Part 3: Improving parton showers with fixed-order calculations

a) Recap
b) Dedicated improvements: Matrix element corrections and NLO matching
c) Iterative improvements: Multi-jet merging

Recap of last lecture

- QCD scattering cross sections factorise.
- The factorisation can be cast into a probabilistic form suitable for a numerical implementation.
- Parton showers tell us how the inclusive cross section is sliced up into exclusive objects, where exclusive means a fixed number of resolved jets.
- Exclusive cross sections are defined through no-emission probabilities.
- All cross sections can be writen as a polynomial of logarithms.
- This log-structure can be illustrated on figures.

Parton showers calculate no-emission probabilities (= Sudakov factors), splitting kernels and emission probabilities:

$$\Pi(\rho_0, \rho_1) = \exp\left(-\int_{\rho_1}^{\rho_0} \frac{d\rho}{\rho} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z)\right) \equiv$$

Probability of no resolvable emission with evolution scale in the range $[\rho_1, \rho_0]$.

$$\int_{\rho_1}^{\rho_0} \frac{dp_\perp^2}{p_\perp^2} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z) \quad \equiv$$

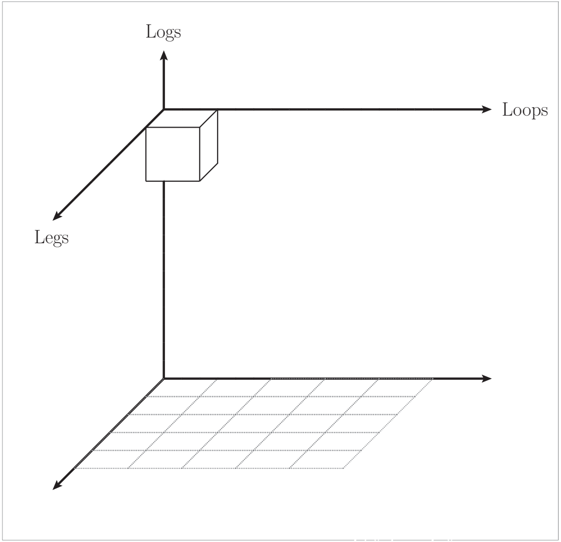
Probability of a resolvable emission in the interval $[\rho_1, \rho_0]$

$$\frac{d\rho}{\rho} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z) \Pi(\rho_0, \rho) \quad \equiv$$

Probability of a exactly one resolvable emission, with evolution scale ρ.

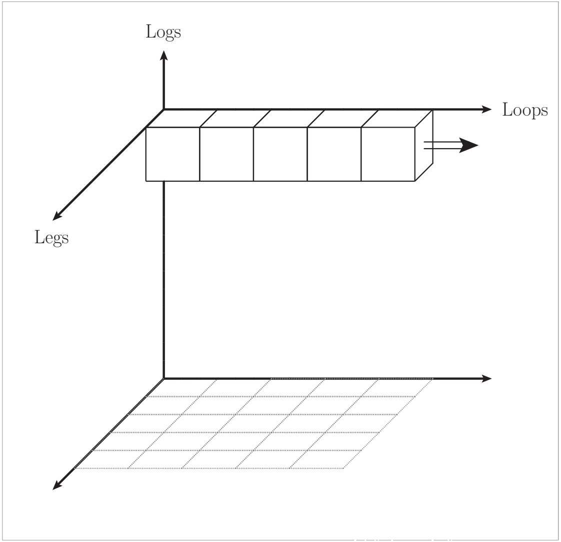
The no-emission probabilities are approximate all-order virtual corrections. These cancel the approximate real emissions exactly.

Recap: PS fixed order input



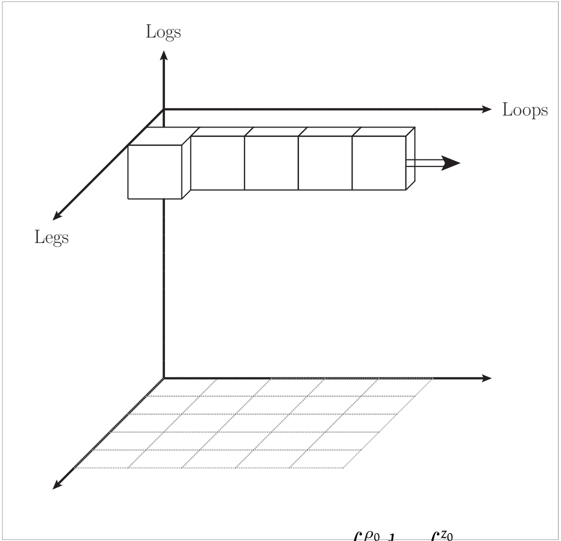
$d\sigma_B(pp \to X)$ $\mathcal{O}_0$

Recap: PS resums LL rows into no-emission probabilities (no PS emission)



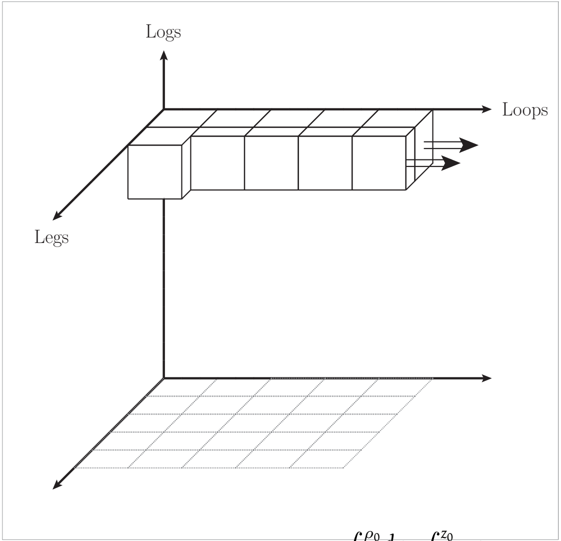
$d\sigma_B(pp \to X) \otimes \Pi_0(\rho_0, \rho_{min})$ $\mathcal{O}_0$

Recap: PS fills layers of LL loop corrections (one PS emission)



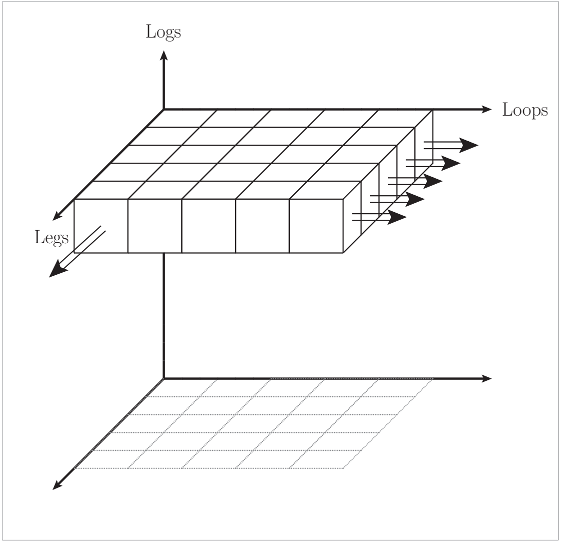
$$d\sigma_B(pp \to X) \otimes \int_{\rho_{min}}^{\rho_0} \frac{d\rho}{\rho} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z)\ \Pi_0(\rho_0, \rho)\ \mathcal{O}_1$$

Recap: PS fills layers of LL loop corrections (no or one PS emission)



$$d\sigma_B(pp \to X) \otimes \Pi_0(\rho_0, \rho_{min}) \; \mathcal{O}_0 \; + \; d\sigma_B(pp \to X) \otimes \int_{\rho_{min}}^{\rho_0} \frac{d\rho}{\rho} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z) \; \Pi_0(\rho_0, \rho) \; \mathcal{O}_1$$

Recap: PS fills layers of LL loop corrections (sum of all PS results)



$$\sigma_{\text{0 or more jets}} = \sigma_{\text{exactly 0 jets}} + \sigma_{\text{exactly 1 jet}} + \sigma_{\text{exactly 2 jets}} + \ldots + \sigma_{\text{n or more jets}}$$

Parton showers vs. fixed order

Parton showers give an approximate multi-parton (jet) cross section which...

- All-order parton showers:
  - $+$ Is always finite.
  - $+$ is good for any number of emissions.
  - $-$ but is only valid for very small relative $p_\perp$.

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  - − but is only valid for very small relative $p_\perp$.

Is your signal affected by (many) jets[1]?
$\Longrightarrow$ Need good calculation for partonic jet seeds!
$\Longrightarrow$ Need something better than plain parton shower.
$\Longrightarrow$ Combine the strengths of showers and fixed-order calculations!

- Fixed-order perturbation theory:
  - + Contains all terms at one order.
  - + Good for high relative $p_\perp$.
  - − Only feasible for few emissions.

Parton showers start from lowest-multiplicity tree-level inputs. The next step is next-to-leading order.

[1] Translation: You need to apply n_{jets}, $p_{\perp jet}$, H_T cuts or use "kinematic endpoint variables" like M_{T2}.

Parton shower improvement schemes

- Matrix element corrections.
  - Oldest scheme: Exponentiate specific real-emission calculations.
  - Usage in HERWIG(++) and PYTHIA(8) slightly different.
  - Very hard to iterate.
- Matrix element matching.
  - Combine a single *adapted* NLO calculation with the parton shower.
  - Hard to iterate.
- Matrix element merging.
  - Slice phase space in two, use ME for hard jets, PS for soft jets.
  - Very easy to iterate.

We will use B_n for the tree-level n-parton differential cross section, and $\widetilde{\mathrm{B}}_n$ or $\overline{\mathrm{B}}_n$ for NLO cross sections that are differential in n-parton phase space.

Remember how we constructed the parton shower:

- Find a factorizing approximation.
- Cast the factorising functions into probabilities.
- Choose branchings probabilistically.

Idea: Find new probabilities that add to the full emission ME.

For this, we need a) an overestimate for the double-differential partonic cross section $P_{\text{full-ME}}$, and b) a corrective probability $P_{\text{ME-correction}}$, so that

$$\sum P_{\text{PS},i} * P_{\text{ME-correction},i} = \sum P_{\text{new}} \equiv P_{\text{full-ME}} \qquad \text{with}$$

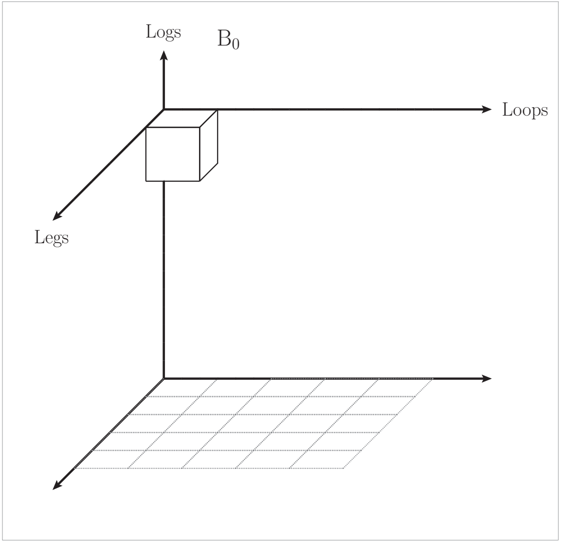
$$P_{\text{ME-correction},i} = \frac{\mathcal{P}_i P_{\text{full-ME}}}{P_{\text{PS},i}} \quad \text{and} \quad \sum_i \mathcal{P}_i = 1$$

Then we can use two steps to correct an emission to the full ME result:

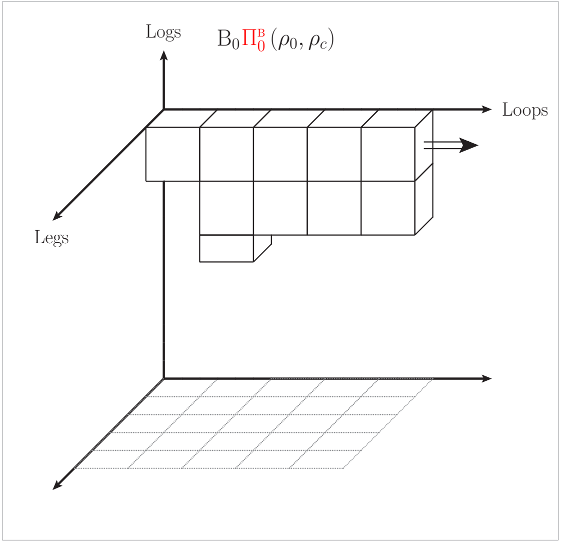
1. Choose a branching according to $P_{PS,i}$
2. Accept with probability $P_{\text{ME-correction},i}$

Summed over all possibilities, this gives the full ME (“Veto algorithm”).

