α_s from R(s) (+ R(s) tests of related τ-based analysis strategies)

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ICTP-SAIFR

Workshop on Determination of Fundamental QCD Parameters

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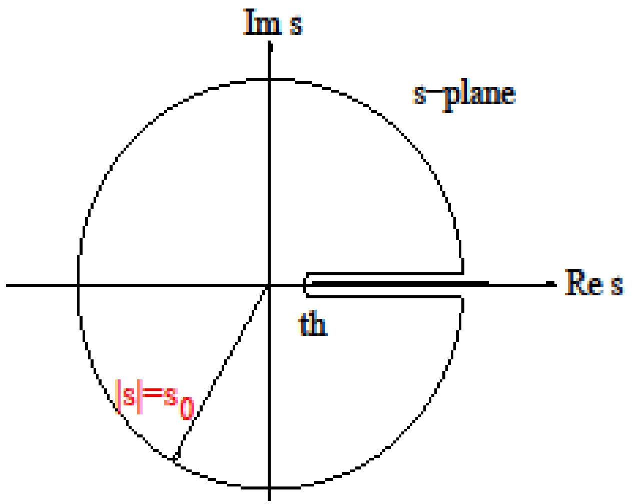
- BGKMNPT: PRD98 (2018) 074030 [1805.08176]
- BGMP: Sci Post Phys Proc 1 (2019) 053 [1811.01591] and PRDXX(2019) [1907.03360]

τ and electroproduction FESRs

- $\Pi(Q^2)$: kinematic-singularity-free scalar polarization (Π_{EM}, $\Pi_{ud;V/A}^{J=0+1}$)
- $\rho(s)$: corresponding spectral function
- $w(s)$: here, analytic inside and on $|s|=s_0$
- $\Pi(Q^2)\equiv\Pi_{OPE}(Q^2)+\Pi_{DV}(Q^2)$ ($\simeq\Pi_{OPE}(Q^2)$ for spacelike $Q^2\gg\Lambda_{QCD}^2$, up to exponentially suppressed corrections)
- Oscillatory (resonance) DV contributions in $\rho(s)$ (+near timelike axis) for s, $|Q^2|$ ~ a few GeV^2 ⇒ potential non-negligible RHS DV contributions (S. Peris talk)

FESR relation (Cauchy's theorem)

$$\int_{s_{th}}^{s_0} ds\, w(s)\rho(s) = \frac{-1}{2\pi i}\oint_{|s|=s_0} ds\, w(s)\Pi(s)$$



- **OPE contributions**

  - D=0 (perturbative) known to 5-loop ($O(\alpha_s^4)$) order

  - D=2 (mass-dependent perturbative): numerically negligible for I=1 τ FESRs, small $O(m_s^2)$, $O(\alpha_{EM})$ contributions included for EM

  - higher D: $[\Pi(Q^2)]_{D\geq 4}^{OPE} \equiv \Sigma_{D\geq 4}\, [C_D/Q^D]$ with effective condensates C_D

  - for polynomial weights $w(y) = w(s/s_0) = \Sigma_{k\geq 0}\, b_k y^k$

    $$\frac{-1}{2\pi i}\oint_{|s|=s_0} ds\, w(y)\, [\Pi(Q^2)]_{D\geq 4}^{OPE} = \Sigma_{k\geq 1}\, (-1)^k\, b_k\, C_{2k+2}/s_0^{\,k}$$

    up to α_s-suppressed log corrections

  - degree N w(y) ↔ unsuppressed OPE contributions to D=2N+2

Qualitative aspects of τ, EM FESR determinations

- Decreasing μ (with fixed precision at μ) ↔ increasing precision at M_Z

$$[\delta\alpha_s(M_Z^2)/\alpha_s(M_Z^2)] \simeq [\alpha_s(M_Z^2)/\alpha_s(\mu^2)]\ [\delta\alpha_s(\mu^2)/\alpha_s(\mu^2)]$$

- Advantage for low-scale τ, EM analyses $[\alpha_s(M_Z^2)/\alpha_s(\mu^2)] \simeq 1/3$ for $\mu \simeq m_\tau]$

- BUT decreasing μ ↔ increasing NP contributions: how large for $\mu \simeq m_\tau$?

- Large α_s-independent part of D=0 OPE integral, $c_w\ [1 + \alpha_s/\pi + \text{w-dependent h.o.}]$,

  ⇒ requirement for control of NP more stringent than naively expected

  e.g. NP to ~ 0.5% of corresponding spectral integral for $\alpha_s(m_\tau^2)$ to ~3%

▪ More re DV contributions

- Poggio, Quinn, Weinberg: DVs localized near timelike axis for intermediate Q^2

- With $\rho_{DV}(s) \equiv \frac{1}{\pi}\, Im\, \Pi_{DV}(s)$, theory side →

  $$\frac{-1}{2\pi i} \oint_{|s|=s_0} ds\ w(s)\ \Pi_{OPE}(Q^2) - \int_{s_0}^{\infty} ds\ w(s)\ \rho_{DV}(s)$$

- (Channel-dependent) asymptotic form [2005 ansatz, Boito et al. PRD97 054007 [1711.10316] for theoretical basis]

  $$\rho_{DV}(s) = \kappa\, e^{-\gamma s} \sin(\alpha+\beta s)$$

- $s_0 \leq m_\tau^2$ kinematic restriction for τ FESRs, no such restriction for EM FESRs

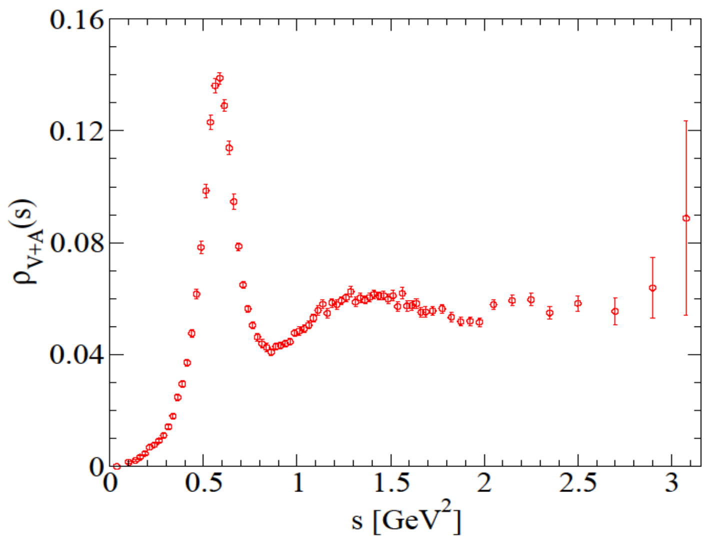
- Exponential damping of $\rho_{DV}(s) \Rightarrow$ significant residual integrated DV reduction from modest s_0 increase (important advantage of EM c.f. τ-based FESRs)

DV contributions in the τ and $e^+e^-\to$hadrons spectra

- The τ, I=1 V+A spectral function, showing "reduced" DVs above s ~1.5-2 GeV ^{2} (reduced c.f. those for V or A alone)

- In the literature: often used to argue for the neglect of DVs in this region

- *However: assessment of relative roles of DV and α_s-dependent perturbative contributions complicated by presence of α_s-independent contribution (e.g. same figure with different (larger) α_s-independent contribution)*



