Determining α_s with Hadronic Event Shapes

Christopher Lee, LANL

ICTP-SAIFR Workshop, São Paulo, Brazil

01 Oct 2019

Los Alamos National Laboratory
EST. 1943

COLLABORATORS

- NNLL' + NLO Angularities

  with Guido Bell, Andrew Hornig[†], Jim Talbert   [†] 1982-2018

  JHEP 01 (2019) 147   [1808.07867]

- Determining α_s and Ω_1 from angularities

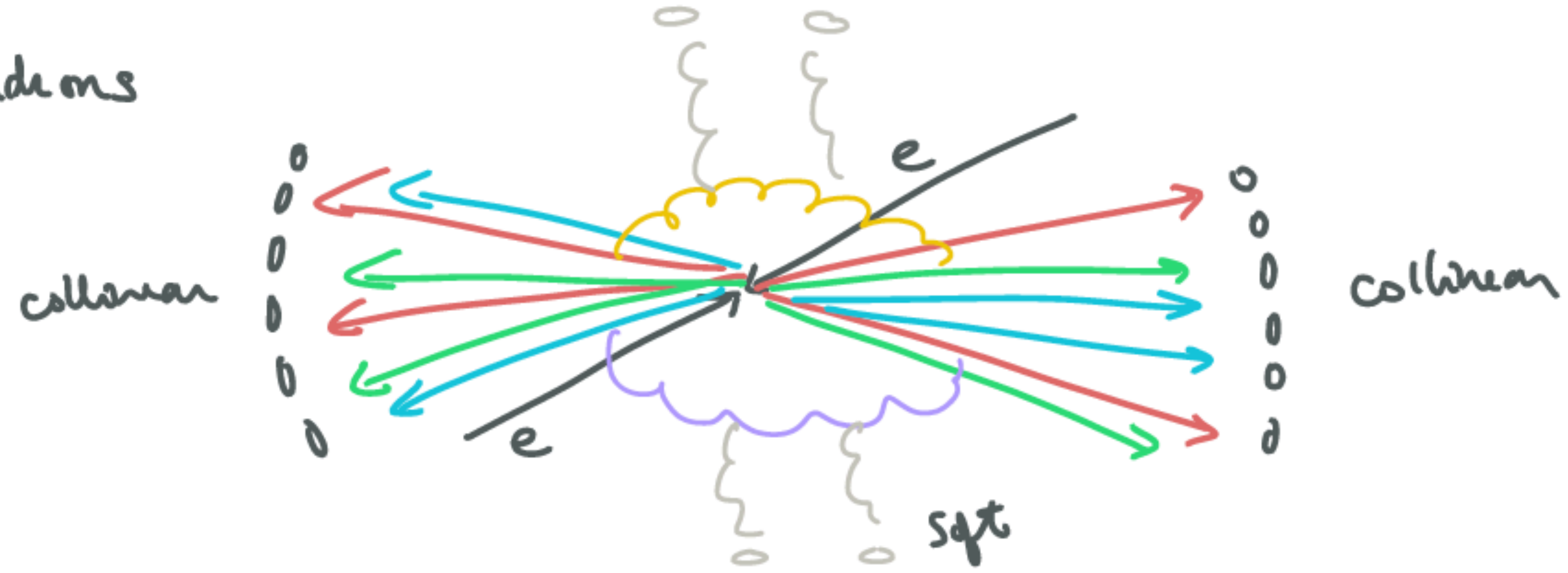
  work in progress

  with G. Bell, Yiannis Makris, Hugo Prager, J. Talbert

Event Shapes

HADRONIC EVENT SHAPES: Global measures of "jetty" structure

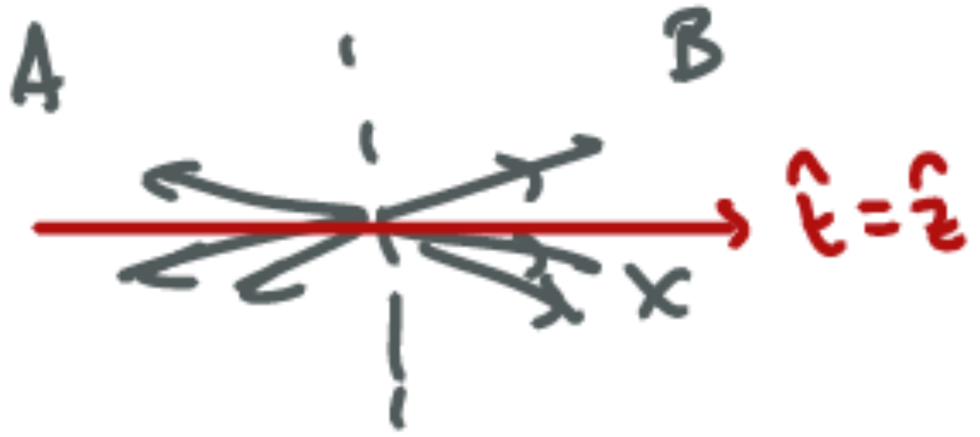
$e^+e^- \to$ hadrons



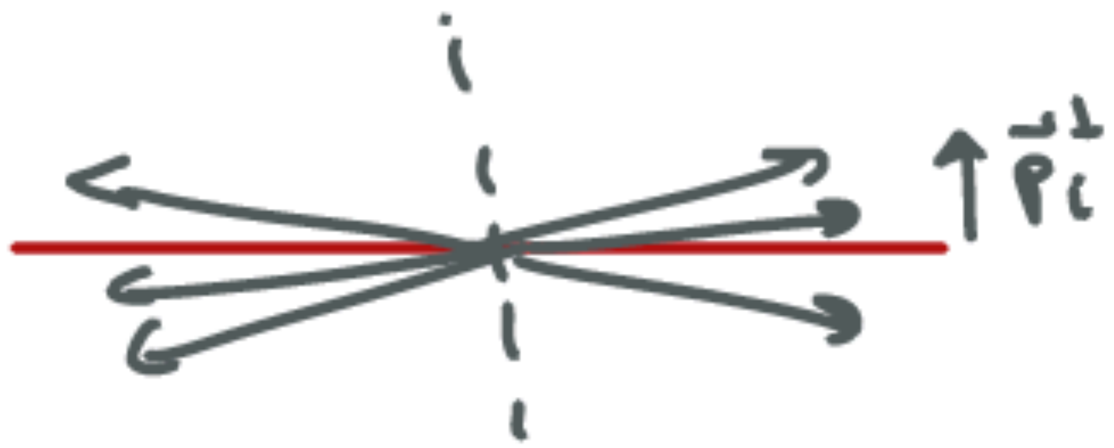
e.g.

THRUST: $T = \frac{1}{Q}\max_{\hat{t}}\sum_{i\in X}|\vec{p}_i\cdot\hat{t}|$

$= \frac{2}{Q}|\vec{p}_z^{\,A}|$ or $\tau = 1-T$



BROADENING: $B = \frac{1}{Q}\sum_{i\in X}|\vec{p}_i^{\,\perp}|$



$T=\frac{1}{2}$

MORE EVENT SHAPES

C-parameter:

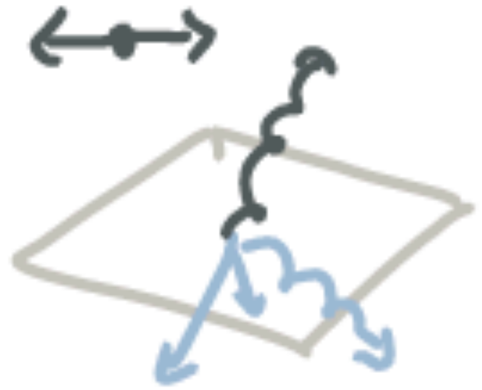
$$\Theta^{\alpha\beta} \equiv \frac{1}{\sum_i |\vec{p}_i|} \sum_i \frac{p_i^\alpha p_i^\beta}{|\vec{p}_i|}$$

Eigenvalues: $\lambda_1, \lambda_2, \lambda_3$

$$C \equiv 3(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3)$$

$$= \frac{3}{2} \sum_{i,j} \frac{|\vec{p}_i||\vec{p}_j| \sin^2\theta_{ij}}{\left(\sum_i |\vec{p}_i|\right)^2} \quad \to 0 \quad \longleftrightarrow$$

$$\to \frac{3}{4}$$



Angularities: [Berger, Kucs, Sterman 2003]

$$\boxed{\begin{aligned} \tau_a &= \frac{1}{Q} \sum_i E_i \sin^a\theta_i \,(1-|\cos\theta_i|)^{1-a} \\ &= \frac{1}{Q} \sum_i |\vec{p}_i^{\perp}|\, e^{-|\eta_i|(1-a)} \end{aligned}}$$

($a < 2$ IRC)

$a \to 0$: thrust τ

$a \to 1$: broadening B

General class:
(+many others)

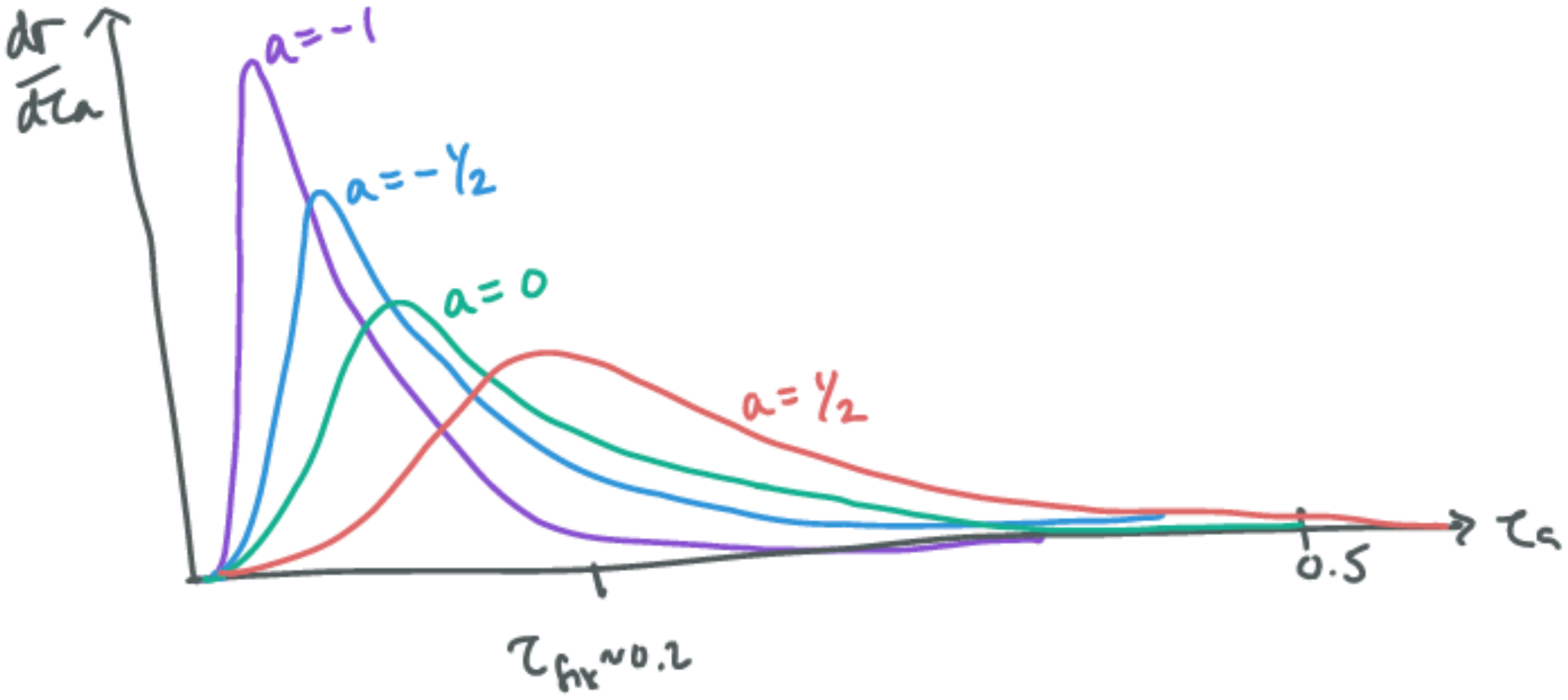
$$e = \frac{1}{Q} \sum_i |\vec{p}_i^{\perp}|\, f_e(\eta_i) \qquad \eta_i = \ln\cot\frac{\theta_i}{2}$$

e.g. $f_{\tau_a}(\eta) = e^{-|\eta|(1-a)}$

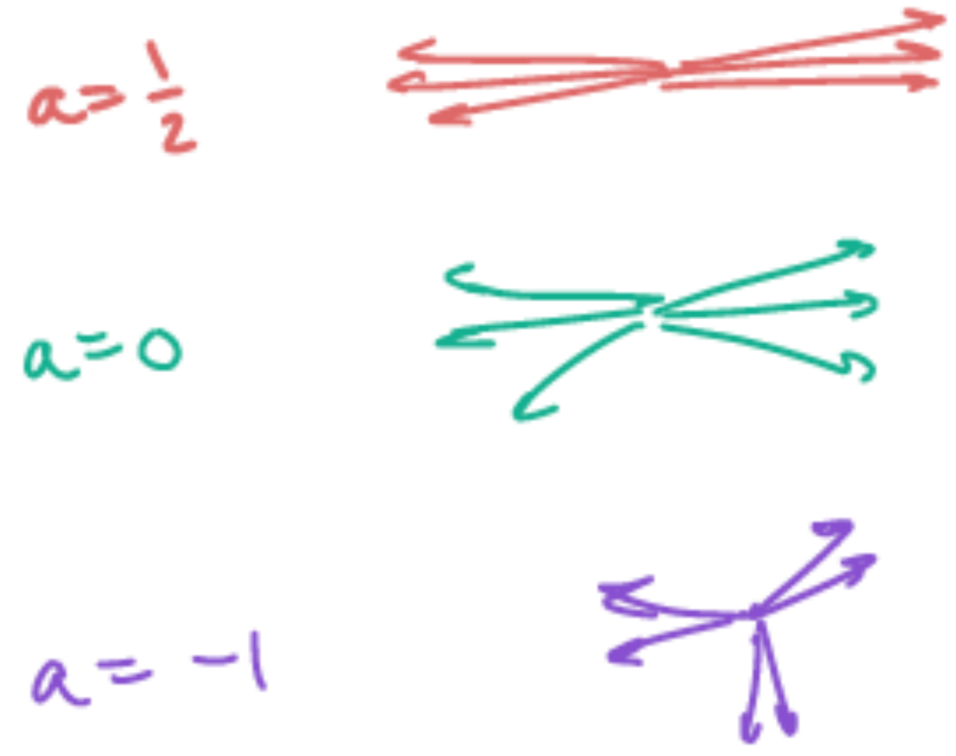
$f_C(\eta) = \dfrac{3}{\cosh\eta}$

VARYING ANGULARITIES

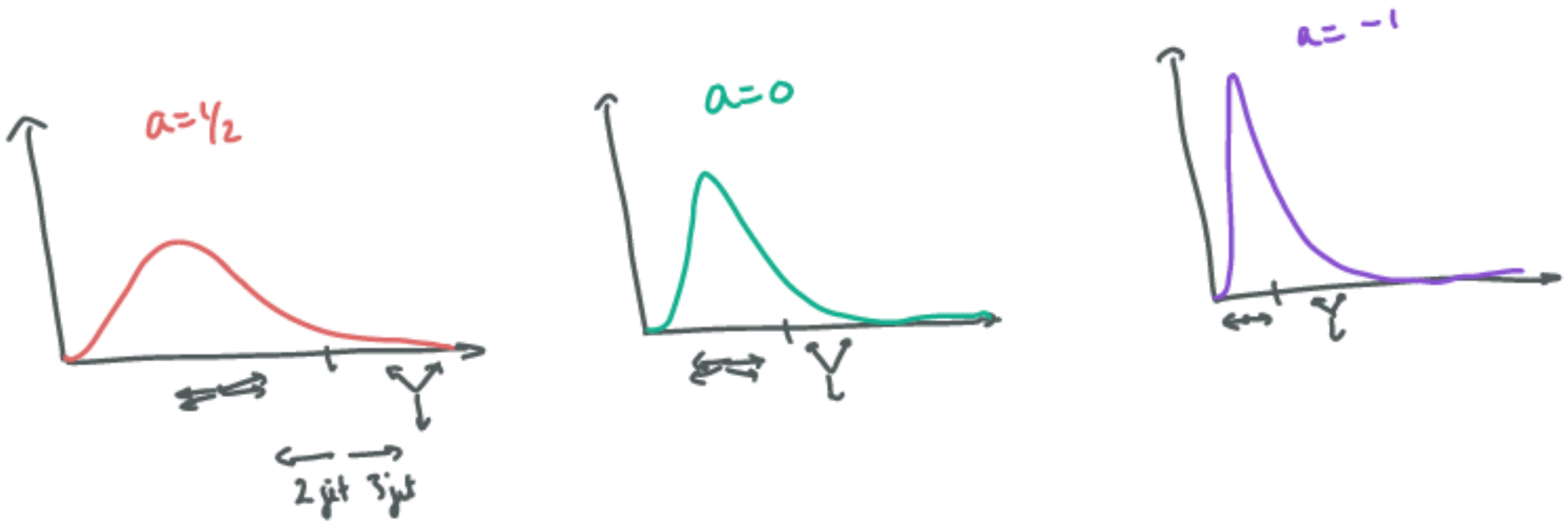
τ_a distributions look like:



typical jet size at τ_{fix}:

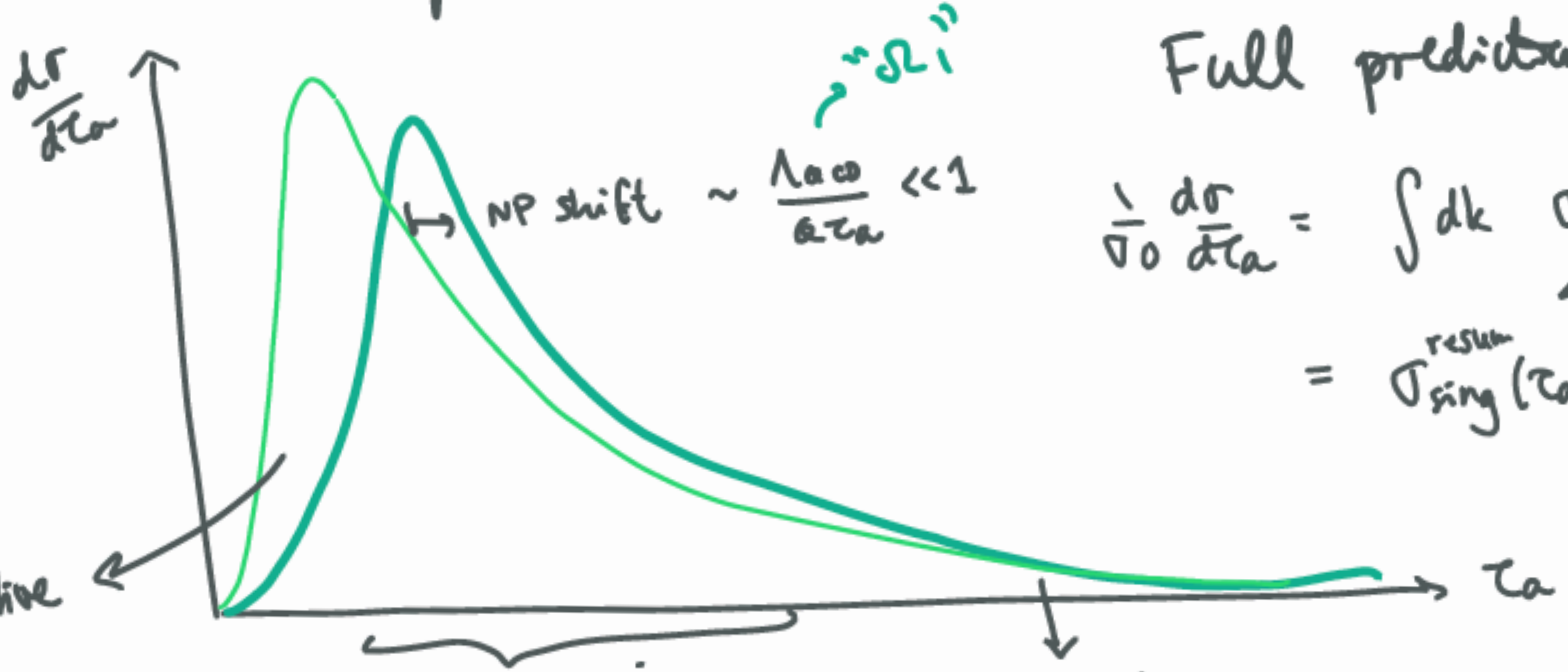


2-jet vs. 3-jet boundary:



EVENT SHAPES & SENSITIVITY TO α_s

τ_a's and similar event shapes probe QCD effects over wide range of scales, perturbative and nonperturbative:



Full prediction:

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau_a} = \int dk\ \underbrace{\sigma_{PT}\left(\tau_a - \frac{k}{Q}\right)}\ S_{NP}(k)$$

$$= \sigma_{sing}^{resum}(\tau_a;\mu_{H,J,S}) + \sigma_{non\text{-}sing}^{F.O.}(\tau_a;\mu_{ns})$$

nonperturbative

$\sigma_{PT}\left(\tau - \frac{k}{Q}\right) \otimes S_{NP}(k)$

$\frac{\Lambda_{QCD}}{Q\tau} \sim 1$

$\downarrow$

SHAPE FUNCTION

expansion in

$\alpha_s(Q\tau)$, $\alpha_s(Q\tau^{\frac{1}{2-a}})$

$\downarrow$

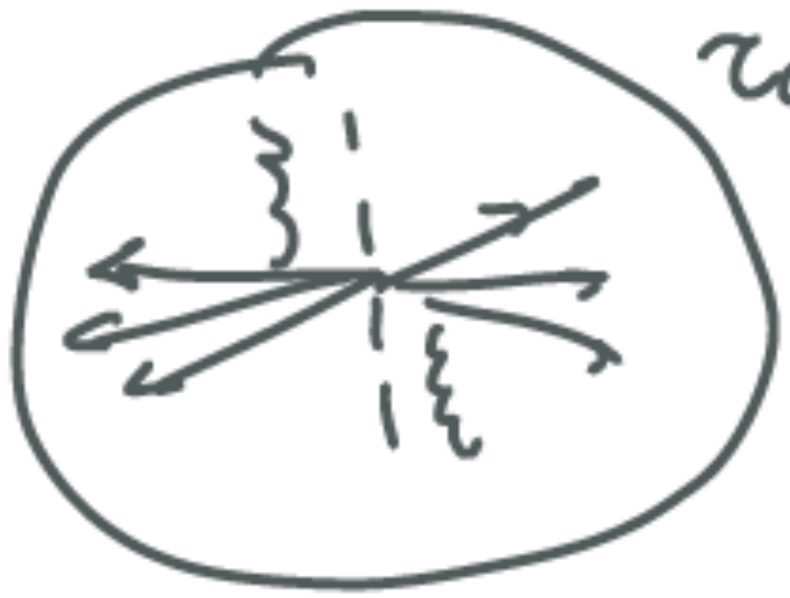
RESUMMATION

expansion in $\alpha_s(Q)$

$\downarrow$

FIXED ORDER

PRECISION EVENT SHAPES



"Global" observables, single number:

"easy" to predict, "easy" to measure

now $N^3LL + \mathcal{O}(\alpha_s^3)$ for τ_0, [Abbate, Fickinger, Hoang, Mateu, Stewart]

C-parameter [Hoang, Kolodrubetz, Mateu, Stewart]

heavy jet mass [Chien, Schwartz]

$NNLL^{(\prime)}$ for B [Becher, Bell; Bragen]

and (now!) $NNLL'$ for τ_a, $a<1$.

especially LEP @ $Q = M_Z$

also SLAC

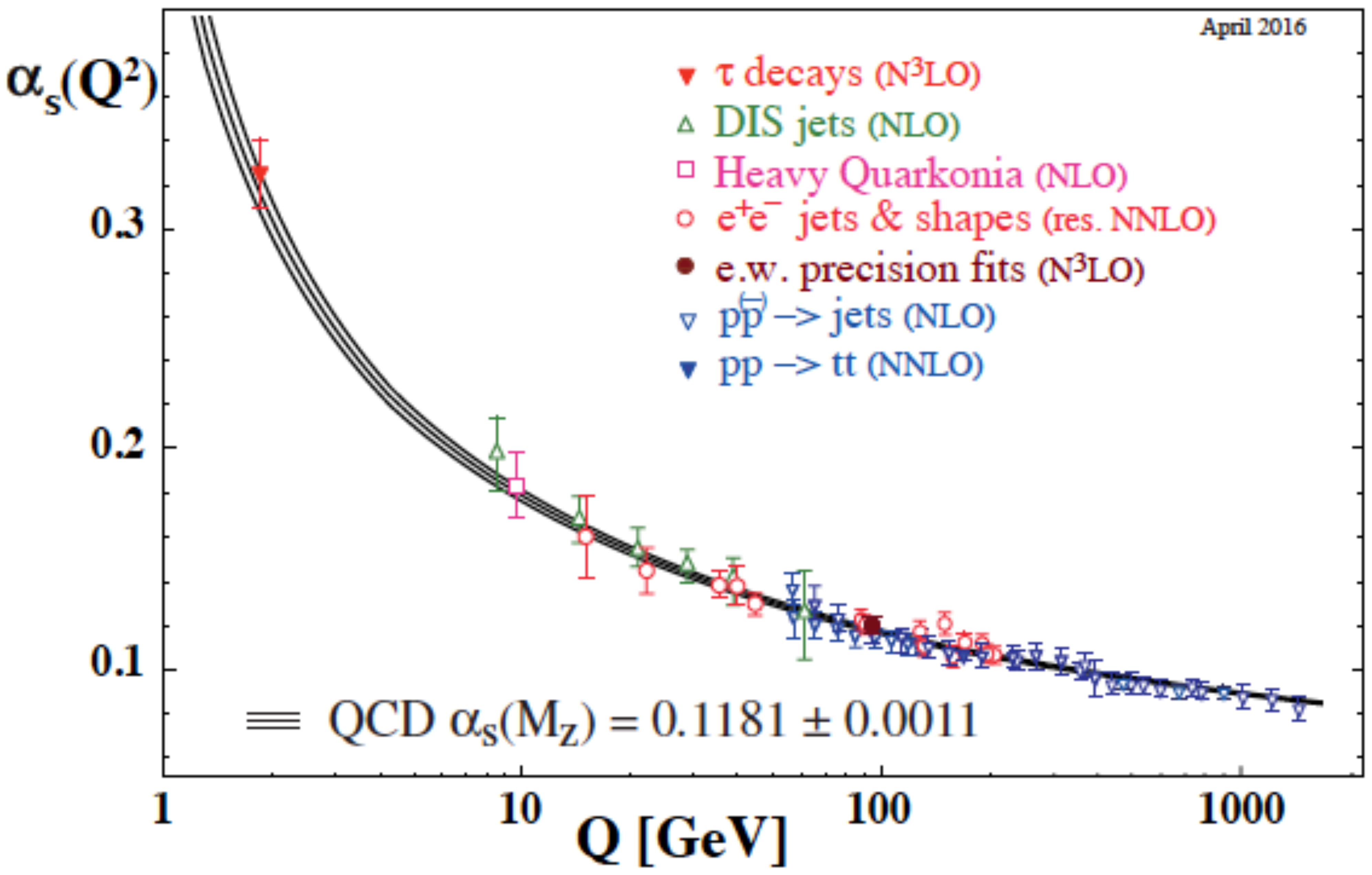
et al.