ME corrections: Start from lowest order cross section.

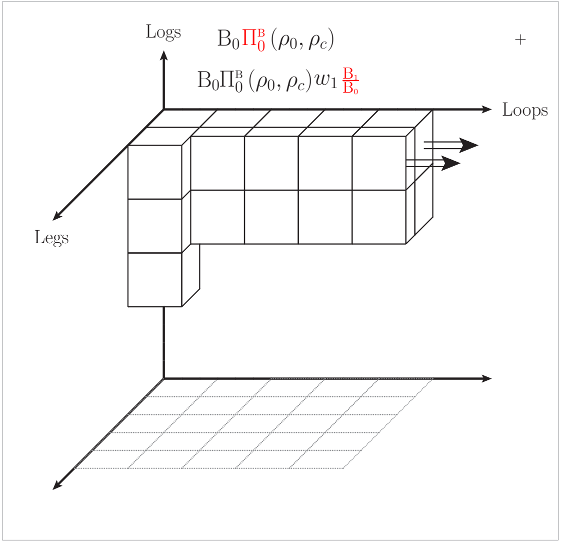


ME corrections: Produce no emissions according to new probability



where $\quad \Pi_0^{\mathsf{B}}(\rho_0, \rho_c) = \exp\left(-\int d\hat{\Phi}\,\frac{\mathrm{B}_1}{\mathrm{B}_0}\Theta(\rho(\hat{\Phi}) - \rho_c)\right) = \exp\left(-\int d\hat{\Phi}\,P_{\mathsf{new}}\Theta(\rho(\hat{\Phi}) - \rho_c)\right)$

ME corrections: Generate emissions according to new probability



This reproduces the full 1-parton radiation pattern, and is finite!

Matrix element corrections

Pro

- Rather natural within parton shower.
- Full ME (incl. interferences) gets exponentiated, not only approximation!
- Very efficient.

Contra

- Difficult to find overestimates, projectors and corrective weights.
- Exponentiation extends over full phase space (need to integrate the 1-parton ME over full phase space).
- Difficult to iterate, since ME-correction for $n+1$-partons has to divide out n-parton ME.

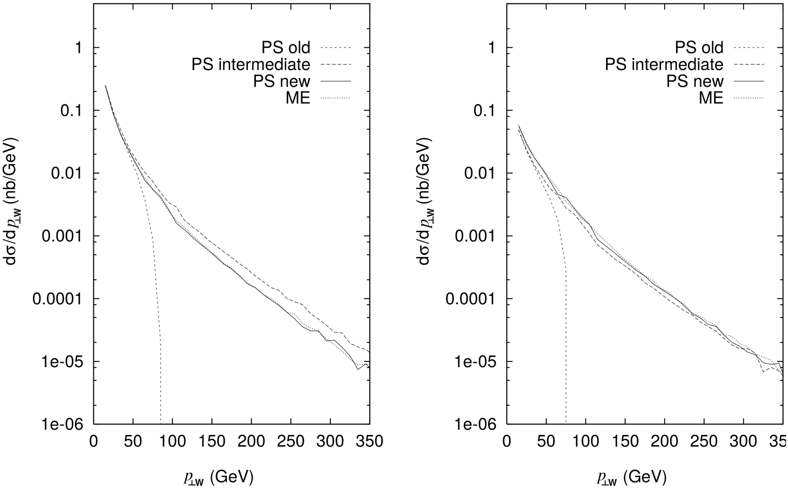
Subtleties

- The hardest emission has to be corrected, not only the first emission.
- Need to use "soft" and "hard" corrections if PS does not cover phase space: Add full ME in the gaps (hard), ME corrections for every "hardest emission" in the evolution (soft).

⇒ Usual attitude: Process dependent, tricky to achieve generality or iterate.

Note: VINCIA iterates MEC's for $e^+e^- \to$ jets, and also aims for pp collisions.

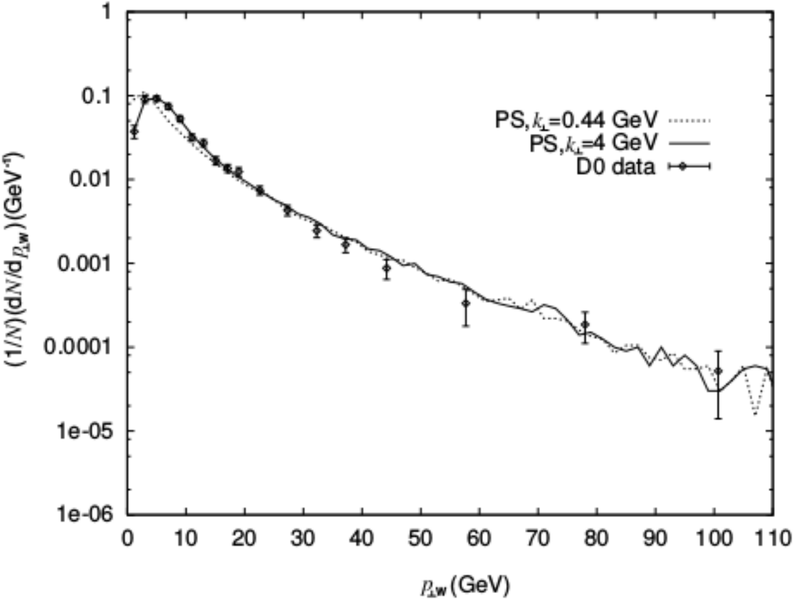
ME corrections results



(a)

(b)

ME corrections results



NLO matching

We know how to construct the correct real emission, so can we achieve NLO accuracy for inclusive $+0$-jet as well?

To get there, remember that the (regularised) NLO cross section is

$$\begin{aligned}
\mathsf{B}_{\mathrm{NLO}} &= \left[\mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_n\right]\mathcal{O}_0 + \int d\Phi_{\mathrm{rad}}\left(\mathrm{B}_{n+1}\mathcal{O}_1 - \mathrm{D}_{n+1}\mathcal{O}_0\right) \\
&= \left[\mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_n\right]\mathcal{O}_0 + \int d\Phi_{\mathrm{rad}}\left(\mathsf{S}_{n+1}\mathcal{O}_0 - \mathrm{D}_{n+1}\mathcal{O}_0\right) \\
&\quad + \int d\Phi_{\mathrm{rad}}\left[\mathsf{S}_{n+1}\mathcal{O}_1 - \mathsf{S}_{n+1}\mathcal{O}_0\right] + \int d\Phi_{\mathrm{rad}}\left(\mathrm{B}_{n+1}\mathcal{O}_1 - \mathsf{S}_{n+1}\mathcal{O}_1\right)
\end{aligned}$$

where S_{n+1} are approximate virtual/real PS corrections.

The term in [...] is the $\mathcal{O}(\alpha_s)$ part of a shower from B_n. $\Rightarrow$ For now discard from $\mathsf{B}_{\mathrm{NLO}}$ $\Longrightarrow$ "Seed" cross section

$$\widehat{\mathsf{B}}_{\mathrm{NLO}} = \left[\mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_n + \int d\Phi_{\mathrm{rad}}\left(\mathsf{S}_{n+1} - \mathrm{D}_{n+1}\right)\right]\mathcal{O}_0 + \int d\Phi_{\mathrm{rad}}\left(\mathrm{B}_{n+1} - \mathsf{S}_{n+1}\right)\mathcal{O}_1$$

This is not the NLO result...but showering the $\mathcal{O}_0$-part will restore this!

$\Longrightarrow$ NLO +PS accuracy!

POWHEG

We have found that NLO +PS is possible if we start from the seed cross section

$$\widehat{\mathsf{B}}_{\mathsf{NLO}} \quad = \quad \left[\mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_n + \int d\Phi_{\mathrm{rad}}\left(\mathsf{S}_{n+1} - \mathrm{D}_{n+1}\right)\right]\mathcal{O}_0 + \int d\Phi_{\mathrm{rad}}\left(\mathrm{B}_{n+1} - \mathsf{S}_{n+1}\right)\mathcal{O}_1$$

where S_{n+1} is the PS approximation of the $n+1$-jet rate.

$\Longrightarrow$ The NLO matching only depends on the first PS step!

The first step can be done externally. Using $\mathsf{S}_{n+1} = \mathsf{B}_{n+1}$, i.e. a MEC for the first splitting, we find

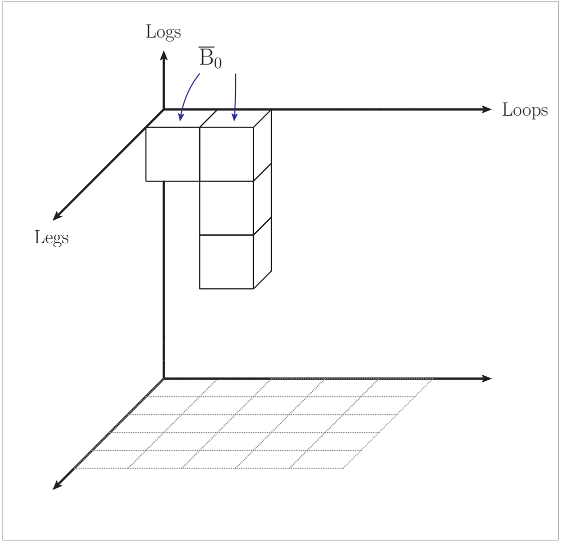
$$\widehat{\mathsf{B}}_{\mathsf{NLO}} \quad = \quad \left[\mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_n + \int d\Phi_{\mathrm{rad}}\left(\mathsf{B}_{n+1} - \mathrm{D}_{n+1}\right)\right]\mathcal{O}_0 \;=\; \overline{\mathrm{B}}_n$$

$\Longrightarrow$ Seed cross section is simply the inclusive NLO result. This is POWHEG.

Roughly, POWHEG combines an ME correction with an NLO weight.

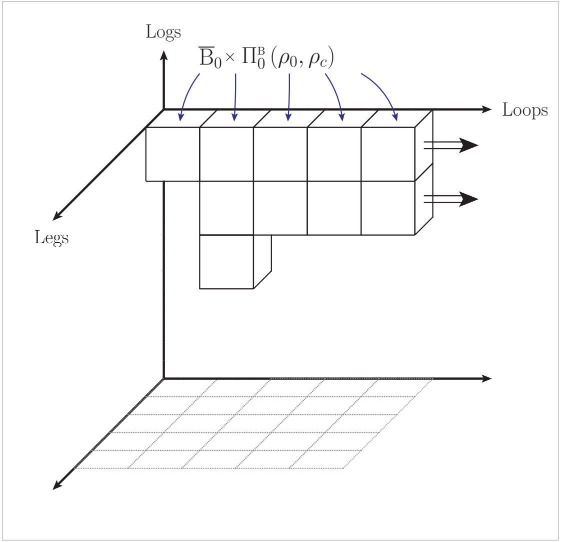
POWHEG-BOX is an ME generator that provides NLO inputs for parton showers. One (ME corrected) emission is done by POWHEG-BOX, other emissions have to be filled in by PS.