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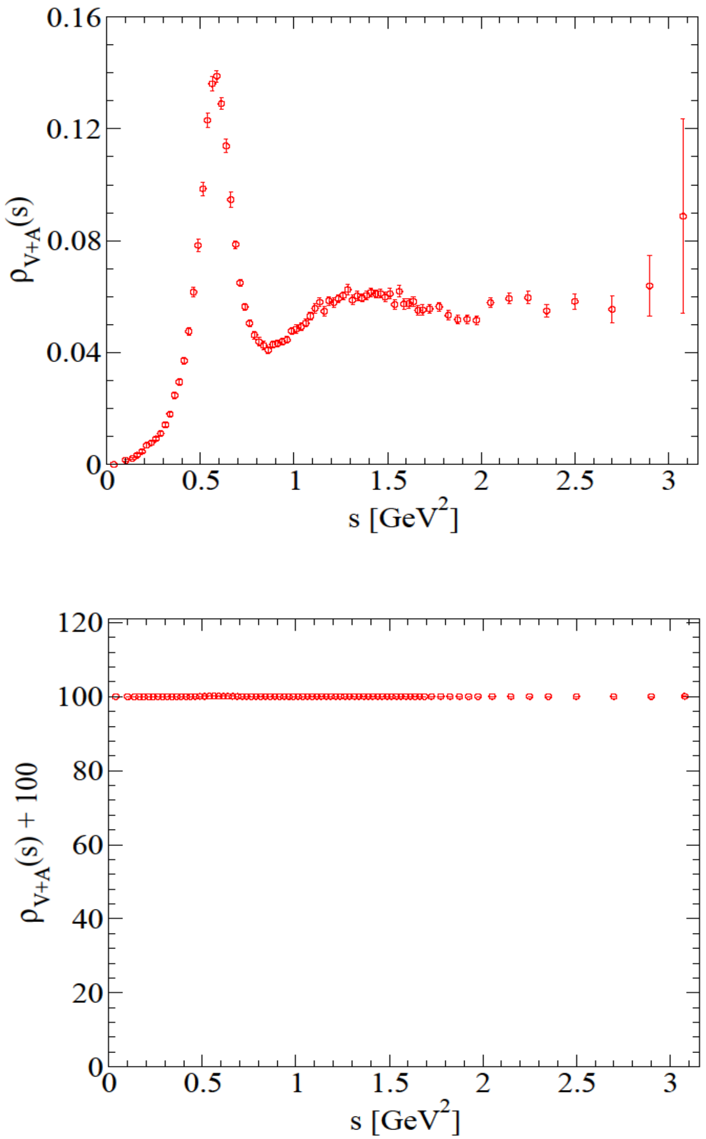

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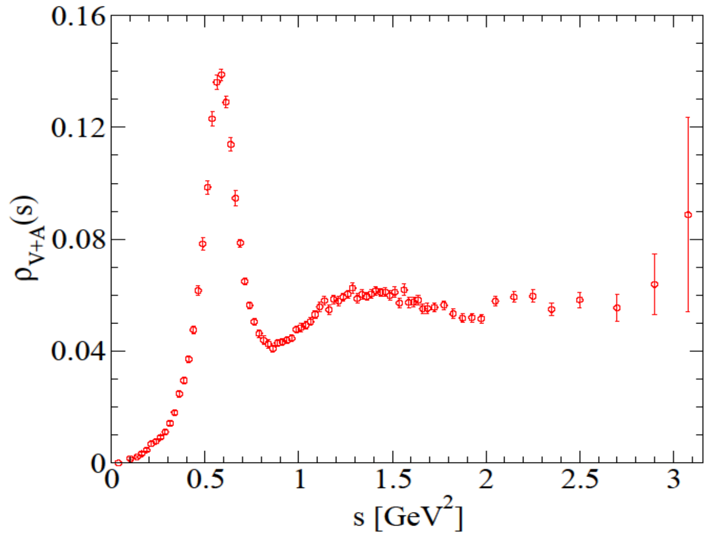

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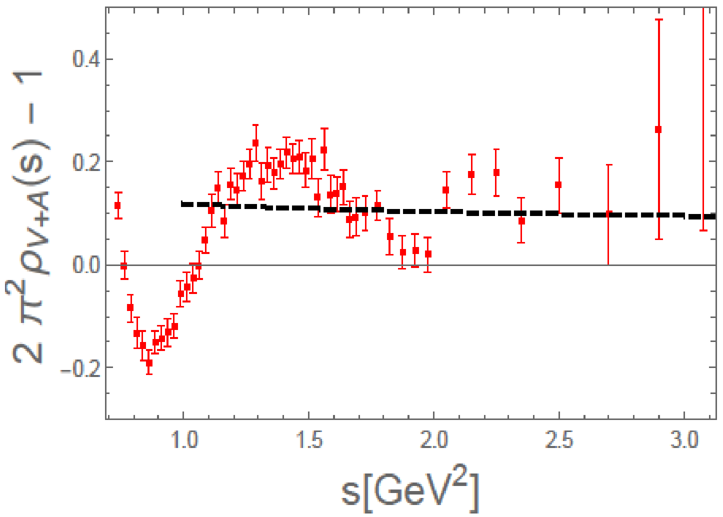
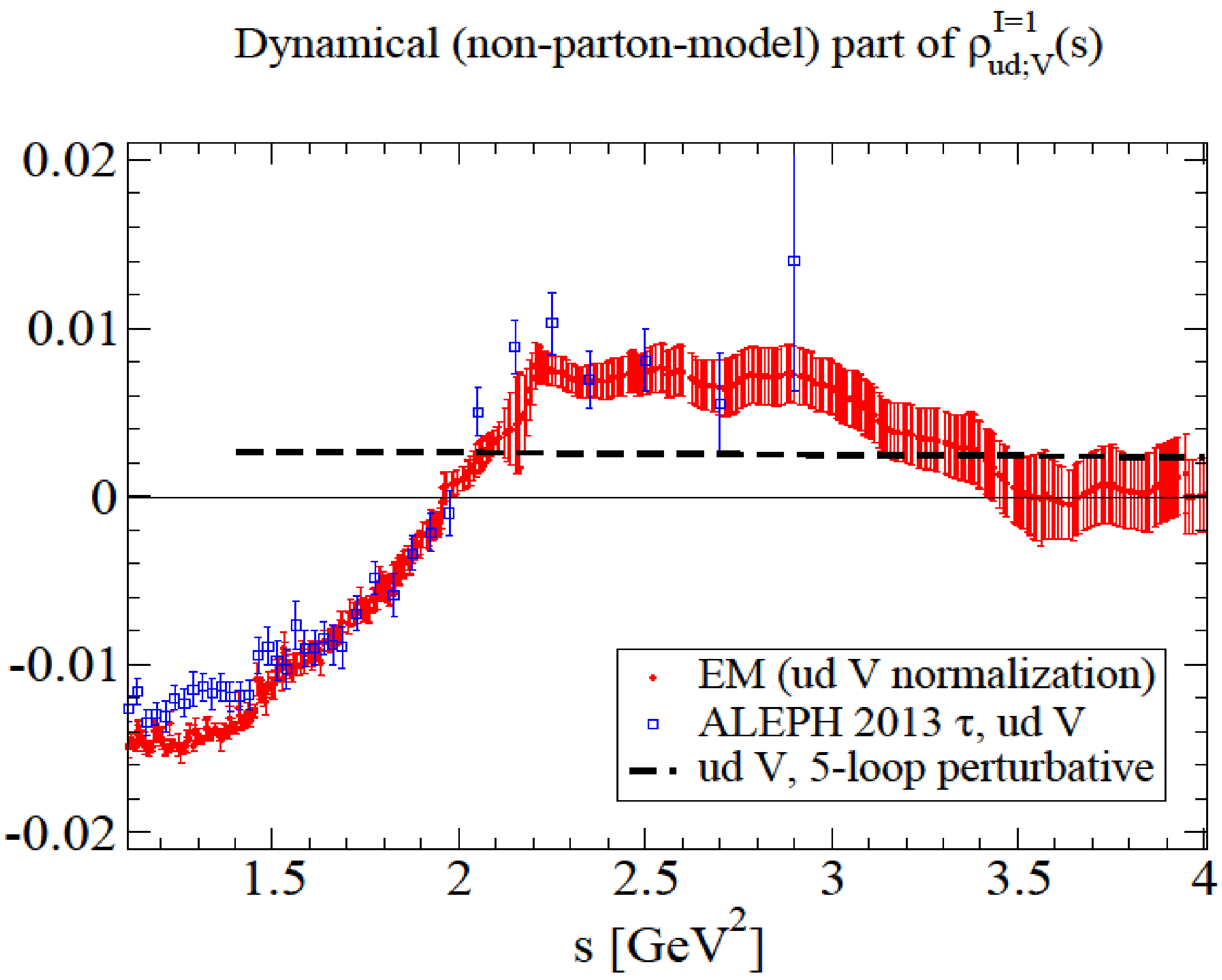
- **C.f. the τ, I=1 V+A figure, now with the non-dynamical, α_s-independent parton model contribution removed**



Evidence for the oscillatory, exponentially damped asymptotic DV behavior in the G-parity separated I=1 part of R(s)