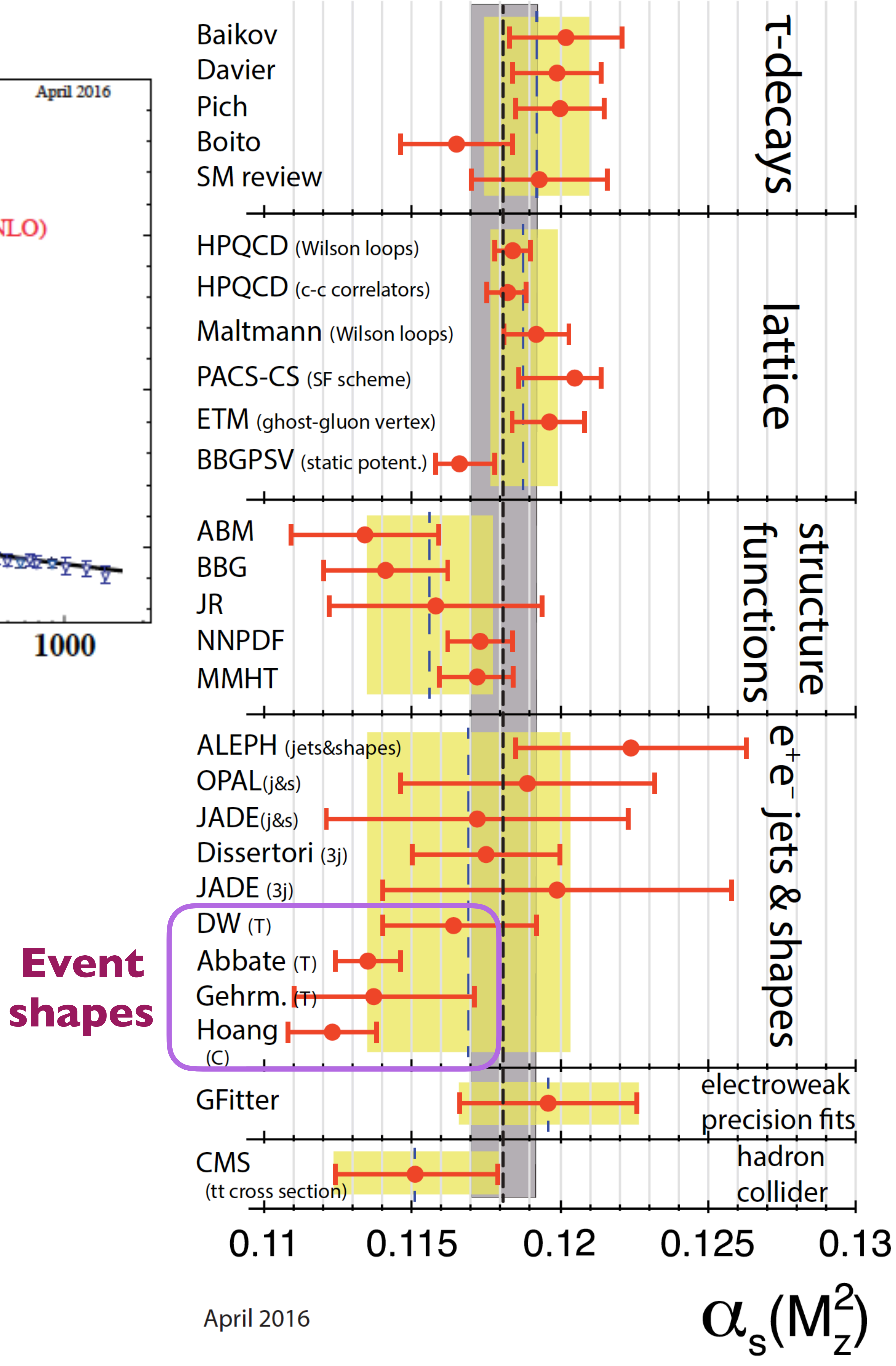
$\longrightarrow$ future linear colliders

[Event shapes in DIS @ HERA, future EIC — D. Kang, CL, I. Stewart 1303.6952 PoS DIS2015,142]

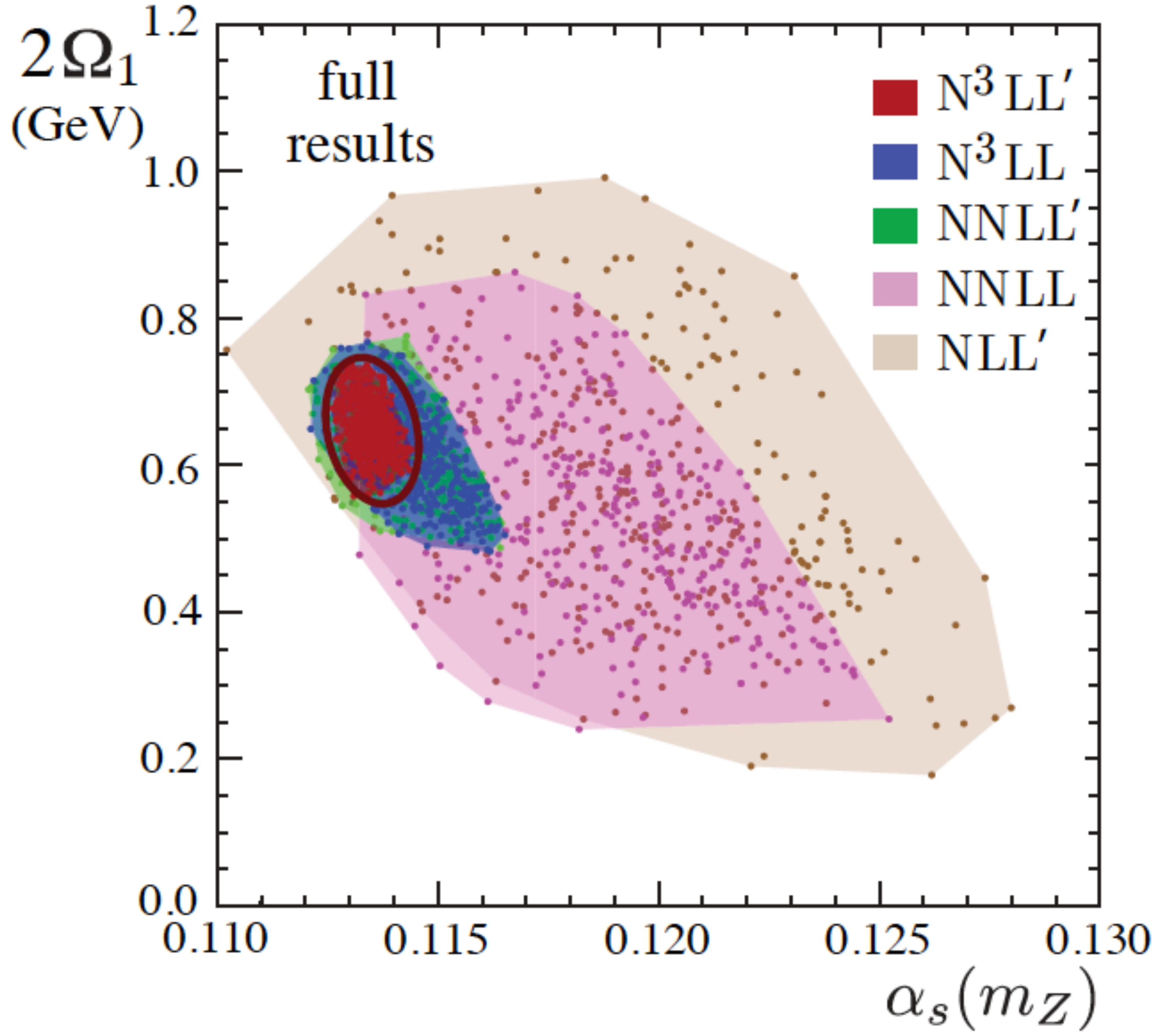
Event shapes and the strong coupling

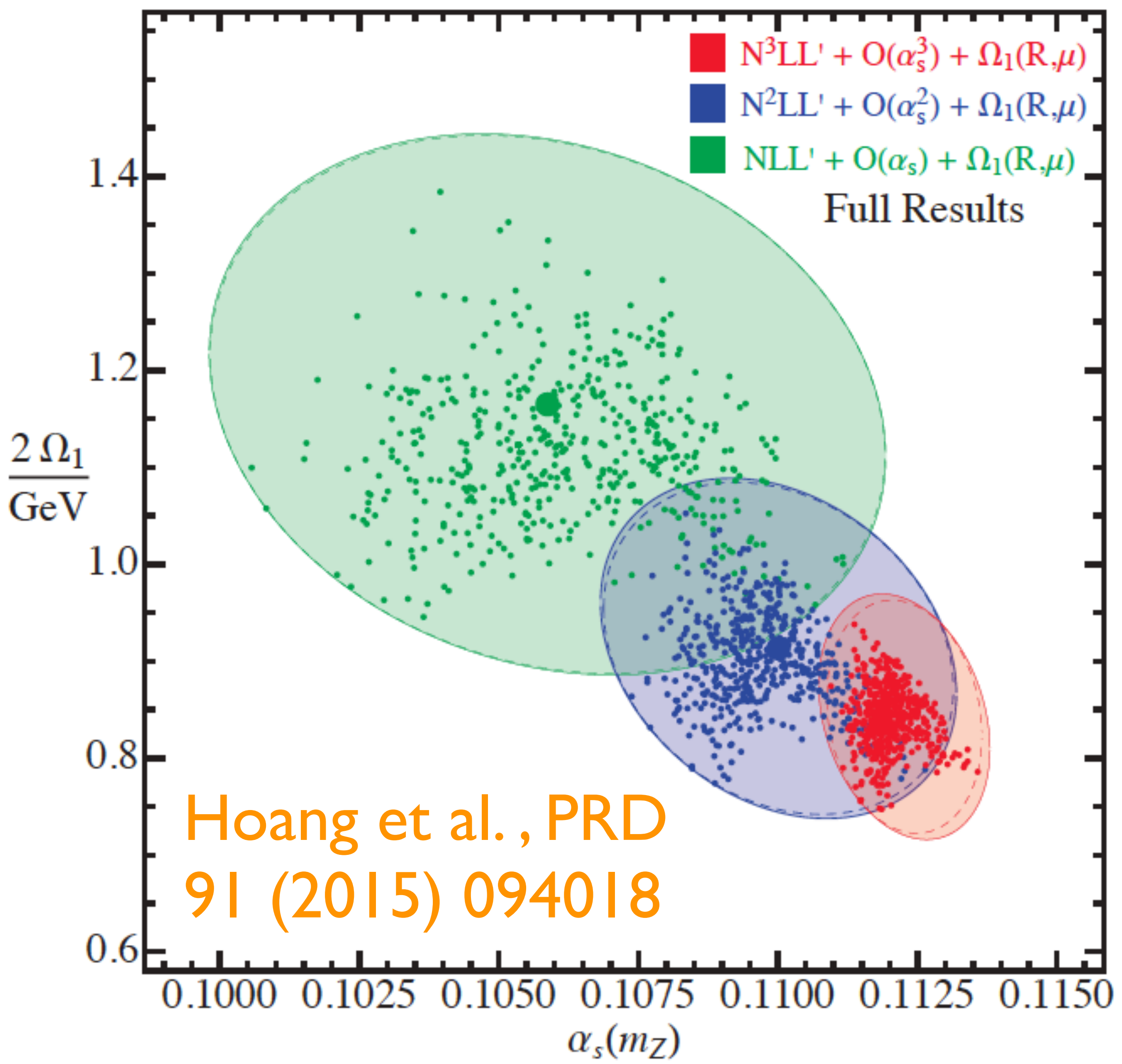
PDG 2016:





Abbate et al., PRD 83 (2011) 074021

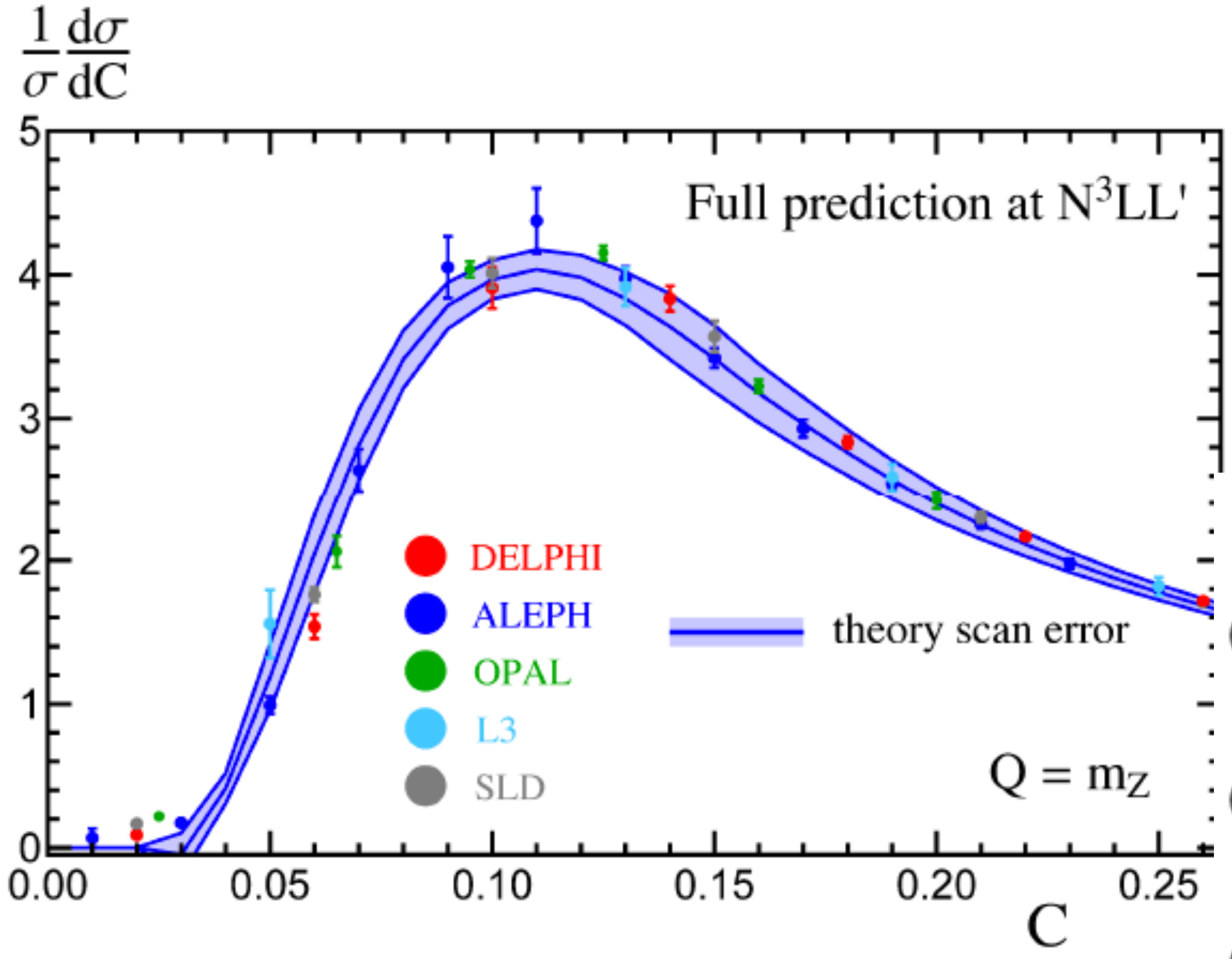


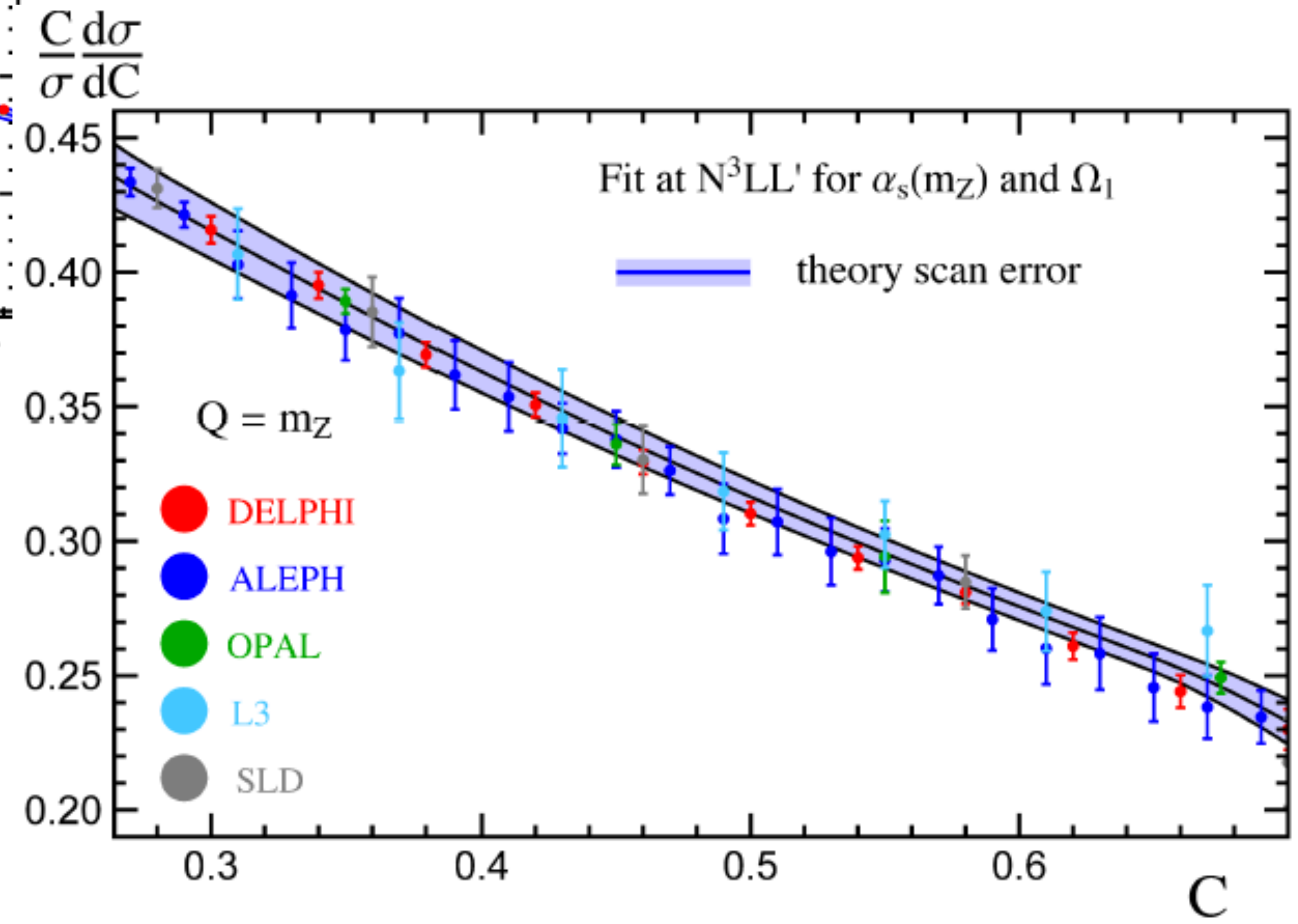


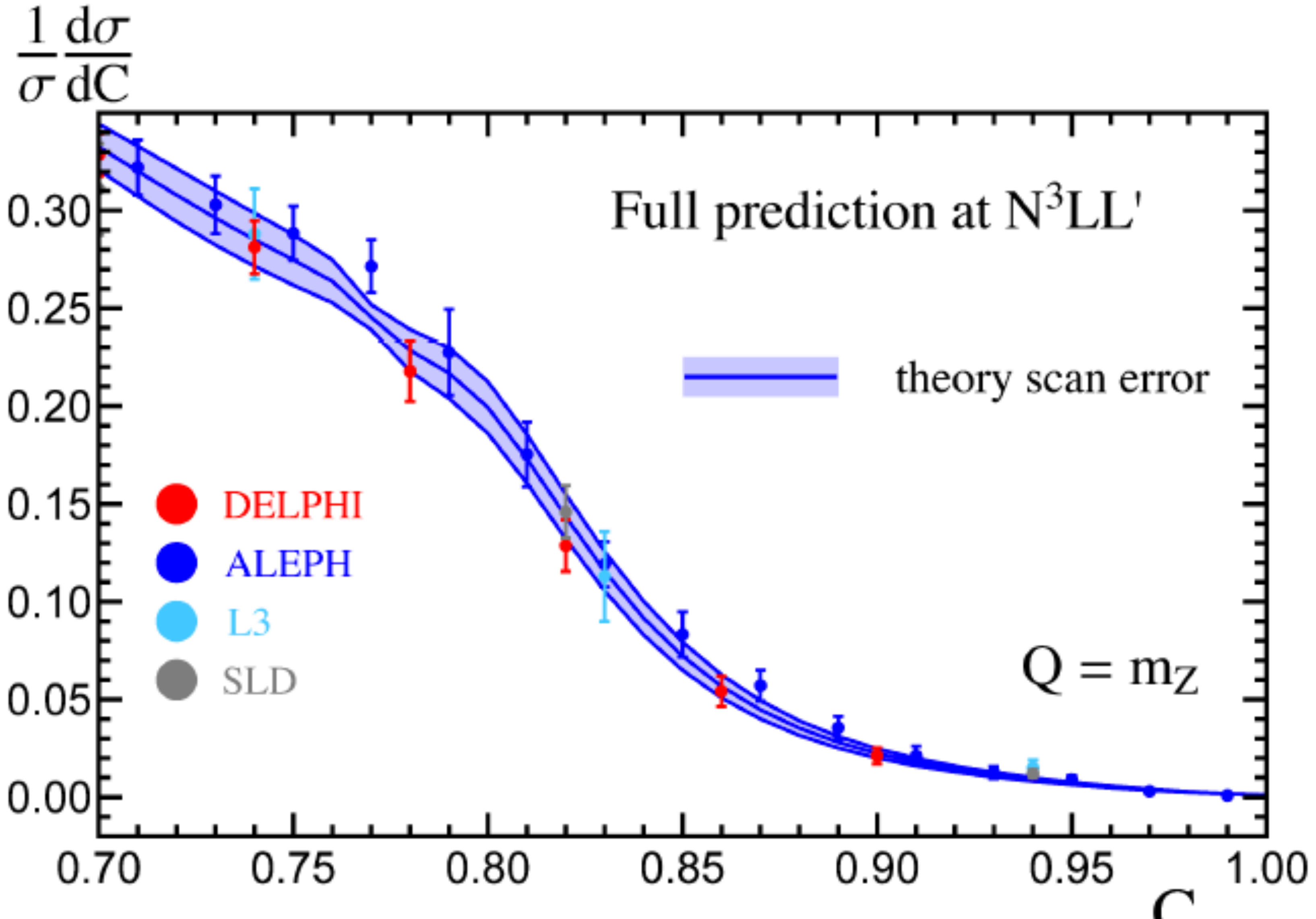
Hoang et al., PRD 91 (2015) 094018

Fit and Predictions: C

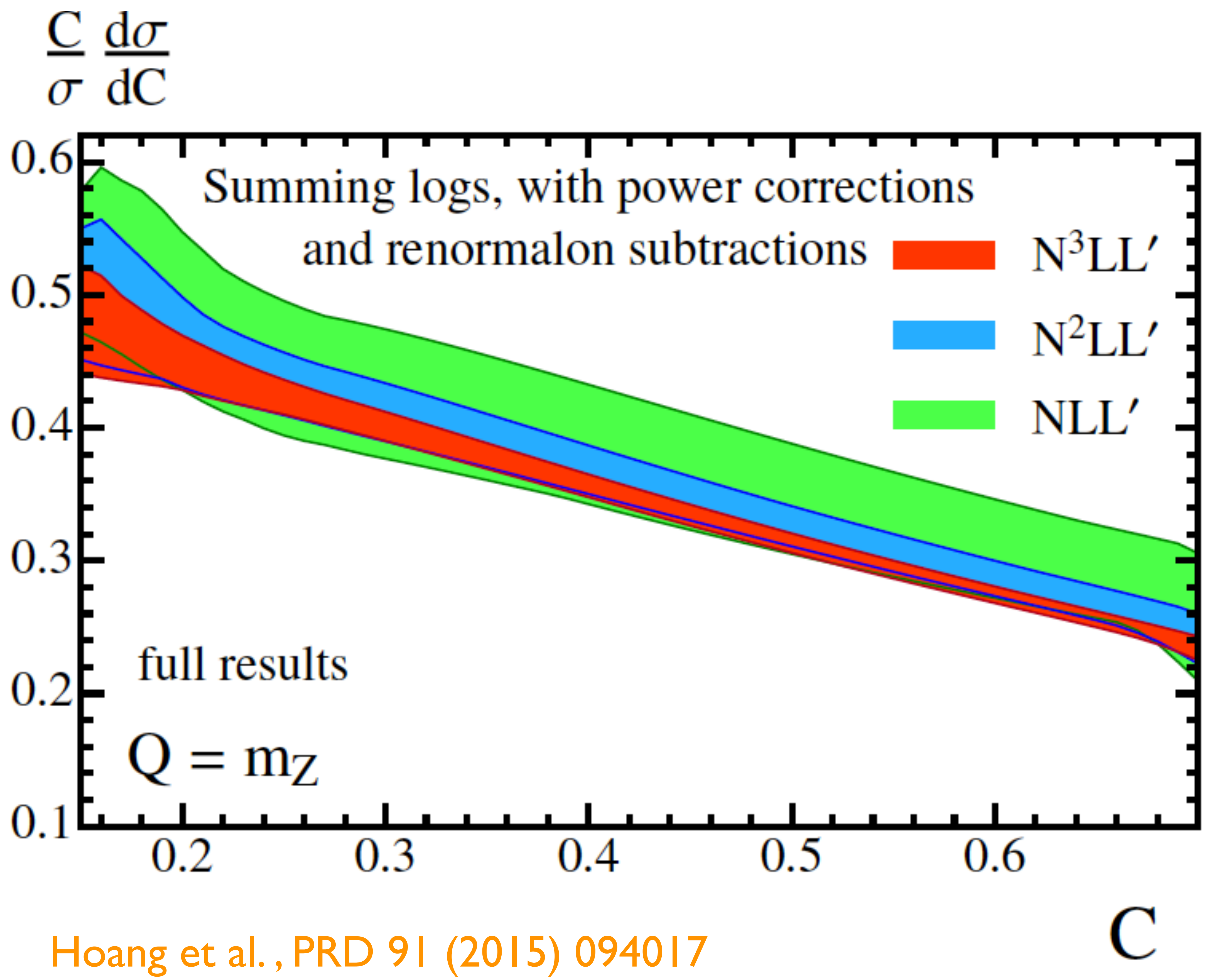
Hoang et al., PRD 91 (2015) 094018



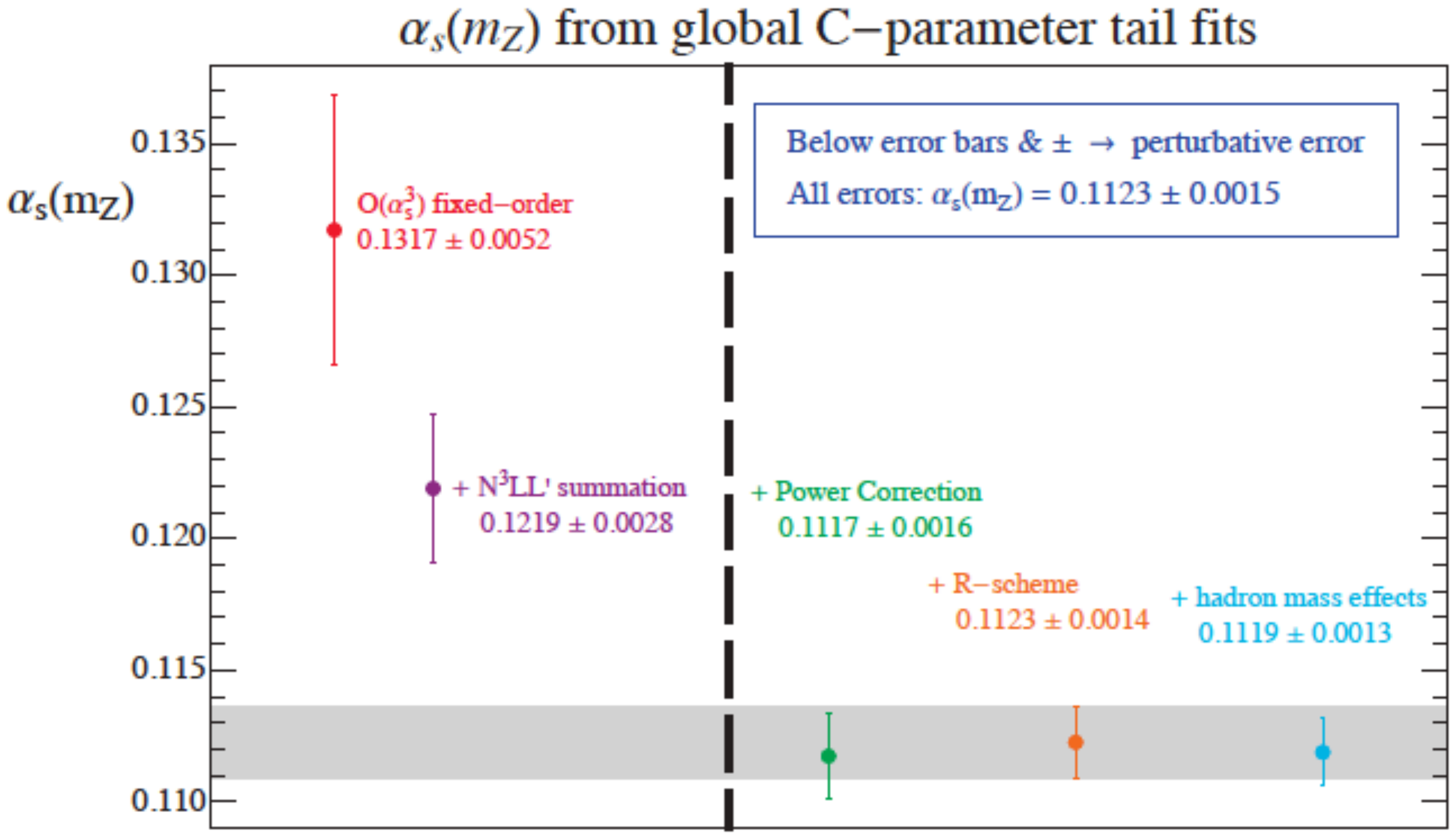




High precision and impact on strong coupling



Hoang et al. , PRD 91 (2015) 094017

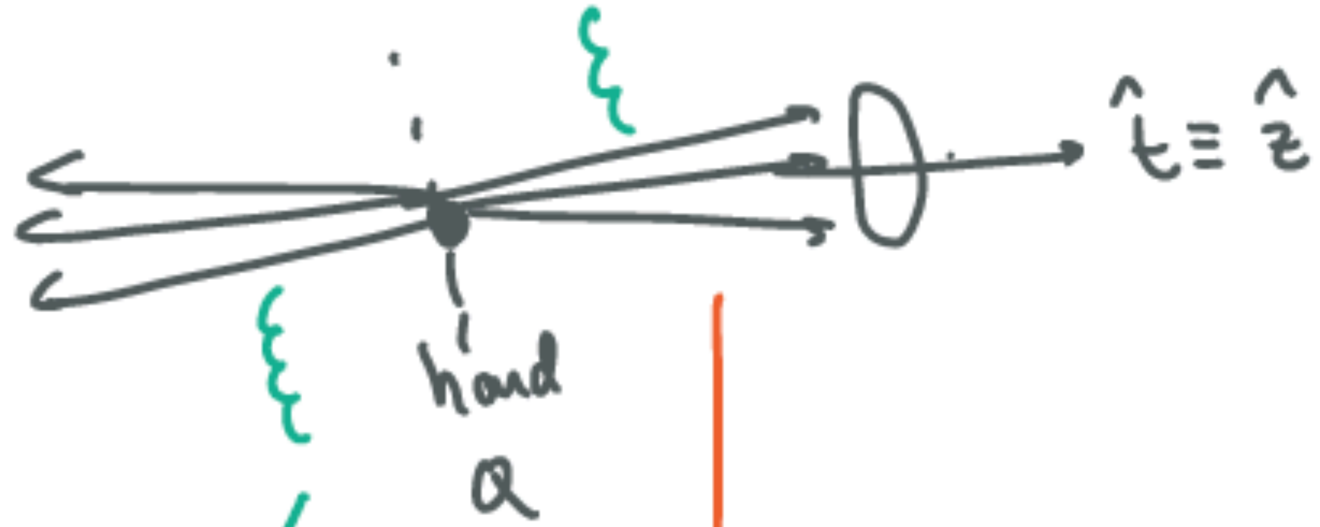


Hoang et al. , PRD 91 (2015) 094018

Factorization & Resummation using EFT

Relevant Physical Scales

Thrust: $M^2 = M_A^2 + M_B^2 = Q^2 \tau$ $\left(\text{if } \ll Q^2\right)$



$n = (1, +\hat z)$

$\bar n = (1, -\hat z)$

light-cone coordinates:

$$p^\mu = (\bar n \cdot p, n \cdot p, \vec p_\perp)$$

jet mass

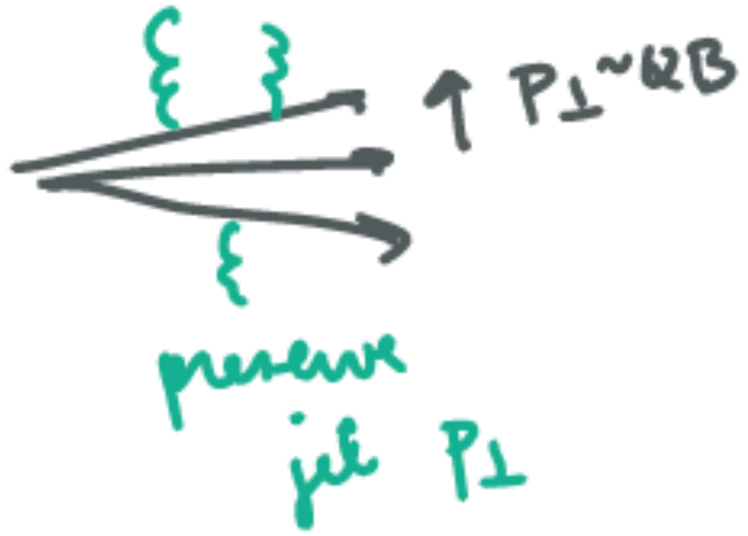
preserve

collinear $p_c \sim \left(Q, \frac{M^2}{Q}, M\right) \sim Q(1, \tau, \sqrt{\tau})$

same

soft $k_s \sim \left(\frac{M^2}{Q}, \frac{M^2}{Q}, \frac{M^2}{Q}\right) \sim Q(\tau, \tau, \tau)$

Broadening:

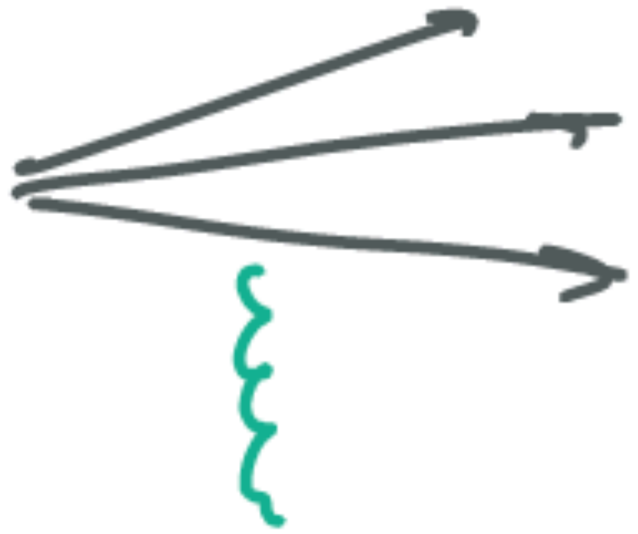


coll $p_c \sim (Q, QB^2, QB)$

same

soft $k_s \sim Q(B, B, B)$

Angularities:



$$\tau_a \sim \frac{p_\perp}{Q}\left(\frac{p^+}{p^-}\right)^{\frac{1-a}{2}}$$

$$\sim \frac{1}{Q}\,(p^+)^{1-\frac{a}{2}}\,(p^-)^{\frac{a}{2}}$$

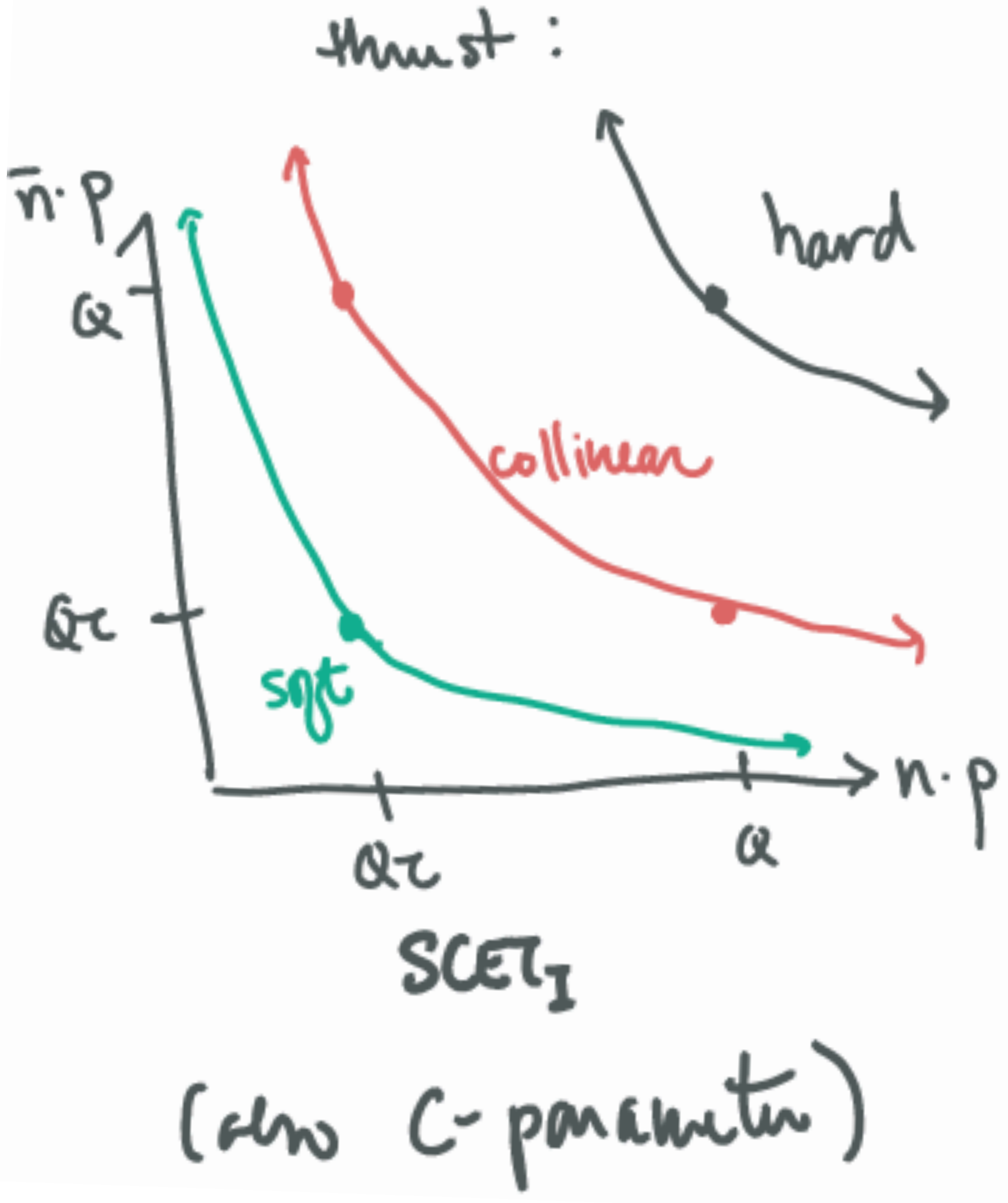
$\Rightarrow$ coll $\quad \tau_a \sim \left(\frac{p^+}{Q}\right)^{1-\frac{a}{2}} \quad \Rightarrow \quad p^+ \sim Q\,\tau_a^{\frac{2}{2-a}}$

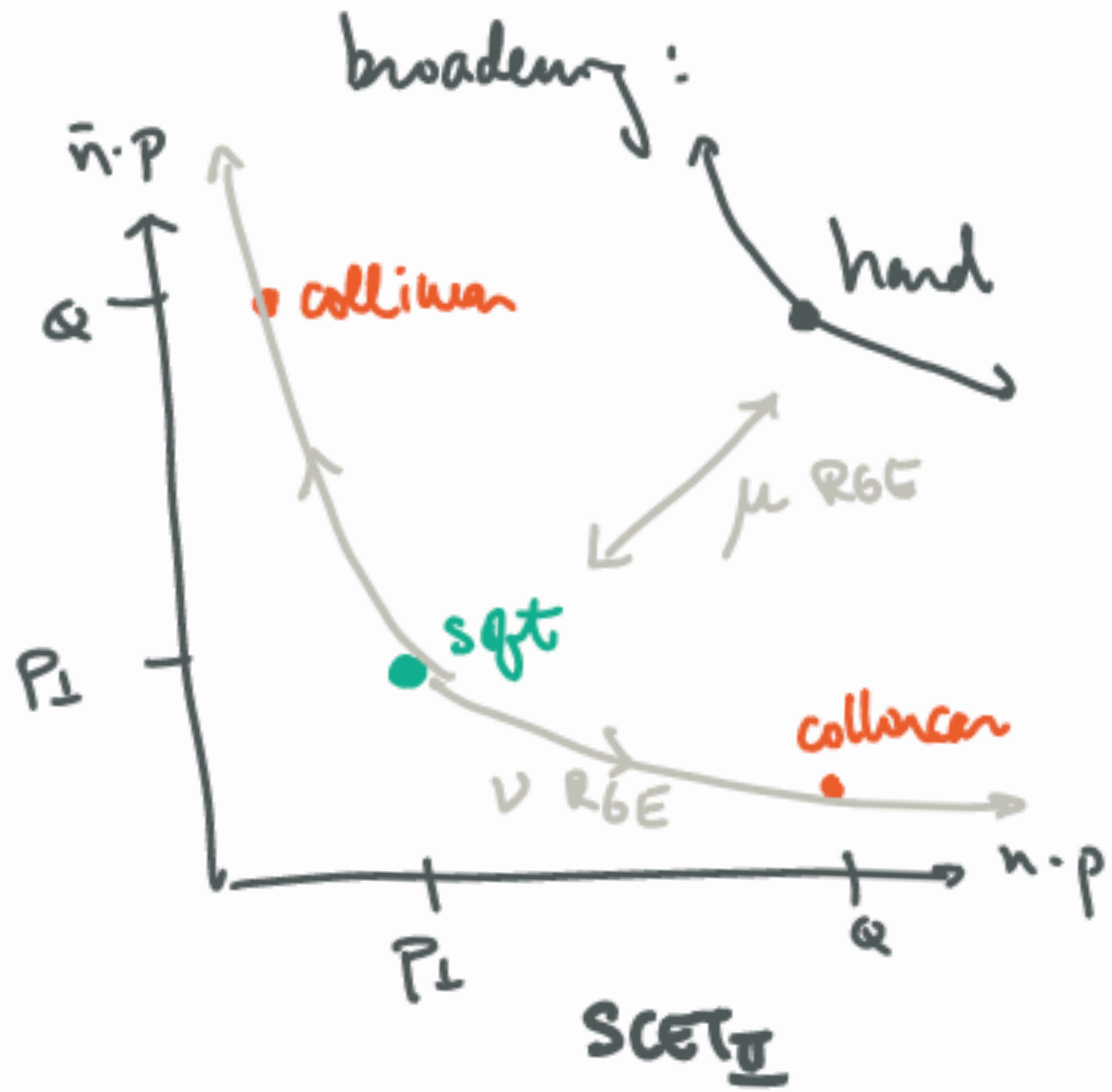
$$p_\perp \sim Q\,\tau_a^{\frac{1}{2-a}}$$

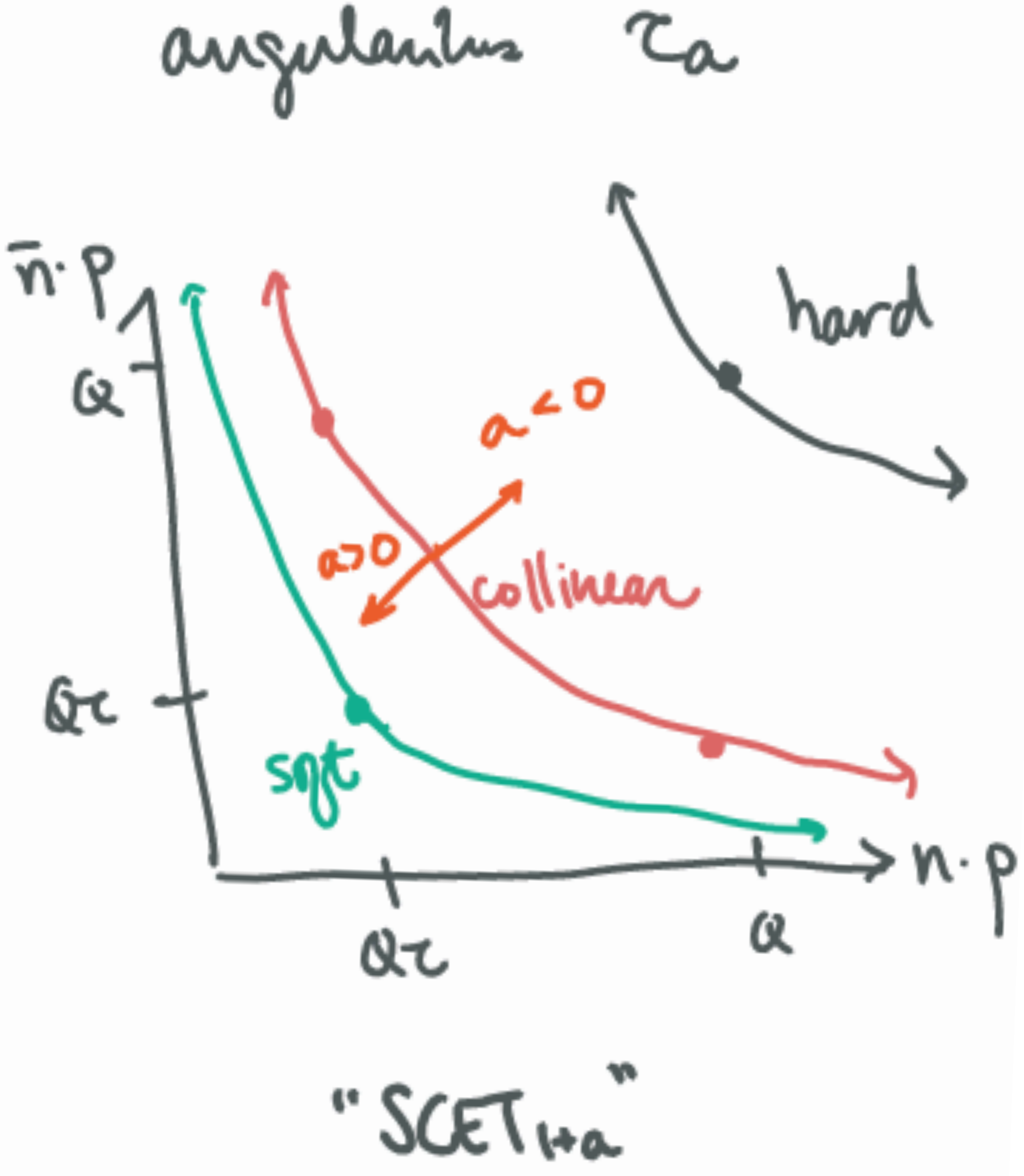
$$p_c \sim Q\left(1,\ \tau_a^{\frac{2}{2-a}},\ \tau_a^{\frac{1}{2-a}}\right)$$

soft $\quad \tau_a \sim \frac{k_s}{Q} \quad \Rightarrow \quad p_s \sim Q(\tau_a, \tau_a, \tau_a)$

SCALES & MODES:







[0901.3780]

SCET IN A NUTSHELL

[Bauer, Fleming, Luke, Pirjol, Stewart 2000-02]

