Acceleration Radiation and the Equivalence Principle

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F. and Wilson, arXiv:1805.01013, submitted to *Phys. Scrip.*

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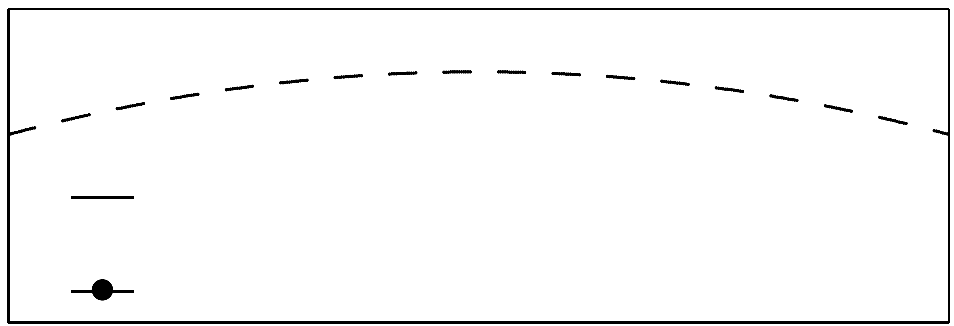
George Matsas, Andre Landulfo (São Paulo)

and 40 years ago: Paul Davies, Bill Unruh, Bob Wald, Steve Christensen, Mike Duff, Chris Isham, Bryce DeWitt, . . .

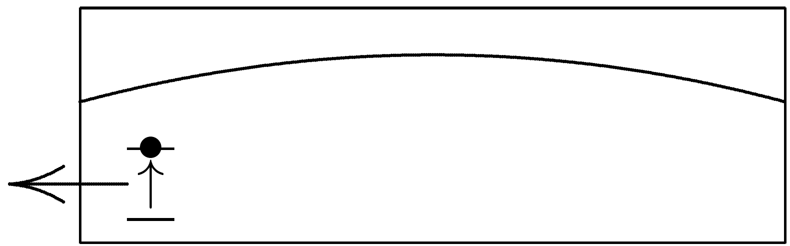
Four Gedankenexperiments with Atoms

Consider a two-level atom in its ground state, inside a resonant cavity acting as a photon detector.



Experiment 1

Suppose the atom is accelerated; is there a chance that the detector will be excited?

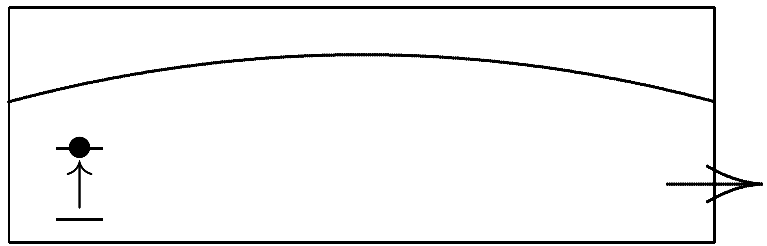


Ans.: Yes, the *Unruh–Wald effect*: The atom may move to its excited state and emit a photon.

Unruh & Wald, *Phys. Rev. D* **29** (1984) 1047.

Experiment 2

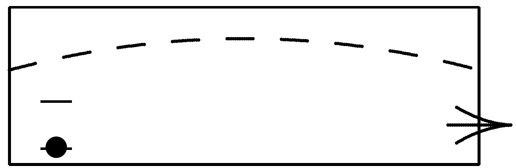
Now suppose the atom is stationary and the detector is accelerated; can the detector be excited?



Ans. 1 (Sagredo): Of course not. Nothing is happening to the atom.

Ans. 2 (Salviati): Yes; I'll explain it later.

For the moment let's accept Sagredo's Answer 1: Nothing happens in Experiment 2.



But suppose I forgot to tell you that these accelerations are caused by a gravitational field. The accelerated bodies are in free fall. The stationary bodies must be *supported* by some force, to avoid falling.