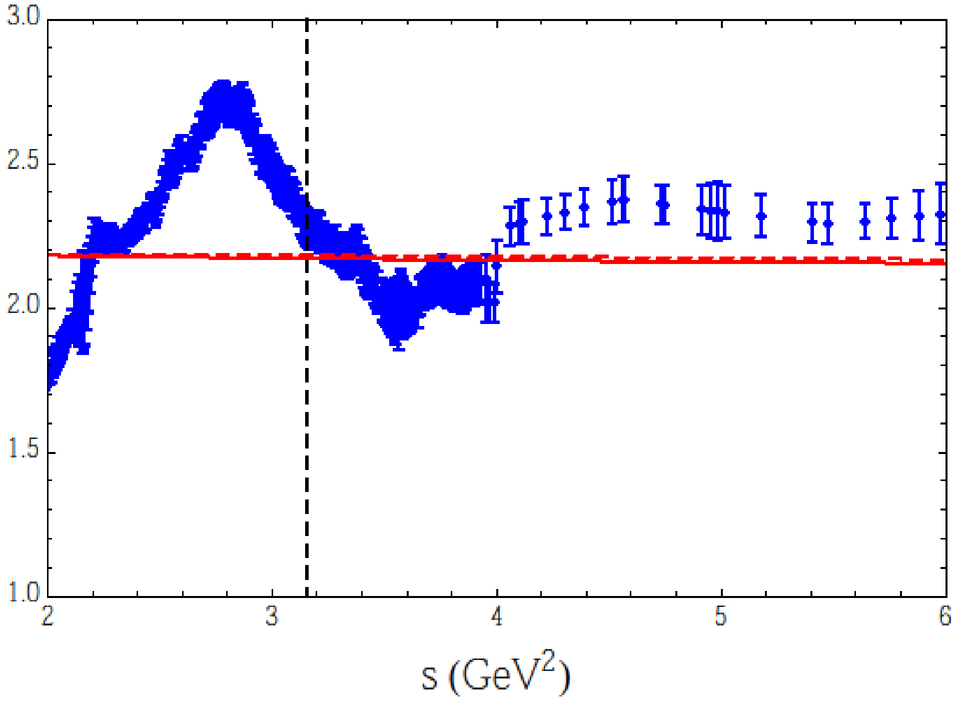


α_s from FESRs with KNT 2018 R(s) data

- $\rho_{EM}(s) = \frac{1}{12\pi^2} R(s)$
- Start with analyses neglecting DVs, $s_0 \sim m_\tau^2$ and above: fit parameters α_s and relevant OPE condensates C_D
- test stability of OPE parameters to inclusion of DVs (extended fits with I=1 DV parameters constrained from τ, new I=0 DV parameters κ_0, α_0 fit with $\beta_0 \simeq \beta_1$, $\gamma_0 \simeq \gamma_1$ assumed)

KNT 2018 R(s) compilation

← exclusive region → ←inclusive region→



KNT: PRD97 (2018) 114025 [1810.02995]

More on the pure-OPE, no-DV fits

- **OPE treatment**
  - D=0 to 5 loops (O(α_s^4)), including O(α_{EM}) contributions
  - $O(m_s^2)$ $D = 2$ to 3 loops
  - avoid weights with term linear in *s* (convergence issues from Beneke, Boito, Jamin renormalon model studies [JHEP 1301 (2013) 125 [1210.8038]]

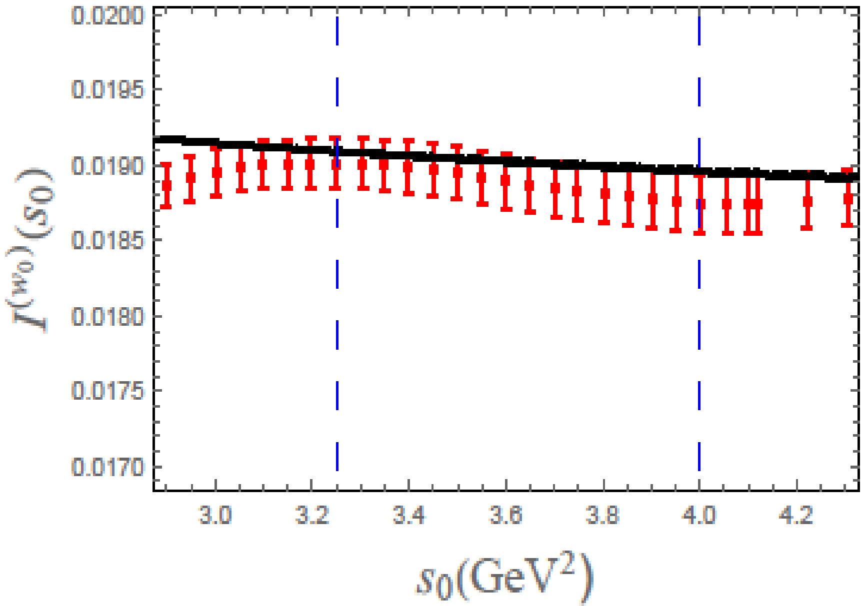
- **Weight choices, w(y) = w(s/s_0)**
  - $w_0(y) = 1$ (no DV suppression near timelike point $s=s_0$, fit parameter α_s)
  - $w_2(y) = 1-y^2$ (single "pinch" DV suppression near $s=s_0$, fit parameters α_s, C_6)
  - $w_3(y) = 1 - 3y^2 + 2y^3$ (double "pinch" near $s=s_0$, fit parameters α_s, C_6, C_8)
  - $w_4(y) = 1 - 2y^2 + y^4$ (double "pinch" near $s=s_0$, fit parameters α_s, C_6, C_{10})

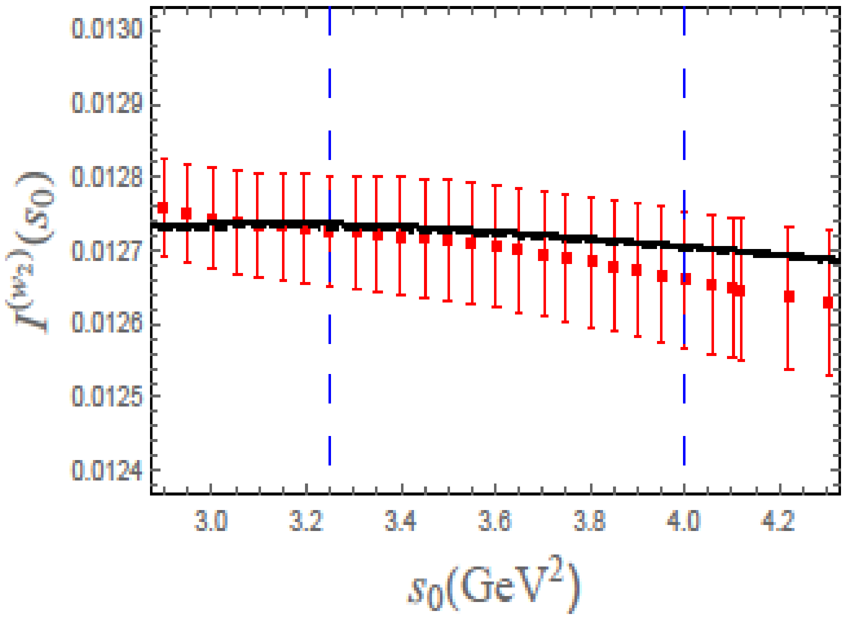
D=0 FOPT, no-DV fit results, w_3, w_4 FESRs, fit windows $s_0^{min} \leq s_0 \leq 4$ GeV^2

s_0^{min} [GeV^2]	χ^2/dof [w_3]	p-value [w_3]	$\alpha_s(m_\tau^2)$ [w_3]	C_6 [GeV^6] [w_3]	χ^2/dof [w_4]	p-value [w_4]	$\alpha_s(m_\tau^2)$ [w_4]	C_6 [GeV^6] [w_4]
3.15	44.8/15	0.00008	0.276(15)	0.0027(20)	45.0/15	0.00008	0.275(15)	0.0027(20)
3.25	31.9/13	0.003	0.292(15)	0.0059(23)	32.0/13	0.002	0.292(15)	0.0060(24)
3.35	26.0/11	0.006	0.296(15)	0.0068(25)	26.0/11	0.006	0.296(15)	0.0069(25)
3.15*	9.8/6	0.13	0.293(15)	0.0055(22)	9.8/6	0.14	0.292(15)	0.0056(22)
3.25*	7.6/5	0.18	0.299(15)	0.0070(25)	7.5/5	0.18	0.299(15)	0.0071(25)
3.35*	5.6/4	0.23	0.305(15)	0.0084(27)	5.6/4	0.23	0.303(15)	0.0086(27)
3.45	12.9/9	0.17	0.303(16)	0.0085(27)	23.9/9	0.17	0.302(16)	0.0087(28)
3.55	11.6/7	0.11	0.301(16)	0.0081(29)	11.6/7	0.11	0.300(16)	0.0082(30)
3.60	11.1/6	0.09	0.298(17)	0.0071(32)	11.0/6	0.09	0.297(17)	0.0072(32)
3.70	5.7/4	0.22	0.292(18)	0.0049(35)	5.7/4	0.22	0.292(18)	0.0050(35)
3.80	2.3/2	0.32	0.289(19)	0.0036(39)	2.3/2	0.32	0.288(19)	0.0037(39)