POWHEG illustration



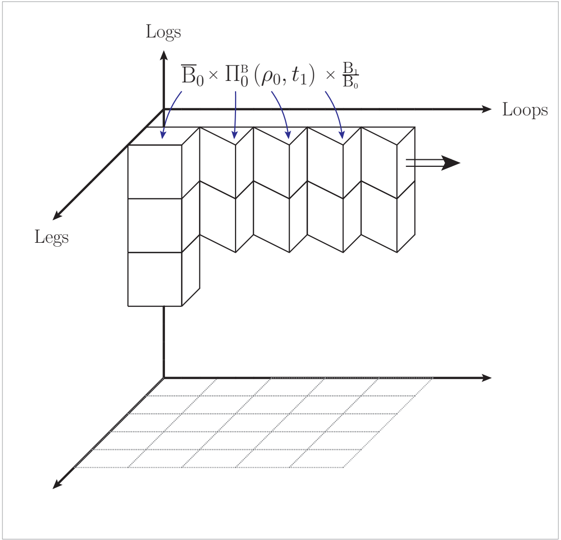
Shower from the seed cross section

POWHEG illustration



Shower from the seed cross section can give no emission,

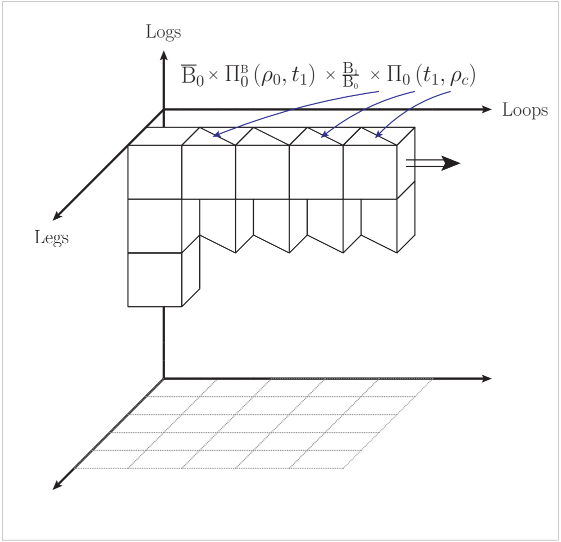
POWHEG illustration



Shower from the seed cross section can give no emission, or one emission.

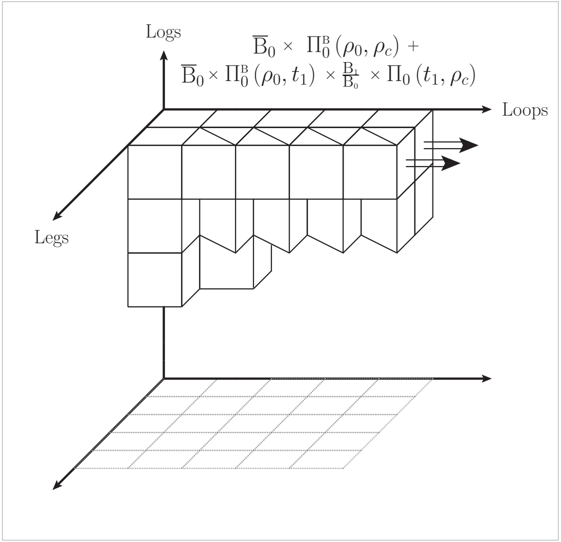
The hardness of the emission is defined differently from parton shower.

POWHEG illustration



The shower needs to be attached to this intermediate result, without introducing overlaps $\Rightarrow$ Truncated, vetoed shower necessary.

POWHEG illustration



The sum of all parts gives an NLO +PS simulation

POWHEG

Pro

- Inherits pros from ME correction.
- Full ME (incl. interferences) gets exponentiated, not only approximation!
- Mostly positive weights!

Contra

- Inherits cons from ME correction.
- Exponentiation extends over full phase space (need to integrate the 1-parton ME over full phase space).
- Difficult to iterate.

Subtleties

- Interface can be very subtle, nearly invalidating the PS independence.
- Truncated, vetoed shower cannot captures full parton shower.
- Can be redefined to consist of "soft" and "hard" corrections, by using $\mathsf{S}_{n+1} = \mathsf{B}_{n+1} F(\Phi)$ instead, at cost of introducing parameters.

MC@NLO

We have found that NLO +PS is possible if we start from the seed cross section

$$\widehat{\mathsf{B}}_{\mathsf{NLO}} = \left[\mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_n + \int d\Phi_{\mathrm{rad}} \left(\mathsf{S}_{n+1} - \mathrm{D}_{n+1}\right)\right] \mathcal{O}_0 + \int d\Phi_{\mathrm{rad}} \left(\mathrm{B}_{n+1} - \mathsf{S}_{n+1}\right) \mathcal{O}_1$$

where S_{n+1} is the PS approximation of the $n+1$-jet rate.

$\Longrightarrow$ The NLO matching only depends on the first PS step!

It is possible to keep $\mathsf{S}_{n+1} = \mathrm{B}_n \otimes \mathsf{K}\Theta(\mu_Q - \rho)$, where the Θ-function limits the subtraction to the PS phase space, and keep

$$\overline{\mathsf{B}}_n^{\mathsf{S}} = \left[\mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_n + \int d\Phi_{\mathrm{rad}} \left(\mathrm{B}_n \otimes \mathsf{K}\Theta(\mu_Q - \rho) - \mathrm{D}_{n+1}\right)\right] \mathcal{O}_0 \qquad \mathbb{S}\text{-events}$$

$$\overline{\mathsf{B}}_n^{\mathsf{H}} = \int d\Phi_{\mathrm{rad}} \left(\mathrm{B}_{n+1} - \mathrm{B}_n \otimes \mathsf{K}\Theta(\mu_Q - \rho)\right) \mathcal{O}_1 \qquad \mathbb{H}\text{-events}$$

This emphasises the PS as an NLO subtraction. The matching now has soft $\mathbb{S}$-events and hard $\mathbb{H}$-events. $\mathbb{H}$-events are a non-logarithmic correction.

MC@NLO

Pro

- Interface to PS very easy.
- Very controlled change of resummation!
- No new shower necessary.

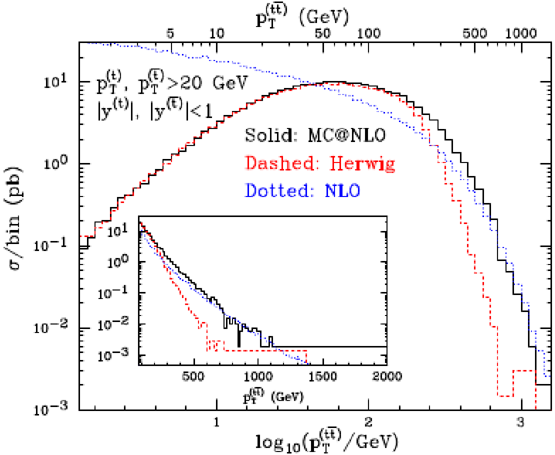
Contra

- $\mathbb{S}$-events alone, or $\mathbb{H}$-events alone are not necessarily positive.
- No clear prescription how to handle/shower $\mathbb{H}$-events.
- Difficult to iterate.

Subtleties

- PS needs to be a full NLO subtraction (requires colour-correct first emissions), or instead use $\mathsf{S}_{n+1} \approx \mathrm{B}_n \otimes \mathsf{K}\Theta(\mu_Q - \rho)$
- If PS is a full NLO subtraction, need to treat anti-probabilistic weights (see e.g. SHERPA, HERWIG++).

NLO matching results and comparisons

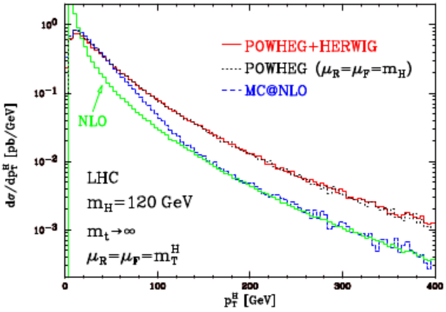


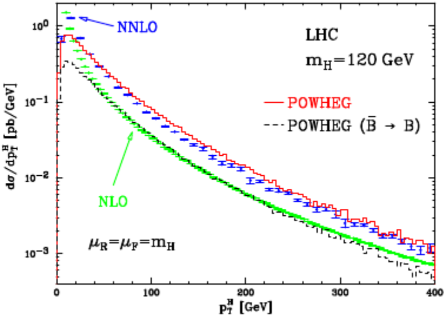
$p_\perp$ of $t\bar{t}$-system at a 14 TeV LHC for $t\bar{t}$-MC@NLO.

PS no-emission probability regulates the divergence. Hard tail given by fixed-order.

Question: When is this observable NLO accurate?

NLO matching results and comparisons



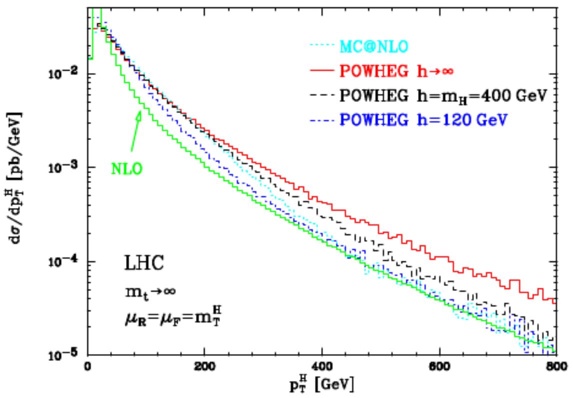


$p_\perp$ of Higgs boson at a 14 TeV LHC for $gg \to H$-POWHEG and $gg \to H$-MC@NLO.

PS no-emission probability regulates the divergence.

What happens in the tail?

Question: Is this observable NLO accurate?



$p_\perp$ of Higgs boson at a 14 TeV LHC for $gg \to H$-POWHEG.

Variations: Use a different PS kernel $\mathrm{S}_{n+1} = \mathrm{B}_{n+1} F(\Phi)$ in POWHEG.

$\Rightarrow$ This is a very big “higher-order” effect!

Good news: We can improve on this!

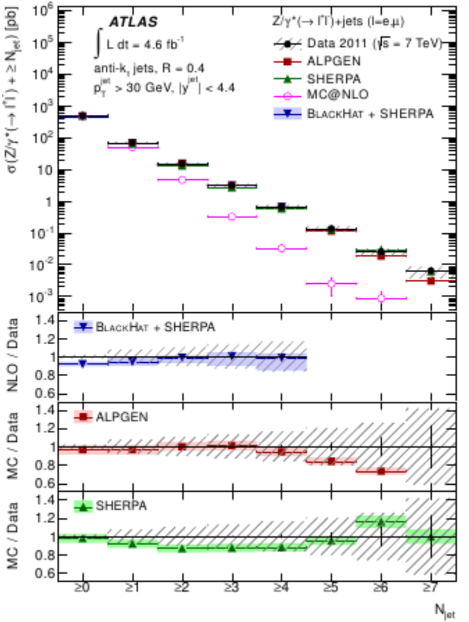
NLO matching results and comparisons

Number of anti-$k_\perp$ jets in Z+jets events in ATLAS.

Zero-jet bin is NLO accurate, one-jet bin is leading order.

NLO matched calculation cannot describe high jet multiplicities.

$\Rightarrow$ No single NLO matched calculation will describe this data.



NLO matching

NLO matching can be obtained by showering the seed cross section

$$\widehat{\mathrm{B}}_{\mathrm{NLO}} = \left[\mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_n + \int d\Phi_{\mathrm{rad}} \left(\mathsf{S}_{n+1} - \mathrm{D}_{n+1}\right)\right] \mathcal{O}_0 + \int d\Phi_{\mathrm{rad}} \left(\mathrm{B}_{n+1} - \mathsf{S}_{n+1}\right) \mathcal{O}_1$$

NLO matching methods differ in the choice of S_{n+1}:

POWHEG uses $\mathsf{S}_{n+1} = \mathsf{B}_{n+1}$ or $\mathsf{S}_{n+1} = \mathsf{B}_{n+1} F(\Phi)$

MC@NLO uses $\mathsf{S}_{n+1} = \mathrm{B}_n \otimes \mathsf{K}\Theta(\mu_Q - \rho)$

Pro

- Promotes the PS for one process to NLO accuracy!

Contra

- **New calculation needed whenever obervable depends on another jet!**
- Multiple matched calculations cannot be combined without major work.

Subtleties

- Interface to PS.
- Treatment of real-emission events.

Part 3c: Iterative improvements

Introduction:

- Inclusive vs. exclusive observables
- Making inclusive cross sections additive

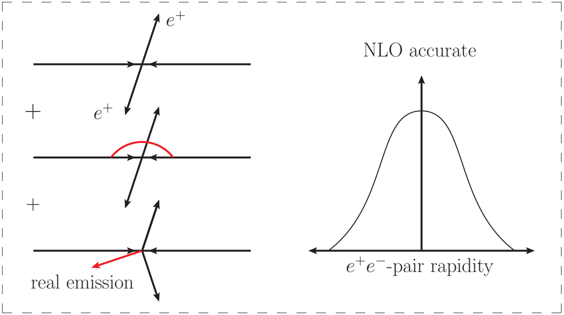
Tree-level merging:

- Overview of the traditional schemes
- Unitarisation

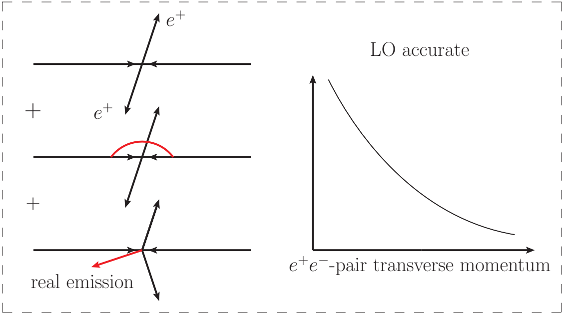
Overview of NLO merging schemes

What is NLO accuracy?