Experiment 3

The atom is falling and the detector is supported. Can the detector be excited?

Ans. 1: Yes. This is still Experiment 1. The nature of the force doesn't affect the atomic and E&M physics.

Ans. 2: No. This is still Experiment 2. By general relativity, it is the atom that is at rest and the cavity that is accelerating.

Experiment 4

The atom is supported and the detector is falling.

(exercise for the reader)

Bonus exercise

Go through all this for radiation from a classical accelerating charge *(an old question that still leads to shouting matches)*.

THE DEBATE CONTINUES

Sagredo seems to be in a dilemma. But he has a possible rebuttal:

Salviati, you are abusing the Equivalence Principle. Free fall in a gravitational field ($ds^2 = z^2\,dt^2 - dz^2$) is not really the same thing as acceleration in flat space-time ($ds^2 = dt^2 - dz^2$). Don't be misled by popularizations that say, "special relativity shows that velocity is relative, and general relativity shows that acceleration is relative."

He is right: *General relativity does not relativize acceleration in the sense that special relativity relativizes velocity.* Experiments 1 and 2 are not exactly equivalent. More quantitatively . . .

Rindler coordinate transformation: $(c = 1)$

$$t = \rho \sinh \tau, \qquad \tau = \tanh^{-1}\left(\frac{t}{z}\right),$$

$$z = \rho \cosh \tau. \qquad \rho = \sqrt{z^2 - t^2}.$$

$$ds^2 = dt^2 - dz^2 = \rho^2\, d\tau^2 - d\rho^2.$$

The worldline $\rho = 1/a$ is the hyperbola $z = \sqrt{t^2 + a^{-2}}$ (uniform acceleration a).

The worldline $z = 1/a$ is the curve

$$\rho = \frac{1}{a \cosh \tau}$$

(a *stationary* body as seen by an accelerated observer).

A better Rindler transformation

Let $\rho = e^{\zeta}$. Then (in 2D space-time)

$$ds^2 = dt^2 - dz^2 = e^{2\zeta}(d\tau^2 - d\zeta^2)$$

(*conformally flat*). The worldline $z = 1/a$ is the curve

$$\zeta = -\ln(a \cosh \tau).$$

2D massless field equation:

$$\frac{\partial^2 \phi}{\partial t^2} = \frac{\partial^2 \phi}{\partial z^2} \iff \frac{\partial^2 \phi}{\partial \tau^2} = \frac{\partial^2 \phi}{\partial \zeta^2}\,.$$

Salviati's papers about atoms

- M. O. Scully, S. Fulling, D. Lee, D. Page, W. Schleich, and A. Svidzinsky, Quantum optics approach to radiation from atoms falling into a black hole, *Proc. Natl. Acad. Sci. U.S.* **115**, in press and on-line.
- A. Svidzinsky, J. Ben-Benjamin, S. A. Fulling, and D. N. Page, Excitation of an atom by a uniformly accelerated mirror through virtual transitions, *Phys. Rev. Lett.* **121**, in press.

Let's Do It with Mirrors

To convince Sagredo that Salviati is correct in principle, we work in 2D space-time and replace the atom by a perfect mirror (Dirichlet boundary, $\phi = 0$ there).

- Moore, *J. Math. Phys.* **11** (1970) 2679.
- DeWitt, *Phys. Reports* **19** (1975) 295.
- F. & Davies, *Proc. Roy. Soc. A* **348** (1976) 393.

This requires a digression:

The Amazing Triviality and Richness of Two-Dimensional Massless Quantum Field Theory

One might think that such an idea could occur only to a field theorist driven mad by spending too many years in two few dimensions.

