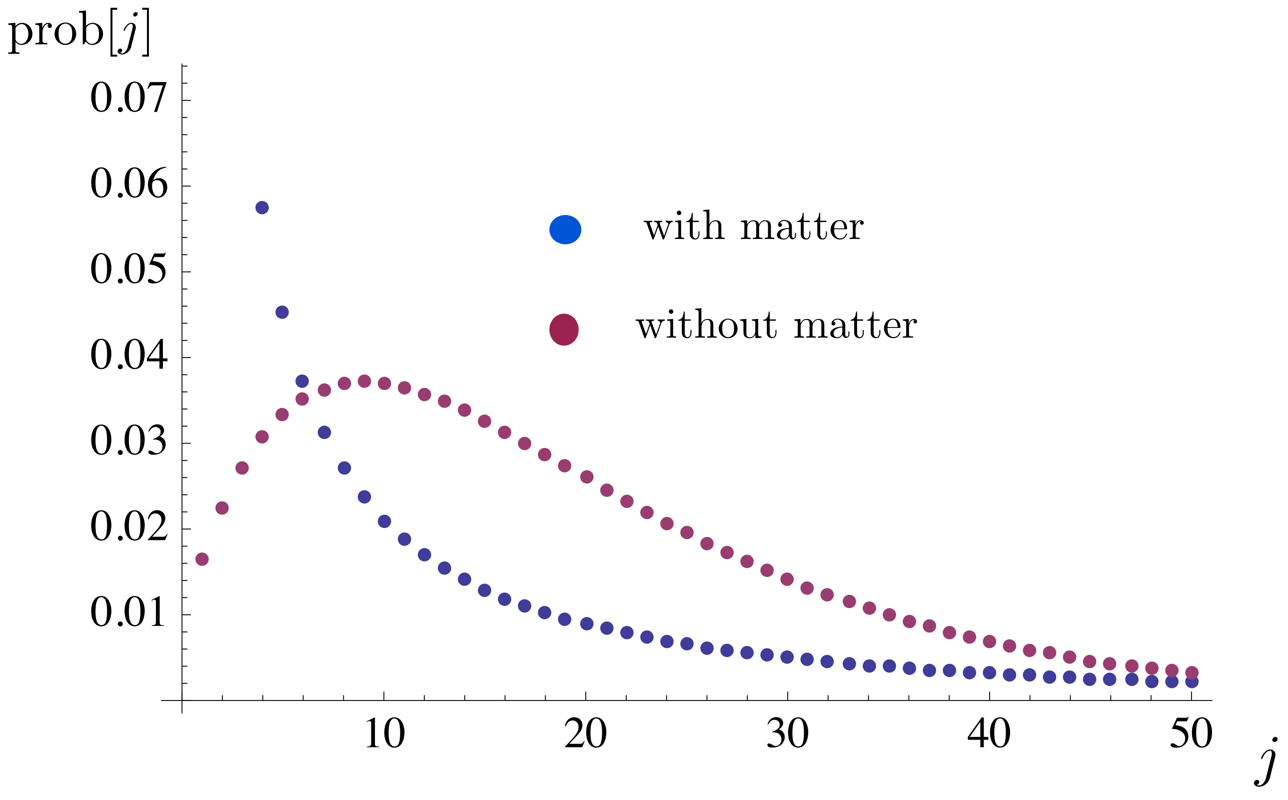
Black Hole Entropy from LQG

Thermal state is a semiclassical low energy state

$$Z[\beta] = \sum_{\{s_j\}} \prod_j e^{-(\beta - \beta_U + \delta\beta_U)s_j E_j}$$

$$\tilde{\beta} \equiv \beta - \beta_U(1-\delta) \ll 1$$

$$\langle N \rangle \approx \sqrt{A/\ell_p^2}, \quad \langle j \rangle \approx \sqrt{A/\ell_p^2}, \quad \Delta j \approx \sqrt{A/\ell_p^2}$$