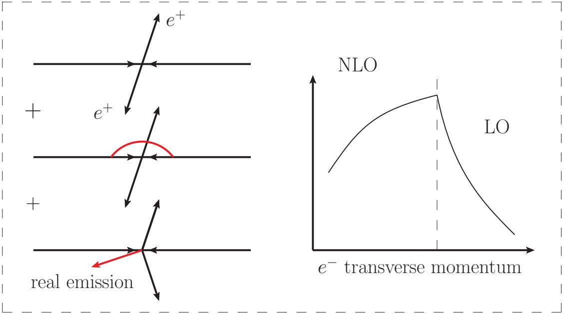
NLO accuracy is achieved when we calculate *corrections* to an observable that was already defined at a lower order.



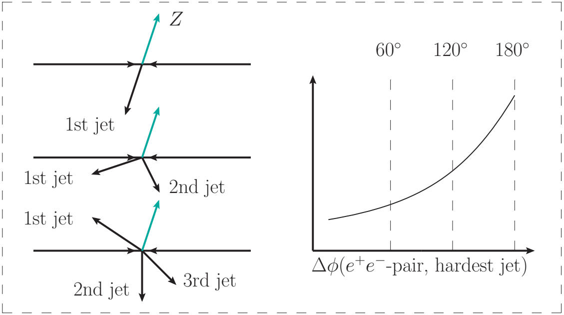
What is NLO accuracy?
...not all outcomes of an NLO calculation are "NLO accurate"



What is NLO accuracy?
NLO up to 45 GeV, LO beyond!



What is NLO accuracy?
How many "next-to's" do you need to describe this at least to lowest order accuracy everywhere?



Goal: Get an accurate prediction of multijet observables (e.g. $\Delta\phi_{\mathsf{Zj}},\ n_{\mathsf{jets}}$)
Idea: Combine predictions for arbitrary many jets into a single calculation!

Problems:

- $\diamond$ Cross sections in fixed-order perturbation theory are inclusive by definition $\Rightarrow$ Overlap:

$$\sigma(pp \to X) \supset \sigma(pp \to X + gluon)$$

  $\Longrightarrow$ Remove overlap between these cross sections!
- $\diamond$ Fixed-order predictions break down for collinear or soft partons.
- $\diamond$ PS gives sensible result in the collinear or soft regions, but breaks down for (many) well-separated jets.
- $\diamond$ Adding PS and fixed-order gives another overlap, since the PS reproduces the LL approximation.
  $\Longrightarrow$ Restrict PS to avoid this overlap!

More precisely, what we want to achieve is:

n hardest jets in an event described by fixed-order calculation.

... that should lead to a good description of high $p_\perp$ multi-jet data.

Any other emissions described by the PS.

... because the PS gets soft/collinear partons right.

For now, simplify and use only tree-level calculations. Remove the singularities with a phase-space cut $t_{\rm MS}$ (called *merging scale*).
$t_{\rm MS} \sim \min\{$all possible jet separations$\}$ works.

More precisely, what we want to achieve is:

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For now, simplify and use only tree-level calculations. Remove the singularities with a phase-space cut t_{MS} (called *merging scale*).
$t_{\mathrm{MS}} \sim \min\{$all possible jet separations$\}$ works.

Then, we can achieve that

- n hardest partons (above t_{MS}) described with tree-level accuracy.
- softer partons (below t_{MS}) described by the PS.

Watch out: a) We want the n *hardest* partons, not just n partons.
b) Dependence on the arbitrary t_{MS} should be small.

Making fixed-order calculations additive

To convert fixed-order calculations into "hardest parton" calculations, remember that the PS generates exclusive cross sections

$$\sigma_{0 \;\; or \;\; more \;\; partons} = \overbrace{\sigma_{exactly \;\; 0 \;\; parton}}^{\text{exclusive due to Sudakov factor}} + \overbrace{\sigma_{exactly \;\; 1 \;\; parton}}^{\text{exclusive due to Sudakov factors}} + \underbrace{\sigma_{2 \;\; or \;\; more \;\; partons}}_{\text{inclusive beyond 2 partons}}$$

Exclusive = Additive
⇒ Convert the inclusive states of the ME calculation into *exclusive hardest parton* states by using the PS.

Different choices how to use the PS give different schemes:

- MLM: Approximate no-emission probabilities by veto on jets.
- CKKW: Analytic Sudakov factors as no-emission probabilities.
- CKKW-L: PS no-emission probabilities directly from PS trial showers.

(POWHEG: Real emission = Exclusive hardest emission because of Sudakov form factor)

MLM jet matching

An intuitive way to make LO calculations additive is the MLM prescription:

- Calculate the tree-level MEs, cut away every state with $t(S_{+n}) > t_{\mathrm{MS}}$.
- Count partons before shower.

$$\langle \mathcal{O} \rangle = \mathrm{B}_0 \; \mathcal{O}(S_{+0p})$$

$$\int \mathrm{B}_1 \qquad \Theta\left(t\left(S_{+1}\right) - t_{\mathrm{MS}}\right) \mathcal{O}(S_{+1p})$$

[1] Glossing over subtleties with the PS interface

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  Veto the event if # shower jets does not match # ME partons[1].

$$\langle \mathcal{O} \rangle = \mathrm{B}_0 \; \mathcal{O}(S_{+0p}) \quad \times \; \textsc{Veto}\;(S_{+np})$$

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[1] Glossing over subtleties with the PS interface

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An intuitive way to make LO calculations additive is the MLM prescription:

- ◇ Calculate the tree-level MEs, cut away every state with $t(S_{+n}) > t_{\mathrm{MS}}$.

- ◇ Count partons before shower.
- ◇ Start the PS on the ME configuration. After shower, count jets. Veto the event if # shower jets does not match # ME partons[1].
- ◇ Combine by adding all accepted events.

$$\langle \mathcal{O} \rangle = \mathrm{B}_0 \; \mathcal{O}(S_{+0p}) \quad \times \; \textsc{Veto}\;(S_{+np})$$

$$+ \int \mathrm{B}_1 \qquad \Theta\left(t\left(S_{+1}\right) - t_{\mathrm{MS}}\right) \mathcal{O}(S_{+1p}) \times \; \textsc{Veto}\;(S_{+mp})$$

The prescription is nicely intuitive. However, the factors $\textsc{Veto}\;(S_{+np})$ do not have a direct correspondence to an object in (fixed- or all-order) QCD.

$\Longrightarrow$ Difficult to argue why scheme performs well, or badly.

[1] Glossing over subtleties with the PS interface

An intuitive way to make LO calculations additive is the MLM prescription:

- Calculate the tree-level MEs, cut away every state with $t(S_{+n}) > t_{\mathrm{MS}}$.
- Reweight with α_s- and PDF-ratios to minimise footprint.
- Count partons before shower.
- Start the PS on the ME configuration. After shower, count jets.
  Veto the event if # shower jets does not match # ME partons[1].
- Combine by adding all accepted events.

$$\langle \mathcal{O} \rangle = \mathrm{B}_0\ \mathcal{O}(S_{+0p}) \quad \times\ \textsc{Veto}\ (S_{+np})$$

$$+ \int \mathrm{B}_1 w_f^0 w_{\alpha_s}^0 \Theta\left(t\left(S_{+1}\right) - t_{\mathrm{MS}}\right) \mathcal{O}(S_{+1p}) \times\ \textsc{Veto}\ (S_{+mp})$$

The prescription is nicely intuitive. However, the factors $\textsc{Veto}\ (S_{+np})$ do not have a direct correspondence to an object in (fixed- or all-order) QCD.
$\Longrightarrow$ Difficult to argue why scheme performs well, or badly.

Observation: Improved t_{MS}-dependence by including PS-style dynamical evaluation of α_s-factors also in the ME.

[1] Glossing over subtleties with the PS interface

How do we minimise the dependence on t_{MS}?

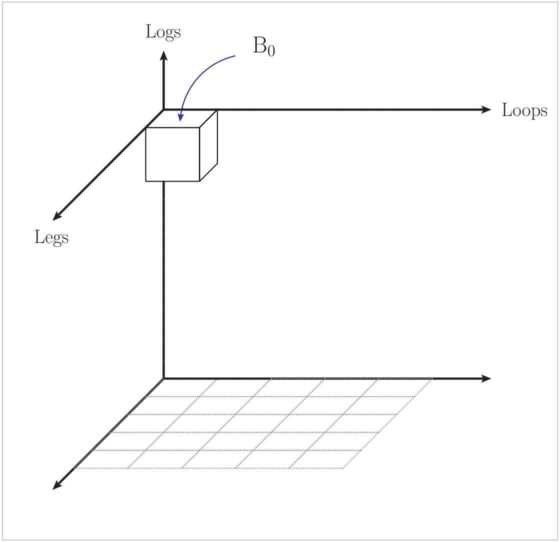
To find a better veto condition, we simply need to follow the PS more closely: A veto on events with extra PS emissions produces a parton shower Sudakov factor, which can make the ME exclusive!

Since we know the functional expression of our veto / Sudakov factor, we know which factors we are missing to reduce the t_{MS} dependence!

Remember: PS emissions use running α_s (PDFs) to capture higher orders!
$\Rightarrow$ So far, running α_s (PDFs) below t_{MS}, fixed values above t_{MS}
$\Rightarrow$ Remove mismatch by using running α_s (PDFs) also in tree-level calculations.

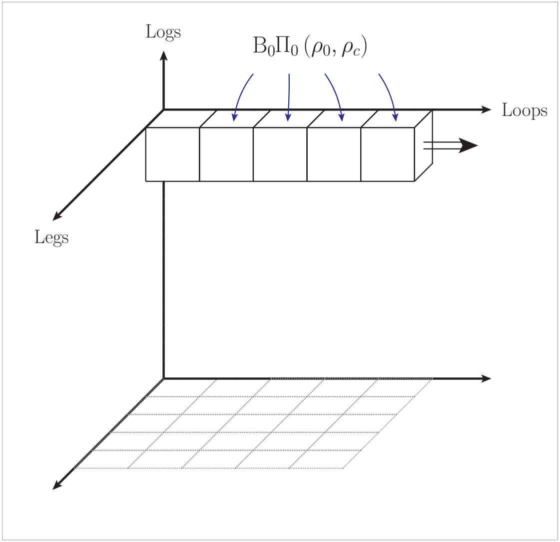
Let's look at an example.

ME+PS merging example



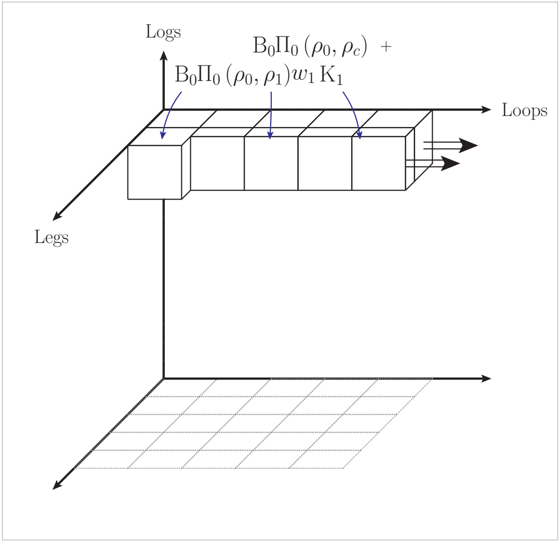
"Normal" shower from the 0-emission cross section can

ME+PS merging example



"Normal" shower from the 0-emission cross section can give no emission,

ME+PS merging example



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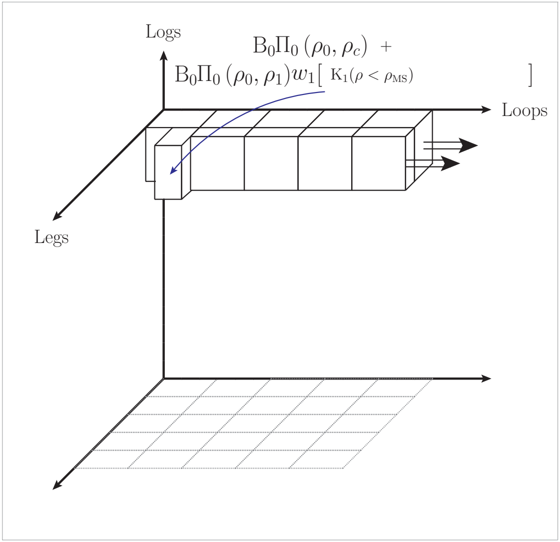
ME+PS merging example



"Normal" shower from the 0-emission cross section can give no emission, or one emission.
Veto all events with $\rho_{\text{emission}} > \rho_{\text{MS}}$.