Sidney Coleman

But $(1+1)$-D models are the source of much of what we understand about QFT in curved space, or in the presence of boundaries or acceleration. This is so, even though $(1+1)$–dimensional QFT is very special, in two ways:

Special Fact 1

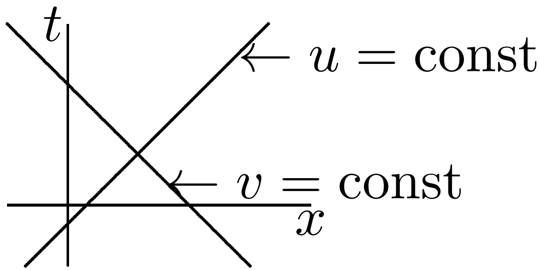
Every 2D manifold is locally *conformally flat*:
There are coordinates (nonunique) where

$$\begin{aligned} ds^2 &= C(\ ,\ )(dt^2 - dx^2) \\ &= C(u,v)\,du\,dv \end{aligned}$$

in *null coordinates*

$$u = t - x, \quad v = t + x.$$

Note: $\quad g_{uv} = \frac{1}{2}C, \quad g^{uv} = \frac{2}{C}\,, \quad \text{diagonals} = 0.$



Lines of constant u or v are *light rays*.

Any other such coordinate system must be just a relabeling of these rays: $\quad u = f(u^*), \quad v = g(v^*),$

$$\begin{aligned} ds^2 &= C(u,v)\,du\,dv \\ &= f'(u^*)g'(v^*)C\big(f(u^*),g(v^*)\big)\,du^*\,dv^* \\ &= C^*(u^*,v^*)\,du^*\,dv^*. \end{aligned}$$

Famous example: Rindler coordinates

Start from flat (Minkowski) space-time: $C(u, v) = 1$.

$$\text{Set} \quad u = -e^{-u^*}, \quad v = e^{+v^*}.$$

$$ds^2 = e^{-u^*+v^*} du^*\, dv^* = e^{2x^*}(dt^{*2} - dx^{*2})$$

since $t^* = \frac{1}{2}(v^* + u^*), \quad x^* = \frac{1}{2}(v^* - u^*)$.

Let $\rho = e^{x^*}$; $ds^2 = \rho^2\, dt^{*2} - d\rho^2$

(ρ = proper distance from the horizon).

Special Fact 2:

The massless wave equation is *conformally invariant*:

$$0 = g^{\alpha\beta}\nabla_\alpha\nabla_\beta\phi = \frac{4}{C}\,\partial_u\partial_v\phi,$$

so

$$\partial_u\partial_v\phi = 0 \iff \partial_{u^*}\partial_{v^*}\phi = 0.$$

A Klein–Gordon mass would ruin this:

$${\partial_{t^*}}^2\phi - {\partial_{x^*}}^2\phi + m^2\phi = 0 \;\Rightarrow\; \partial_u\partial_v\phi + C(u,v)m^2\phi = 0.$$

Furthermore, the normal modes are elementary: built out of plane waves $\quad e^{ip_u u}$ and $e^{-ip_v v} \quad$ or

$$e^{-i(\omega t - kx)} \quad (\omega = |k|) \quad \text{and conjugates.}$$

$e^{ip_u u} = e^{ip_u (t-x)}$ is right-moving;
$e^{ip_v v} = e^{ip_u (t+x)}$ is left-moving.

How to build normal modes depends on global geometry and boundary conditions. For a reflecting boundary at $x = 0$ we'll need $\phi \propto e^{-i\omega t} \sin(kx)$, for instance.

Warning: The physics *does* depend on C!
The physical world as a whole is not conformally invariant.
Measuring instruments know about the metric tensor!

If that were not so, all 2D models would be just like static, flat space, and there would be no "effects" (Unruh, Moore, Hawking). Only Casimir would survive (for *two* reflecting boundaries).

Observables?

Particles are problematical. E.g., the Minkowski vacuum is *not* the ground state of the Fock space built on Rindler modes. It is a mixed state of nonzero temperature proportional to the acceleration of a trajectory of fixed x^*. *Unruh:* This is real physics. A detector at x^* observes a thermal bath at that temperature. *But for a general conformal coordinate system the quanta have no clear physical interpretation.*

Instead ...