Semiclassical and low energy regime

$$\ell_p^2 \ll \bar{a} = \ell_p^2 \langle j \rangle \ll A$$

Thiemann, Sahlmann, Winkler (2001)
Ashtekar et al. (2001)

Han et al. (2012) see spin foam talk

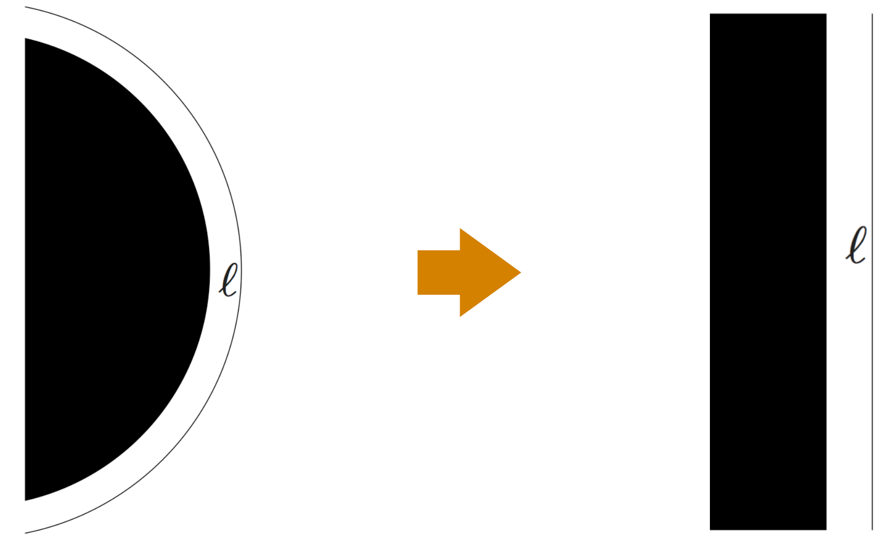
Q: can we get a more explicit manifestation that we are indeed in the semiclassical regime?

Semiclassical correspondence

relationship with the Euclidean path integral

$$Z[\beta] = \sum_N \sum_{\{s_j\}} e^{-\sum_j (\beta - \beta_U + \delta\beta_U) s_j E_j}$$

$$\approx \sum_N \sum_{\{s_j\}} e^{-(\beta - 2\pi\ell) \sum_j s_j \frac{a_j}{8\pi \ell_p^2 \ell}}$$



$$Z_{PI}[\beta] \equiv \int Dg_\beta^{(4)} \exp\Big[-\frac{1}{16\pi\ell_p^2} \int_M R[g^{(4)}] - \frac{1}{8\pi\ell_p^2} \int_{\partial M} (K - K_0) \Big]$$

$$\approx \exp\Big[-(\beta - 2\pi\ell) \frac{A[g^{(2)}]}{8\pi\ell_p^2 \ell} \Big]$$

[Banados-Teitelboim-Zanelli, Carlip-Teitelboim, etc]

[E. Frodden thesis]

$$Z[\beta] \approx Z_{PI}[\beta]$$

Conclusions

- The quasilocal approach captures the relevant physics for black hole thermodynamics.
- It is complementary to the *isolated horizon framework* of Ashtekar et. al.
- It provides an effective energy notion proportional to the horizon area.
- It holds for stationary horizons that are not necessarily asymptotically flat (no need to normalize killing fields in the derivation of the quasilocal first law)
- This energy notion can be used in the statistical mechanical description of the quantum horizon degrees of freedom.