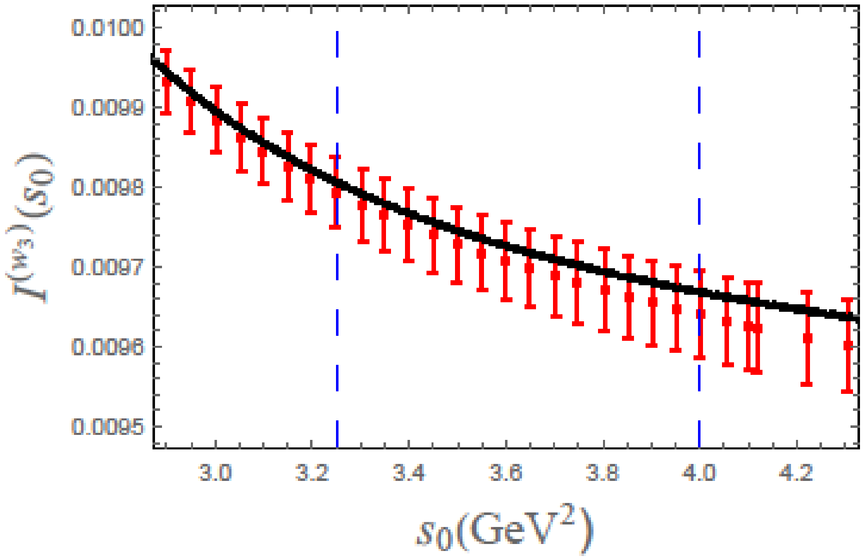
Theory vs experiment matches, s_0^{min}=3.25 GeV^2, no-DV fits

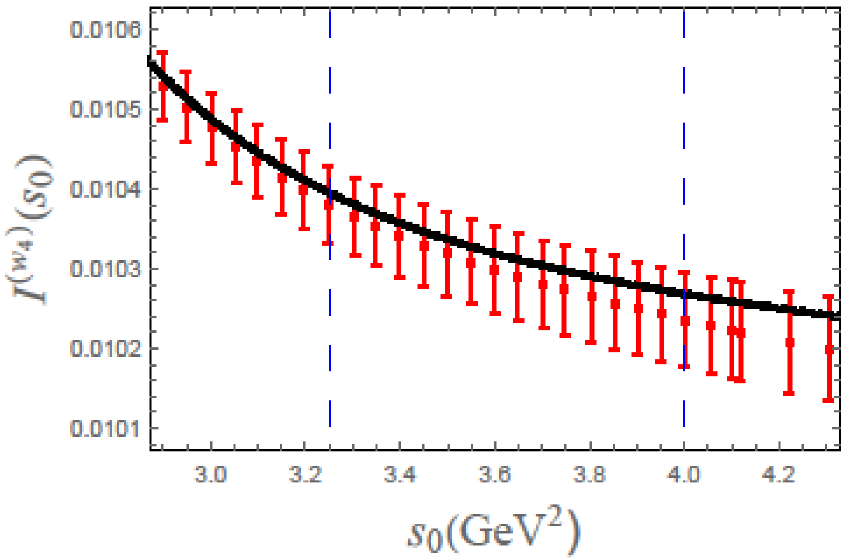
- Left: w_0, right: w_2
  solid/dashed lines:
  FOPT/CIPT D=0 fits





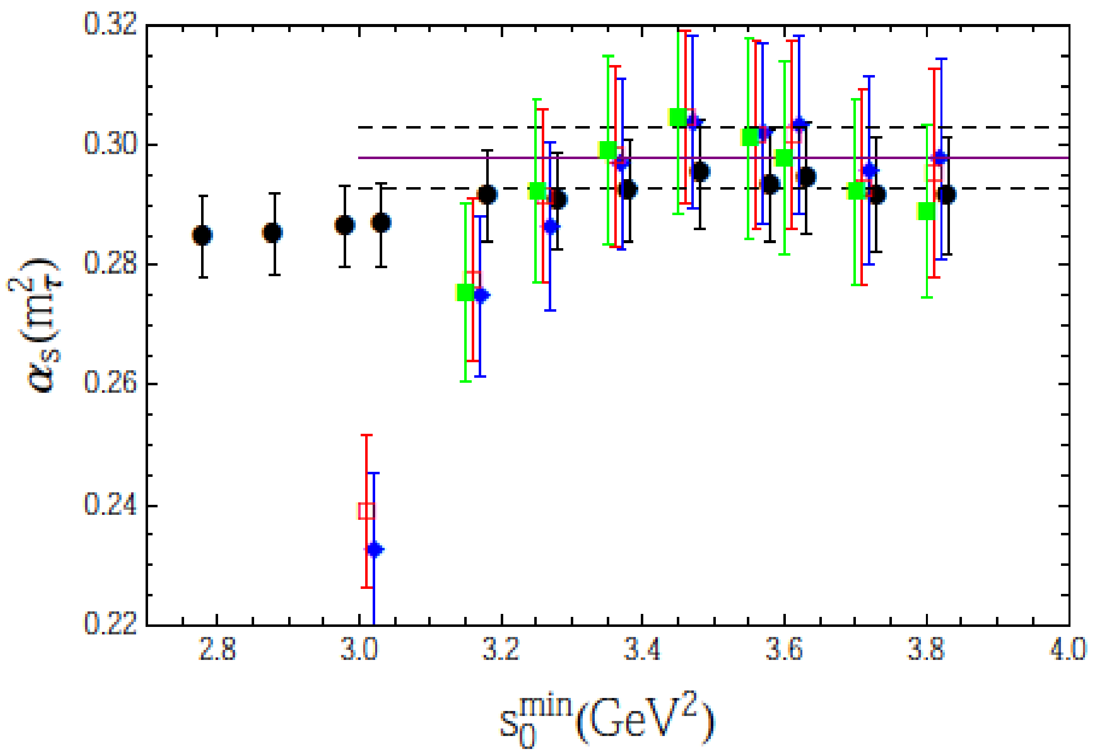
- Left: w_3, right: w_4
  solid/dashed lines:
  FOPT/CIPT D=0 fits





$\alpha_s(m_\tau^2)$ vs s_0^{min}, various weights, with and without DVs

- **Blue:** w_0 FESR, no DVs
- **Red:** w_2 FESR, no DVs
- **Green:** w_3 FESR, no DVs
- **Black:** w_0 FESR, with DVs



Addition of DVs stabilizes fits at lower s_0

Final averaged EM results for α_s

FOPT: $\alpha_s^{(3)}(m_\tau^2) = 0.298(17) \leftrightarrow \alpha_s^{(5)}(M_Z^2) = 0.1158(22)$

CIPT: $\alpha_s^{(3)}(m_\tau^2) = 0.304(19) \leftrightarrow \alpha_s^{(5)}(M_Z^2) = 0.1166(25)$

- c.f. analogous ALEPH 2013 I=1, τ-data-based analysis, including DVs

  [D. Boito et al., Phys. Rev. D91 (2015) 034003 [1410.3528]]

  FOPT: $\alpha_s^{(3)}(m_\tau^2) = 0.296(10) \leftrightarrow \alpha_s^{(5)}(M_Z^2) = 0.1155(14)$

  CIPT: $\alpha_s^{(3)}(m_\tau^2) = 0.310(14) \leftrightarrow \alpha_s^{(5)}(M_Z^2) = 0.1174(17)$

- EM errors currently data dominated

- *Note 0.014 → 0.006 reduction in FOPT-CIPT $\alpha_s(m_\tau^2)$ difference in higher-scale EM vs τ analysis (hence reduced theory uncertainty)*