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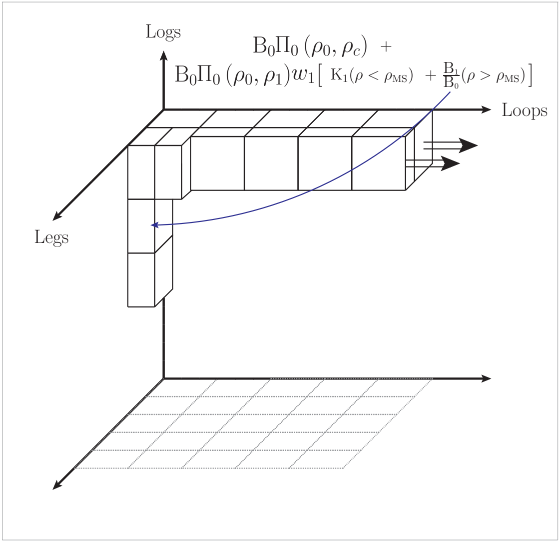

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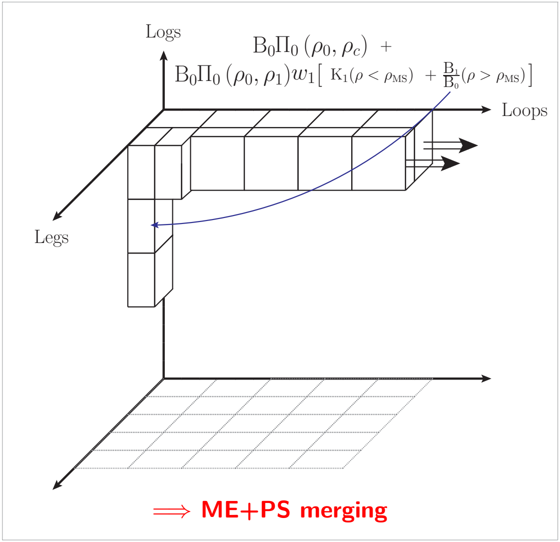
ME+PS merging example



"Normal" shower from the 0-emission cross section can give no emission, or one emission.
Veto all events with $\rho_{\text{emission}} > \rho_{\text{MS}}$. Add the reweighted 1-emission ME above ρ_{MS}.

Merging algorithms step-by-step

We have defined a ME+PS merging by

1. Regularise MEs with t_{MS} cut.
2. Make MEs exclusive by attaching PS no-emission probabilities $\Pi_i(\rho_i, \rho_{i+1})$.
3. Reweight MEs with factors w_i to include α_s and PDF running.
4. Shower these inputs.
   Veto event if the PS produced an additional "hard" emission.
5. Add up all processed phase space points.

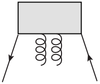
Note: To calculate the necessary no-emission probabilities $\Pi_i(\rho_i, \rho_{i+1})$ and α_s+PDF weights w_i, we need to define the scales $\rho_0, \rho_1, \ldots, \rho_n$.

This information can be extracted by constructing a parton shower history for each tree-level phase space point.

PS histories not only define the ordering of emissions (i.e. the scale sequence $\rho_0, \rho_1, \ldots, \rho_n$) but also complete, physical intermediate states.
Complete int. states can be used for trial showers...and much more.

Parton shower histories

Construction of PS histories for input phase space points is crucial in ME+PS merging.



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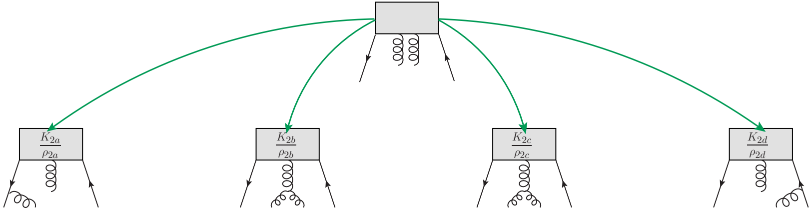

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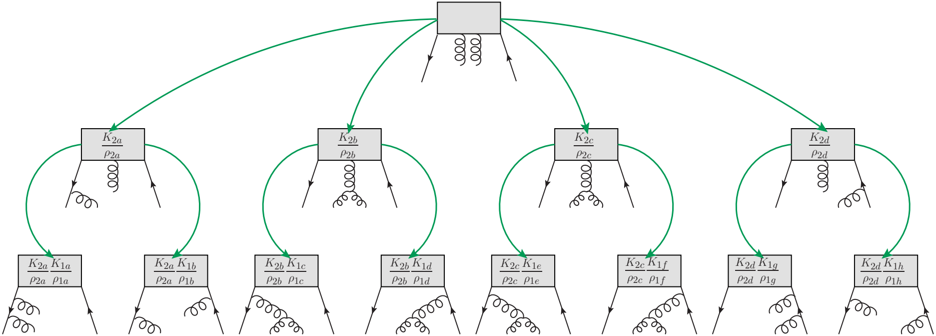
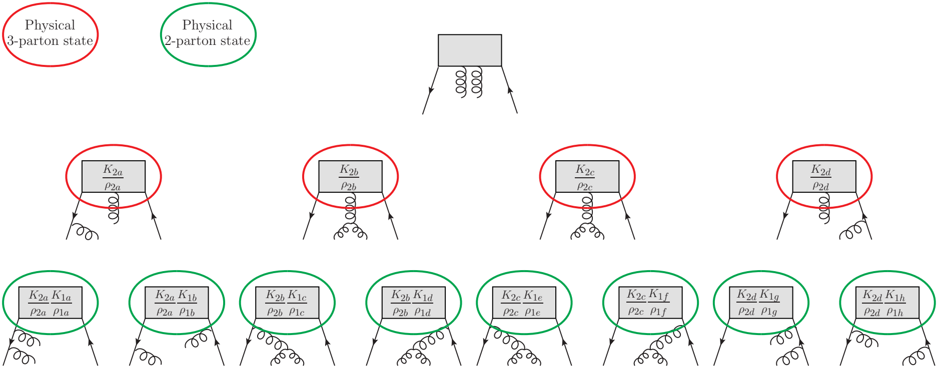


Different merging algorithms choose a PS history differently:

◇ CKKW only constructs the scales of one history, with the $k_\perp$ clustering algorithm.

Parton shower histories

Construction of PS histories for input phase space points is crucial in ME+PS merging.

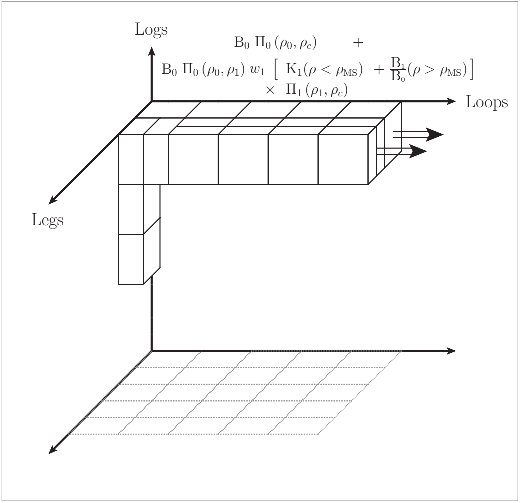


Different merging algorithms choose a PS history differently:

- ◇ METS chooses full intermediate states probabilistically at each step.
- ◇ CKKW-L constructs all histories, chooses path of full int. states probabilistically.

Physical intermediate states $S_{\text{n-jet}}$ allow trial showers: Run PS on $S_{\text{n-jet}}$.
If $\rho_{emission} > \rho_{n+1}$, veto $\Longrightarrow$ Generated no-emission probability $\Pi_n(\rho_n, \rho_{n+1})$

Multileg merging can be iterated!

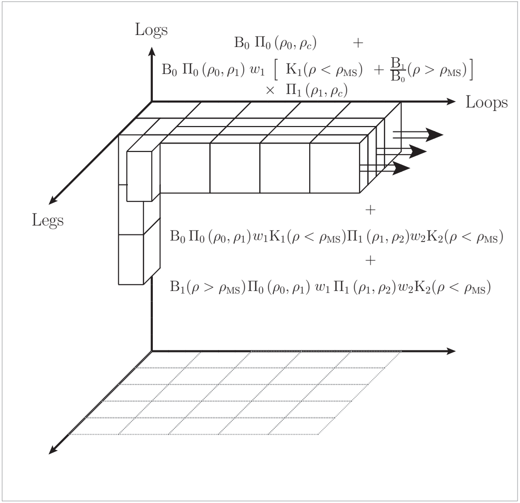


Previous zero+one leg merging result.

Now also veto all events with $\rho_{\text{emission}} > \rho_{\mathrm{MS}}$ when showering 1-emission MEs

...which can produce one hard + no soft jet

Multileg merging can be iterated!



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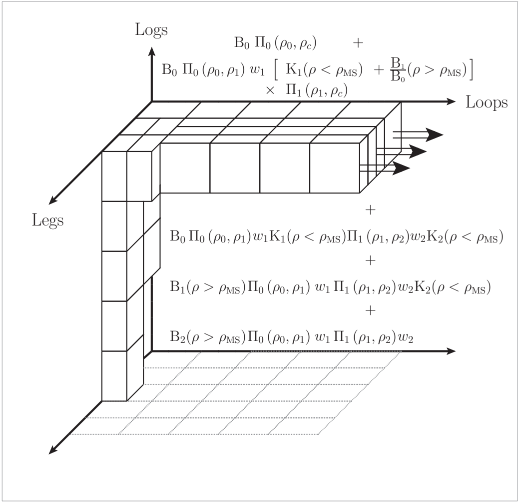

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Now also veto all events with $\rho_{\text{emission}} > \rho_{\text{MS}}$ when showering 1-emission MEs

...which can produce one hard + no soft jet, or one hard + one soft jet.

Then add the reweighted ME for two hard jets. Iterate.

Multileg merging

Merging methods differ in the choice

...with which no-emission probability to make MEs exclusive.

...how to decide on a sequence of states used in reweighting.

Pro

- Process independent.
- Combine multiple tree-level cross section with each other and with PS resummation.
- Good prediction for exclusive observables.

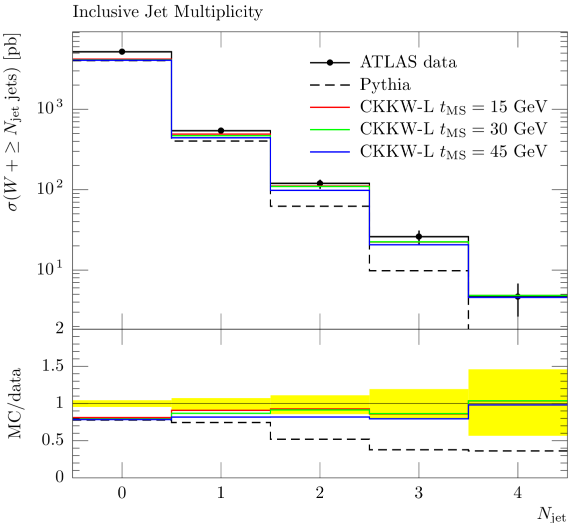
Contra

- Not NLO (yet, see later)
- **Changes inclusive cross sections.**

Subtleties

- Treatment of non-shower like configurations.
- Non-shower type configurations might (depending on the scheme) require truncated showers.

Multijet merged results (CKKW-L)



Bug vs. Feature in ME+PS

The ME includes terms that are not compensated by the PS approximate virtual corrections (i.e. no-emission probabilities).

These terms from the ME are what we need to describe multiple hard jets!

But if we simply add samples, the "improvements" will degrade the inclusive cross section: σ_{inc} will contain $\ln(t_{\rm MS})$ terms.