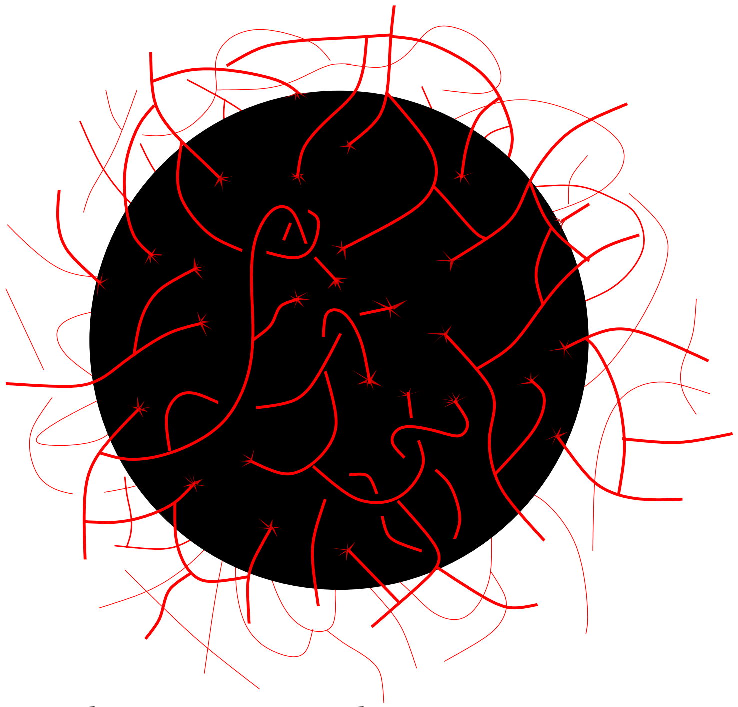
Conclusions

➡ If matter contributions are neglected, the quantum geometry degeneracy of the area spectrum implies *low spin dominance*.

Punctures taken as distinguishable.
[Rovelli 96, Ashtekar-Baez-Corichi-Krasnov 98, Pithis 2012]

$$S = \frac{A}{4\ell_p^2} + \sigma(\gamma)N$$

The chemical potential $\mu \neq 0$.



No clear how to establish the correspondence with the semiclassical low energy limit.

Conclusions

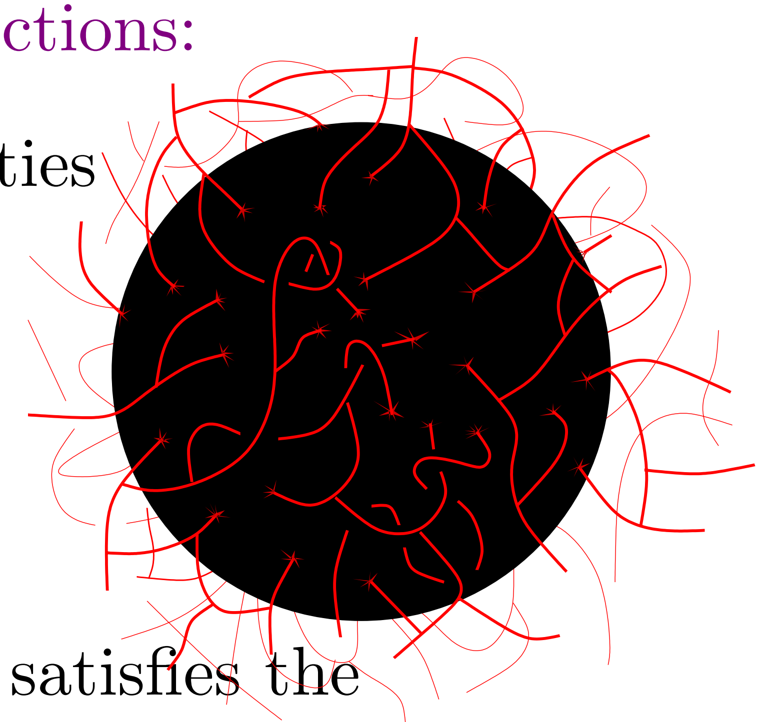
We can include the effects of matter in the quasilocal treatment by the introduction of an extra degeneracy.

$D \approx \exp(\lambda A/(4\ell_p^2))$ large spin dominance

Assuming punctures are indistinguishable.