Stress-energy-momentum tensor

In a standard Cartesian frame,

$$T_{\alpha\beta} = \begin{pmatrix} T_{tt} & T_{tx} \\ T_{xt} & T_{xx} \end{pmatrix},$$

$$T^t_t = \text{energy density}, \quad T^x_x = \text{pressure},$$

off-diagonals $\propto$ energy flux and momentum density.

$$T_{\alpha\beta} = \partial_\alpha\phi\partial_\beta\phi - \tfrac{1}{2} g_{\alpha\beta}\, \partial_\lambda\phi\partial^\lambda\phi.$$

In null coordinates,

$$T_{uu} = \partial_u \phi \, \partial_u \phi, \quad T_{vv} = \partial_v \phi \, \partial_v \phi,$$

$$T_{uv} = \partial_u \phi \, \partial_v \phi - \partial_u \phi \, \partial_v \phi = 0 \quad \text{(classically)}.$$

$$T_{uu} = \text{rightward flux};$$
$$T_{vv} = \text{leftward flux}.$$

Note that $T^{uu} = \frac{4}{C^2} T_{vv}$, etc.

Conservation law

$$\nabla_\alpha T^{\alpha\beta} = 0.$$

$$\partial_u T_{vv} = -\partial_v T_{uv} - C^{-1}\partial_v C\, T_{uv} = 0,$$

$$\partial_v T_{uu} = 0 \quad \text{(classically)}.$$

Spoiler alert: In QFT with curvature, $T_{uv} \neq 0$ arises from the dead!

$$T_{uv} = -\frac{CR}{96\pi}\,,$$

$$R = \text{Ricci curvature scalar} = \frac{4}{C^3}(C\,\partial_u\partial_v C - \partial_u C\,\partial_v C)$$
$$= -\frac{4}{C}\,\partial_u\partial_v(\ln C) = -\Box_g(\ln C).$$

Recall that $\partial_u T_{vv} =$ (operator on)T_{uv}, etc. Thus the curvature R acts as a *source* for T_{uu} and T_{vv} (Unruh). R itself is not dynamical (for a given geometry), but it influences how $T_{\alpha\beta}$ propagates.

"Vacuum" expectation value of the stress tensor

For a given conformal null chart (u, v), construct the normal modes and Fock space. Take expectation value in the Fock vacuum:

$$\langle T_{vv} \rangle = \langle (\partial_v \phi)^2 \rangle, \quad \text{etc.}$$

Divergences must be removed **in a covariant manner** that reduces to the known right answer in flat, or initially flat, space-time. *This prescription uses the metric structure of flat space.* It is *not* conformally invariant.

Also, we must require **the conservation law**, $\nabla_\alpha T^{\alpha\beta} = 0$. (This resolves some ambiguities in the renormalization prescription.) This requirement forces a *trace anomaly*,

$$T^\alpha_\alpha = \frac{4}{C}\, T_{uv} = -\frac{R}{24\pi} \neq 0.$$

(Trace $= 0$ was expected for a conformally invariant theory.)

Cf. axial anomaly in QED, forced by conservation of the renormalized current.

The answer (excluding Casimir term)

$$\langle T_{\alpha\beta}\rangle = \theta_{\alpha\beta} - \frac{1}{48\pi}\, R\, g_{\alpha\beta}\ ,$$

$$\theta_{uu} = \frac{1}{24\pi}[C^{-1}\partial_u{}^2 C - \tfrac{3}{2}(\partial_u C)^2], \qquad g_{uu} = 0,$$

$$\theta_{vv} = \frac{1}{24\pi}[C^{-1}\partial_v{}^2 C - \tfrac{3}{2}(\partial_v C)^2], \qquad g_{vv} = 0,$$

$$\theta_{uv} = 0, \qquad g_{uv} = \tfrac{1}{2}C.$$

Manifestly covariant formulation: Barceló et al., *Phys. Rev. D* **12** (2012) 084001.