PART II: R(s)-based tests of the "truncated OPE" (tOPE) approach (used for most results included in the PDG assessment of α_s from τ)

- [E.g., Pich-Lediberder PLBB289, 165; ALEPH; OPAL; Pich, Rodriguez-Sanchez PRD94, 034027 [1605.06830]]
- τ, I=1 V, A, V+A channel analyses using (at least) doubly pinched weights, neglecting DVs (with V+A argued safest)
- Final results from $s_0 = m_\tau^2$ only (minimizes residual DV contributions)
- Kinematic weight case $w_\tau(y)$=1-3y^2+2y^3 (spectral integral from inclusive BFs) insufficient as theory side involves 3 OPE parameters α_s, C_6, C_8
- Additional (higher-degree-weight) FESRs to fit C_6, C_8
- **Complication: new degree 4 w(y) brings in the new OPE parameter C_{10}, new degree 5 w(y) the new OPE parameter C_{12}, etc. ⇒ # of OPE parameters always exceeds # s_0=m_τ^2 spectral integrals without further assumptions/OPE truncation**

- **With conventional Pich-Le Diberder spectral weights $w_{km}(y)=y^m(1-y)^{2+k}(1+2y)$**

D≥4 OPE contributions (dimensionless)

Weight	D=4	D=6	D=8	D=10	D=12	D=14	D=16
$w_{00}=w_\tau$		$-3C_6/s_0^3$	$-2C_8/s_0^4$				
w_{10}	C_4/s_0^2	$-3C_6/s_0^3$	$-5C_8/s_0^4$	$-2C_{10}/s_0^5$			
w_{11}	$-C_4/s_0^2$	$-C_6/s_0^3$	$3C_8/s_0^4$	$5C_{10}/s_0^5$	C_{12}/s_0^6		
w_{12}		C_6/s_0^3	C_8/s_0^4	$-3C_{10}/s_0^5$	$-5C_{12}/s_0^6$	$-C_{14}/s_0^7$	
w_{13}			$-C_8/s_0^4$	$-C_{10}/s_0^5$	$3C_{12}/s_0^6$	$5C_{14}/s_0^7$	C_{16}/s_0^8

- 5 $s_0=m_\tau^2$ spectral integrals; 4 OPE fit parameters: α_s, C_4, C_6, C_8
- D=10, 12, 14, 16 contributions dropped (the tOPE assumption) on grounds of assumed scaling with additional factors of $\sim(\Lambda_{QCD}^2/m_\tau^2)$

- **With Pich, Rodriguez-Sanchez "optimal" weights $w_{2k}(y)=1-(k+2)y^{k+1}+(k+1)y^{k+2}$**

D≥4 OPE contributions (dimensionless)

Weight	D=4	D=6	D=8	D=10	D=12	D=14	D=16
$w_{21}=w_\tau$		$-3C_6/s_0^3$	$-2C_8/s_0^4$				
W_{22}			$4C_8/s_0^4$	$3C_{10}/s_0^5$			
W_{23}				$-5C_{10}/s_0^5$	$-4C_{12}/s_0^6$		
W_{24}					$6C_{12}/s_0^6$	$5C_{14}/s_0^7$	
W_{25}						$-7C_{14}/s_0^7$	$-6C_{16}/s_0^8$

- 5 $s_0=m_\tau^2$ spectral integrals; 4 OPE fit parameters: α_s, C_6, C_8, C_{10}
- D=12, 14, 16 contributions dropped (the tOPE assumption) on grounds of assumed scaling with additional factors of $\sim (\Lambda_{QCD}^2/m_\tau^2)$

tOPE assumptions, potential issues, and possible tests

- **Basic tOPE assumptions**
  - $s_0=m_\tau^2$ large enough that residual integrated DVs negligible (at least for doubly pinched w(y))
  - integrated OPE series behaves as if (rapidly) converging with D for $s_0=m_\tau^2$, out to at least D=16
- **Potential tOPE issues**
  - $s_0=m_\tau^2$ only: precludes variable-s_0 tests of validity of assumed neglect of residual DVs
  - **Even if residual DVs negligible, OPE asymptotic (at best) ⇒ assumed scaling with increasing D (and related tOPE neglect of unsuppressed higher D terms) certainly incorrect in general**
- **Potential tests of tOPE assumptions**
  - exponential damping of $\rho_{DV}(s)$, decrease of higher D non-perturbative contributions with increasing s_0 ⇒ if assumptions good for some s_0^*, should be even better for $s_0>s_0^*$
  - Kinematic constraint $s_0\leq m_\tau^2$ precludes test with $s_0>m_\tau^2$ in τ, but not EM case

An R(s)-based strategy for testing tOPE assumptions

- If residual integrated DVs not negligible, tOPE assumptions incorrect and tOPE ruled out, so assume DVs negligible for $s_0 \sim m_\tau^2$ and above and test OPE truncation assumption

- Find $s_0^* \gtrsim m_\tau^2$ admitting a successful $s_0 = s_0^*$ tOPE optimal weight or w_{km} spectral weight fit

- With resulting tOPE fit parameters, test theory predictions for the $s_0 > s_0^*$ spectral integrals

- **Because of strong correlations between (i) spectral integrals for different s_0, (ii) theory integrals for different s_0, (iii) fitted OPE parameters and $\rho_{EM}(s)$ data (hence theory and spectral integrals) form single difference combinations**

$$\Delta I_w^{th/exp}(s_0; s_0^*) \equiv I_w^{th/exp}(s_0) - I_w^{th/exp}(s_0^*)$$

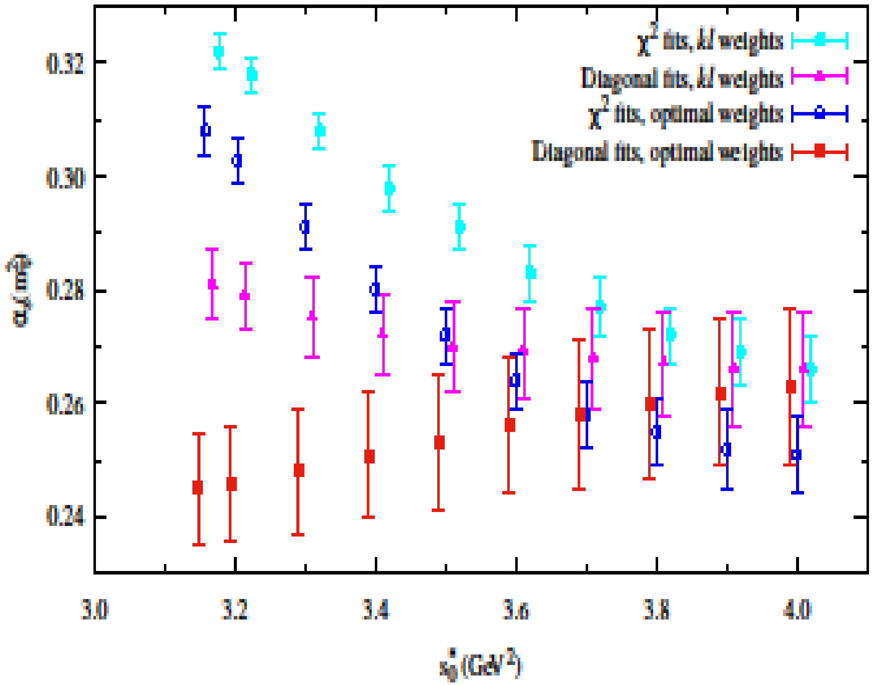
and display test results in double difference theory-minus-experiment form

$$\Delta^{(2)}(s_0; s_0^*) \equiv \Delta I_w^{th}(s_0; s_0^*) - \Delta I_w^{exp}(s_0; s_0^*)$$