Then, up to small quantum corrections:

$\lambda = \frac{1}{4}$ (no regularization ambiguities no species problem)



$S = \frac{A}{4\ell_p^2}$

The thermal state of the horizon satisfies the semiclassical and low energy condition

$$\ell_p^2 \ll \bar{a} = \ell_p^2 \langle j \rangle \ll A$$

The chemical potential vanishes $\mu = 0$

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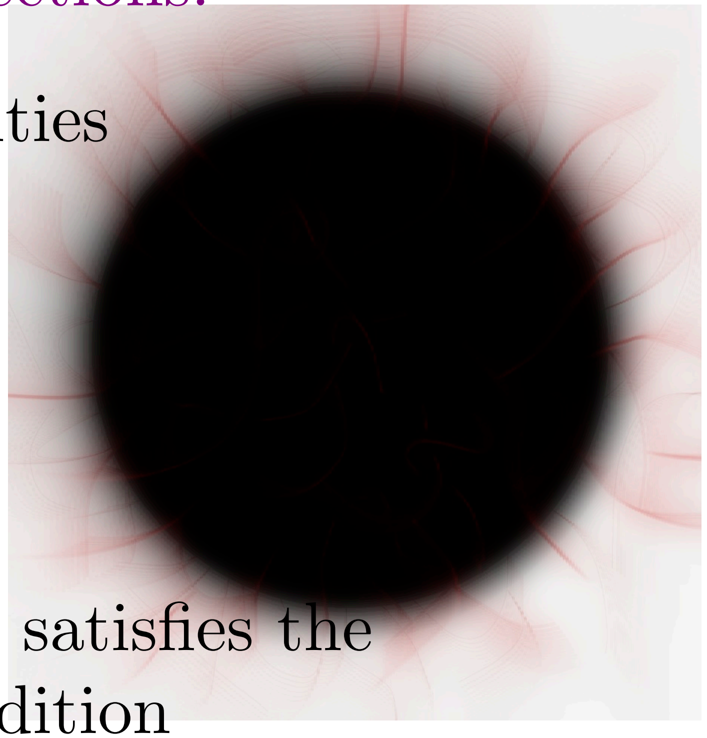

The thermal state of the horizon satisfies the semiclassical and low energy condition

$\ell_p^2 \ll \bar{a} = \ell_p^2 \langle j \rangle \ll A$

The chemical potential vanishes $\mu = 0$

Conclusions (independent of quantum statistics)

$\tilde{\beta} \equiv \beta - \beta_U(1-\delta) \ll 1$

➡ We can include the effects of matter in the quasilocal treatment by the introduction of an extra degeneracy.

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The thermal state of the horizon satisfies the semiclassical and low energy condition

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Outlook

➡ Can we have a microscopic description of the *holographic* matter degeneracy in LQG?

- Studies of matter coupling in LQG and the relationship with QFT

  [Ashtekar et al. and Thiemann et al. $\approx$ 2001]

- Chern-Simons quantum horizon for self dual gravity?

  [Smolin (1995), Krasnov (1996), Ashtekar-Baez-Corichi-Krasnov (2000), Engle-Noui-AP (2010)]

Analytic continuation to self dual variables

[Frodden-Geiller-Noui-AP (2012), Bodendorfer-Stottmeister-Thurn (2012), Pranzetti (2013)]

