Sub-leading microstate counting of AdS black hole entropy

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Integrability in Gauge and String Theory
Sao Paulo August 27, 2020

2008.03239, LPZ, Y. Xin
2007.12604, A. González Lezcano, J. Hong, J. Liu, LPZ
JHEP 03 (2020) 057, F. Benini, D. Gang and LPZ
JHEP 03 (2020) 164, D. Gang, N. Kim and LPZ
PRL 120, 221602 (2018), J. Liu, LPZ, V. Rathee and W. Zhao

Why a black hole talk at IGST confence?

- The AdS/CFT: Field theory observables answer quantum gravity puzzles.
- The Universality and the curse of the Black Hole Entropy formula

$$S = \frac{k_B c^3}{\hbar}\frac{A}{4G}$$

- Corrections/Physics: Gravity as an effective theory (Higher curvature), Quantum corrections (Newton's coupling)
- String theory: ℓ_s and g_s. Field theory: 't Hooft coupling and rank.
- Goal of this talk: Systematically controlling the corrections in the field theory side may require integrability techniques and deeper understanding of certain Bethe Ansätze.

AdS Black Hole Entropy

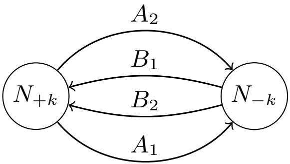
- AdS_4/CFT_3: Leading large-N limit of the topologically twisted index of ABJM reproduces the entropy of magnetically charged AdS_4 black holes [Benini-Hristov-Zaffaroni '15].
- Rotating BH's in various dimensions through superconformal indices; AdS_5 [Cabo-Bizet et al '18, Choi et al '18, Benini-Milan '18].
- Agreement beyond the large N limit by matching the coefficient of $\log N \sim \log\left(\frac{L}{\ell_P}\right)$
  [Liu-PZ-Rathee-Zhao '17, Gang-Kim-PZ '19, Benini-Gang-PZ '19, PZ-Xin '20].

Outline

- The Topologically Twisted Index of 3d $\mathcal{N} = 2$ Chern-Simons matter Theories beyond large N
- Magnetically Charged Asymptotically AdS_4 Black Holes
- Logarithmic Corrections in Quantum Supergravity: $\log\left(\frac{L}{\ell_P}\right)$
- Entropy Counting for AdS$_5$ black holes and the SCI beyond large N
- Open problems

ABJM Theory

- ABJM: A $U(N)_k \times U(N)_{-k}$ Chern-Simons-matter theory.
- Matter sector in bifundamental representations.
- SCFT $\mathcal{N} = 6$ supersymmetry generically but for $k = 1, 2$, the symmetry is enhanced to $\mathcal{N} = 8$.
- Global Symmetry in $\mathcal{N} = 2$ notation is $SU(2)_{1,2} \times SU(2)_{3,4} \times U(1)_T \times U(1)_R$.



General form of the Index

- Background: $S^2 \times S^1$ with background A^R

$$ds^2 = R^2(d\theta^2 + \sin^2\theta d\phi^2) + \beta^2 dt^2, \quad A^R = \frac{1}{2}\cos\theta d\phi.$$

- The index can be expressed as a contour integral:

$$Z(n_a, y_a) = \sum_{\mathfrak{m}\in\Gamma_{\mathfrak{h}}} \oint_{\mathcal{C}} Z_{int}(x, \mathfrak{m}; n_a, y_a).$$

- Z_{int} meromorphic form, Cartan-valued complex variables $x = e^{i(A_t + i\beta\sigma)} = e^{iu}$, lattice of magnetic gauge fluxes $\Gamma_{\mathfrak{h}}$.
- Flavor magnetic fluxes n_a and fugacities $y_a = e^{i(A_t^a + i\beta\sigma^a)}$.
- Localization: $Z_{int} = Z_{\rm class} Z_{\rm one-loop}$.
- E.G.: $Z^{CS}_{class} = x^{k\mathfrak{m}}$, $Z^{gauge}_{1-loop} = \prod_{\alpha\in G}(1 - x^\alpha)(idu)^r$, r – rank of the gauge group, α – roots of G and $u = A_t + i\beta\sigma$.