Expand QCD Lagrangian in collinear & soft limits:

$$\psi(x) \longrightarrow \psi_n(x) = \sum_{\tilde{p}} e^{-i\tilde{p}\cdot x}\, \psi_{n,\tilde{p}}(x)$$

$$\tilde{p} = \underbrace{\bar{n}\cdot\tilde{p}}_{O(Q)}\, \frac{n}{2} + \underbrace{\tilde{p}_\perp}_{O(Q\lambda)}$$

$$A(x) \longrightarrow A_n^c(x) + A_s(x)$$

$$A_n^c = \sum_{\tilde{q}} e^{-i\tilde{q}\cdot x} A_{n,\tilde{q}}(x)$$

$$(\bar{n}\cdot A_n,\, n\cdot A_n,\, A_n^\perp) \sim Q(1,\lambda^2,\lambda)$$

$$A_s \sim Q\lambda^2$$

LO: $\mathcal{L}_{QCD}(\psi, A) \longrightarrow \mathcal{L}_{SCET}(\psi_n, A_n, A_s)$

$$\downarrow$$

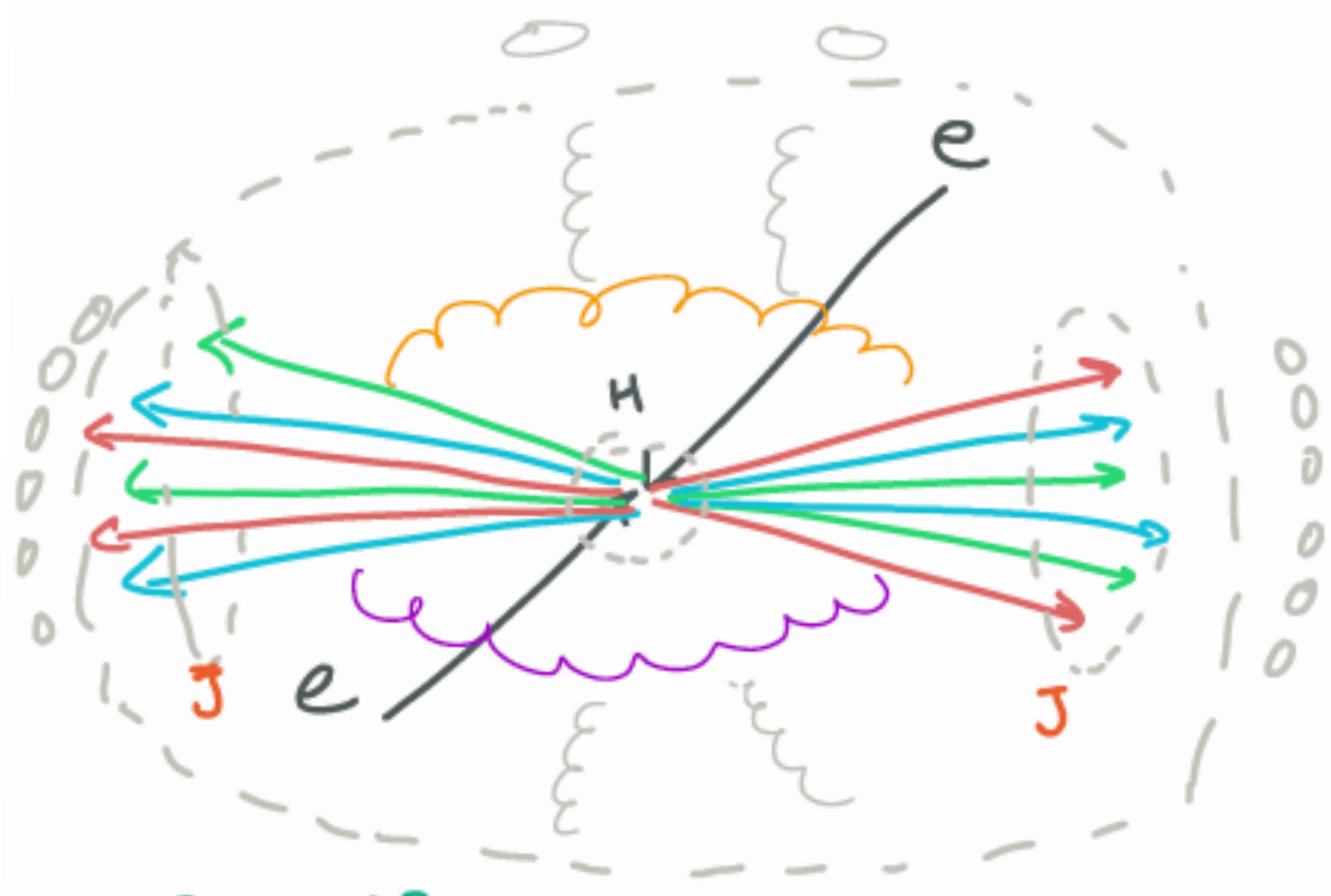
soft-collinear decoupling $\Rightarrow$ $\mathcal{L}_{n_1}^{QCD} + \mathcal{L}_{n_2}^{QCD} + \cdots + \mathcal{L}_s^{QCD}$

$$\psi_n \to Y_n \psi_n$$

$$Y_n = P\exp\left[\int_0^\infty ds\, n\cdot A_s(ns)\right]$$

[Bauer, Pirjol, Stewart 2003]

Factorization in a Nutshell



$S = S_{PT} \otimes S_{NP}$

For $\tau_a \ll 1$:

$$\frac{d\sigma}{d\tau_a} =$$

σ_0

$\times \{$

$+ \cdots \}$

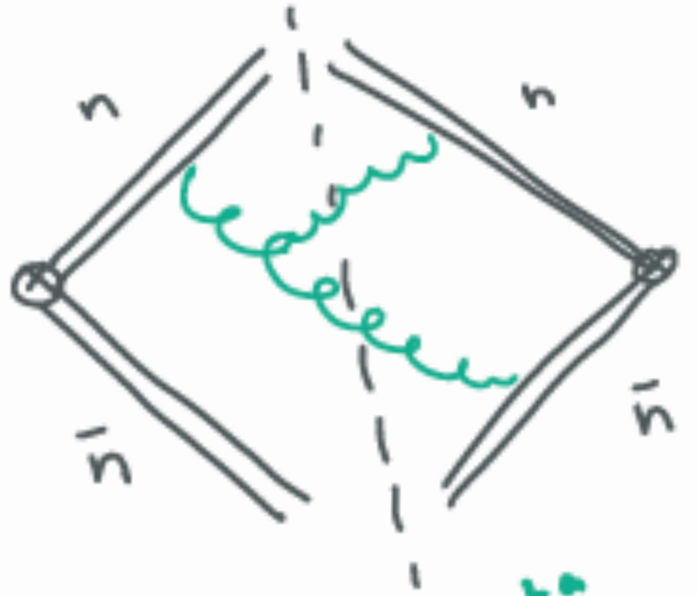
$H(Q^2, \mu)$

$$\times \int dt_J \, dt_{\bar{J}} \, dk_s \; \delta\left(\tau_a - \frac{t_J + t_{\bar{J}}}{Q^{2-a}} - \frac{k_s}{Q}\right)$$

$\times \{$

$) \quad t_J = \sum_i |p_{\perp i}|^{2-a} = t_{\bar{J}} \; ($

$\} \times$



$J_n^a(t_J, \mu) \; J_{\bar{n}}^a(t_{\bar{J}}, \mu)$

$$k_s = \sum_i (n \cdot k_i)^{1 - \frac{a}{2}} (\bar{n} \cdot k_i)^{a/2}$$

$\Downarrow$

defined as matrix elements of operators in SCET $\leftarrow$ $S_a(k_s, \mu)$

$$= \sigma_0 \, H(Q^2, \mu) \int dt_J \, dt_{\bar{J}} \, dk_s \, \delta\left(\tau_a - \frac{t_J + t_{\bar{J}}}{Q^{2-a}} - \frac{k_s}{Q}\right) J(t_J, \mu) \; J(t_{\bar{J}}, \mu) \; S(k_s, \mu)$$

Hard, Jet, Soft Functions @ $\mathcal{O}(\alpha_s)$

e.g. at 1-loop:

$$H(Q^2,\mu) = 1 + \frac{\alpha_s(\mu)}{4\pi}\left[\left(-8\ln^2\frac{\mu}{Q} - 12\ln\frac{\mu}{Q}\right)C_F + c_H^1\right]$$

[Manohar hep-ph/0309176
Bauer, CL, AM, Wise /0309278]

$$\int J(t_J,\mu) = 1 + \frac{\alpha_s(\mu)}{4\pi}\left\{\frac{C_F}{2-a}\left(\frac{4}{1-a}\ln^2\frac{t_J}{\mu^{2-a}} - 6\ln\frac{t_J}{\mu^{2-a}}\right) + c_J^1(a)\right\}$$

[a=0 Bauer & Manohar hep-ph/0312109
a≠0 Hornig, CL, Ovanesyan 0901.3780]

$$\int S(k_S,\mu) = 1 + \frac{\alpha_s(\mu)}{4\pi}\left\{-\frac{8C_F}{1-a}\ln^2\frac{k}{\mu} + c_S^1(a)\right\}$$

[a=0 Fleming et al. 0711.2079
a≠0 Hornig, CL, Ovanesyan 0901.3780]

Evolution & Resummation

RG Evolution of each piece as μ is changed

and constraint that $\frac{d\sigma}{d\tau}$ is independent of μ

can be used to constrain log-terms to arbitrarily high order in $\alpha_s(\mu)$

and resum these logs to all orders.

Disentangle convolution in τ_a:

Laplace transform $\qquad \tilde{\sigma}(\nu_a) = \frac{1}{\sigma_0}\int_0^\infty d\tau_a \, e^{-\nu_a \tau_a} \frac{d\sigma}{d\tau_a}$

$$\Rightarrow \quad \tilde{\sigma}(\nu_a) = H(Q^2,\mu)\, \tilde{J}^2\left(\frac{\nu_a}{Q^{2-a}},\mu\right) \tilde{S}\left(\frac{\nu_a}{Q},\mu\right)$$

Each function satisfies RG eq.:

$$\mu \frac{d}{d\mu} H = \gamma_H(\mu)\, H \qquad \gamma_H(\mu) = -\kappa_H \Gamma_{\text{cusp}}[\alpha_s(\mu)] \ln \frac{\mu}{Q} + \gamma_H[\alpha_s(\mu)]$$

$$\mu \frac{d}{d\mu} \tilde{J} = \gamma_J(\mu)\, \tilde{J} \qquad \gamma_J(\mu) = -\kappa_J \Gamma_{\text{cusp}}[\alpha_s(\mu)] \ln \frac{\mu^{2-a} e^{\gamma_E \nu}}{Q^{2-a}} + \gamma_J[\alpha_s(\mu)]$$

$$\mu \frac{d}{d\mu} \tilde{S} = \gamma_S(\mu)\, \tilde{S} \qquad \gamma_S(\mu) = -\kappa_S \Gamma_{\text{cusp}}[\alpha_s(\mu)] \ln \frac{\mu e^{\gamma_E \nu}}{Q} + \gamma_S[\alpha_s(\mu)]$$

constants

universal cusp anom dim

"non-cusp" pieces

$\kappa_H = 4$

$\kappa_J = -\frac{2}{1-a}$

$\kappa_S = \frac{4}{1-a}$

subject to constraint $\boxed{\gamma_H + 2\gamma_J + \gamma_S = 0}$

Then

$$\tilde\sigma(\nu_a) = H(Q^2,\mu_H)\left(\frac{\mu_H}{Q}\right)^{-K_H\eta_\Gamma(\mu_H,\mu)} e^{-K_H K_H(\mu_H,\mu)+\eta_{\gamma_H}(\mu_H,\mu)}$$

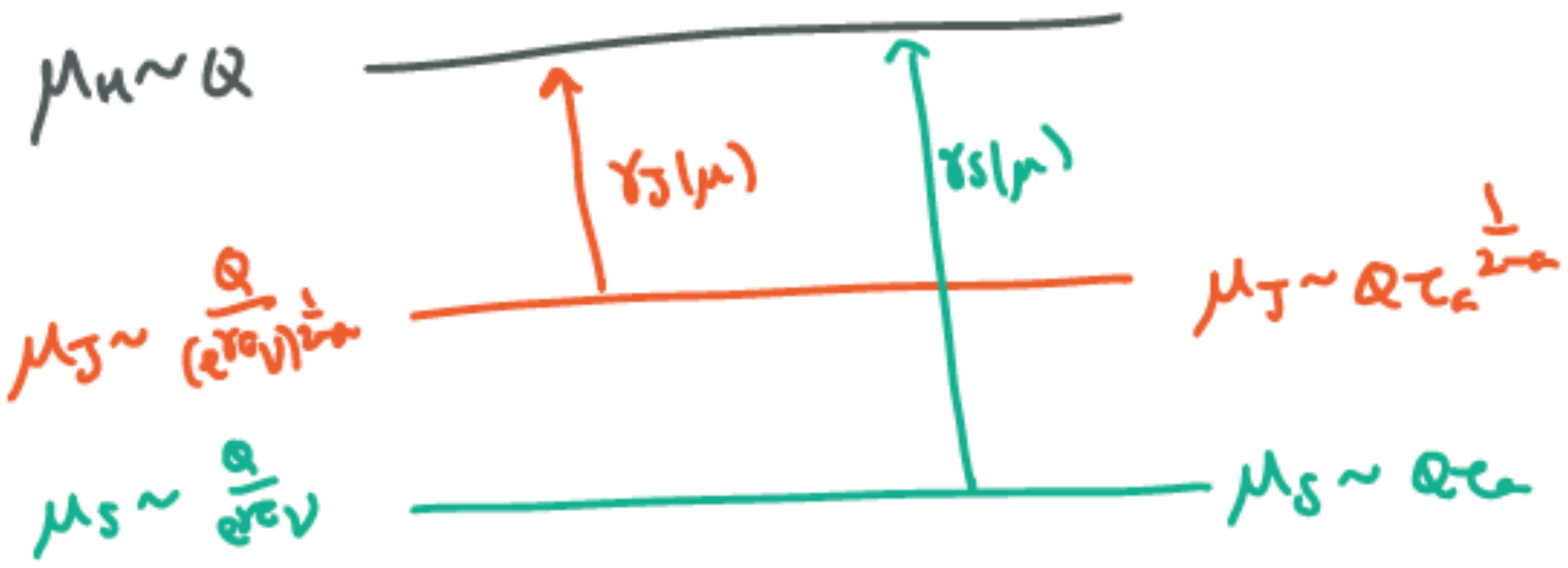
$$\cdot\left[\tilde J\left(\frac{\nu_a}{Q^{2-a}},\mu_J\right)\left(\frac{\mu_J^{2-a}}{Q^{2-a}}e^{\gamma_E}\nu\right)^{-K_J\eta_\Gamma(\mu_J,\mu)} e^{-K_J K_J(\mu_J,\mu)+\eta_{\gamma_J}(\mu_J,\mu)}\right]^2$$

$$\cdot\ \tilde S\left(\frac{\nu_a}{Q},\mu_S\right)\left(\frac{\mu_S}{Q}e^{\gamma_E}\nu\right)^{-K_S\eta_\Gamma(\mu_S,\mu)} e^{-K_S K_S(\mu_S,\mu)+\eta_{\gamma_S}(\mu_S,\mu)}$$

choose $\mu_{H,J,S}$ where logs are small

evolve to μ

Sums large logs of $\frac{\mu_F}{Q_F}$



Then inverse transform $\tilde\sigma(\nu_a) \longrightarrow \frac{d\sigma}{d\tau_a}$.

COUNTING ACCURACY OF LOG RESUMMATION

Straightforward to define in Laplace space:

$\tilde{\sigma}(\nu)$ at the "canonical" scales $\mu_H = Q \quad \mu_J = \frac{Q}{(e^{\gamma_E}\nu)^{\frac{1}{2-a}}} \quad \mu_S = \frac{Q}{e^{\gamma_E}\nu}$

$$= \underbrace{H(\mu_H^c)\,\tilde{J}^2(\mu_J^c)\,\tilde{S}(\mu_S^c)}_{\text{just constants}}\; e^{\overbrace{K(\mu_H^c,\mu_J^c,\mu_S^c)}^{\text{count logs in this exponent}}}$$

just constants

Count logs in this exponent

$$K(\mu_H^c,\mu_J^c,\mu_S^c) \equiv -\kappa_H K_\Gamma(\mu_H^c,\mu) - 2\kappa_J K_\Gamma(\mu_J^c,\mu) - \kappa_S K_\Gamma(\mu_S^c,\mu) + \eta_{\gamma_H}(\mu_H^c,\mu) + 2\eta_{\gamma_J}(\mu_J^c,\mu) + \eta_{\gamma_S}(\mu_S^c,\mu)$$

$$K_\Gamma(\mu_0,\mu) = \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'}\,\Gamma_{\text{cusp}}[\alpha_s(\mu')]\ln\frac{\mu'}{\mu_0}$$

$$\Gamma_{\text{cusp}}[\alpha_s(\mu')] = \Gamma_0\frac{\alpha_s(\mu')}{4\pi} + \Gamma_1\left(\frac{\alpha_s(\mu')}{4\pi}\right)^2 + \dots$$

$$\sim \Gamma_0\frac{\alpha_s}{4\pi}\ln^2\frac{\mu}{\mu_0}$$
$$+\,\Gamma_0\beta_0\left(\frac{\alpha_s}{4\pi}\right)^2\ln^3\frac{\mu}{\mu_0} + \Gamma_1\left(\frac{\alpha_s}{4\pi}\right)^2\ln^2\frac{\mu}{\mu_0}$$
$$+\,\dots \qquad + \Gamma_2\left(\frac{\alpha_s}{4\pi}\right)^3\ln^2\frac{\mu}{\mu_0}$$

LL   NLL   NNLL

$$\eta_\gamma(\mu_0,\mu) = \int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\,\gamma[\alpha_s(\mu')]$$

$$\sim \gamma_0\frac{\alpha_s}{4\pi}\ln\frac{\mu}{\mu_0}$$
$$+\,\gamma_0\beta_0\left(\frac{\alpha_s}{4\pi}\right)^2\ln^2\frac{\mu}{\mu_0} + \gamma_1\left(\frac{\alpha_s}{4\pi}\right)^2\ln\frac{\mu}{\mu_0}$$
$$+\,\dots \qquad + \dots \qquad + \gamma_2\left(\frac{\alpha_s}{4\pi}\right)^3\ln\frac{\mu}{\mu_0}$$

NLL   NNLL N^3LL

...

for Laplace-space $\tilde{\sigma}(\nu_a)$:

	Γ_{cusp}, β_n	γ_F	c_F	c_F	
LL	α_s	1	1	—	—
NLL	α_s^2	α_s	1	α_s	NLL′
NNLL	α_s^3	α_s^2	α_s	α_s^2	NNLL′

→ better for matching onto $\mathcal{O}(\alpha_s), \mathcal{O}(\alpha_s^2)$ full QCD nonsingular terms

See 1401.4460 for corresponding counting for $\sigma_c(\tau_a)$, $\frac{d\sigma}{d\tau_a}$

gist: $\tilde{\sigma}(\nu_a) \leftrightarrow \sigma_c(\tau_a) \longleftrightarrow \frac{d\sigma}{d\tau_a}$

same accuracy

same accuracy only at $N^k LL'$ orders:
need add'l terms at $N^k LL$ orders

Ingredients for NNLL' Angularities

KNOWNS & UNKNOWNS (until now)

$H(Q^2,\mu)$: $\mathcal{O}(\alpha_s^3)$

$\Gamma_{\rm cusp}$: $\mathcal{O}(\alpha_s^3)$

$J(t_J,\mu)$: $\begin{cases} \mathcal{O}(\alpha_s) & \text{any } a \\ \mathcal{O}(\alpha_s^3) & a=0 \end{cases}$

0901.3780

Bauer & Manohar 2003 $\mathcal{O}(\alpha_s)$

Becher & Neubert 2006 $\mathcal{O}(\alpha_s^2)$

Brüser, Liu, Stahlhofen 2018 $\mathcal{O}(\alpha_s^3)$

$S(k_s,\mu)$: $\begin{cases} \mathcal{O}(\alpha_s) & \text{any } a \\ \mathcal{O}(\alpha_s^2) & a=0 \end{cases}$

0901.3780

Fleming et al 2007 $\mathcal{O}(\alpha_s)$

Kelley et al 2011 $\mathcal{O}(\alpha_s^2)$

Monni et al

$\Rightarrow$ could only go to NLL$^{(\prime)}$ for general τ_a (NNLL$'$ or "N^{3}LL$'$" for τ_0 and C)

Now, we can obtain S to $\mathcal{O}(\alpha_s^2)$ for any a thus γ_J^1 (α_s^2) by consistency

and c_J^2 and $r_2(\tau_a)$ by EVENT2.

New 2-Loop Ingredients:

- $S_a(k_s,\mu) = \frac{1}{N_c}\mathrm{Tr}\langle 0| Y_{\bar n} Y_n^\dagger\, \delta(k_s - Q\hat{e}_a)\, Y_n Y_{\bar n}^\dagger |0\rangle$

  $\downarrow$ measurement function $\hat{u}$

- Bell, Rahn & Talbert's procedure in 1805.12414 and implementation in SoftSERVE

  (Soft function Simulation and Evaluation of Real and Virtual Emissions)

- general representation of measurement functions $\hat{u}$ using dimensional analysis and IRC safety
- obtain wide range of soft functions for various measurement functions numerically to $O(\alpha_s^2)$

$\Rightarrow$ gives us $\gamma_S^1(a)\,[\alpha_s(\mu)]$ $\Rightarrow$ $\gamma_H^1 + 2\gamma_J^1 + \gamma_S^1 = 0$ then gives us $\gamma_J^1(a)$

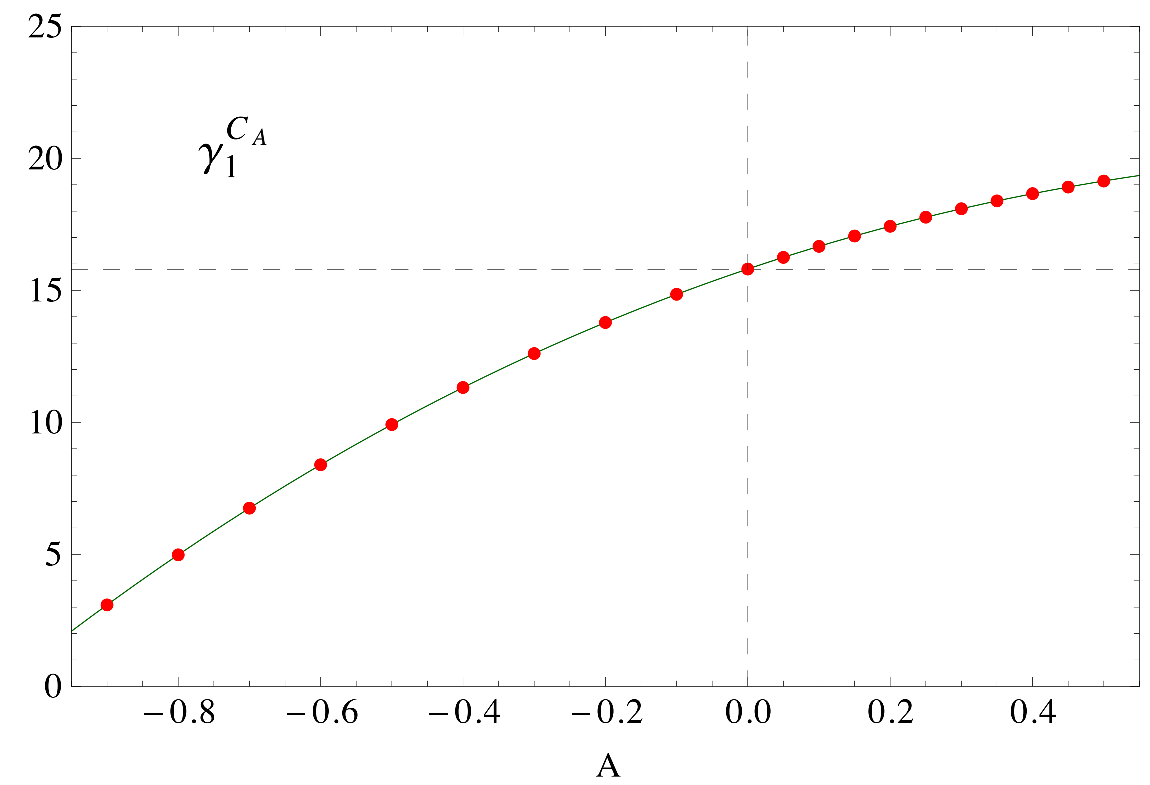
$\Rightarrow$ and singular constant $c_S^2(a)$ $\Rightarrow$ allows us to use EVENT2 to get $c_J^2(a)$.

