Problem set 5: Drag force

In this homework you will compute the energy loss of a heavy quark plowing, at constant velocity, through a plasma using holography.

A simple model for this process is described by the equation of motion

$$\frac{d\vec{p}}{dt} = -\mu\vec{p} + \vec{F}\,,$$

where $\vec{p}$ is the quark's momentum, μ is a drag coefficient, and $\vec{F}$ is an external force. The necessary force to keep a steady motion is $\vec{F} = \mu\vec{p}$. As in yesterday's problems we may model the quark as the endpoint of a string extending in the bulk of AdS_5. Recall that

$$ds^2 = \frac{L^2}{u^2}\left(-f(u)dt^2 + d\mathbf{x}^2 + \frac{du^2}{f(u)}\right)\,, \qquad f(u) = 1 - \frac{u^4}{u_H^4}\,.$$

a) Imagine that, after a transient, the quark moves with constant velocity v along the direction x_1. Fix the gauge $\tau = t$ and $\sigma = u$. The string profile is then going to be given by $x_1(\tau, \sigma)$. What is the boundary condition at $u = 0$ for $x_1(\tau, \sigma)$? How would you write an Ansatz for $x_1(\tau, \sigma)$ at generic u?

b) Compute the induced metric on the string world-sheet and the Nambu-Goto action.

c) This action has an obvious constant of motion. Write this down and express the equation of motion for $x_1(\tau, \sigma)$ in terms of this integration constant.

d) By inspection of the equation of motion find the explicit value for the integration constant. Actually, there are two possible values, one corresponding to a physical motion (string trailing behind the quark) and one corresponding to an unphysical motion of the string (string trailing in front of the quark).

e) Discuss the physical meaning of the constant of motion and relate it to the drag force experienced by the quark on the boundary.

f) Find the string profile.

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