- The topologically twisted index for ABJM theory:

$$Z(y_a, n_a) = \prod_{a=1}^{4} y_a^{-\frac{1}{2}N^2 n_a} \sum_{I \in BAE} \frac{1}{\det \mathbb{B}} \times \frac{\prod_{i=1}^{N} x_i^N \tilde{x}_i^N \prod_{i \neq j} \left(1 - \frac{x_i}{x_j}\right)\left(1 - \frac{\tilde{x}_i}{\tilde{x}_j}\right)}{\prod_{i,j=1}^{N} \prod_{a=1,2} (\tilde{x}_j - y_a x_i)^{1-n_a} \prod_{a=3,4} (x_i - y_a \tilde{x}_j)^{1-n_a}}.$$

- Contour integral → Evaluation (Poles): $e^{iB_i} = e^{i\tilde{B}_i} = 1$

$$e^{iB_i} = x_i^k \prod_{j=1}^{N} \frac{(1 - y_3 \frac{\tilde{x}_j}{x_i})(1 - y_4 \frac{\tilde{x}_j}{x_i})}{(1 - y_1^{-1} \frac{\tilde{x}_j}{x_i})(1 - y_2^{-1} \frac{\tilde{x}_j}{x_i})},$$

$$e^{i\tilde{B}_j} = \tilde{x}_j^k \prod_{i=1}^{N} \frac{(1 - y_3 \frac{\tilde{x}_j}{x_i})(1 - y_4 \frac{\tilde{x}_j}{x_i})}{(1 - y_1^{-1} \frac{\tilde{x}_j}{x_i})(1 - y_2^{-1} \frac{\tilde{x}_j}{x_i})}.$$

- The $2N \times 2N$ matrix $\mathbb{B}$ is the Jacobian relating the $\{x_i, \tilde{x}_j\}$ variables to the $\{e^{iB_i}, e^{i\tilde{B}_j}\}$ variables

Algorithmic Summary:

- Given the chemical potentials Δ_a according to $y_a = e^{i\Delta_a}$, and variables $x_i = e^{iu_i}$, $\tilde{x}_j = e^{i\tilde{u}_j}$, the equations (poles):

$$0 = k u_i - i \sum_{j=1}^{N} \left[\sum_{a=3,4} \log\left(1 - e^{i(\tilde{u}_j - u_i + \Delta_a)}\right) - \sum_{a=1,2} \log\left(1 - e^{i(\tilde{u}_j - u_i - \Delta_a)}\right) \right] - 2\pi n_i ,$$

$$0 = k \tilde{u}_j - i \sum_{i=1}^{N} \left[\sum_{a=3,4} \log\left(1 - e^{i(\tilde{u}_j - u_i + \Delta_a)}\right) - \sum_{a=1,2} \log\left(1 - e^{i(\tilde{u}_j - u_i - \Delta_a)}\right) \right] - 2\pi \tilde{n}_j .$$

- The topologically twisted index: (i) solve BA equations for $\{u_i, \tilde{u}_j\}$; (ii) insert the solutions into the expression for Z.
- Numerical answer, but exact expression in N.

The large-N limit

- In the large-N limit, the eigenvalue distribution becomes continuous, and the set $\{t_i\}$ may be described by an eigenvalue density $\rho(t)$.

$$u_i = iN^{1/2}\,t_i + \pi - \tfrac{1}{2}\delta v(t_i), \qquad \tilde{u}_i = iN^{1/2}\,t_i + \pi + \tfrac{1}{2}\delta v(t_i),$$

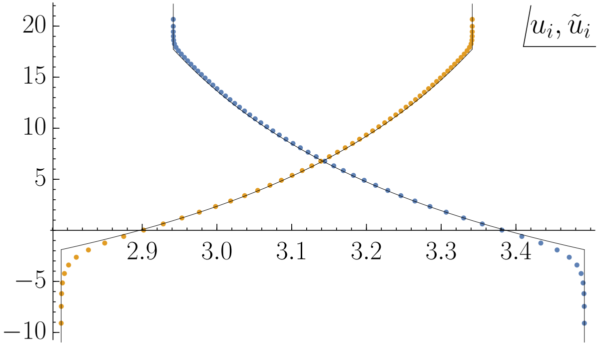


Figure: Eigenvalues for $\Delta_a = \{0.4, 0.5, 0.7, 2\pi - 1.6\}$ and $N = 60$.