Scenario 1. The Rindler example

Space is flat; $C^* = e^{2x^*} = e^{v^*} e^{-u^*}$. $\quad T_{uv} \propto R = 0$.

$$T_{v^* v^*} = \theta_{v^* v^*} = -\frac{1}{48\pi} = T_{u^* u^*}\,.$$

$$\begin{aligned}
T_{t^* t^*} &= T_{u^* u^*} + T_{v^* v^*} + 2T_{u^* v^*} = -\frac{1}{24\pi}\,, \\
T_{x^* x^*} &= T_{u^* u^*} + T_{v^* v^*} - 2T_{u^* v^*} = -\frac{1}{24\pi}\,, \\
T_{t^* x^*} &= T_{v^* v^*} - T_{u^* u^*} = 0.
\end{aligned}$$

In a local orthonormal frame aligned with the curvilinear coordinates,

$$T_{t^*}^{t^*} = -\frac{1}{12\pi}\, e^{-2x^*} = -T_{x^*}^{x^*} \,.$$

The stress is traceless and the energy is negative. This Rindler-space vacuum energy is singular at the horizon, $x^* = -\infty$. In the "true" Minkowski vacuum state, it is cancelled by the Unruh thermal bath.

Thus the natural Rindler vacuum is singular on the horizon and is different from the usual vacuum, which appears *thermal* to an accelerated observer. The Rindler (or Boulware) vacuum energy is *negative* to cancel the thermal energy: $T_{u^* u^*} = T_{v^* v^*} = -1/48\pi$.

- Rindler, *Am. J. Phys.* **34** (1966) 1174.
- F., *Phys. Rev. D* **7** (1973) 2850.
- Boulware, *Phys. Rev. D* **11** (1975) 1404.
- Unruh, *Phys. Rev. D 14* (1976) 870.

Scenario 2. An accelerating mirror

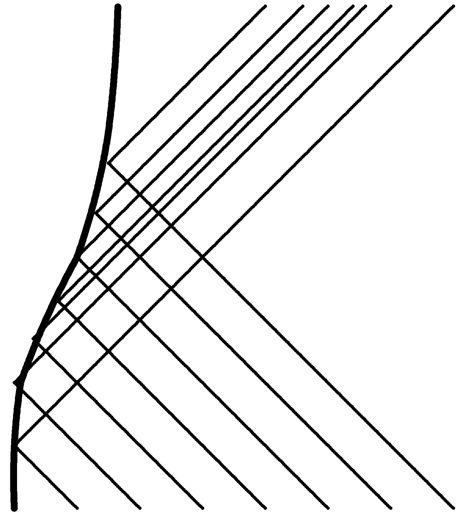
In flat space, place a mirror, or "perfect conductor", on a trajectory $x = z(t)$, with $|\dot{z}| < 1$. Let's consider only the space to its right.

Boundary condition: $\phi\big(t, z(t)\big) = 0$ for all t.

Choose new coordinates (t^*, x^*) to hug the boundary. In starred coordinates, $\phi(t^*, 0) = 0$. When $0 = x^* = \frac{1}{2}(v^* - u^*)$, we have $v^* = u^* = t^*$.
We need mode functions $\propto e^{-i\omega t^*} \sin(\omega x^*)$.

Also we need $u = f(u^*)$, $v = g(v^*)$. What are f and g? Require that the mirror is initially static, and that the starred and unstarred coordinates initially coincide. Then there is a unique "retarded" solution, with g = identity function. The incoming rays are equally spaced, while the reflected rays' spacing is distorted by the factor $f'(u^*)$. I suppress the calculational details, but the magic formula to be solved for f is

$$\tfrac{1}{2}\big(t^* - f(t^*)\big) = z\big[\tfrac{1}{2}\big(t^* + f(t^*)\big)\big].$$



Now suppose the mirror is at rest in the far future, too. In the future, generically the starred coordinates won't "settle down" and become Cartesian again! A Bogolubov calculation with the normal modes then shows that particles are created: $|\text{out-vac}\rangle \neq |\text{in-vac}\rangle$.