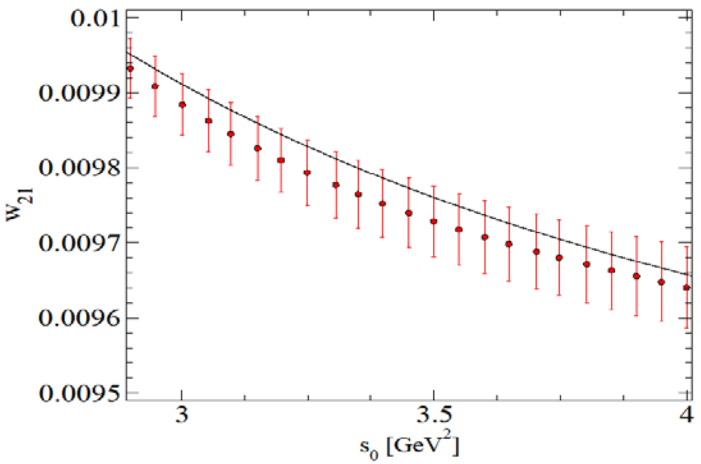
tOPE test results

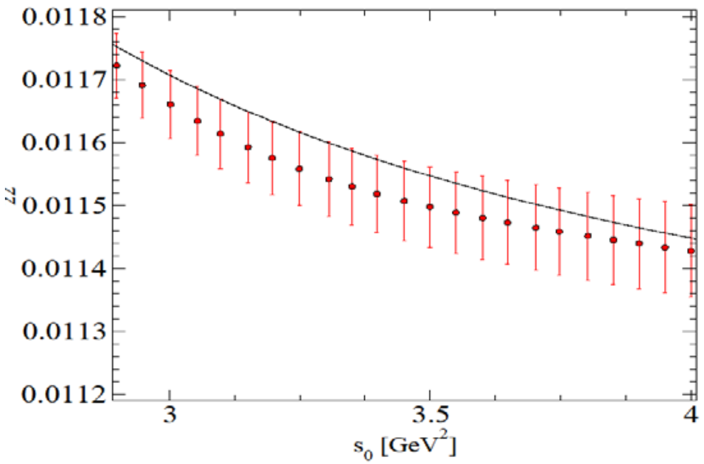
- $s_0^* = m_\tau^2$: very low correlated-fit p-values, incompatible correlated, diagonal fit $\alpha_s(m_\tau^2)$, incompatible w_{km}, optimal weight fit $\alpha_s(m_\tau^2)$
- **$s_0^* > m_\tau^2$ for acceptable correlated EM fit** (correlated, diagonal $\alpha_s(m_\tau^2)$ then compatible, but correlated w_{km}, optimal weight not)

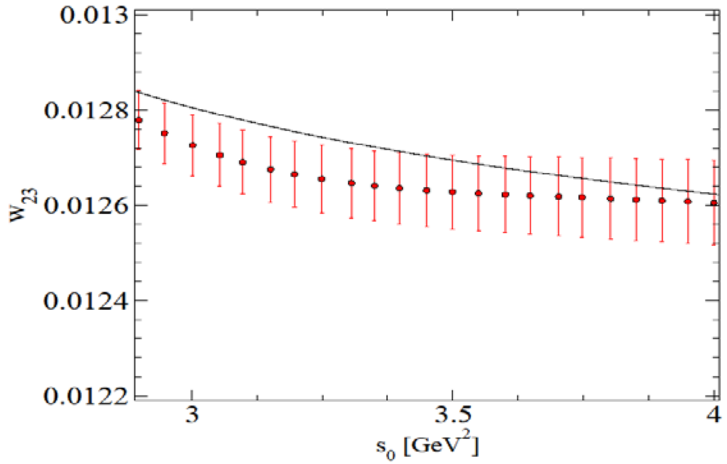
Weight type	s_0^* [GeV²]	p-value [corr fit]	$\alpha_s(m_\tau^2)$ [corrd]	$\alpha_s(m_\tau^2)$ [diag]	
w_{km}	m_τ^2	$7\text{x}10^{-21}$	0.322(3)	0.281(6)	X
Optimal	m_τ^2	$2\text{x}10^{-15}$	0.308(4)	0.245(10)	X
w_{km}	3.7	0.16	0.277(5)	0.268(9)	
Optimal	3.6	0.41	0.264(5)	0.256(12)	

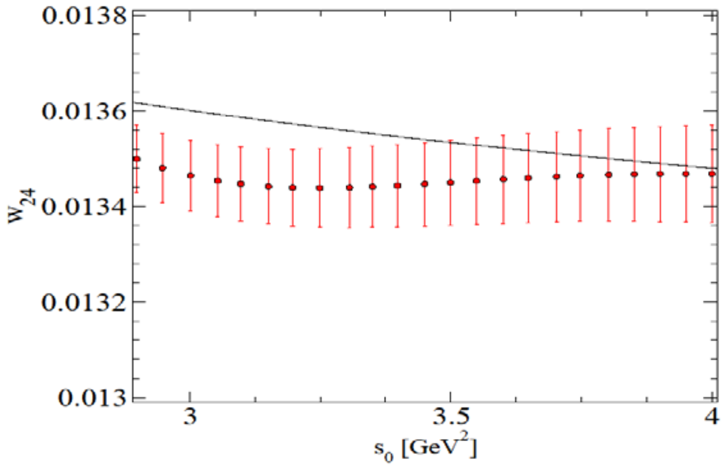


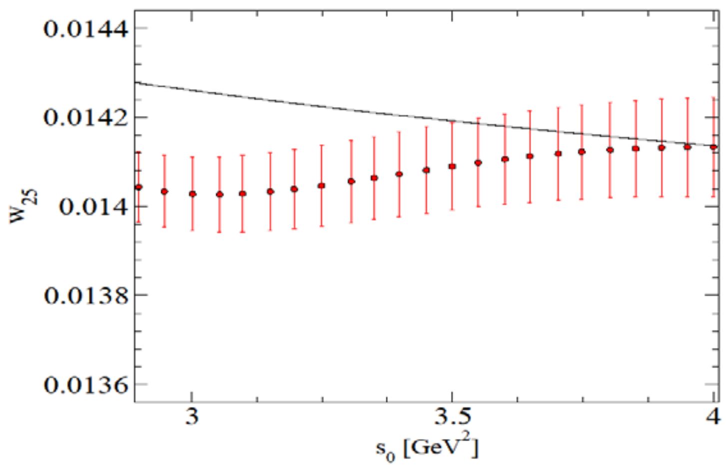
Correlated s_0^*=3.6 GeV^2 optimal weight fit theory-experiment matches