INCLUSIVE CROSS SECTIONS DO NOT KNOW ABOUT (CUTS ON) HIGHER MULTIPLICITIES. INCLUSIVE IS INCLUSIVE!

Traditional approach: Don't use a too small value for the merging scale.

$\rightarrow$ Uncancelled terms numerically not important.

Unitary approach[1]:

Use a (PS) unitarity inspired approach exactly cancel the dependence of the inclusive cross section on $t_{\rm MS}$.

[1] JHEP1302(2013)094 (Leif Lönnblad, SP), JHEP1308(2013)114 (Simon Plätzer)

We can use parton shower unitarity to rewrite CKKW-L as

$$\begin{aligned}\langle\mathcal{O}\rangle &= \mathrm{B}_0\Pi_{S_{+0}}(\rho_0,\rho_{\mathrm{MS}})\mathcal{O}(S_{+0j})\\ &+ \int \mathrm{B}_1\Theta\left(t\left(S_{+1}\right)-t_{\mathrm{MS}}\right)w_f^0 w_{\alpha_s}^0\Pi_{S_{+0}}(\rho_0,\rho_1)\mathcal{O}(S_{+1j})\end{aligned}$$

We can use parton shower unitarity to rewrite CKKW-L as

$$\begin{aligned}\langle\mathcal{O}\rangle \quad &= \mathrm{B}_0 - \int d\rho\; w_f^0 w_{\alpha_s}^0 \mathrm{B}_0 \mathsf{K}_0(\rho)\Pi_{S_{+0}}(\rho_0,\rho)\Theta\left(t\left(S_{+1}\right)-t_{\mathrm{MS}}\right)\mathcal{O}(S_{+0j}) \\ &+ \int \mathrm{B}_1\Theta\left(t\left(S_{+1}\right)-t_{\mathrm{MS}}\right) w_f^0 w_{\alpha_s}^0 \Pi_{S_{+0}}(\rho_0,\rho_1)\mathcal{O}(S_{+1j})\end{aligned}$$

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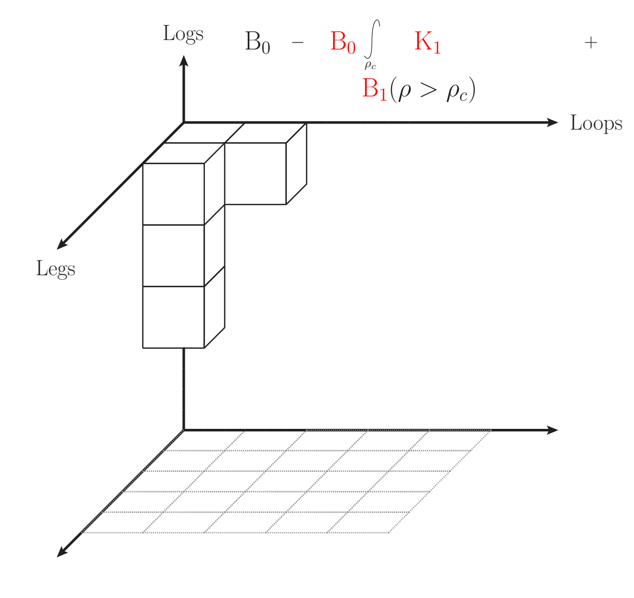
and replace

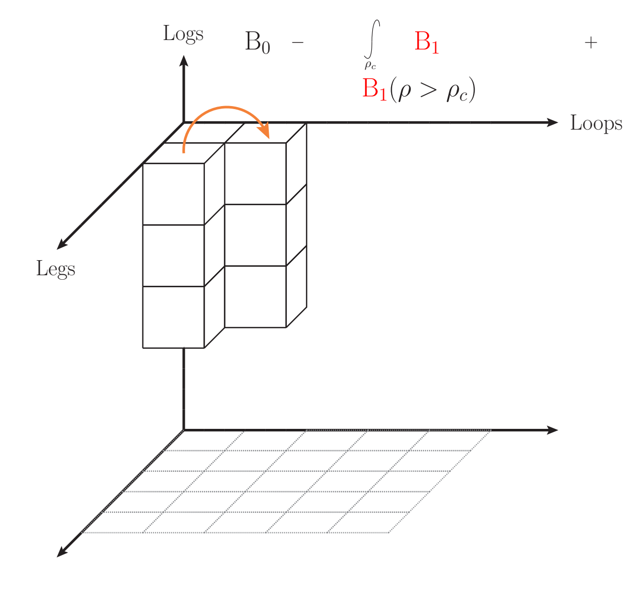
$$\langle \mathcal{O} \rangle = \mathrm{B}_0 - \int d\rho \; w_f^0 w_{\alpha_s}^0 \mathrm{B}_1 \Pi_{S_{+0}}(\rho_0, \rho) \Theta\left(t\left(S_{+1}\right) - t_{\mathrm{MS}}\right) \mathcal{O}(S_{+0j}) + \int \mathrm{B}_1 \Theta\left(t\left(S_{+1}\right) - t_{\mathrm{MS}}\right) w_f^0 w_{\alpha_s}^0 \Pi_{S_{+0}}(\rho_0, \rho_1) \mathcal{O}(S_{+1j})$$

$\Longrightarrow$ Unitarised ME+PS!

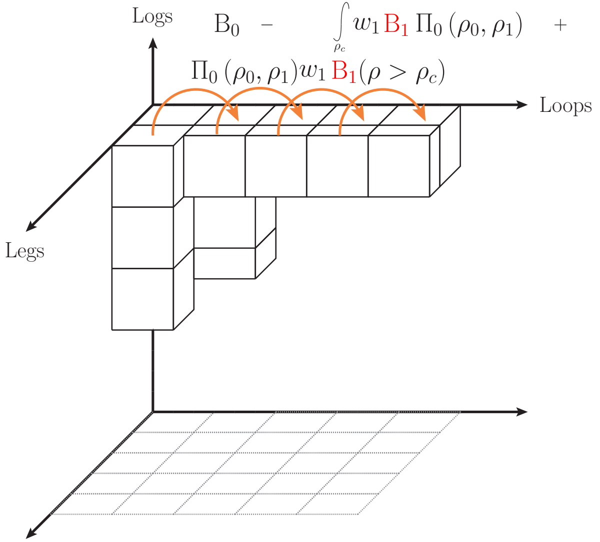
ME+PS, σ changes because virtual cannot cancel real correction! ($t_{\mathrm{MS}} \to$ PS cut-off ρ_c for simplicity)



Remember KLN ⇒ Construct approximate virtuals by integrating real! (LoopSim)



This also works when integrating reweighted exclusive real corrections! (UMEPS)



Unitarised ME+PS merging (UMEPS)

This sketch can directly be extended to the case when we have

$\widehat{\mathrm{B}}_2$ = LO cross section, weighted with w_f, w_{α_s} and Π's
$\int \widehat{\mathrm{B}}_{n\to m}$ = integrated LO cross section, weighted with w_f, w_{α_s} and Π's.

For example two-jet merging:

$$\begin{aligned}\langle\mathcal{O}\rangle \quad = \quad \int d\phi_0 \Bigg\{ & \mathcal{O}(S_{+0j}) \left[\mathrm{B}_0 \;-\; \int \widehat{\mathrm{B}}_{1\to 0} \;-\; \int \widehat{\mathrm{B}}_{2\to 0} \right] \\ & + \int \mathcal{O}(S_{+1j}) \left[\widehat{\mathrm{B}}_1 \;-\; \int \widehat{\mathrm{B}}_{2\to 1} \right] \\ & + \int\!\!\int \mathcal{O}(S_{+2j})\; \widehat{\mathrm{B}}_2 \;\Bigg\}\end{aligned}$$

Integrated configurations are available anyway since we need them to perform the reweighting with no-emission probabilities!

$\Rightarrow$ Do integration simply by replacing input state $S_{\text{n-jet}}$ by $S_{\text{n-1-jet}}$.

Unitarised ME+PS merging (UMEPS)

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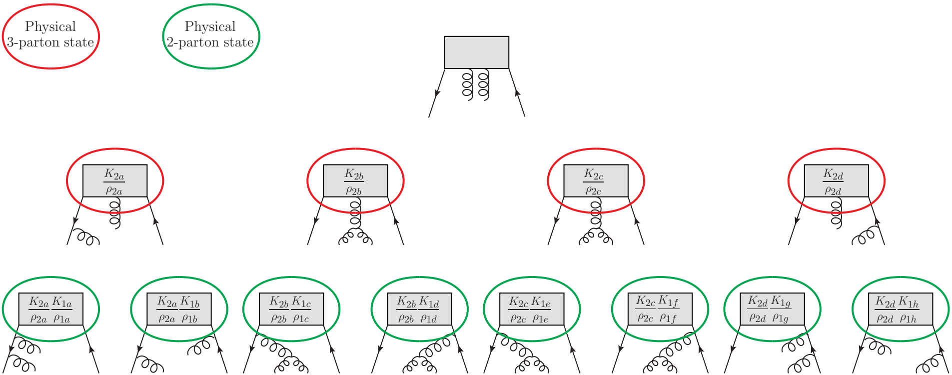

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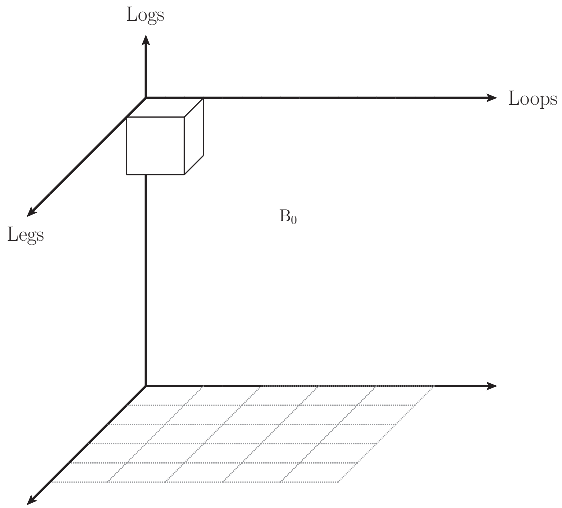
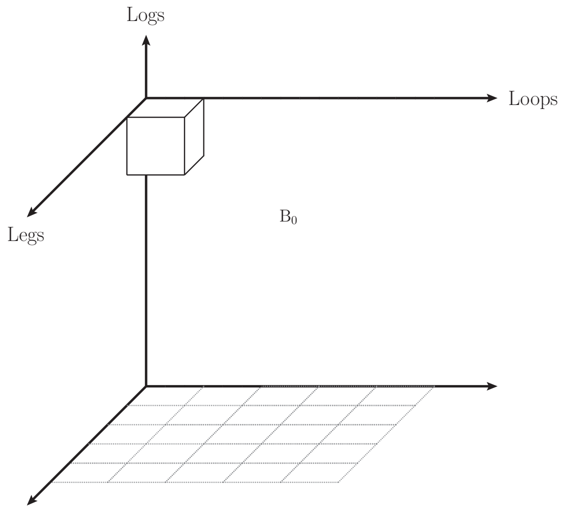
⇒ Do integration simply by replacing input state $S_{\text{n-jet}}$ by $S_{\text{n-1-jet}}$.

UMEPS step-by-step



Start from the 0-parton ME

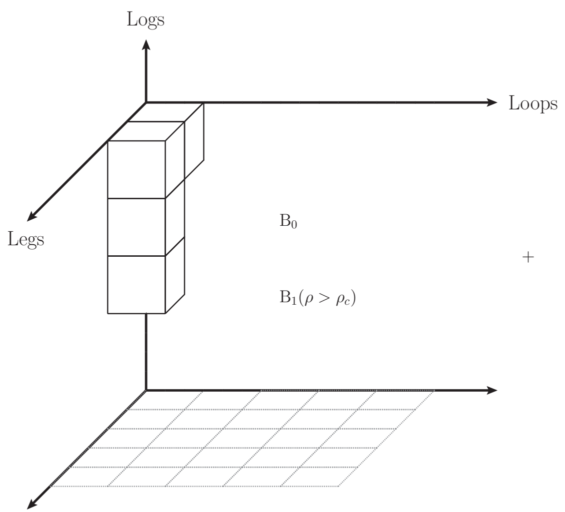
UMEPS step-by-step: 0-jet inclusive ✓



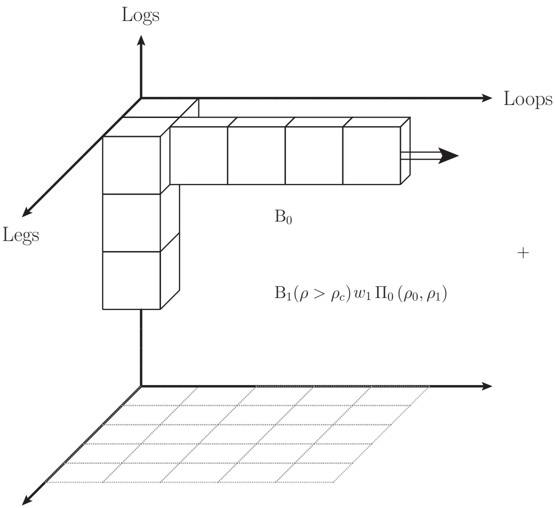
Start from the 0-parton ME

…and do nothing above t_{MS}.