- Description of the eigenvalue distribution.

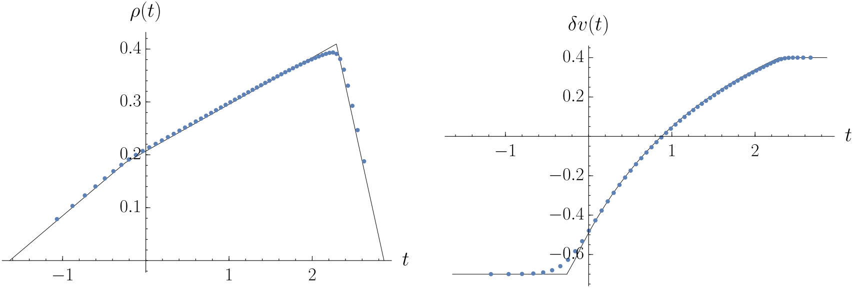


Figure: The eigenvalue density $\rho(t)$ and the function $\delta v(t)$ for $\Delta_a = \{0.4, 0.5, 0.7, 2\pi - 1.6\}$ and $N = 60$, compared with the leading order expression.

$$\mathrm{Re}\log Z = -\frac{N^{3/2}}{3}\sqrt{2\Delta_1\Delta_2\Delta_3\Delta_4}\sum_a \frac{n_a}{\Delta_a}$$

Beyond Large N: Numerical Fits

Δ_1	Δ_2	Δ_3	f_1	f_2	f_3
$\pi/2$	$\pi/2$	$\pi/2$	3.0545	-0.4999	-3.0466
$\pi/4$	$\pi/2$	$\pi/4$	$4.2215 - 0.0491n_1$ $-0.1473n_2 - 0.0491n_3$	$-0.4996 + 0.0000n_1$ $+0.0000n_2 + 0.0000n_3$	$-4.1710 - 0.2943n_1$ $+0.0645n_2 - 0.2943n_3$
0.3	0.4	0.5	$7.9855 - 0.2597n_1$ $-0.5833n_2 - 0.6411n_3$	$-0.4994 - 0.0061n_1$ $-0.0020n_2 - 0.0007n_3$	$-9.8404 - 0.9312n_1$ $-0.0293n_2 + 0.3739n_3$
0.4	0.5	0.7	$6.6696 - 0.1904n_1$ $-0.4166n_2 - 0.4915n_3$	$-0.4986 - 0.0016n_1$ $-0.0008n_2 - 0.0001n_3$	$-7.5313 - 0.6893n_1$ $-0.1581n_2 + 0.2767n_3$

- Numerical fit for:

$$\mathrm{Re}\log Z = \mathrm{Re}\log Z_0 + f_1 N^{1/2} + f_2 \log N + f_3 + \cdots$$

- The values of N used in the fit range from 50 to $N_{\max}$ where $N_{\max} = 290, 150, 190, 120$ for the four cases, respectively.

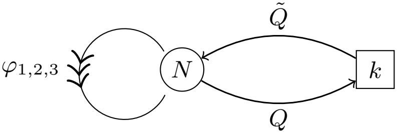
- In the large-N limit, the $k=1$ index takes the form

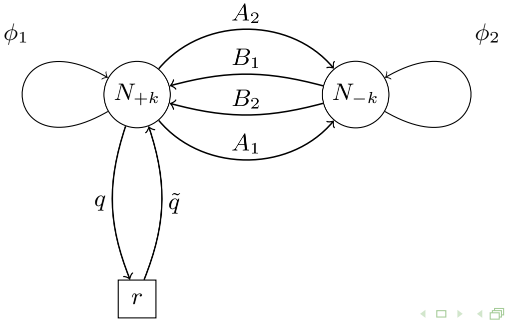
$$F = -\frac{N^{3/2}}{3}\sqrt{2\Delta_1\Delta_2\Delta_3\Delta_4}\sum_a \frac{n_a}{\Delta_a} + N^{1/2} f_1(\Delta_a, n_a)$$
$$-\frac{1}{2}\log N + f_3(\Delta_a, n_a) + \mathcal{O}(N^{-1/2}),$$

where $F = \text{Re}\log Z$.