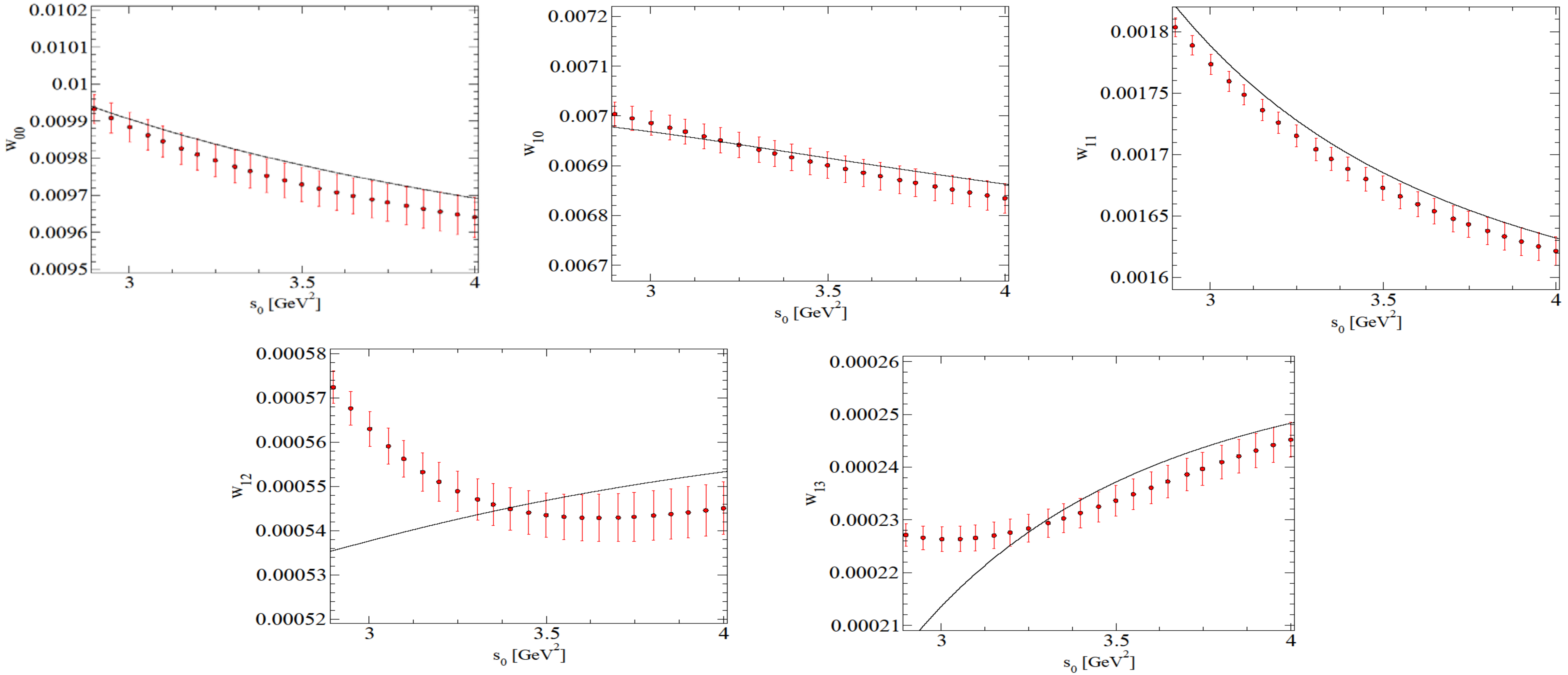




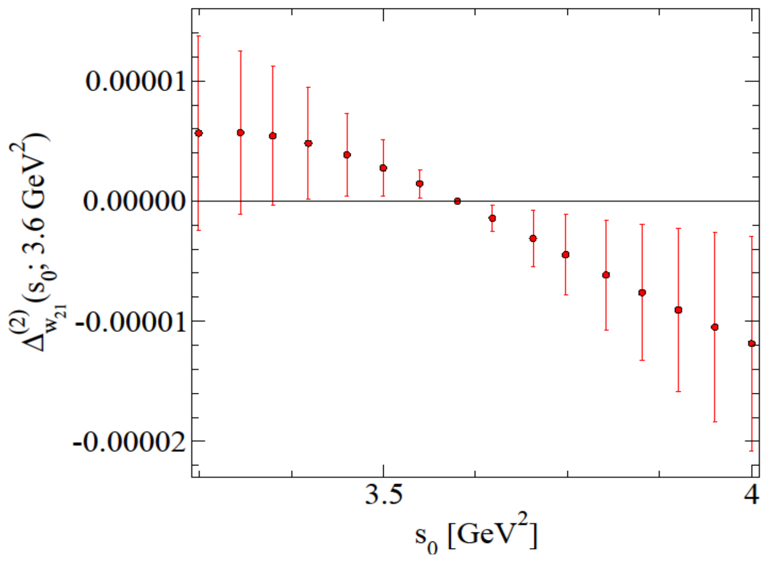


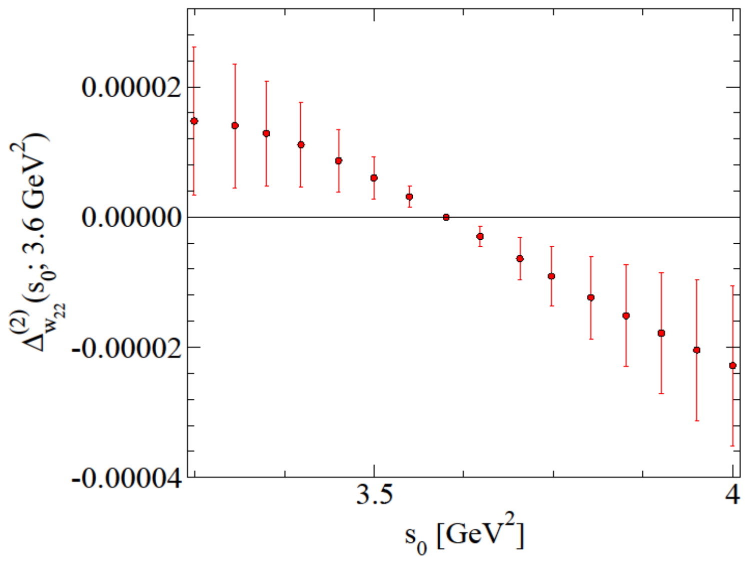


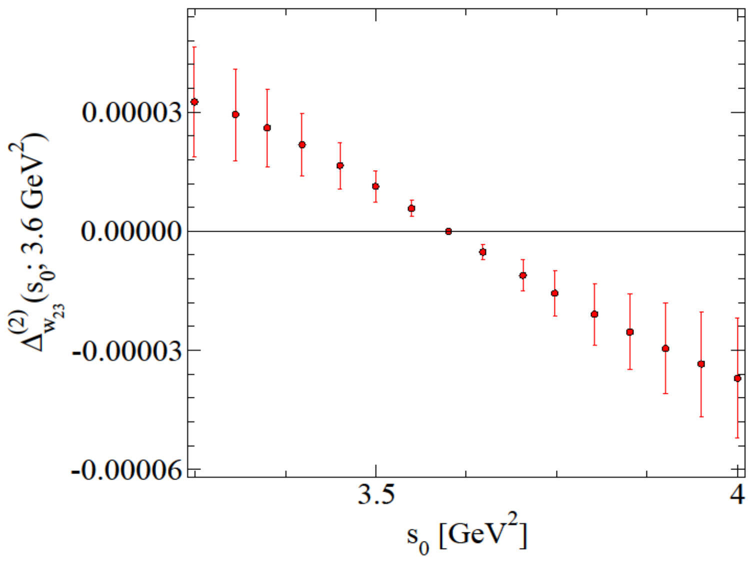
Correlated s_0^*=3.7 GeV^2 w_{km} fit theory-experiment matches

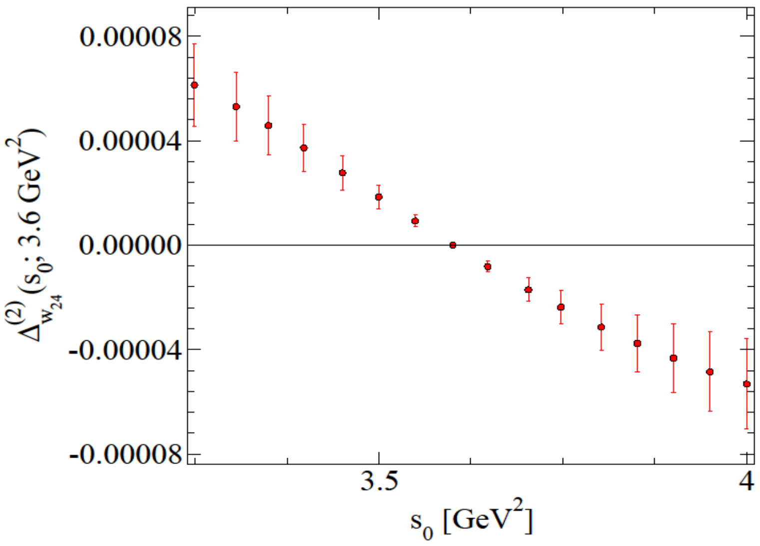


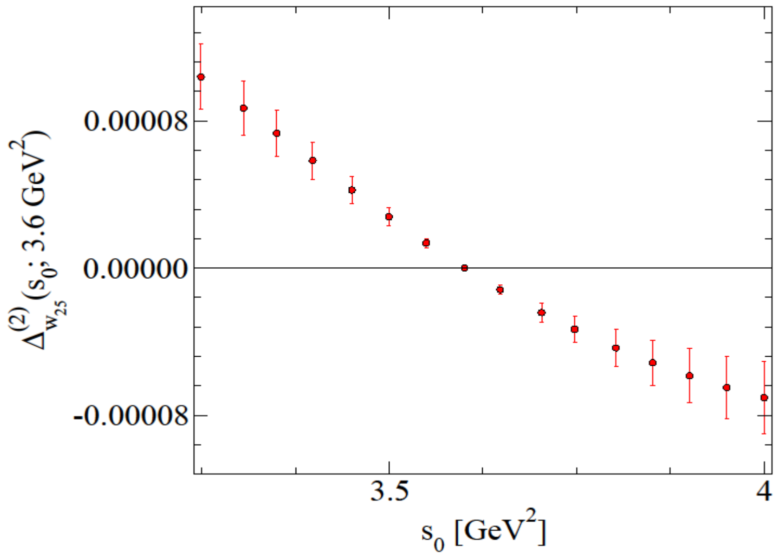
$\Delta^{(2)}(s_0; s_0^*)$ correlated s_0^*=3.6 GeV^2 optimal weight fit results