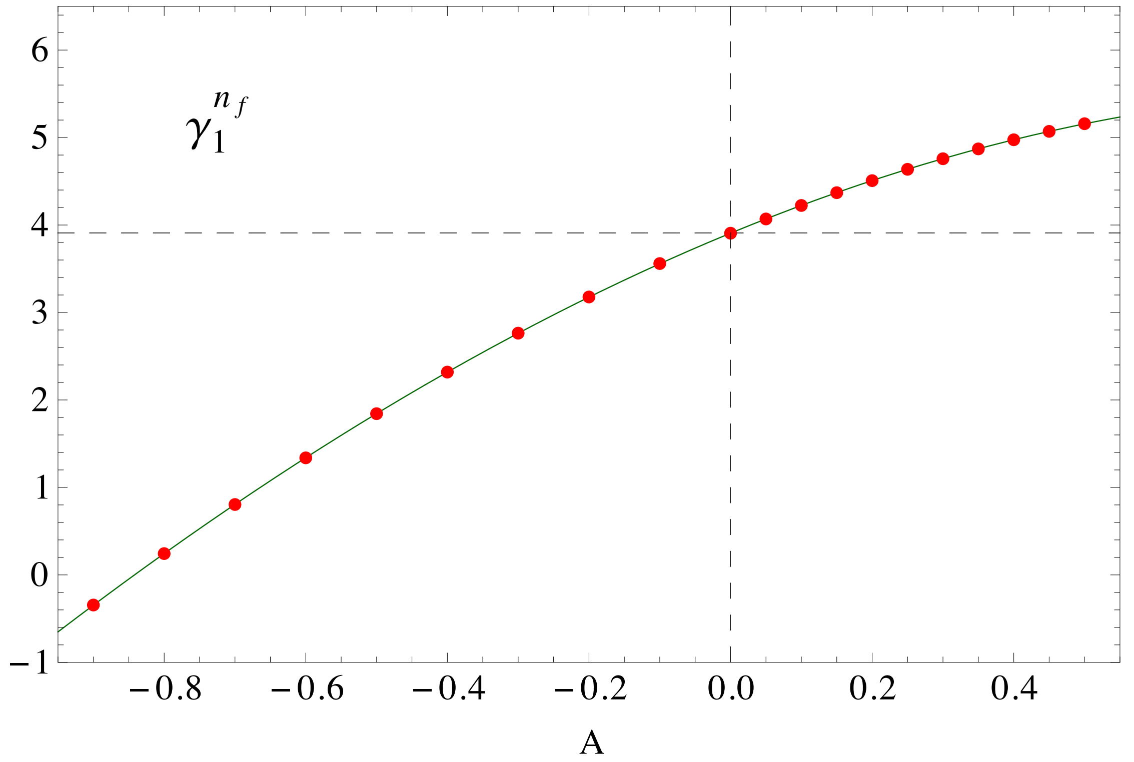
From Bell, Rahn & Talbert:

$$\gamma_S^1(a) = \frac{2}{F_a}\left[\gamma_1^{C_A}(a)\, C_F C_A + \gamma_1^{n_f}(a)\, C_F T_F n_f\right]$$

$$\gamma_1^{C_A} = -\frac{808}{27} + \frac{11\pi^2}{9} + 28\zeta_3 - \Delta\gamma_1^{C_A}(a)$$

$$\gamma_1^{n_f} = \frac{224}{27} - \frac{4\pi^2}{9} - \Delta\gamma_1^{n_f}(a)$$



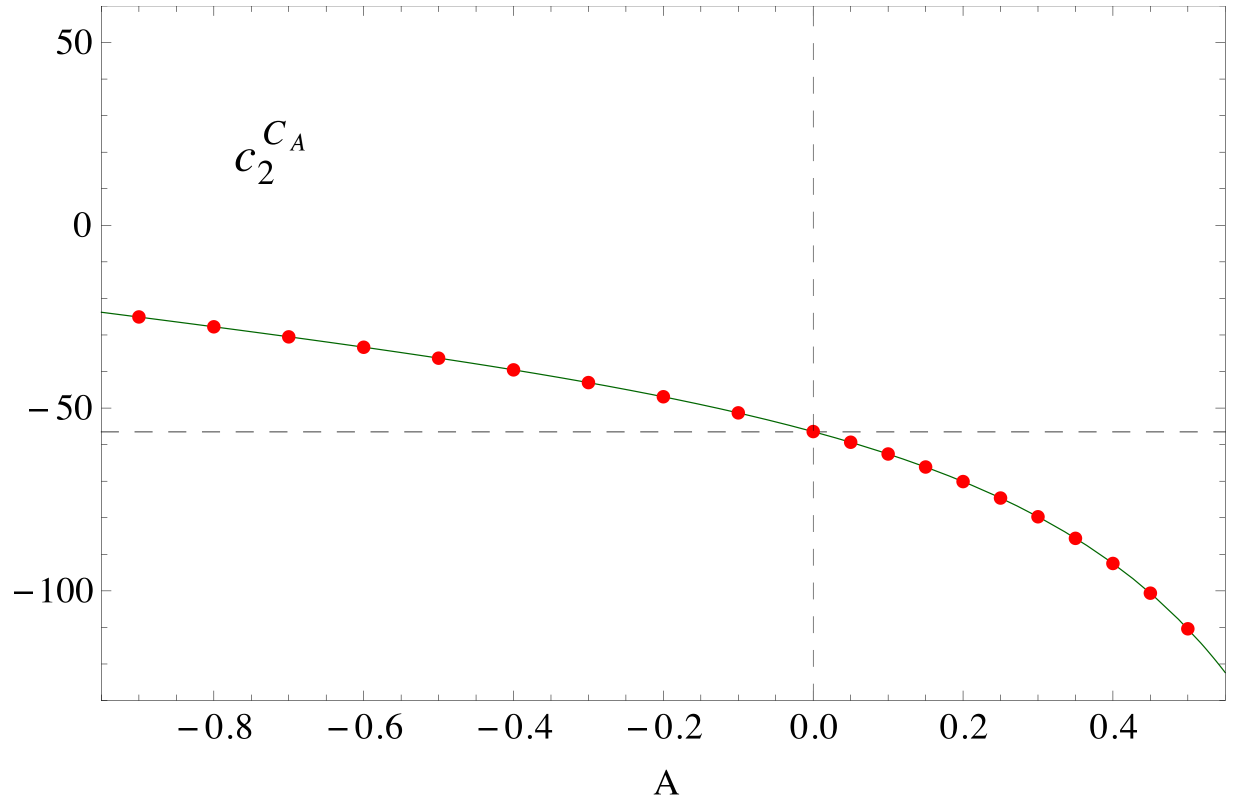


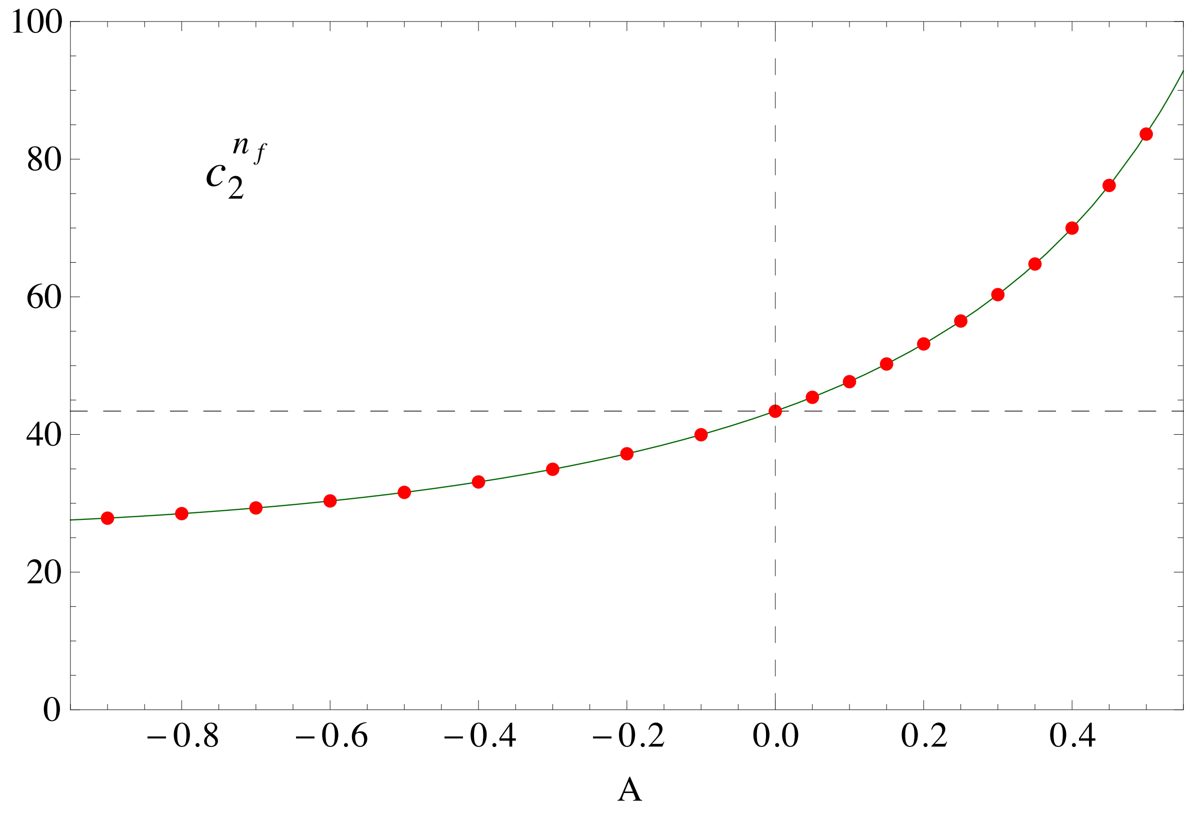
$$\Delta\gamma_1^{C_A} = \int_0^1 dx \int_0^1 dy\, \frac{32x^2(1+xy+y^2)\left[x(1+y^2)+(x+y)(1+xy)\right]}{y(1-x^2)(x+y)^2(1+xy)^2} \ln\left[\frac{(x^a+xy)(x+x^a y)}{x^a(1+xy)(x+y)}\right]$$

$$\Delta\gamma_1^{n_f} = \int_0^1 dx \int_0^1 dy\, \frac{64x^2(1+y^2)}{(1-x^2)(x+y)^2(1+xy)^2} \ln\left[\frac{(x^a+xy)(x+x^a y)}{x^a(1+xy)(x+y)}\right]$$

and $c_S^2(a) = c_2^{C_A}(a)\, C_F C_A + c_2^{n_f}(a)\, C_F T_F n_f + \frac{\pi^4}{2(1+a)^2} C_F^2$

comes from SoftSERVE





C_J^2 and r_2 from EVENT2

(follows Hoang-Kluth 2008)

EVENT2 (Catani & Seymour) gives us $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2)$ distributions for measurements in $e^+e^- \to \{q,\bar{q},g\}$:

$$\sigma_{QCD}(\tau_a) = \frac{1}{\sigma_0}\int_0^{\tau_a} d\tau_a' \frac{d\sigma_{QCD}}{d\tau_a'} = 1 + \frac{\alpha_s(Q)}{2\pi}\left[C_{12}\ln^2\tau_a + C_{11}\ln\tau_a + C_{10} + r_c^1(\tau_a)\right]$$

$$+\left(\frac{\alpha_s(Q)}{2\pi}\right)^2\Big[C_{24}\ln^4\tau_a + C_{23}\ln^3\tau_a + C_{22}\ln^2\tau_a + C_{21}\ln\tau_a \underbrace{+C_{20}}_{\text{singular constant}} + \underbrace{r_c^2(\tau_a)}_{\text{nonsingular remainder}}\Big]$$

Now, $$\frac{1}{\sigma_0}\frac{d\sigma_{QCD}}{d\tau_a} = \underbrace{A\,\delta(\tau_a) + [B(\tau_a)]_+}_{\text{sing.}} + \underbrace{r(\tau_a)}_{\text{non-sing.}}$$

SCET should predict: $\frac{1}{\sigma_0}\frac{d\sigma_{sing}}{d\tau_a} = A\,\delta(\tau_a) + [B(\tau_a)]_+$

EVENT2 should give: $\frac{1}{\sigma_0}\frac{d\sigma_{EV2}}{d\tau_a} = [B(\tau_a)]_+ + r(\tau_a)$ $\quad(\tau_a > 0)$

$[B(\tau_a)]_+$ → plus distribution w/ $\int_0^1 d\tau_a\,[B(\tau_a)]_+ = 0$

For $\tau_a > 0$ we get: $\frac{1}{\sigma_0}\frac{d\sigma_{\rm ev2}}{d\tau_a} - \frac{1}{\sigma_0}\frac{d\sigma_{\rm sing}}{d\tau_a} = r(\tau_a)$

so if we compute: $\lim_{\tau_a \to 0} \int_{\tau_a}^{1} d\tau_a' \, r(\tau_a') = r_c(1)$

we get total integral of $r(\tau_a)$.

But how do we get A? (and thus c_{20}?)

We know: $\sigma_{\rm had}^{\rm tot} = A + r_c(1)$. $\longrightarrow$ we know this!

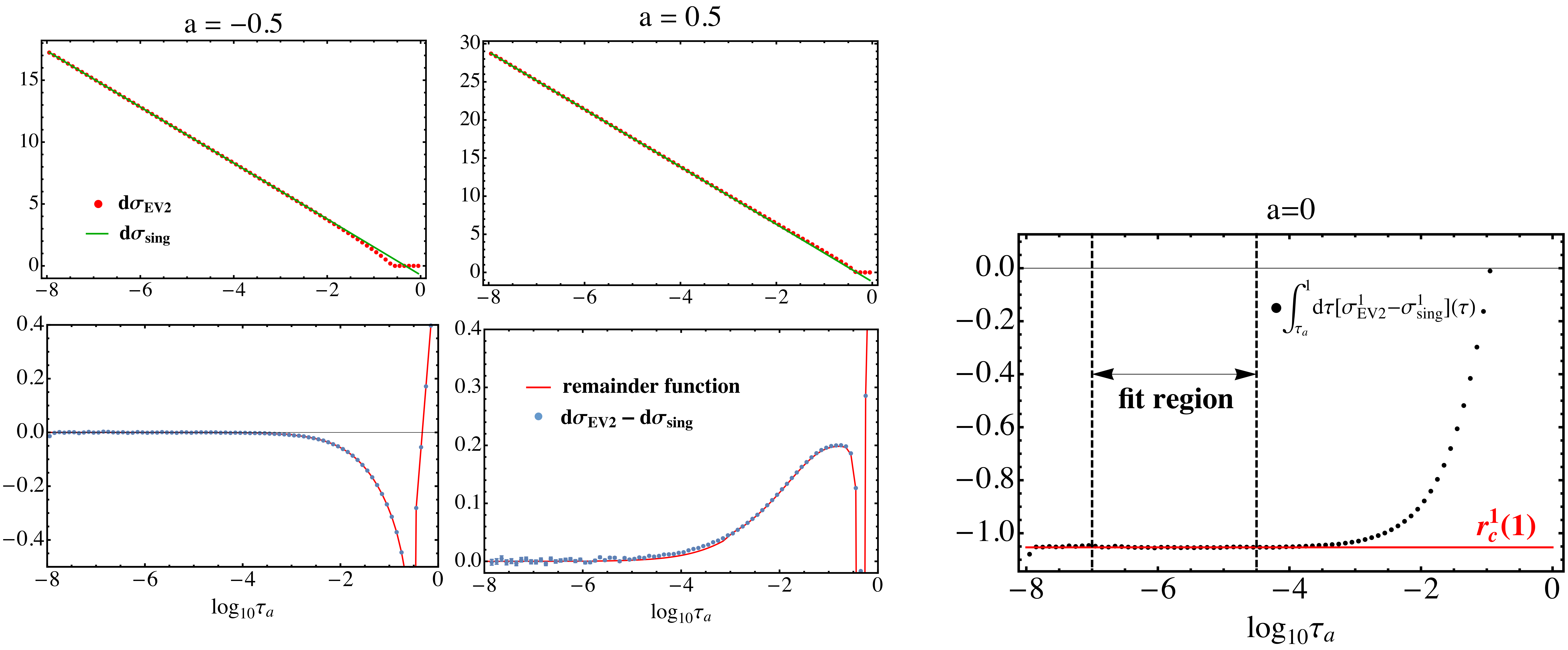
$$\sigma_{\rm had}^{\rm tot} = 1 + \frac{\alpha_s(Q)}{2\pi} \cdot \frac{3}{2} C_F + \left(\frac{\alpha_s(Q)}{2\pi}\right)^2 \left[-\frac{3}{8} C_F^2 + \left(\frac{123}{8} - 11\zeta_3\right) C_F C_A + \left(-\frac{11}{2} + 4\zeta_3\right) C_F T_F n_f \right]$$

so $A = \sigma_{\rm had} - r_c(1)$ in particular $c_{20} = \sigma_{\rm had}^{(2)} - r_c^{(2)}(1)$

$\downarrow$

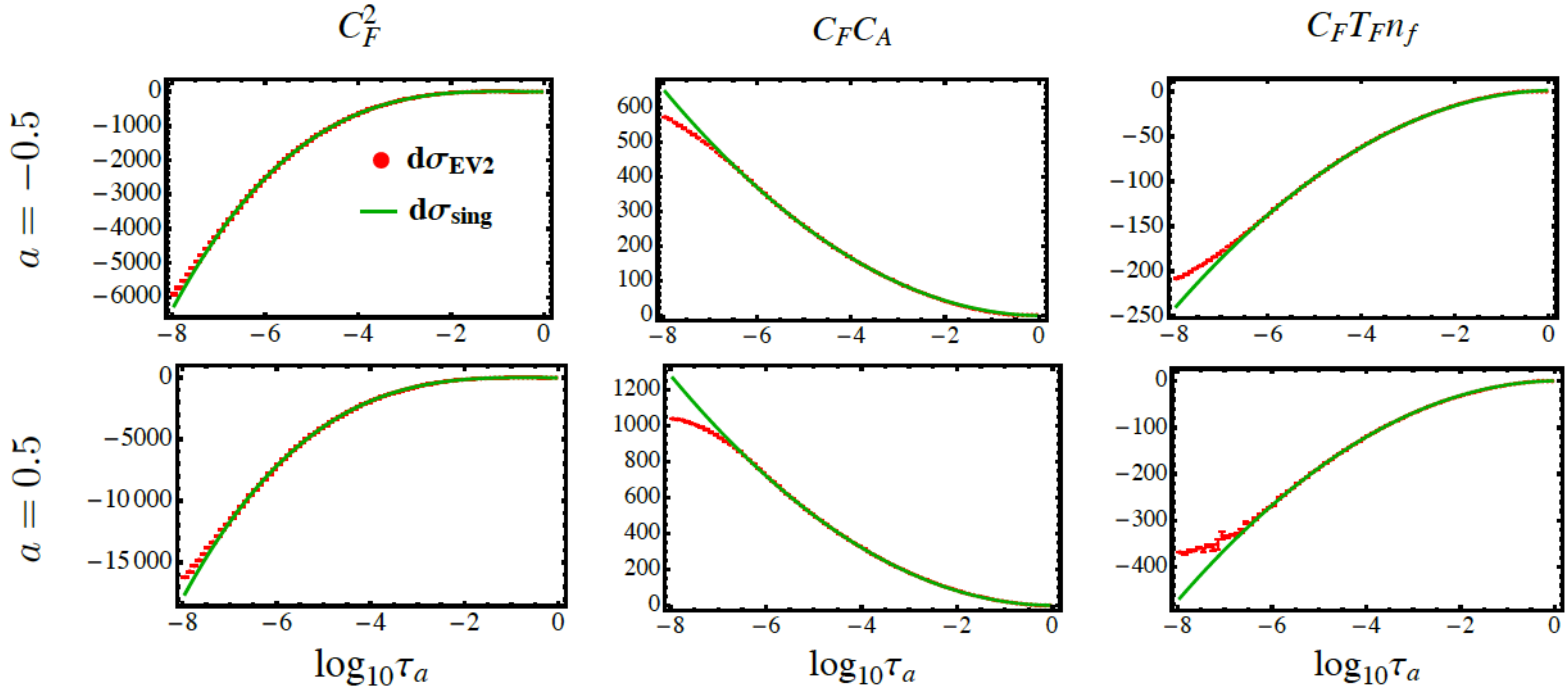
since we know c_H^2, $c_{\tilde{S}}^2$ then we can solve for $c_{\tilde{J}}^2$.

EVENT2 @ LO

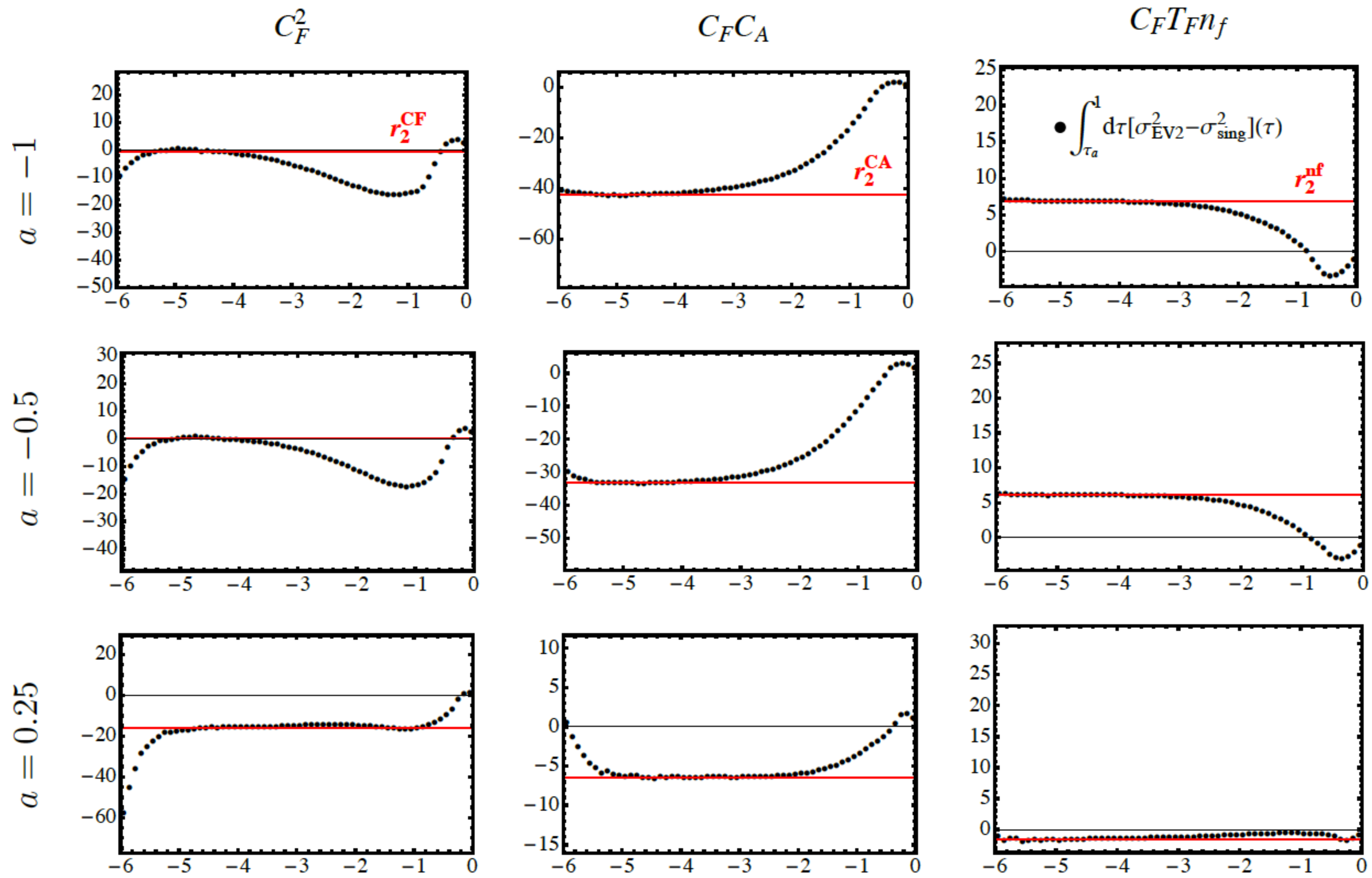


a = −0.5
a = 0.5
$d\sigma_{\text{EV2}}$
$d\sigma_{\text{sing}}$
remainder function
$d\sigma_{\text{EV2}} - d\sigma_{\text{sing}}$
$\log_{10}\tau_a$
$\log_{10}\tau_a$
a=0
$\int_{\tau_a}^{1} d\tau[\sigma^1_{\text{EV2}} - \sigma^1_{\text{sing}}](\tau)$
fit region
$r_c^1(1)$
$\log_{10}\tau_a$

EVENT2 @ NLO distributions



EVENT2 @ NLO: singular constant extraction



EVENT2 @ NLO: singular constant extraction

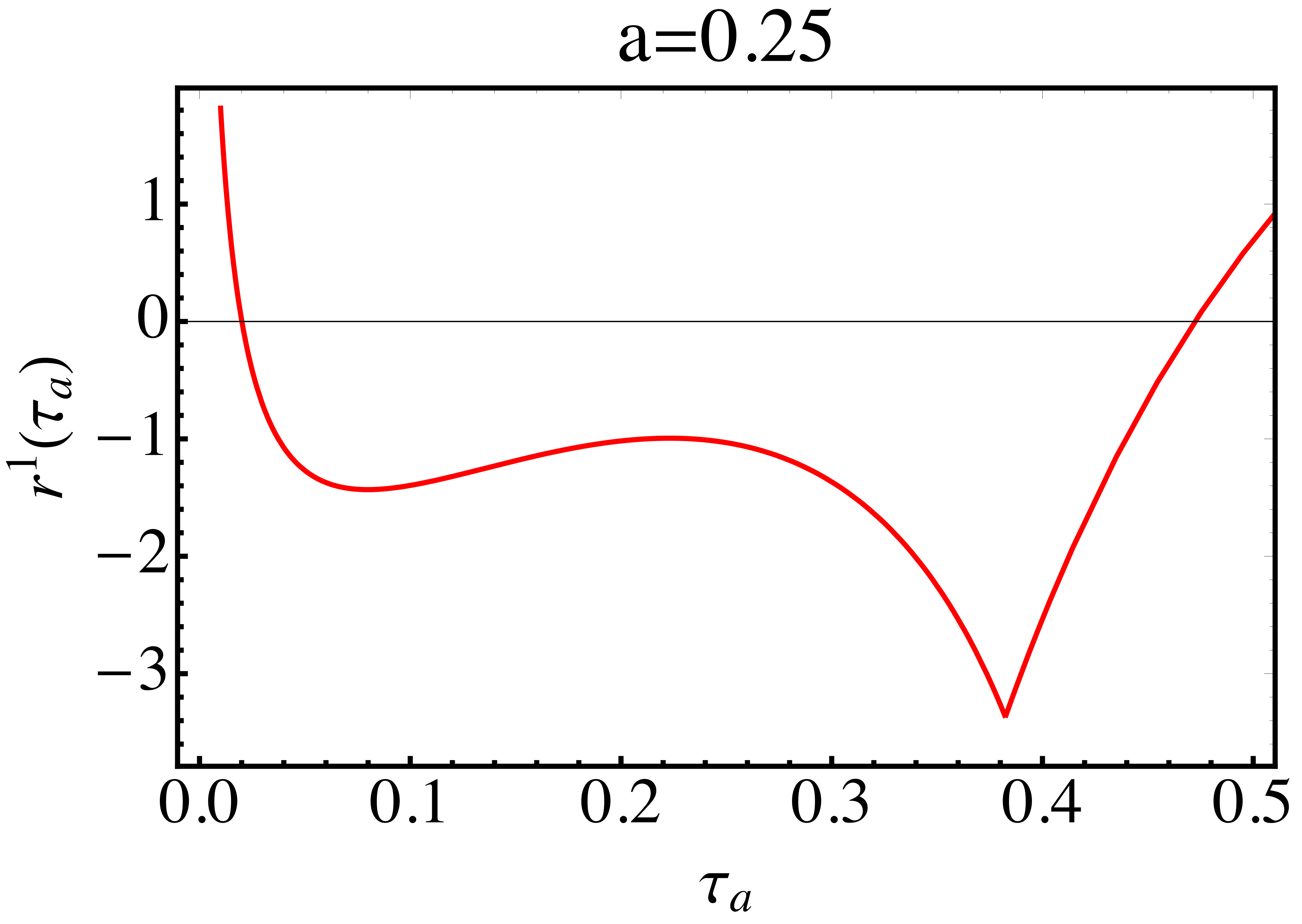
a	-1.0	-0.75	-0.5	-0.25	0.0	0.25	0.5
r_2^{CF}	$-0.16^{+0.37}_{-0.28}$	$0.19^{+0.34}_{-0.24}$	$0.24^{+0.35}_{-0.18}$	$-0.66^{+0.36}_{-0.29}$	$-4.03^{+0.38}_{-0.27}$	$-15.9^{+0.4}_{-0.7}$	$-49.9^{+3.2}_{-8.4}$
r_2^{CA}	$-42.3^{+0.2}_{-0.5}$	$-38.0^{+0.2}_{-0.5}$	$-33.2^{+0.1}_{-0.3}$	$-27.3^{+0.1}_{-0.2}$	$-19.3^{+0.1}_{-0.2}$	$-6.42^{+0.20}_{-0.11}$	$18.1^{+1.5}_{-0.5}$
r_2^{nf}	$6.76^{+0.08}_{-0.03}$	$6.57^{+0.08}_{-0.03}$	$6.03^{+0.07}_{-0.03}$	$4.92^{+0.06}_{-0.02}$	$2.78^{+0.03}_{-0.02}$	$-1.42^{+0.02}_{-0.06}$	$-9.92^{+0.23}_{-0.87}$