UMEPS step-by-step: 0-jet inclusive ✗, 1-jet inclusive ✓



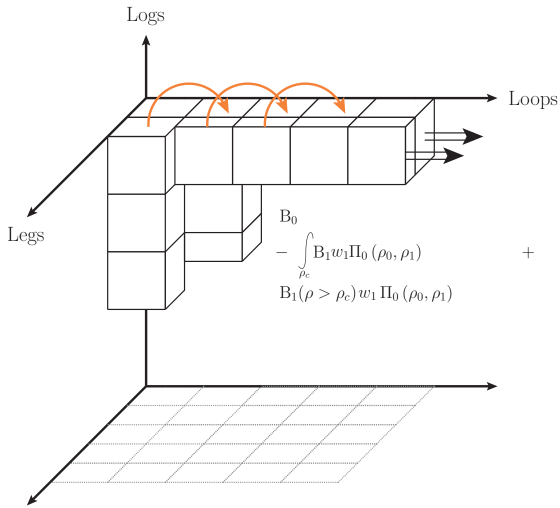
Then start from the 1-parton ME



Then start from the 1-parton ME

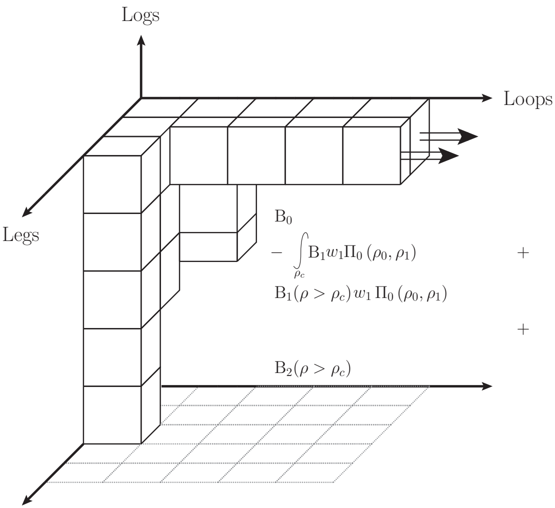
...and multiply no-emission probabilities and α_s (PDF) weights.

UMEPS step-by-step: 0-jet inclusive ✓, 1-jet inclusive ✓



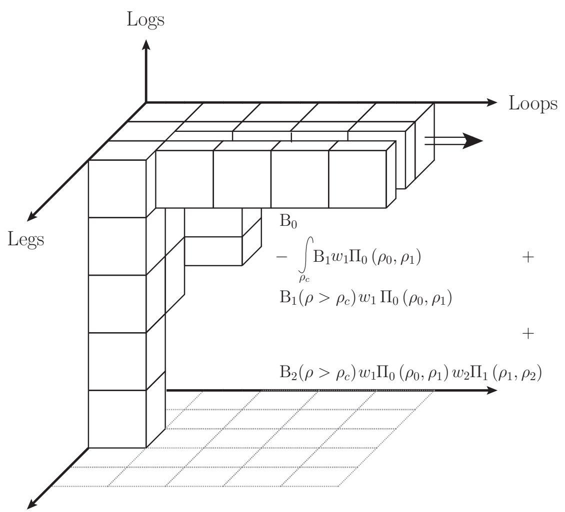
Now restore the 0-jet inclusive cross section.

...by subtracting the integrated reweighted 1-jet cross section.



Then start from the 2-parton ME

UMEPS step-by-step: 0-jet inclusive ✗, 1-jet inclusive ✗, 2-jet inclusive ✓



Then start from the 2-parton ME

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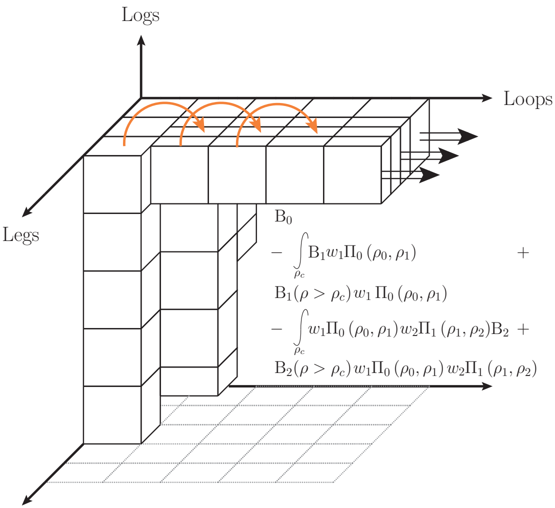
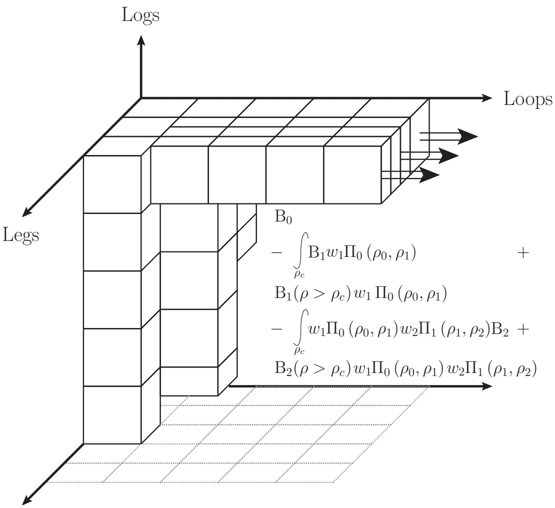
UMEPS step-by-step: 0-jet inclusive ✓, 1-jet inclusive ✓, 2-jet inclusive ✓



Now restore the 0-jet and 1-jet inclusive cross sections

...by subtracting the integrated reweighted 2-jet cross section.

UMEPS step-by-step: 0-jet inclusive ✓, 1-jet inclusive ✓, 2-jet inclusive ✓



$$
\begin{aligned}
&B_0 \\
&- \int_{\rho_c} B_1 w_1 \Pi_0(\rho_0, \rho_1) \qquad + \\
&B_1(\rho > \rho_c)\, w_1\, \Pi_0(\rho_0, \rho_1) \\
&- \int_{\rho_c} w_1 \Pi_0(\rho_0, \rho_1)\, w_2 \Pi_1(\rho_1, \rho_2) B_2 \; + \\
&B_2(\rho > \rho_c)\, w_1 \Pi_0(\rho_0, \rho_1)\, w_2 \Pi_1(\rho_1, \rho_2)
\end{aligned}
$$

…and continue further, adding and subtracting…

Unitarised paradigm, summary

Pro

- Inherits Pros from multileg merging.
- Does not change any of the inclusive cross sections by having better approximate $\mathcal{O}(\alpha_s^{+1})$ corrections.

Contra

- Not NLO (yet, see later)
- Subtraction means counter events with negative weight.

Subtleties

- Inherited from multileg merging.

Differences merging/matching

NLO matching is NLO-correct.

$\Longrightarrow$ Good uncertainty estimate, limited applicability.

Merging can be used to combine "any number" of LO calculations.

$\Longrightarrow$ Questionable uncertainty, broad applicability.

We can be lucky if

...NLO matched calculation describes very exclusive data.

...merged calculations describe normalisations.

It would be unreasonable to expect

Luck in one process = Luck in another process

$\Rightarrow$ Both strategies are incomplete and need to be combined for a satisfactory result.

NLO merging: Strategy

Any leading-order method **X** only ever contains approximate virtual corrections.

We want to use the full NLO multijet results whenever possible, e.g. have

- NLO accuracy for inclusive W + 0 jet observables
- NLO accuracy for inclusive W + 1 jet observables
- NLO accuracy for inclusive W + 2 jet observables

...all at the same time. And the method should be process-independent.

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NLO accuracy for inclusive W + 2 jet observables

...all at the same time. And the method should be process-independent.

To do NLO multi-jet merging for your preferred LO scheme **X**, do:

- ◇ Subtract approximate **X** $\mathcal{O}(\alpha_s)$-terms, add multiple NLO calculations.
- ◇ Ensure that real-emission parts of fixed-order calculations do not overlap.
- ◇ Ensure that fixed-order and shower calculations do not overlap ...just as we did at leading order.

$\Rightarrow$ **X**@NLO

The meaning of "NLO " will become clear below.

NLO merging schemes

FxFx[1]: Combine MC@NLO's by MLM jet matching@NLO
Pro: Probably fewest counter events.
Con: Restricted t_{MS} range. Accuracy unclear.

MEPS@NLO[2]: Combine MC@NLO's by METS@NLO
Pro: Improved Sudakovs.
Con: Restricted t_{MS} range.

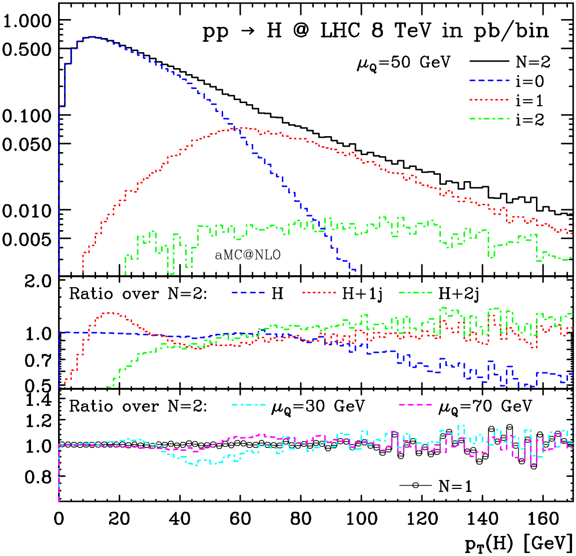
UNLOPS[3]: Combine MC@NLO's or POWHEG's by UMEPS @NLO
Pro: Unitarity by approximate NNLO terms.
Con: Naively, many counter events.

MiNLO[4]: Get zero-jet NLO by CKKW-reweighted 1-jet POWHEG after integration
Pro: Improved resummation, unitary.
Con: Process-dependent, only two NLO's can be combined.