- The leading $\mathcal{O}(N^{3/2})$ term [BHZ '15], reproduces the Bekenstein-Hawking entropy of extremal AdS_4 magnetic black holes.
- The $-\frac{1}{2}\log N$ term [Liu-PZ-Rathee-Zhao].
- Open problem: The $N^{1/2}$ term describes higher curvature corrections in gravity $R^2_{string} \sim \sqrt{N/k} \sim N^{1/2}$.

- Class 3d $\mathcal{N}=2$ Chern-Simons matter theories: $S^7 \to V^{5,2}, N^{0,1,0}$
- The $-\frac{1}{2}\log N$ term [2008.03239, PZ-Xin]





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Logarithmic terms: An IR window into UV physics

- Logarithmic corrections are determined macroscopically from the massless particle spectrum, insensitive to the UV completion of the theory.
- A consistency check for any enumeration of quantum black hole microstates: Given the field theory answer, use gravity to check its veracity through the logarithmic correction; a litmus test.
- Sen and collaborators have checked many black holes in string theory and challenged loop quantum gravity [Precision Strominger-Vafa].
- Microscopic realization of Kerr/CFT [Strominger].
- Free energy of ABJM [Bhattacharyya-Grassi-Mariño-Sen '12]: $F_{S^3} \sim \mathrm{Ai}(N, k_a) \mapsto -\frac{1}{4}\log N$.

Logarithmic terms in one-loop effective actions

- One-loop effective action is equivalent to computations of determinants.
- Determinant of A through nonvanishing eigenvalues:

$$\frac{1}{2}\,\ln \mathrm{det}' A = \frac{1}{2}\sum_n{}'\,\ln \kappa_n$$

- The heat kernel contains information about both, the non-zero modes and the zero modes, ϵ is a UV cutoff.

$$-\frac{1}{2}\,\ln \mathrm{det}' A = \frac{1}{2}\int_\epsilon^\infty \frac{d\tau}{\tau}\left(\mathrm{Tr}K(\tau) - n_A^0\right)$$

- At small τ, the Seeley-DeWitt expansion for the heat kernel

$$\mathrm{Tr}K(\tau) = \frac{1}{(4\pi)^{d/2}}\sum_{n=0}^{\infty}\tau^{n-d/2}\int d^dx\,\sqrt{g}\,a_n(x,x).$$

Logarithmic terms in one-loop effective actions

- Since, non-zero eigenvalues of a standard Laplace operator A scale as L^{-2}, it is natural to redefine $\bar{\tau} = \tau/L^2$.

$$-\frac{1}{2}\,\ln\det{}'A = \frac{1}{2}\int_{\epsilon}^{\infty}\,\frac{d\bar{\tau}}{\bar{\tau}}\,\Big(\sum_{n=0}^{\infty}\,\frac{1}{(4\pi)^{d/2}}\,\bar{\tau}^{\,n-d/2}\,L^{2n-d}\,\int d^dx\,\sqrt{g}\,a_n(x,x) - n_A^0\Big).$$

- The logarithmic contribution to $\ln\det{}'A$ comes from the term $n = d/2$,

$$-\frac{1}{2}\,\ln\det{}'A = \left(\frac{1}{(4\pi)^{d/2}}\,\int d^dx\,\sqrt{g}\,a_{d/2}(x,x) - n_A^0\right)\log L + \ldots.$$

- On very general grounds (diffeomorphism), the coefficient $a_{d/2}$ vanishes in odd-dimensional spacetimes.

Logarithmic terms: Key Facts

- Robustness: Independent of UV cutoff ϵ.
- In odd-dimensional spaces the coefficient of the $\log$ can only come from zero modes or boundary modes.
- Corrections to entropy from one-loop part of the partition function:

$$S_1 = \lim_{\beta\to\infty}(1-\beta\partial_\beta)\left(\sum_D(-1)^D(\tfrac{1}{2}\log\det{}' D) + \Delta F_0\right),$$

- D stands for kinetic operators corresponding to various fluctuating fields and $(-1)^D = -1$ for bosons and 1 for fermions.
- The zero modes are accounted for separately by

$$\Delta F_0 = \log\int [d\phi]|_{D\phi=0},$$

where $\exp(-\int d^dx\sqrt{g}\phi D\phi) = 1$.