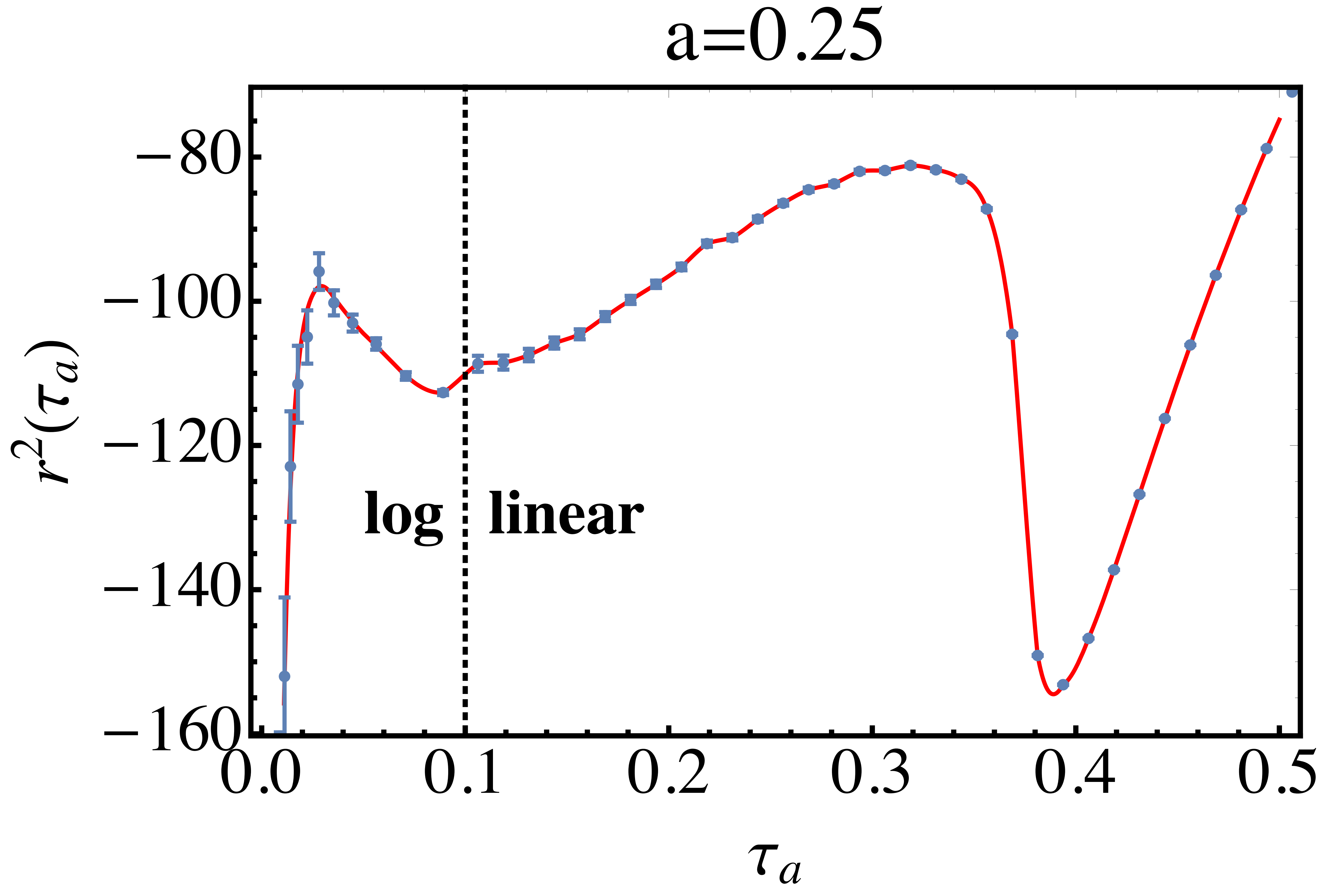
Table 3. Fit values for the coefficients of the integral $r_c^2(1)$ of the nonsingular QCD distribution as defined in Eq. (3.19). The central values and their uncertainties have been extracted from the plots in Fig. 6 as described in the text.

a	-1.0	-0.75	-0.5	-0.25	0.0	0.25	0.5
$c_{\tilde{J}}^2$	$66.0^{+5.2}_{-3.4}$	$42.3^{+5.1}_{-3.3}$	$17.3^{+3.2}_{-2.5}$	$-9.34^{+2.76}_{-2.48}$	$-36.3^{+2.7}_{-2.4}$	$-57.6^{+3.8}_{-3.2}$	$-79.8^{+39.7}_{-24.9}$

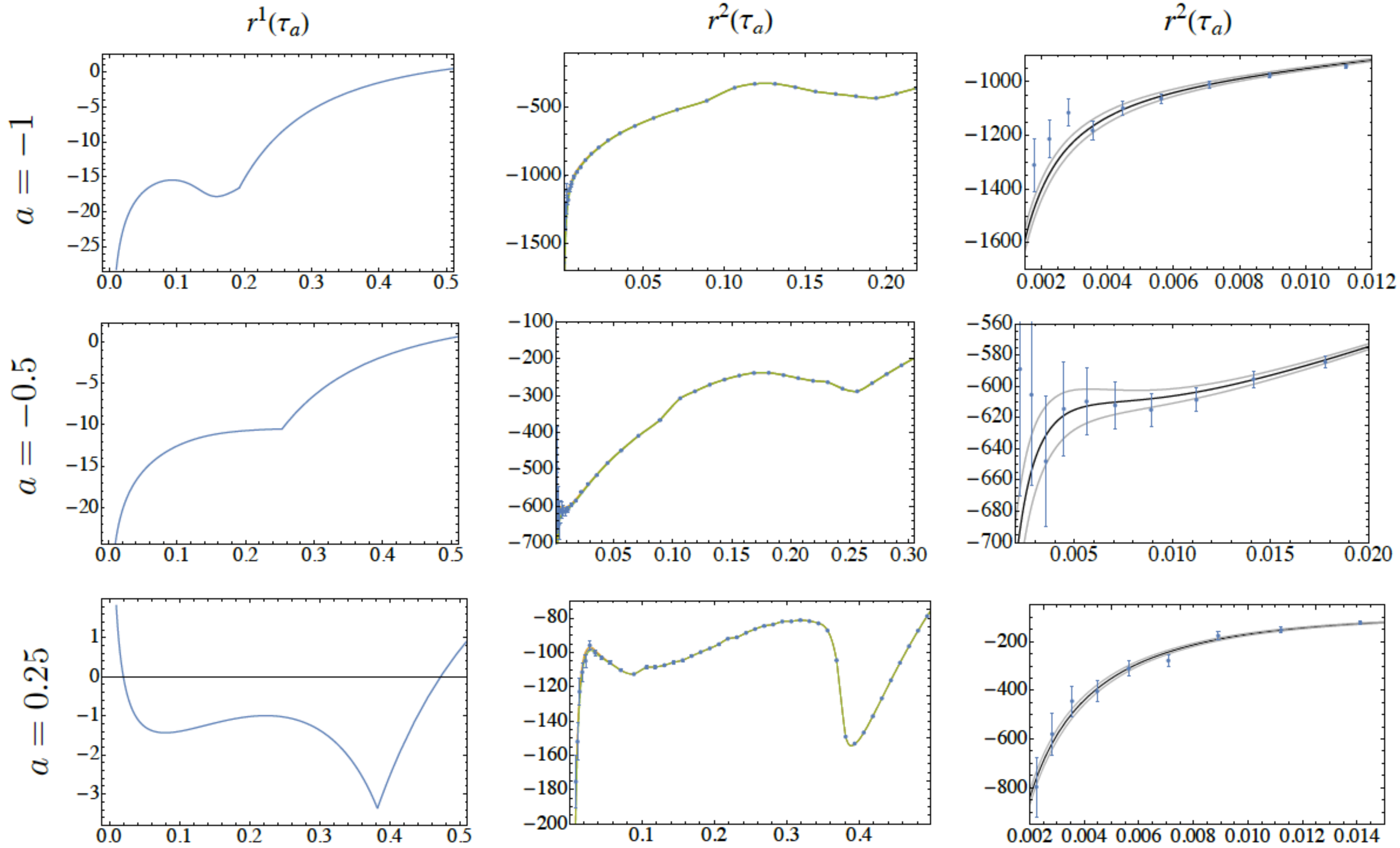
Table 4. Extracted values of the two-loop jet function constants $c_{\tilde{J}}^2$.

EVENT2: remainder functions





EVENT2: remainder functions



Nonperturbative Corrections

NP Shape Function S_{NP}

Key properties:

- Ω_1 has a field theory def:

$$\Omega_1 = \frac{1}{N_c} \mathrm{Tr} \langle 0 | Y_{\bar n} Y_n^\dagger \hat{\mathcal{E}}_T Y_n Y_{\bar n}^\dagger | 0 \rangle$$

"transverse energy flow"

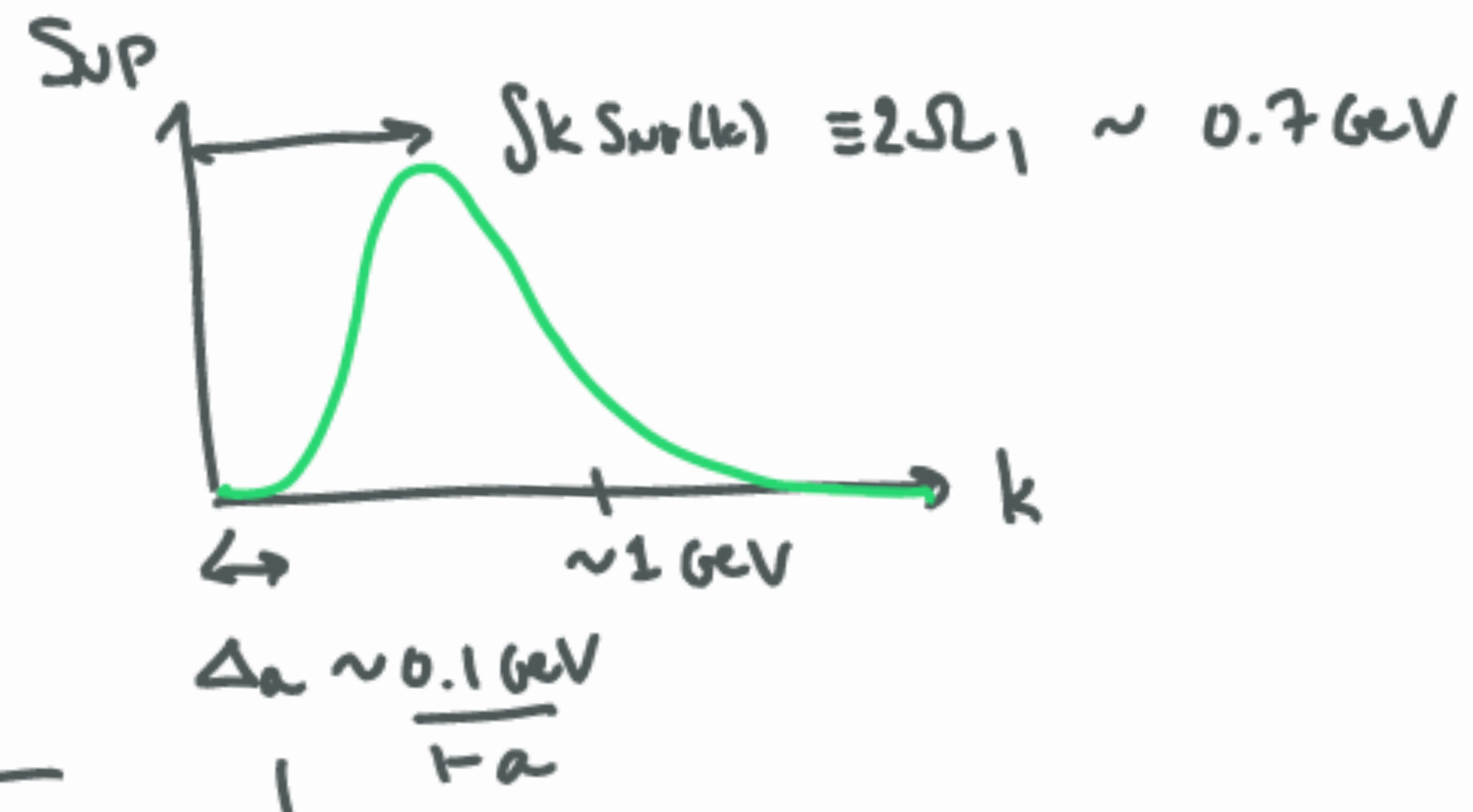
$\Downarrow$

$$\langle \tau_a \rangle = \langle \tau_a \rangle_{PT} + \frac{2\Omega_1}{Q(1-a)}$$

universal!

(appears in thrust, C parameter)

…l, Sterman (2006)



technical details:

- needs renormalon subtraction
- we adopt "R-gap" scheme

Hoang & Stewart
Hoang & Kluth
Hoang, Jain, Scimemi, Stewart
"R-evolution"

scaling $\frac{1}{Q} \cdot \frac{1}{1-a}$ is a prediction of QCD factorization

Breaking Degeneracies

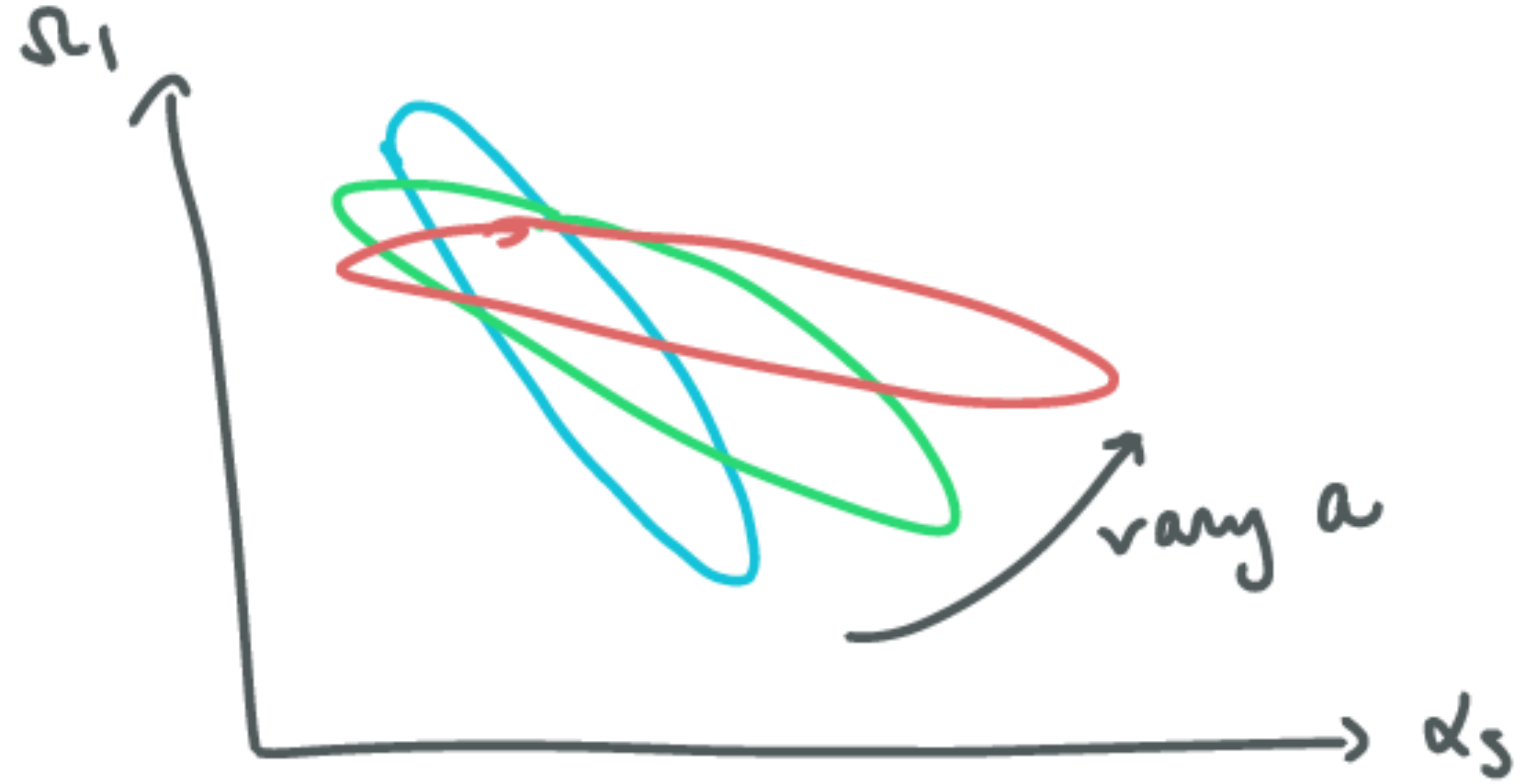
There is a degeneracy between α_s & Ω_1;

can be broken by varying Q or a:

$$\Delta \langle e \rangle_s \sim \frac{2\Omega_1}{Q(1-a)}$$

Varying $-2 < a < 0.5$ equivalent to varying Q by factor of 6.

ideally:



PROOF OF UNIVERSAL SHIFT

[CL, Sterman 2006]

$$\Delta\langle e\rangle_s = \frac{1}{Q}\int_{-\infty}^{\infty} d\eta\; f_e(\eta)\;\frac{1}{N_c}\mathrm{Tr}\langle 0|\bar T[Y_{\bar n}^\dagger Y_n^\dagger]\,\hat{\mathcal{E}}_T(\eta)\,T[Y_n Y_{\bar n}]\,|0\rangle$$

$\hat{\mathcal{E}}_T(\eta)|X\rangle$

$= \sum_{i\in X}|\vec p_i^{\,\perp}|\,\delta(\eta-\eta_i)\,|X\rangle$

Lorentz boosts: $\Lambda_{\eta'}^{-1}\Lambda_{\eta'}$ $\quad\Lambda_{\eta'}^{-1}\Lambda_{\eta'}$

$$Y_n = P\exp\left[ig\int_0^\infty ds\; n\cdot A_s(ns)\right] \longrightarrow Y_n$$

$$|0\rangle \longrightarrow |0\rangle$$

$$\hat{\mathcal{E}}_T(\eta) \longrightarrow \hat{\mathcal{E}}_T(\eta+\eta')$$

Pick η' to be anything!

$$\Rightarrow \Delta\langle e\rangle_s = \underbrace{\frac{1}{Q}\int_{-\infty}^{\infty} d\eta\, f_e(\eta)}_{=\,c_e}\;\underbrace{\frac{1}{N_c}\mathrm{Tr}\langle 0|\bar T[Y_{\bar n}^\dagger Y_n^\dagger]\,\hat{\mathcal{E}}_T(0)\,T[Y_n Y_{\bar n}]|0\rangle}_{\Omega_1}$$

(massless parton case)

generalizes single emission models e.g. Dokshitzer-Webber 95-96

Final Form of Theoretical Predictions:

$$\sigma(\tau_a) = \int dk \left[\sigma_{\rm sing}\left(\tau_a - \frac{k}{Q}\right) + r\left(\tau_a - \frac{k}{Q}; \mu_{ns}\right) \right] \left[e^{\underset{\substack{\downarrow \\ \text{renormalon subtractions} \\ \text{from } \sigma_{PT}}}{-2\delta a(\mu_s, R)}\frac{d}{dk}} S_{NP}\left(k - \underset{\substack{\downarrow \\ \text{renormalon-free gap}}}{\Delta a(\mu_s, R)}\right) \right]$$

$$\sigma^c_{\rm sing}(\tau_a) = H(Q^2, \mu_H)\, e^{\tilde K(\mu_H, \mu_J, \mu_S; Q) + K_\gamma(\mu_H, \mu_J, \mu_S)} \left(\frac{1}{\tau_a}\right)^{\Omega(\mu_J, \mu_S)}$$
$$\times \tilde J^2\left(\partial_\Omega + \ln \frac{\mu_J^{2-a}}{Q^{2-a}\tau_a}, \mu_J\right) \tilde S\left(\partial_\Omega + \ln\frac{\mu_S}{Q\tau_a}, \mu_S\right) \left[\frac{e^{\gamma_E \Omega}}{\Gamma(1-\Omega)}\right]$$

where* $\tilde K = -\kappa_H \tilde K_\Gamma(\mu, \mu_H; Q) - 2(2-a)\kappa_J \tilde K_\Gamma(\mu, \mu_J; Q) - \kappa_S \tilde K_\Gamma(\mu, \mu_S; Q)$

$$\tilde K_\Gamma(\mu, \mu_F; Q) \equiv \int_{\mu_F}^{\mu} \frac{d\mu'}{\mu'} \Gamma_{\rm cusp}[\alpha_s(\mu')] \ln\frac{\mu'}{Q} \qquad \Omega = -2\kappa_J \eta_\Gamma(\mu, \mu_J) - \kappa_S \eta_\Gamma(\mu, \mu_S)$$

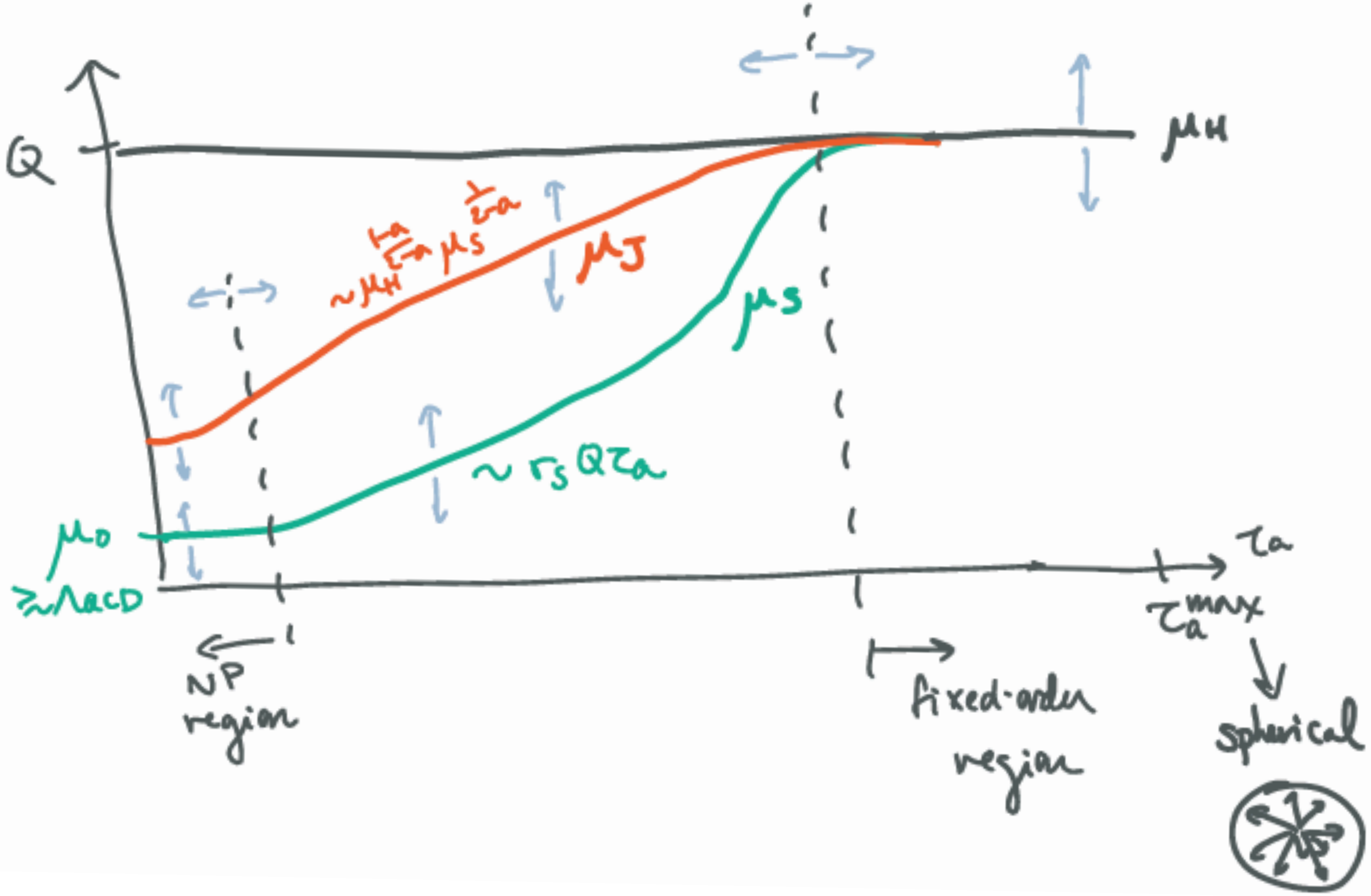
(* recombine K_Γ w/ $\left(\frac{\mu_F}{Q}\right)^{\omega_F}$ to obtain explicitly μ-invariant form at every $N^k LL$ order)

Cross Section Results

UNCERTAINTIES AND SCALE PROFILES

[1006.3080 + ⋯]

Freedom to choose μ_H, μ_J, μ_S (and μ_{ns}, R) allows not only log resummation but robust estimates of perturbative theory uncertainties in each region:

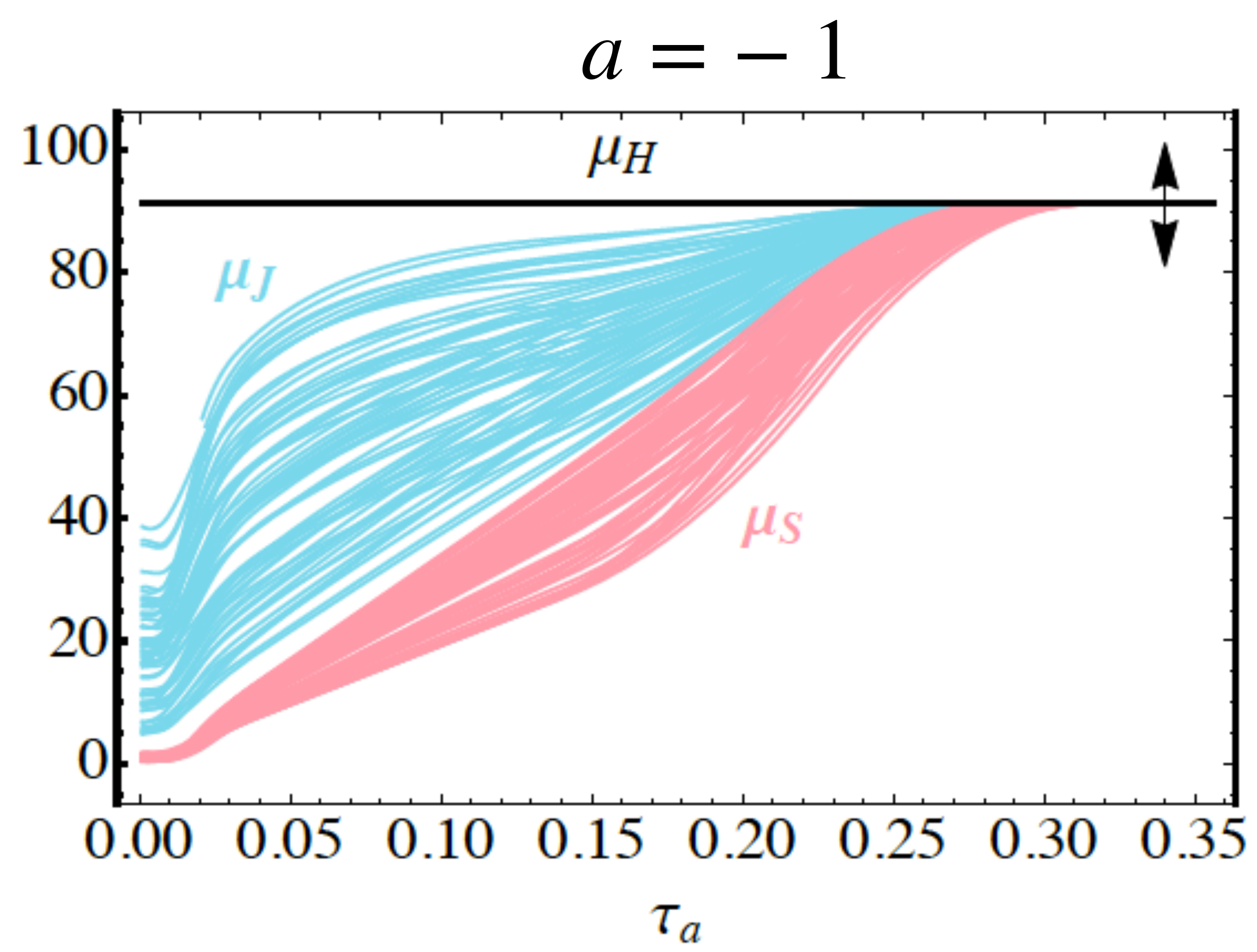


- allow scales to merge in fixed order region before $\tau_a = \tau_a^{max}$ (< 1)
- stable NP region to convolve shape function
- variation of all parameters to fully probe theory uncertainty

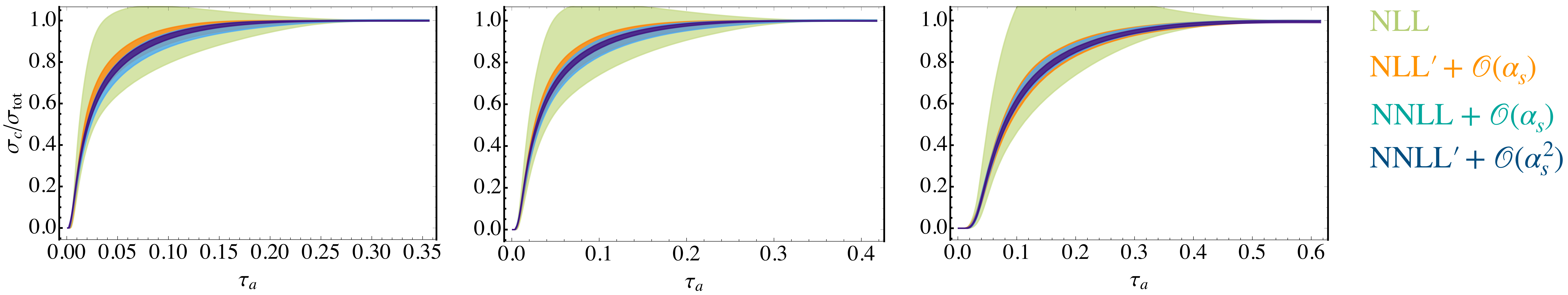
Cross section predictions

Bell, Hornig, CL, Talbert (2018)

Random scan over sets of scale profile functions:



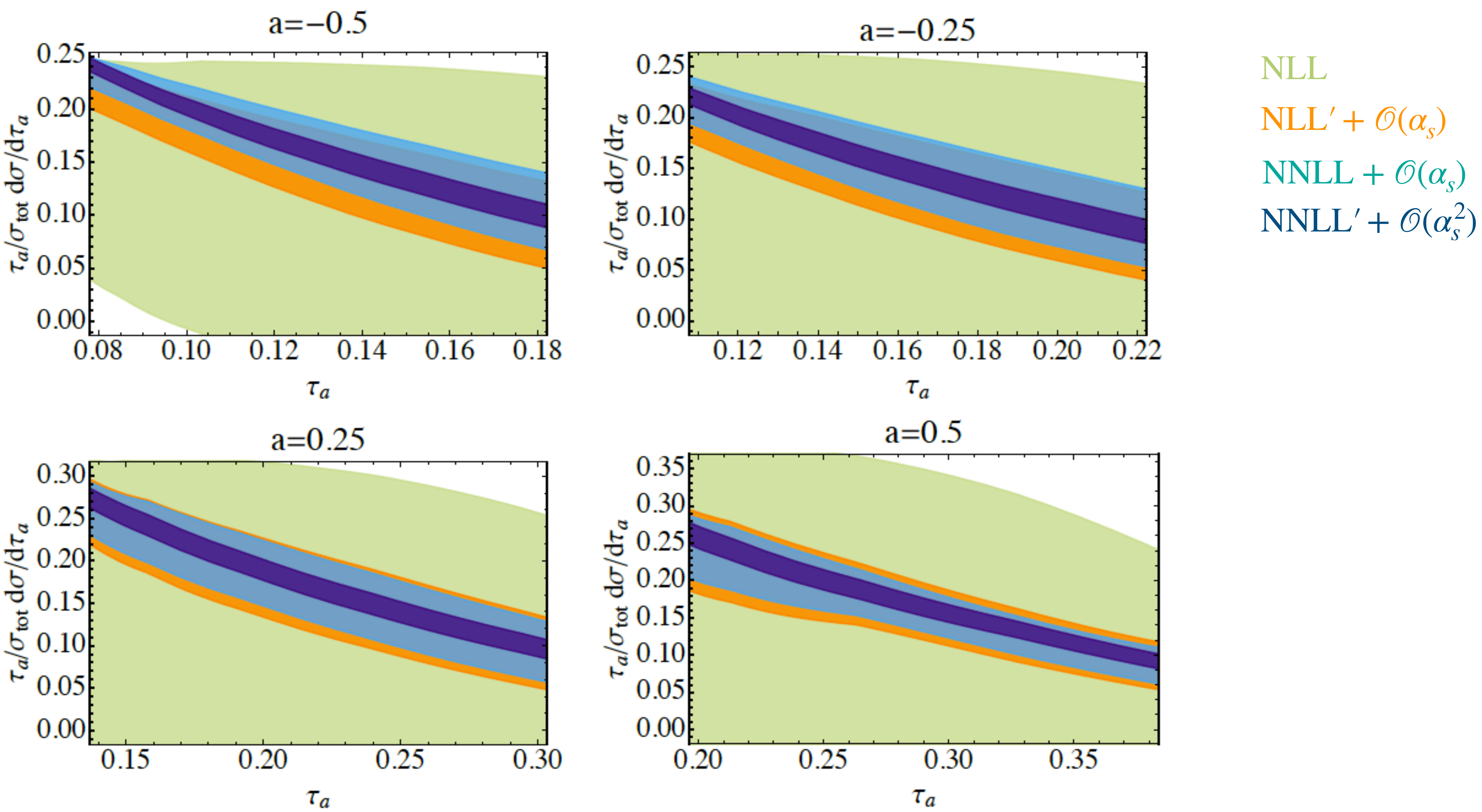
Resulting envelope of cross section predictions (integrated distributions):



Cross section predictions

Bell, Hornig, CL, Talbert (2018)

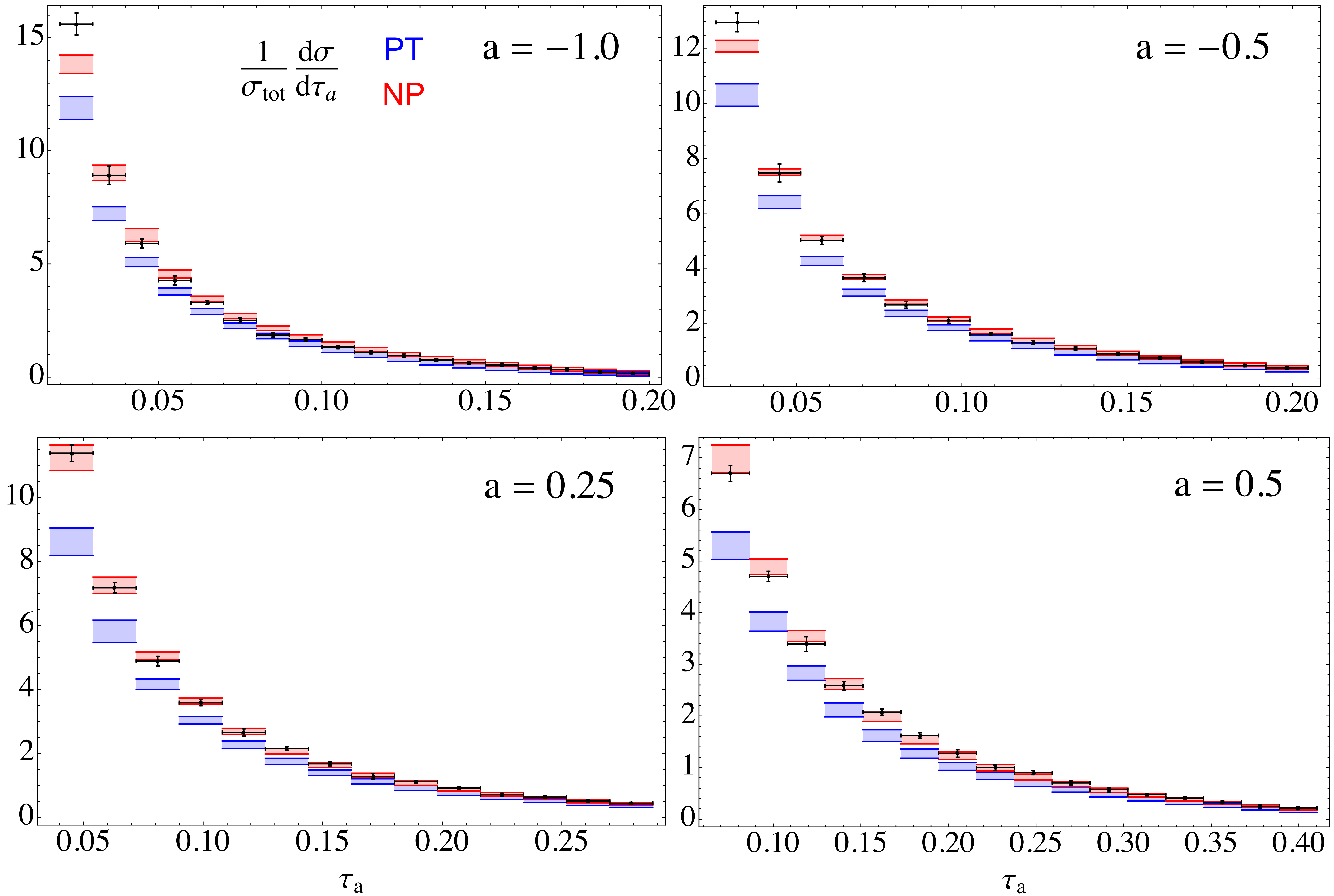
Differential distributions:



Comparison to data

L3 Collaboration (2011) $Q = M_Z$

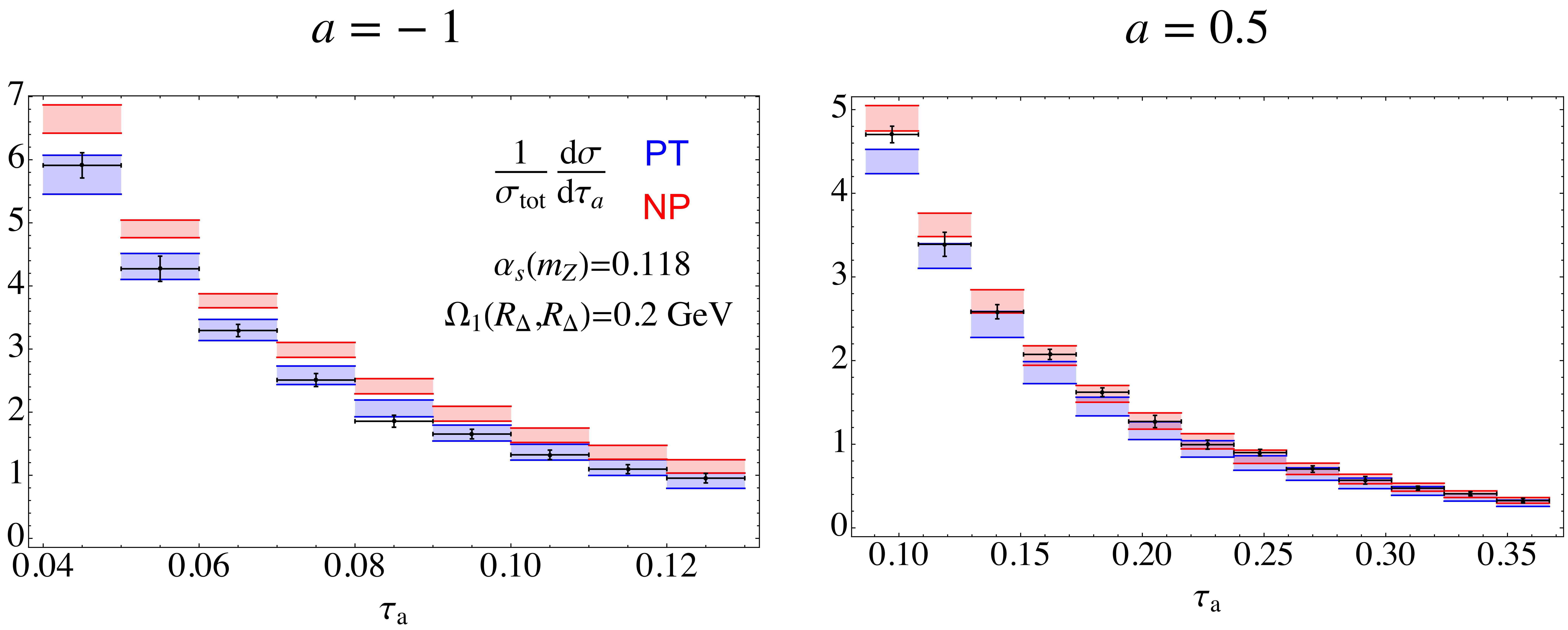
$\alpha_s(M_Z) = 0.11\,,\Omega_1(R_\Delta, R_\Delta) = 0.4$ GeV



With PDG world average

L3 Collaboration (2011) $Q = M_Z$

$\alpha_s(M_Z) = 0.118\,, \Omega_1(R_\Delta, R_\Delta) = 0.2$ GeV



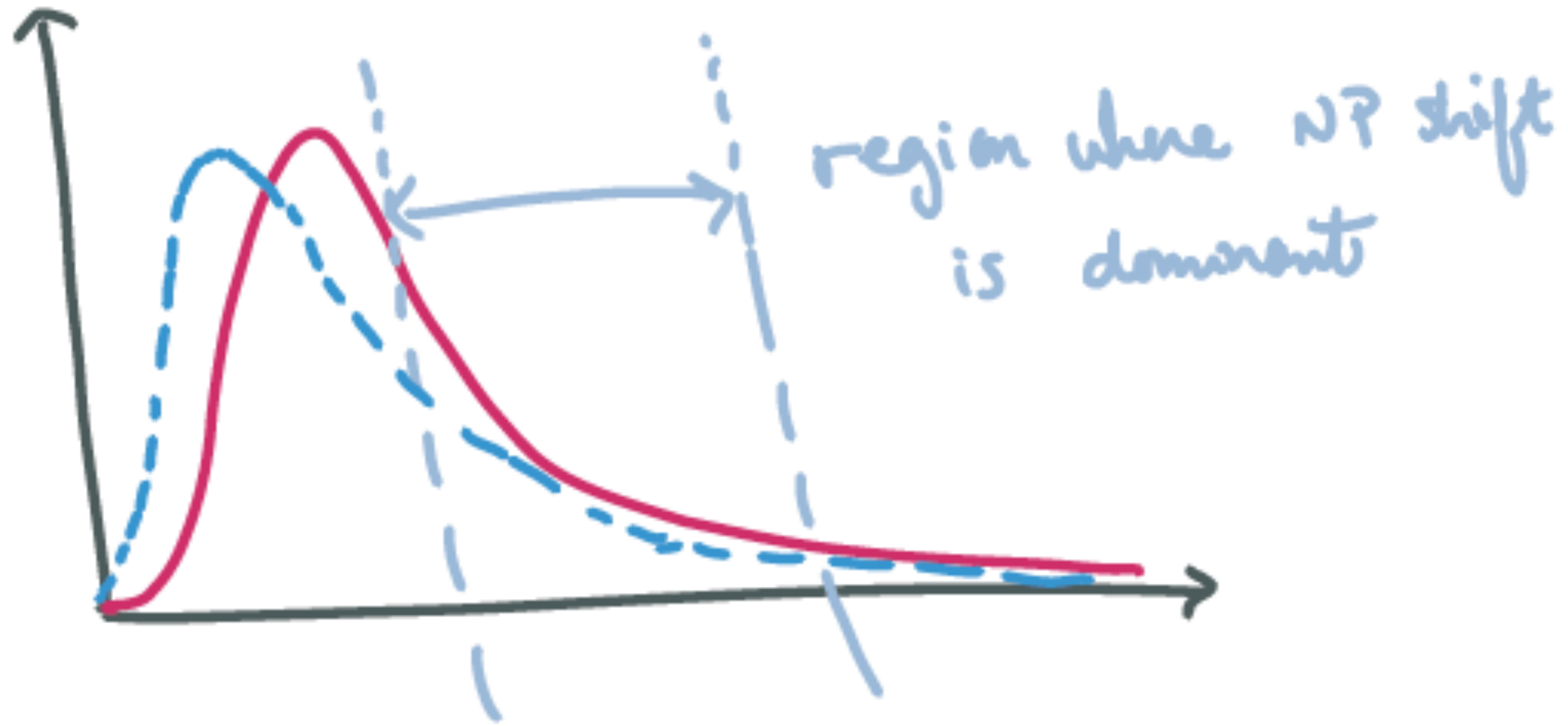
Fitting the strong coupling with angularities

CAVEATS: preliminary exercises (not "results"!)

- using "minimal overlap" assumption for exp. uncertainties
- assuming no correlation between α's (clearly untrue, but just as exercise)
- α_s, Ω_1 fits shown today only using simple shift in theory predictions not shape function:

  i.e. $\sigma_{PT}(\tau) \longrightarrow \sigma_{PT}(\tau - c_e \frac{\Omega_1}{Q})$

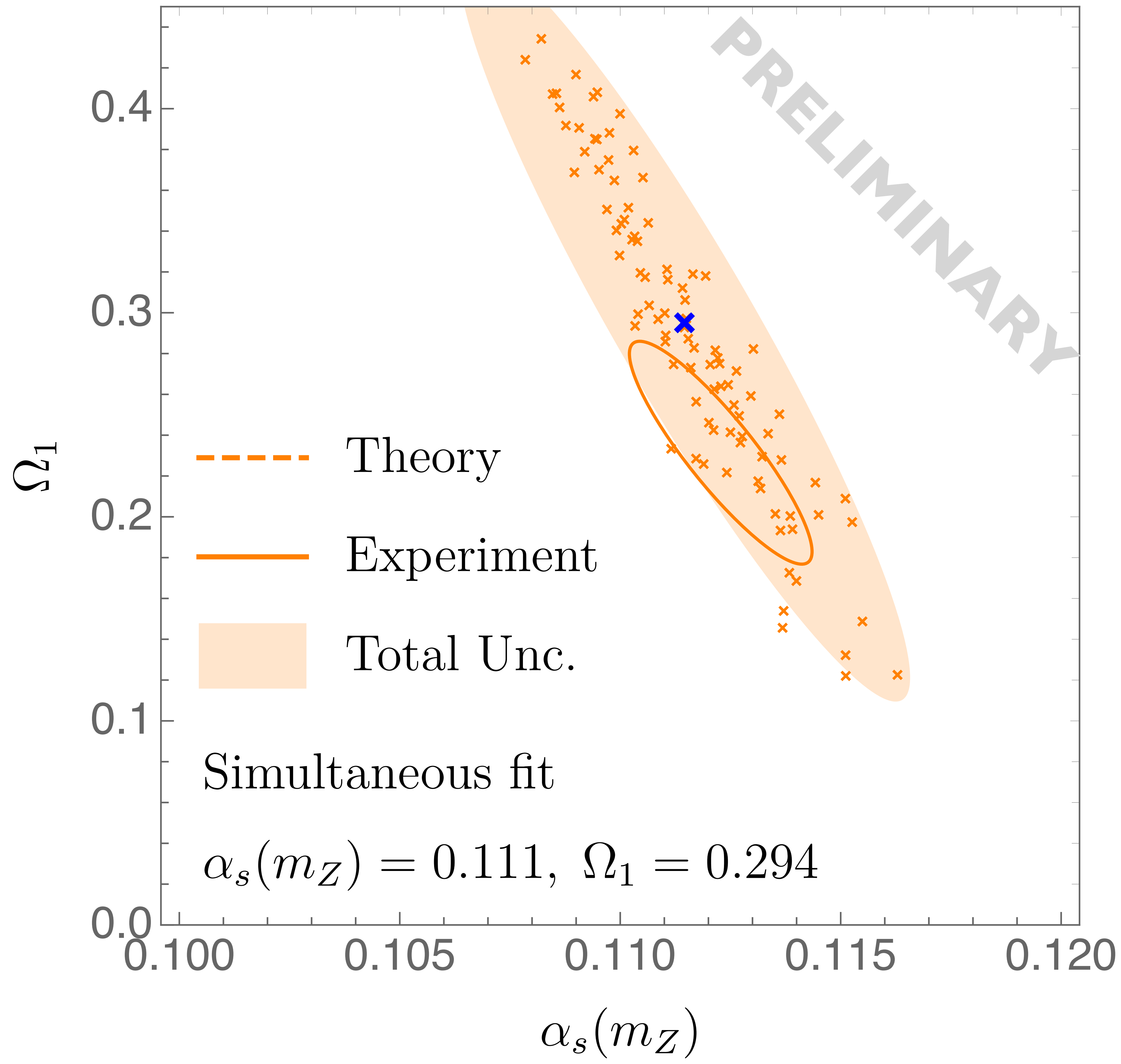
- dependence on fit window needs further study



Preliminary look at fit

Bell, CL, Makris, Prager, Talbert (in progress)

- fitting all angularities simultaneously (*but assuming no correlations):
- obtaining theoretical uncertainties from the random scan of scale profile functions



- compare to other event shape extractions using SCET: $[\mathrm{NNLL}' + \mathcal{O}(\alpha_s^2)]$

Thrust: Abbate et al. (2010)

$$\alpha_s(M_Z) = 0.1143 \pm .0022$$
$$\Omega_1 = 0.316 \pm .072\ \mathrm{GeV}$$

***C*-parameter:** Hoang et al. (2015)

$$\alpha_s(M_Z) = 0.1102 \pm .0038$$
$$\Omega_1 = 0.443 \pm 0.138\ \mathrm{GeV}$$

Preliminary look at fit

Bell, CL, Makris,
Prager, Talbert
(in progress)

- fitting each angularity individually:
- using a "fixed" set of bins as the fitting region (5th + 8 bins in data; usually from 2nd to 3rd after peak)

SUMMARY & OUTLOOK

- New angularity predictions at NNLL' + NLO $[\mathcal{O}(\alpha_s^2)]$

- Preliminary fitting ~~results~~ exercises suggest consistency with "recent" τ & C extractions
  - → still need to account for renormalon-free shape function in fit
  - → and correlations across a values
  - → would like to improve far-tail accuracy to NNLO $[\mathcal{O}(\alpha_s^3)]$

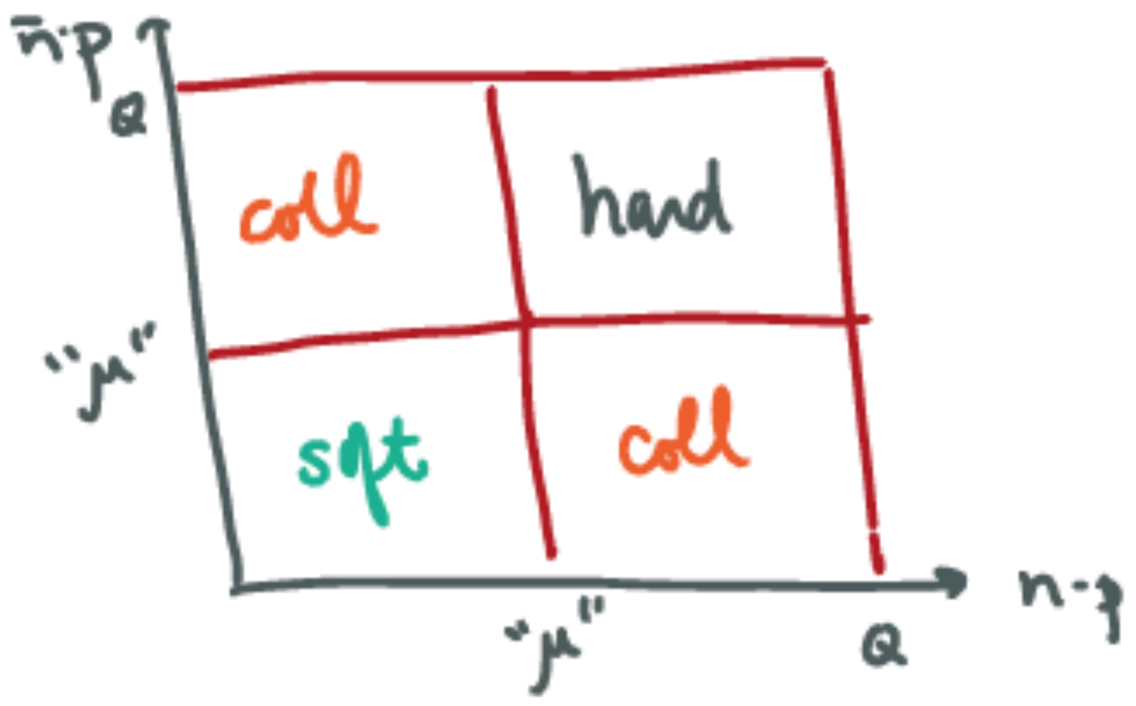
- Studies so far show the value of examining multiple τ_a's and the importance of additional data, either reanalyzed from LEP or at a future collider!