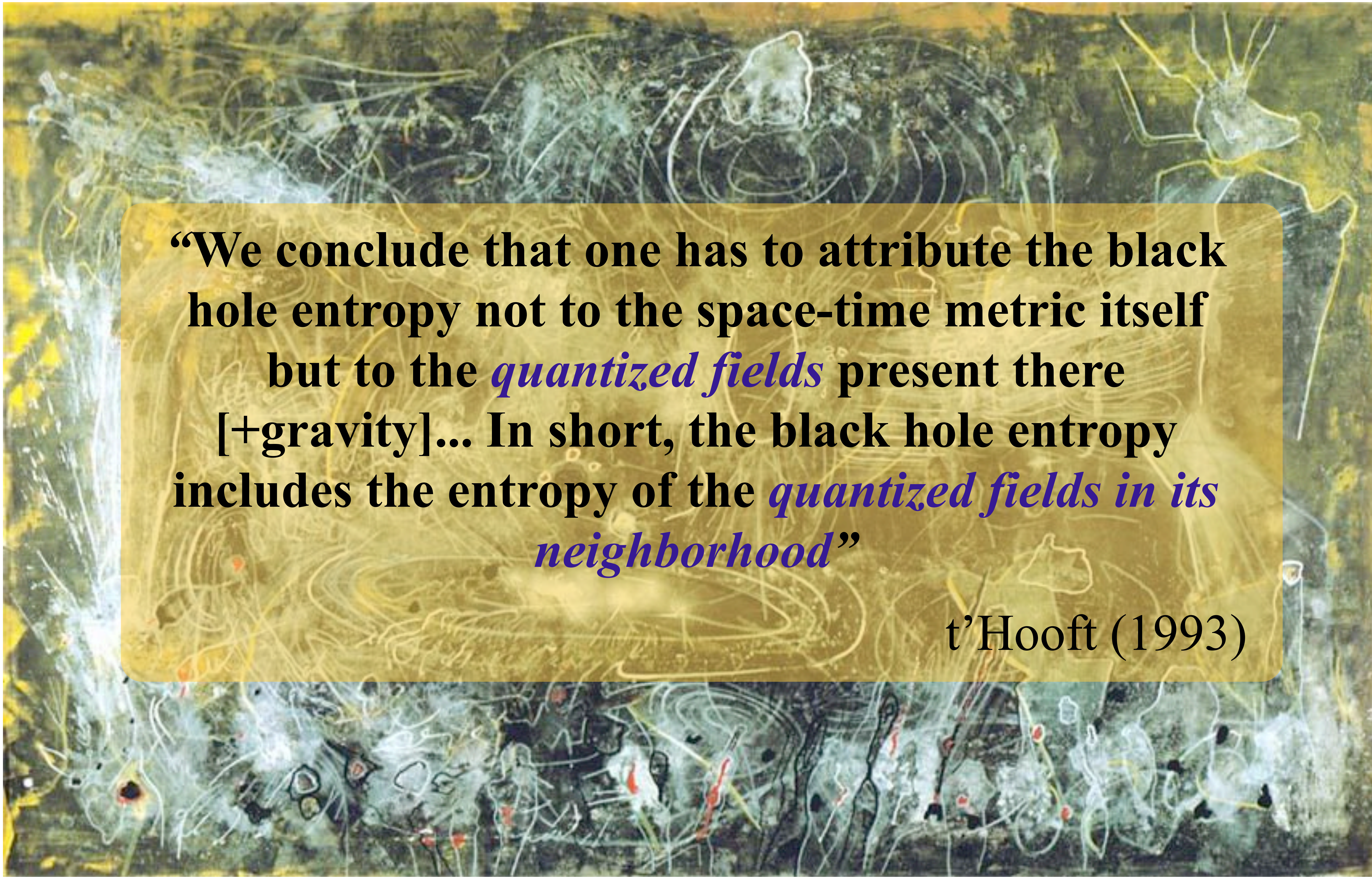
$$j \quad \Rightarrow \quad is - \frac{1}{2}$$

self dual representations satisfying reality condition $\hat{\Sigma} \cdot \hat{\Sigma} > 0$

$$D_k(j_1, \ldots, j_N) \quad \Rightarrow \quad i^{-p} D_k(is_1 - \frac{1}{2}, \ldots, is_p - \frac{1}{2}) = \frac{2}{k+2} \sum_{d=1}^{k+1} \sin^2\left(\frac{\pi d}{k+2}\right) \prod_{\ell=1}^{p} \frac{\sinh\left(\frac{2\pi d s_\ell}{k+2}\right)}{\sin\left(\frac{\pi d}{k+2}\right)}$$

What about matter?

The vacuum in QFT



"We conclude that one has to attribute the black hole entropy not to the space-time metric itself but to the *quantized fields* present there [+gravity]... In short, the black hole entropy includes the entropy of the *quantized fields in its neighborhood*"

t'Hooft (1993)

Thank you very much!

Roberto Matta "Integrale du Silence" (1990)