In $T_{\alpha\beta}$ terms: If you ignore my warning and renormalize naively in the starred chart, you get $\langle T_{\alpha\beta}\rangle = 0$, which is wrong at late times. If you do it right, $\langle T_{\alpha\beta}\rangle = \theta_{\alpha\beta}$, and $\theta_{u^*u^*}$ is nonzero at points in the null future of points $(t, z(t))$ where the mirror is accelerating. Scalar photons have been emitted!

Recall that in flat space $\partial_v \langle T_{uu} \rangle = 0$, so right-moving radiation present (on a ray of constant u) in the asymptotic region must have been present (on that ray) forever.

But in curved space $\partial_v \langle T_{uu} \rangle =$ functional of R. Curvature is the *source* that produces nonzero energy-momentum at ∞ from zero at the star! There is also a *left-moving*, *negative* flux $\langle T_{vv} \rangle < 0$ to preserve global energy conservation.

Davies, F., & Unruh, *Phys. Rev. D* **13** (1976) 2720.

How can things depend on coordinates like that?

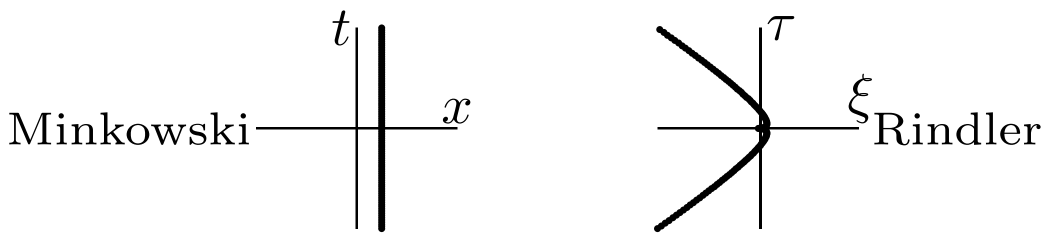
So it seems that any conformal coordinate system (u, v) $[ds^2 = C\, du\, dv]$ defines a "vacuum" state.

The point is not which coordinate system you use. Coordinates are just a tool to make calculations feasible. The point is *which initial state* of the field is assumed (perhaps tacitly) to be interacting with the atom (or mirror).

A Stationary Mirror Viewed by an Accelerating Observer

- Davies, *J. Phys. A* **8** (1975) 609.
- Davies & F., *Proc. Roy. Soc. A* **356** (1977) 237.
- F. & Wilson, The equivalence principle at work in radiation from unaccelerated atoms and mirrors, arXiv:1805.01013, to appear in a festschrift for Wolfgang Schleich in *Physica Scripta*.

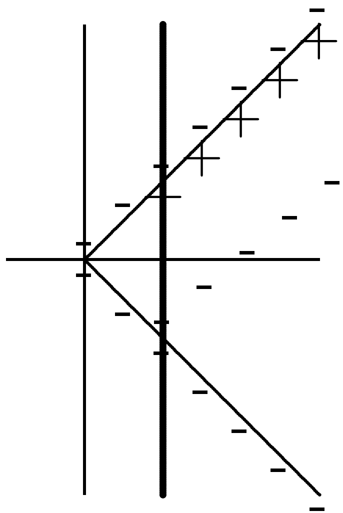
The mirror path is just $x = 1$. Of course it does not emit anything.



But from the point of view of an accelerated observer (or resonant cavity), the path is $\xi = -\ln(\cosh\tau)$, we know. This path was studied by Davies and Fulling in 1977, except that their background metric was Cartesian, not Rindler. Davies studied it in Rindler already in 1975 by Bogolubov-transformation methods, but later retracted his conclusions (wrongly). Wilson and I applied the conformal-transformation method.