Backups

Regulating & Defining EFTs

"μ" is a regulator defining the boundaries between hard, jet and soft regions

With hard cutoffs:



dim reg:
$(g^2 \to g^2\mu^{2\epsilon})$

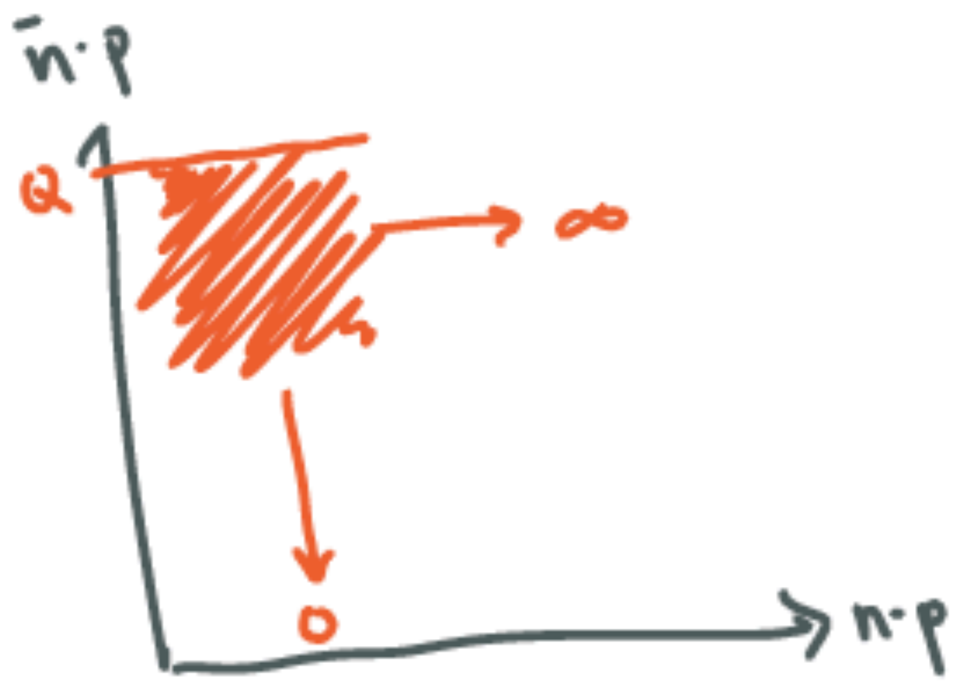
$H(Q^2,\mu)$ +

↓

UV matching

↓

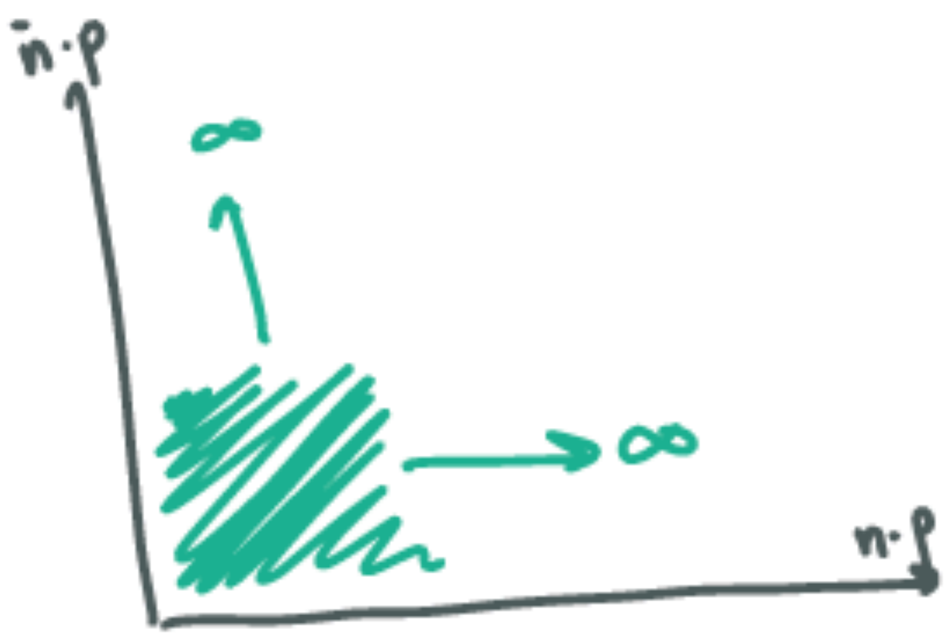
accounts for $3 \to \infty$ mismatch w/ QCD



collinear theory

(− zero-bin / soft subtraction)

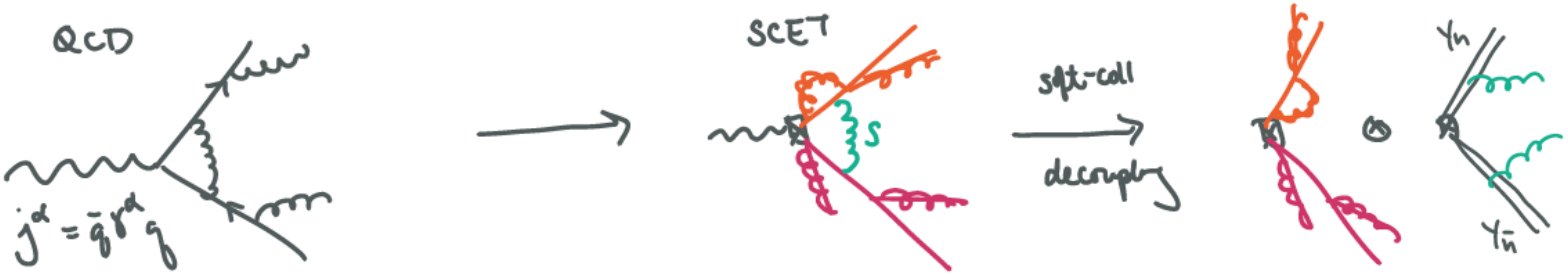
+



soft theory

effective theories only accurate inside shaded regions

OPERATOR MATCHING



$$\langle j^\alpha_{QCD} \rangle = C_2(Q,\mu)\, \langle O_2^\alpha \rangle(\mu)$$

$$O_2^\alpha = [\xi_n W_n]\, Y_n \gamma^\alpha Y_{\bar{n}}\, [W_{\bar{n}} \xi_{\bar{n}}]$$

$$W_n(x) = P \exp\left[ig \int_x^\infty ds\, \bar{n}\cdot A_n(\bar{n}s) \right]$$

hard matching coeff.

= QCD − SCET

HADRON MASS EFFECTS

[Mateu, Stewart, Thaler 2012]
[Salam, Wicke 2001]

For measurements on massive hadrons $m_h \sim \Lambda_{QCD}$ $\Rightarrow$ should not ignore.

Define event shapes in terms of rapidity y

and "transverse velocity" $r = \frac{p_\perp}{m_\perp} = \frac{p_\perp}{\sqrt{p_\perp^2 + m^2}}$

$$e = \frac{1}{Q} \sum_i m_i^\perp f_e(r_i, y_i) \qquad \text{e.g.} \quad f_\tau(r,y) = \sqrt{r^2 + \sinh^2 y} - \sinh|y|$$

Generalize $\hat{\mathcal{E}}_T(\eta) \longrightarrow$ "transverse velocity operator" $\quad \hat{\mathcal{E}}_T(r,y)\,|X\rangle = \sum_{i\in X} m_i^\perp\, \delta(r-r_i)\,\delta(y-y_i)\,|X\rangle$

$$\Rightarrow \quad \Omega_1 \longrightarrow \Omega_1(r) = \langle 0|\,\bar{T}[Y_{\bar n}^\dagger Y_n^\dagger]\, \underset{\substack{\downarrow \text{boost} \\ 0}}{\hat{\mathcal{E}}_T(r,y)}\, T[Y_n Y_{\bar n}]\,|0\rangle$$

$$\Delta\langle e\rangle_s = c_e\, \Omega_1^{g_e} \qquad \text{where} \quad \begin{aligned} c_e &= \int_{-\infty}^{\infty} dy\, f_e(1,y) \\ \Omega_1^{g_e} &= \int_0^1 dr\, g_e(r)\, \Omega_1(r) \\ g_e(r) &= \frac{1}{c_e} \int_{-\infty}^{\infty} dy\, f_e(r,y) \end{aligned}$$

$\downarrow$

event shapes w/ same g_e $\longleftarrow$

in same "universality class"

hadron masses $\Rightarrow$ $\sim 1\%$ change in c.s. or α_s

RENORMALON REMOVAL

$$S(k,\mu) = \int dk'\; S_{PT}(k-k',\mu)\; S_{NP}(k'-\bar{\Delta}_a,\mu)$$

each has a "renormalon" ambiguity

$$\Rightarrow \sim \sum_{n=0}^{\infty} (n!)\,\alpha_s^{n+1} \xrightarrow[\text{transform}]{\text{Borel}} \sim \frac{8C_F}{\pi\beta_0}\,\frac{1}{t-\frac{1}{2}}\,\mu\,\delta'(k) \xrightarrow[\text{inverse Borel transform}]{} \int_0^{\infty} dt\, e^{-t\frac{4\pi\beta_0}{\alpha_s}}\,\frac{8C_F}{\pi\beta_0}\,\frac{1}{t-\frac{1}{2}}\,\mu\,\delta'(k)$$

ambiguity $\sim \frac{16C_F}{\beta_0}\Lambda_{QCD}\,\delta'(k) \Leftarrow$

$\Rightarrow$
- negative cross sections
- poor convergence in PT

$\Rightarrow$ Shift terms between $S_{PT}(k-k')\,S_{NP}(k'-\bar{\Delta}_a)$

$$\Downarrow$$

$$\hat{S}_{PT}(k-k'-\delta_a)\,S_{NP}(k'-\Delta_a)$$

$$\bar{\Delta}_a \equiv \Delta_a(\mu) + \delta_a(\mu)$$

where δ_a determined by condition $\boxed{R e^{\gamma_E}\frac{d}{d\ln\nu}\left[e^{-2\nu\delta_a}\tilde{S}_{PT}(\nu,\mu)\right]_{\nu=\frac{1}{Re^{\gamma_E}}}=0}$ (Laplace space)

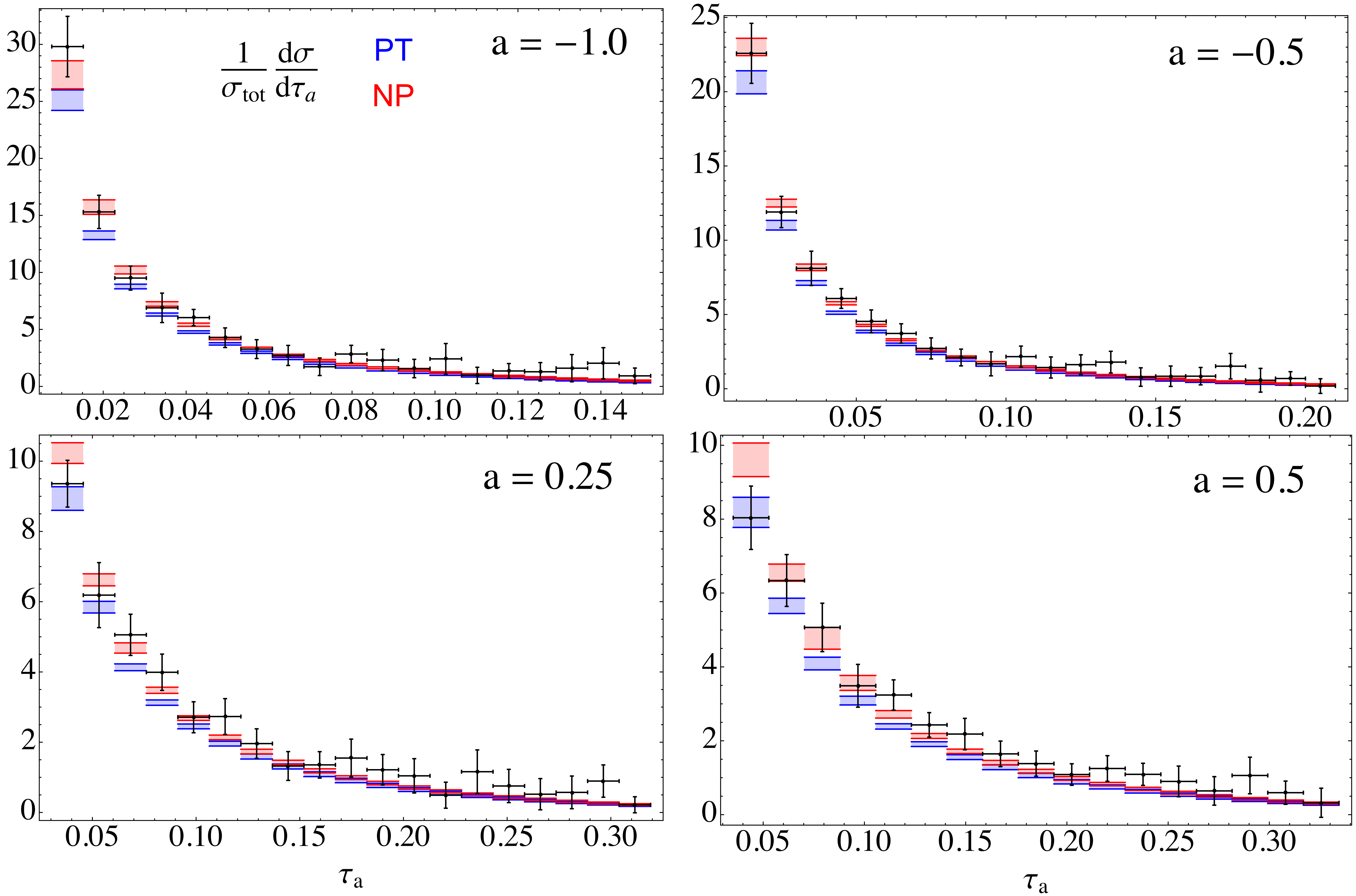
"Rgap" scheme

$\Rightarrow$ Δ_a (and thus Ω_1) evolve in μ and R ("R-evolution")

Comparison to data

L3 Collaboration (2011) $Q = 197$ GeV

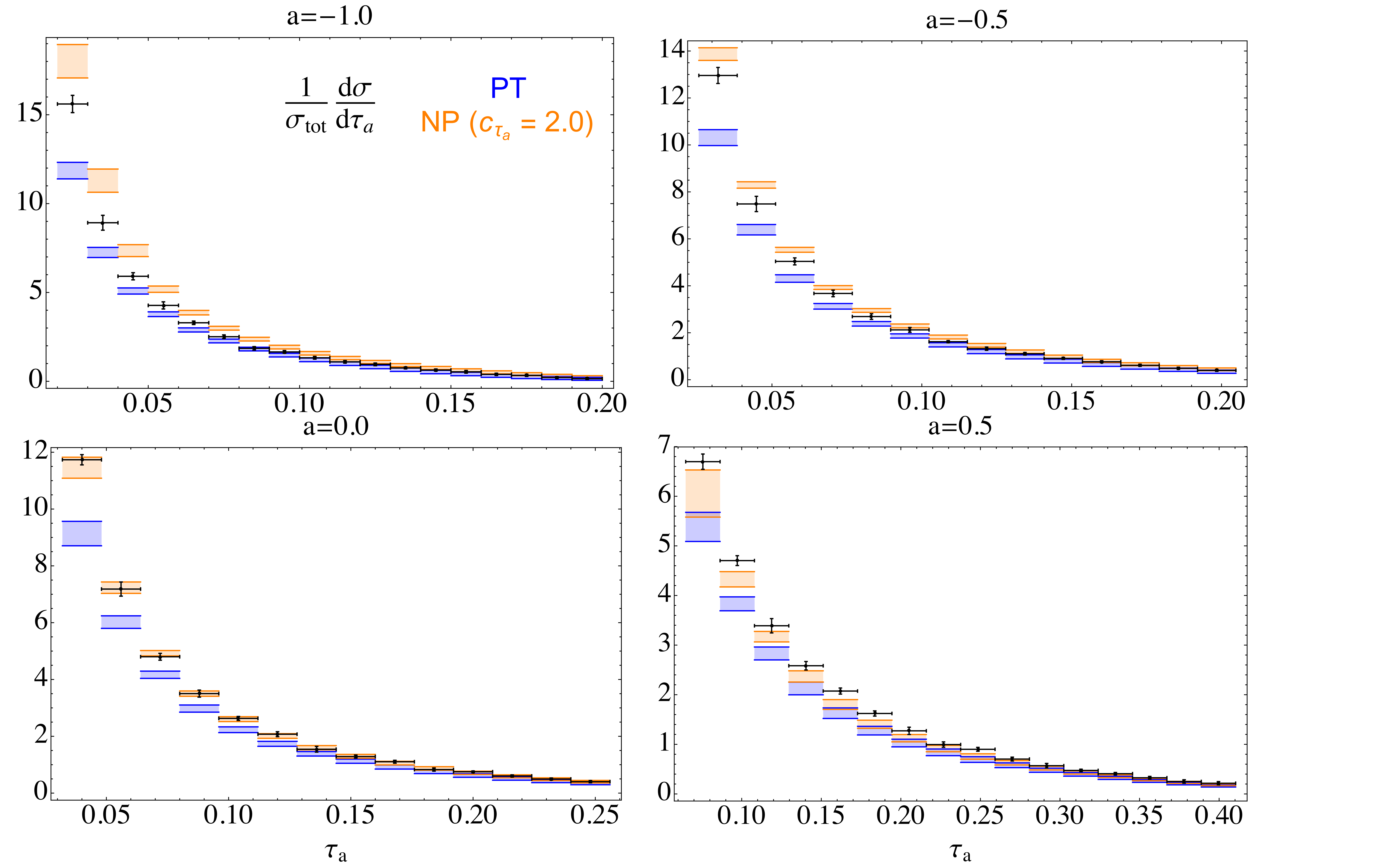
$\alpha_s(M_Z) = 0.11\,, \Omega_1(R_\Delta, R_\Delta) = 0.4$ GeV



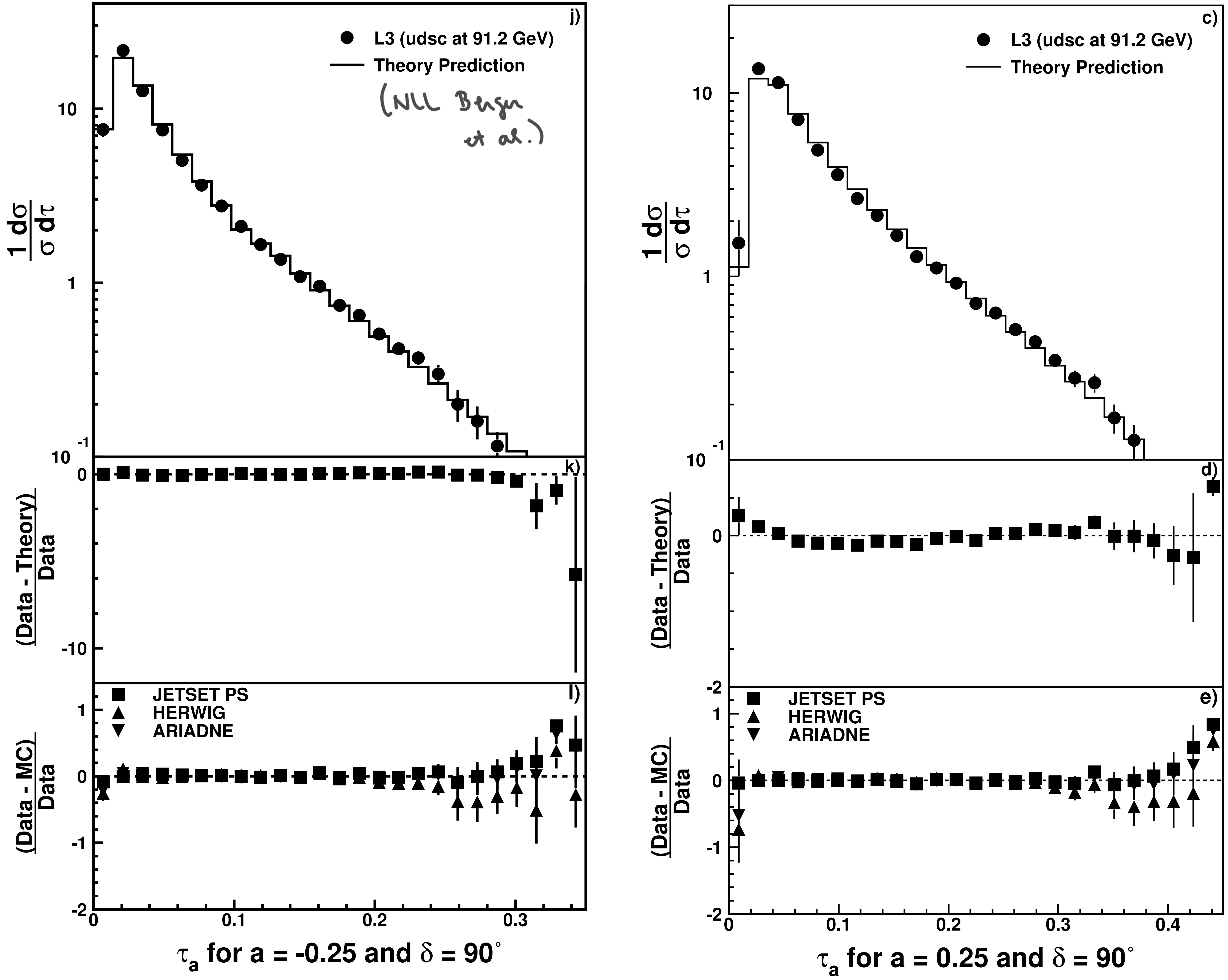
With “wrong” NP scaling

L3 Collaboration (2011) $Q = M_Z$

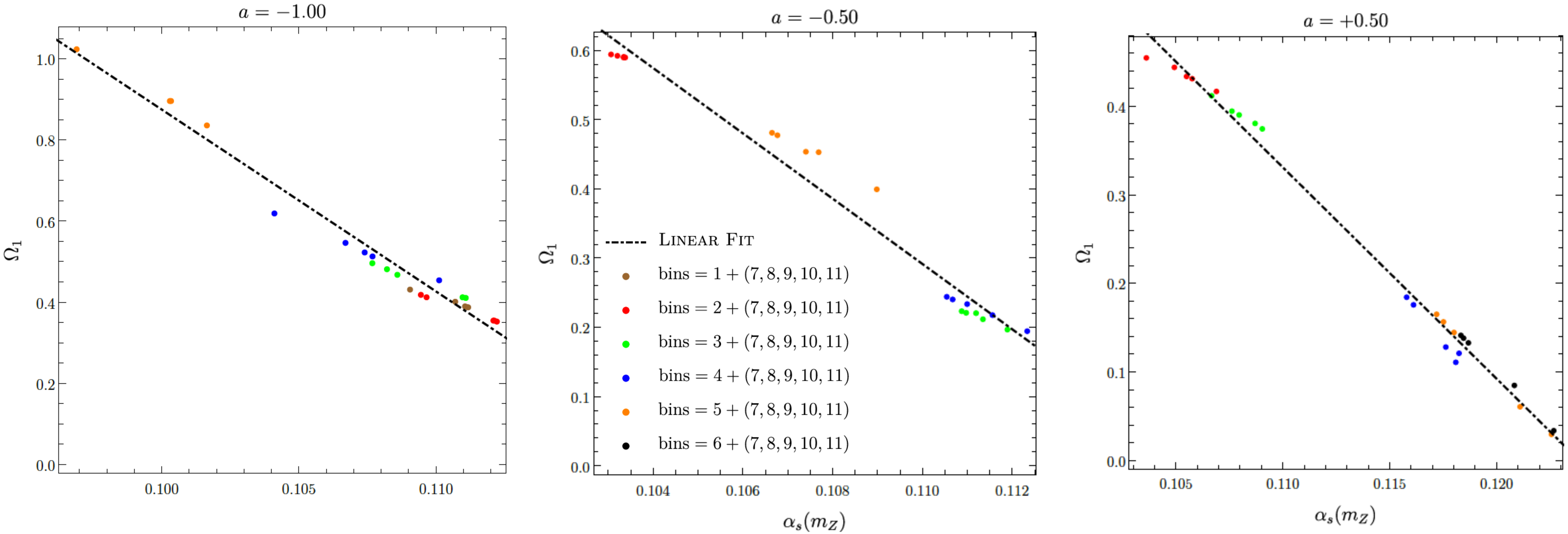
$\alpha_s(M_Z) = 0.11\,, \Omega_1(R_\Delta, R_\Delta) = 0.4$ GeV



Data from L3 Collaboration [JHEP 10 (2011), 143]



Dependence on fit window



cf. thrust
(Abbate et al.)
& C-parameter
(Hoang et al.)