[1] JHEP1212(2012)061 (Frixione, Frederix), [2] JHEP1304(2013)027 (Höche, Krauss, Schönherr, Siegert)

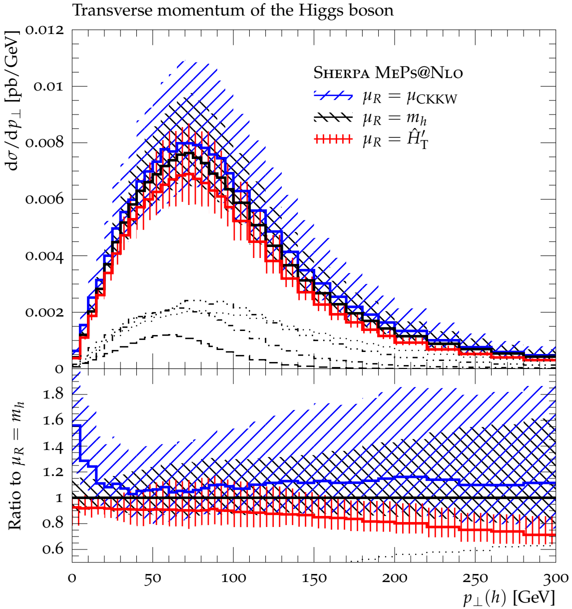
[3] JHEP1303(2013)166 (Lönnblad, SP), JHEP1308(2013)114 (Plätzer), [4] JHEP1305(2013)082 (Hamilton, Nason, Oleari, Zanderighi)

FxFx plots

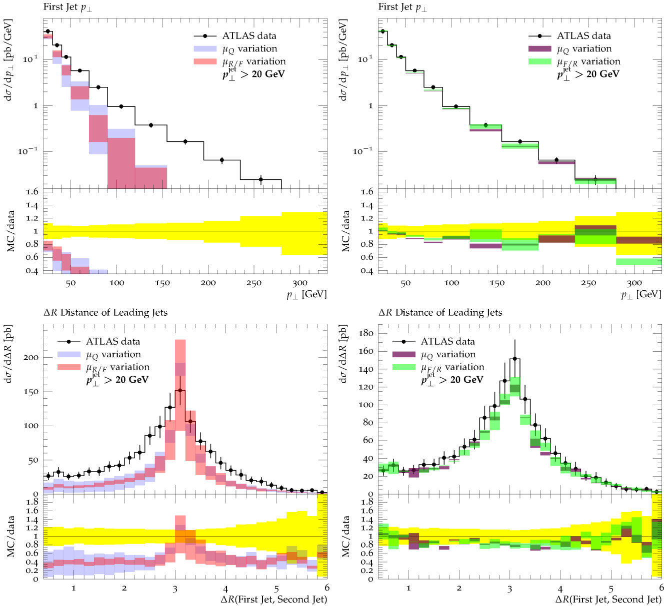


pp → H @ LHC 8 TeV in pb/bin
μ_Q=50 GeV
N=2
i=0
i=1
i=2
aMC@NLO
Ratio over N=2: H H+1j H+2j
Ratio over N=2: μ_Q=30 GeV μ_Q=70 GeV
N=1
p_T(H) [GeV]

MEPS@NLO plots



UNLOPS results (W+jets)



Inclusive sample containing (W + no resolved)@NLO, (W + one resolved)@NLO and (W + two resolved)@LO.

NLO merged results (H+jets)

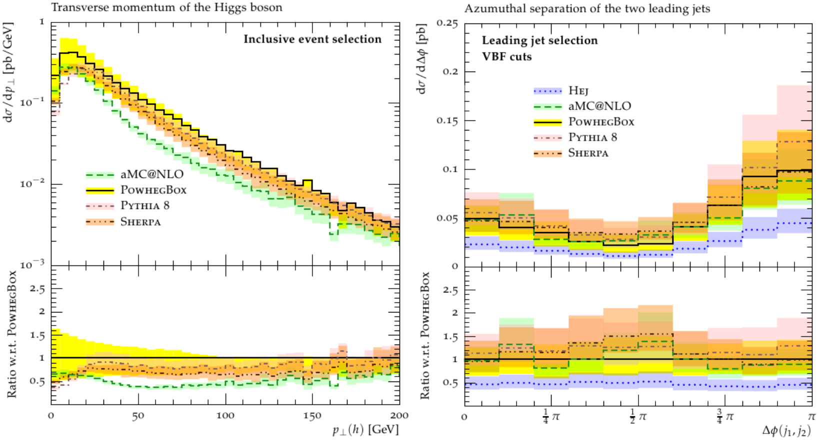
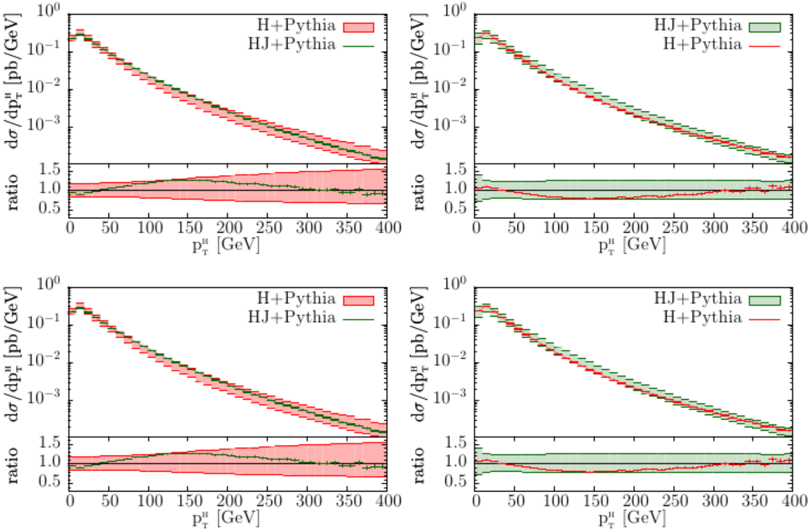


Figure: $p_{\perp,H}$ and $\Delta\phi_{12}$ for gg$\rightarrow$H after merging (H+0)@NLO, (H+1)@NLO, (H+2)@NLO, (H+3)@LO, compared to other generators.

$\Rightarrow$ The generators come closer together if enough fixed-order matrix elements are employed. The uncertainties after cuts are still very large.

MiNLO plots


H+Pythia
HJ+Pythia
HJ+Pythia
H+Pythia
H+Pythia
HJ+Pythia
HJ+Pythia
H+Pythia
$d\sigma/dp_T^H$ [pb/GeV]
ratio
p_T^H [GeV]

NLO merging methods have (mostly) been derived from LO schemes. Thus, we face many confusing acronyms.

Goal: Combine as many NLO calculations as are available into one inclusive calculation.

Pro

- Best Monte Carlo predictions for broad variety of processes at LHC.

Contra

- Not NNLO (yet, see later)
- All schemes contain counter events with negative weight.

Subtleties

- Inherited from the multileg merging scheme used to derive the method.
- All schemes differ in the treatment of yet higher orders.

Next steps: NNLO matching

Idea: Use a NLO merging scheme, assume that the 0-jet inclusive cross section after merging is $\sigma^{\text{NLO merged}} = \sigma_0^{\text{NLO}} = 1 + c_1\alpha_s$, and that we know $\sigma_0^{\text{NNLO}} = 1 + c_1\alpha_s + c_2\alpha_s^2$.

Then note

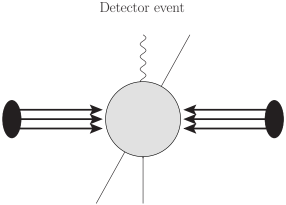
$$\frac{\sigma^{\text{NNLO}}}{\sigma^{\text{NLO merged}}}\sigma^{\text{NLO merged}} = (1 + c_2\alpha_s^2 + \mathcal{O}(\alpha_s^3))(1 + c_1\alpha_s) = \sigma^{\text{NNLO}} + \mathcal{O}(\alpha_s^3)$$

$\Rightarrow$ A unitary NLO merging scheme can easily be upgraded to NNLO!

MiNLO was upgraded (NNLO for Higgs) with a multiplicative K-factor.
$\Rightarrow$ POWHEG philosophy at NNLO

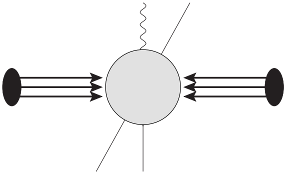
UNLOPS was upgraded (NNLO for Drell-Yan) by defining two classes of states - "0-jet exclusive" and "1-jet inclusive", and putting new NNLO only for "0-jet exclusive" states.
$\Rightarrow$ MC@NLO philosophy at NNLO

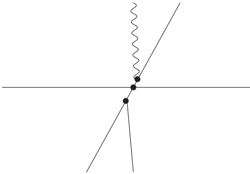
Back to the big picture: Some questions...



Say an event contains one boson and *three or four* jets. Where do these particles come from?

Back to the big picture: Some questions...





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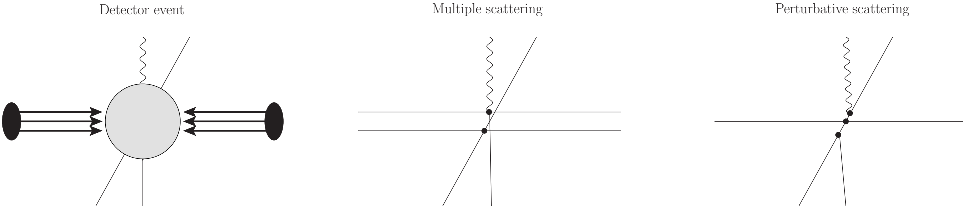
By now, we know quite well how to get these jets by dressing a complicated hard scattering. But when does this apply?

What if two jets merge? What if the boson and a jet are collinear? What if the jets have a low transverse momentum? What if pairs are back-to-back?

Back to the big picture: Some questions...



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When colliding composite objects, many constituent scatterings "compete" for the collision energy – **and multiple scattering can look like single complicated scatterings!**

Summary of Part 3: Improving parton showers with fixed-order calculations

- Parton showers can systematically improved with fixed-order calculations.
- Three major schools exist
  - Matrix element corrections: Oldest scheme, dating back to 80's. Available for simple processes in all parton showers. Iteratively used for e^+e^- in VINCIA (even at NLO).
  - Matrix element matching: "PS" used as extended subtraction for NLO calculations. Two schools: MC@NLO and POWHEG. Differences in exponentiation and in treatment of real corrections.
  - Matrix element merging: Emphasis on combining many multijet ME's. Make fixed-order calculations additive by making them exclusive through no-emission probabilities. Then minimise the impact of arbitrary slicing parameters. Three schools: MLM, CKKW(-L) and UMEPS. Differences in generation (approximation of) no-emission probabilities, and in the treatment of non-showerlike configurations. NLO merging: Combination of multiple NLO calculations. Take leading-order merging **X**, remove approximate $\mathcal{O}(\alpha_s)$ terms and add the full NLO. Inherits philosophy from LO merging scheme. NLO merging should be the workhorse for LHC Run II. NNLO matching: Recent extension of NLO merging methods.

References

Matrix element corrections
Pythia (PLB 185 (1987) 435, NPB 289 (1987) 810, PLB 449 (1999) 313, NPB 603 (2001) 297)
Herwig (CPC 90 (1995) 95)
Vincia (Phys.Rev. D78 (2008) 014026, Phys.Rev. D84 (2011) 054003, Phys.Rev. D85 (2012) 014013, Phys.Lett. B718 (2013) 1345-1350, Phys.Rev. D87 (2013) 5, 054033, JHEP 1310 (2013) 127)

POWHEG
JHEP 0411 (2004) 040 JHEP 0711 (2007) 070 POWHEG-BOX (JHEP 1006 (2010) 043)

MC@NLO
Original (JHEP 0206 (2002) 029) Herwig++ (Eur.Phys.J. C72 (2012) 2187)
Sherpa (JHEP 1209 (2012) 049) aMC@NLO (arXiv:1405.0301)

NLO matching results and comparisons
Plots taken from Ann.Rev.Nucl.Part.Sci. 62 (2012) 187 Plots taken from JHEP 0904 (2009) 002

Tree-level merging MLM (Mangano, http://www-cpd.fnal.gov/personal/mrenna/tuning/nov2002/mlm.pdf. Talk presented at the Fermilab ME/MC Tuning Workshop, Oct 4, 2002, Mangano et al. JHEP 0701 (2007) 013)
Pseudoshower (JHEP 0405 (2004) 040)
CKKW (JHEP 0111 (2001) 063, JHEP 0208 (2002) 015)
CKKW-L (JHEP 0205 (2002) 046, JHEP 0507 (2005) 054, JHEP 1203 (2012) 019)
METS (JHEP 0911 (2009) 038, JHEP 0905 (2009) 053)

Parton shower histories: Andre, Sjöstrand (PRD 57 (1998) 5767)

Unitarised merging
Pythia (JHEP 1302 (2013) 094) Herwig (JHEP 1308 (2013) 114) Sherpa (arXiv:1405.3607)

Intermediate step: MENLOPS
POWHEG (JHEP 1006 (2010) 039) Sherpa (JHEP 1108 (2011) 123)

FxFx: Jet matching @ NLO: JHEP 1212 (2012) 061

Merging MC@NLO calculations with MEPS@NLO
JHEP 1304 (2013) 027 JHEP 1301 (2013) 144 Plots taken from arXiv:1401.7971

UNLOPS = UMEPS @NLO: JHEP 1303 (2013) 166 Plots taken from arXiv:1405.1067

MiNLO: Original (JHEP 1210 (2012) 155) Improved (JHEP 1305 (2013) 082)

MiNLO-NNLOPS: JHEP 1310 (2013) 222 arXiv:1407.2940

UN^{2}LOPS: arXiv:1405.3607 arXiv:1407.3773

GENEVA: JHEP 1309 (2013) 120 JHEP 1406 (2014) 089