We need to calculate the Moore–DeWitt radiation when this mirror is introduced into a cavity that is already in equilibrium in a Rindler-like ground state. There are two steps of conformal transformation: from Cartesian (u, v) to Rindler (u^*, v^*), then from Rindler to coordinates $(\overline{u}, \overline{v})$ that hug the mirror in such a way that any radiation is outward (causal).

Correspondingly there are two factors in $C = e^{2x^*} \frac{du^*}{d\overline{u}}$ and hence two terms in $T_{u^*u^*}$. The first is the Rindler–Boulware flux. The second is the radiation from the mirror, which **is not zero** and cancels the RB flux near the future horizon! It also contains a burst of negative radiation from the point where the mirror crosses the past horizon: a displaced clone of the RB flux.



One can say that
the initial Rindler
negative energy
is reflected off the
mirror!

How can a static object (mirror or atom) radiate?

Take your pick:

1. The static entity is not at rest with respect to the Rindler time translations that help define the Boulware "vacuum". Therefore, it dynamically disrupts that state.
2. The cavity (and the quantum field state it contains at equilibrium) is not at rest with respect to the static object. The latter is thus interacting with a time-dependent state.

Conclusion

Atom and mirror analyses establish a **qualitative equivalence principle**, a rough symmetry between frames in relative acceleration. Its main practical implication is that a uniformly accelerated frame can be treated as a "static" frame in a gravitational field. Near a black hole (or in any nontrivial static gravitational field) the accelerated frame is the physically simpler one, as compared to a frame in free fall (which does not experience static conditions).

Appendix: Bogolubov Transformations

Consider $H = \frac{1}{2}(p^2 + \omega^2 q^2)$. Let

$$a = \frac{1}{\sqrt{2\omega}}(\omega q + ip).$$

Then $H = \omega(a^\dagger a + \frac{1}{2})$.

What happens if I use the wrong ω?

$$b = \frac{1}{\sqrt{2\Omega}}(\Omega q + ip).$$

Then $H = \frac{1}{2\Omega}(b^\dagger b + \frac{1}{2})(\omega^2 + \Omega^2) + \frac{1}{4\Omega}(b^2 + b^{\dagger 2})(\omega^2 - \Omega^2)$.

To get back to the right number operators,

$$b = \frac{1}{2}\left(\sqrt{\frac{\Omega}{\omega}} + \sqrt{\frac{\omega}{\Omega}}\right) a + \frac{1}{2}\left(\sqrt{\frac{\Omega}{\omega}} - \sqrt{\frac{\omega}{\Omega}}\right) a^\dagger.$$

This is a Bogolubov transformation.

Now

$$0 = b|b \text{ ground state}\rangle = (\alpha a + \beta a^\dagger)|b \text{ ground state}\rangle,$$

which can be solved for

$$|b \text{ ground state}\rangle = \\ |a \text{ ground state}\rangle + |2\ a \text{ quanta}\rangle + |4\ a \text{ quanta}\rangle + \cdots.$$

Now imagine this for infinitely many degrees of freedom instead of just one, with mode mixing allowed.

In quantum field theory in curved space-time there are two standard types of calculation of this sort.

(1) We start with the vacuum state at some time and calculate the particle content at some later time.
Examples:

(a) expanding cosmological models (Parker)
(b) a body collapsing to form a black hole (Hawking)

(2) We start with a particular representation of the field algebra with a preferred vacuum state and compare a different representation where the natural vacuum state is different. *Examples:*
(a) uniformly accelerated detectors (Unruh)
(b) "superradiance" of the Kerr solution (rotating black hole), where the identification of positive-frequency modes inside the ergosphere is different from that at infinity (Starobinsky, Unruh)

Thank you for your hospitality!