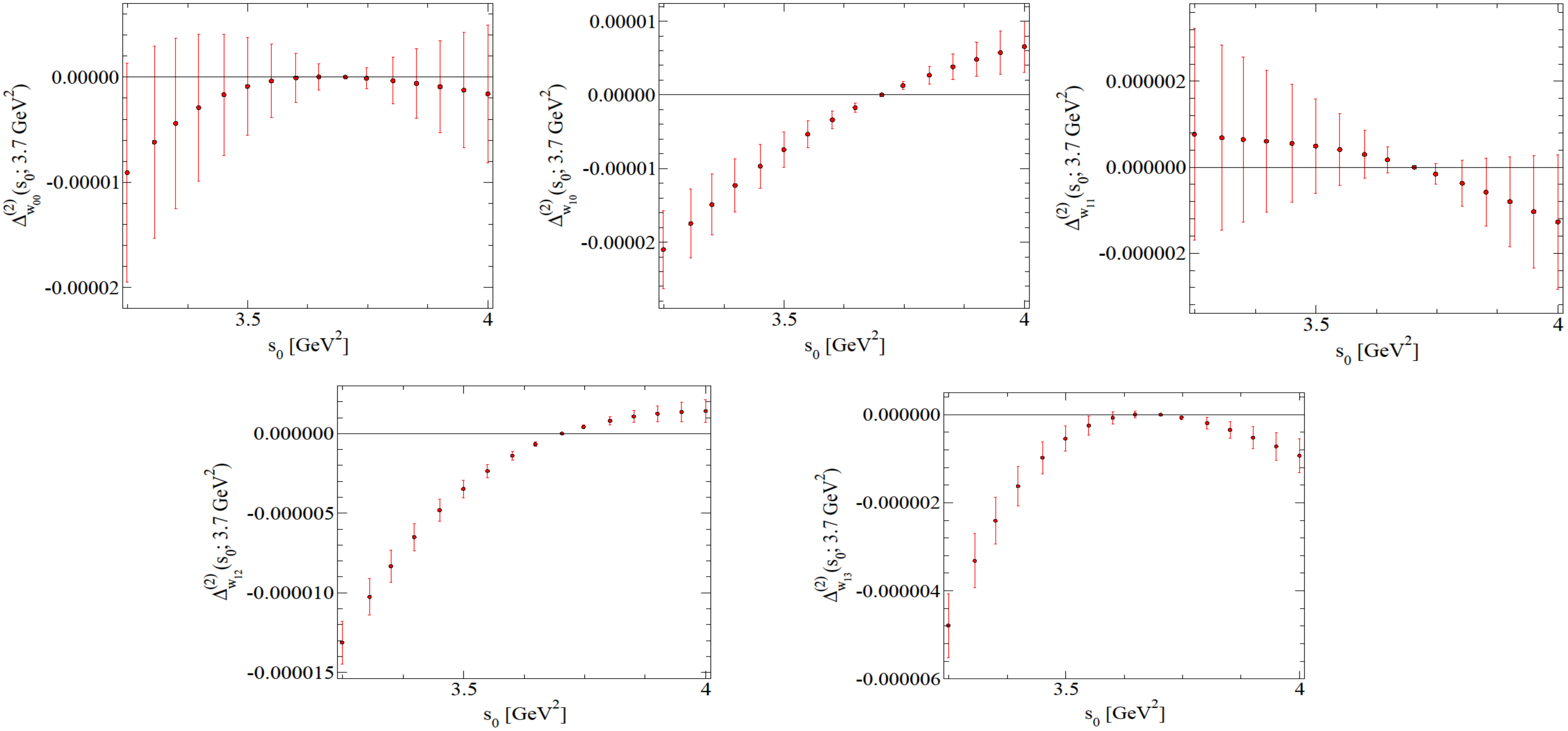








$\Delta^{(2)}(s_0; s_0^*)$ correlated s_0^*=3.7 GeV^2 w_{km} fit results



Conclusions of the R(s)-based tOPE strategy tests

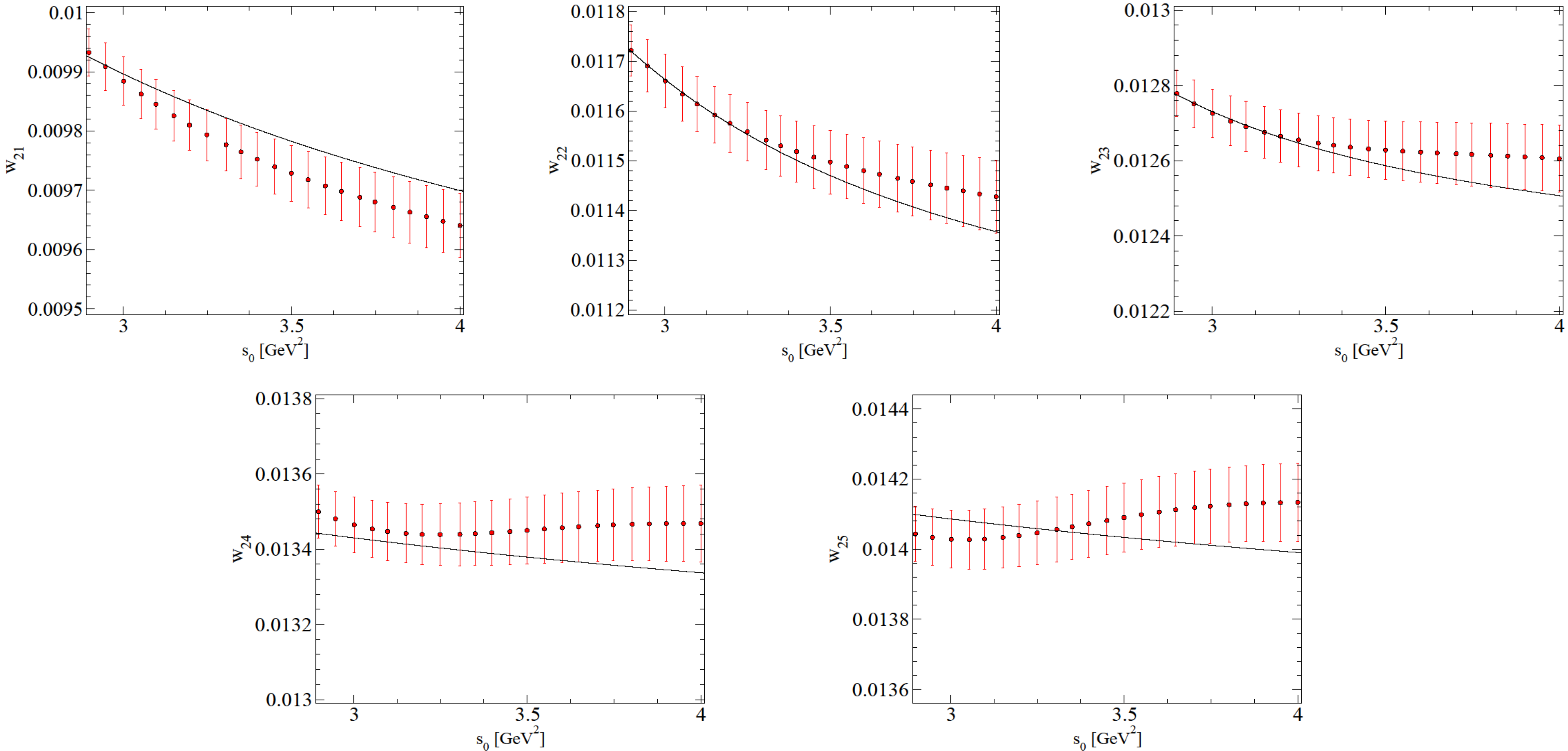
- Good χ^2 from single-s_0 tOPE fit demonstrably insufficient to ensure reliability of neglect of DVs and/or prematurely truncated OPE theory representation

- **$\Rightarrow$ even if integrated DVs negligible for conventional τ analyses (doubtful: see e.g. S. Peris talk), α_s results from tOPE implementations unreliable**

- Strong correlations between different-s_0 spectral integrals, different-s_0 OPE integrals, and fitted OPE and spectral integrals make it easy to be misled re level of theory-experiment agreement: **double-difference-type tests crucial**

BACKUP SLIDES

$s_0^* = m_\tau^2$ diagonal tOPE optimal weight fit theory-experiment matches



$\Delta^{(2)}(s_0; s_0^*=m_\tau^2)$ diagonal optimal weight fit results