Quantum Supergravity: Key Facts

- The structure of the logarithmic term in 11d Sugra:

$$\log Z[\beta, \dots] = \sum_{\{D\}} (-1)^D (\beta_D - 1) n_D^0 \log L + \Delta F_{\text{Ghost}}.$$

- Subtract the zero modes (-1) and add them appropriately due to integration over zero modes (β_D).
- The ghost contributions are treated separately.

Quantizing C_3 and zero modes

- Ghost contribution to one-loop effective action:

$$\Delta F = \Delta F_{\text{Ghost2form}}.$$

- The 2-form ghost A_2 in 11d has action

$$S_2 = \int A_2 \wedge \star(\delta d + d\delta)^2 A_2,$$

- The logarithmic term in the one-loop contribution to the entropy is

$$(2 - \beta_2) n_2^0 \log L,$$

- Recall β_2 comes from integrating the zero modes in the path integral, and the minus sign takes care of the Grassmann odd nature of A_2.

Final Result

- In the non-extremal case the topology of the black hole is homotopic to its horizon S^2 due to the contractible (t, r) directions.
- The Euler characteristic of the non-extremal black hole $\chi_{\mathsf{BH}} = 2 \mapsto n_2^0 = 2$.
- Recall that the $\log L$ correction to the partition function is $((2-\beta_2)n_2^0 \log L$; using that $\beta_2 = 7/2$ and $n_2^0 = 2(1-g))$ leads to:

$$\log Z[\beta, \dots] = -3(1-g)\log L + \cdots .$$

- The AdS/CFT dictionary: $L/\ell_P \sim N^{1/6}$

$$S = \cdots - \frac{1}{2}\log N + \cdots ,$$

- Perfectly agrees with the microscopic result!!!

Universality of Logarithmic Corrections: Gravity

- Similar results for asymptotically $\mathrm{AdS}_4 \times M^7$ black holes with $M^7 = \{S^7, Q^{1,1,1}, M^{1,1,1}, V^{5,2}, N^{0,1,0}\}$
- Every seven-dimensional, compact Einstein manifold of positive curvature has vanishing first Betti number, $R_{mn} = 6m^2 g_{mn} \Rightarrow \Delta_1 \geq 6m^2$.
- A universal result matched from gravity:

$$S = S_{BH} \quad -\frac{1}{2}\log N + \cdots ,$$

Microscopic Counting of AdS_5 Black Hole Entropy

- Electrically charged, rotating, asymptotically AdS_5 black holes in IIB [Gutowski-Reall '04, Chong-Cvetic-Lu-Pope '05]
- Early attempt at a microscopic explanation using the $\mathcal{N} = 4$ SYM index on $S^1 \times S^3$ did not found enough degrees of freedom [Kinney-Maldacena-Minwalla-Raju '05]
- Three seemingly different resolutions for the microscopic entropy of AdS_5 black holes:
  - Cabo-Bizet-Cassani-Martelli-Murthy (Localization);
  - Choi-Kim-Kim-Nahmgoong (Physical Partition Fn plus constraint)
  - Benini-Milan (SCI, Bethe).
- An embarrassment of richness? Send the *Logarithmic Police !*

The SCI: Luigi's Talk

$$\mathcal{I}(p,q;v) = \text{Tr}_{\mathcal{H}(S^1\times S^3)}\left[(-1)^F\, e^{-\beta\{\mathcal{Q},\mathcal{Q}^\dagger\}} v_a^{Q_a} p^{J_1+\frac{r}{2}} q^{J_2+\frac{r}{2}}\right],$$

- Counts $\frac{1}{16}-$BPS states for $\mathcal{N}=4$ SYM theory and $\frac{1}{4}-$BPS states for generic $\mathcal{N}=1$ SCFT's.
- Q_a are the charges of states that commute with the super charge $\mathcal{Q}$
- r is the R-charge
- The fugacities p and q are associated to the two angular momenta $J_{1,2}$ of S^3.

Matrix Integral Presentation

- The $\mathcal{N}=4$ SYM theory u_i are holonomies along S^1

$$\begin{aligned}\mathcal{I}\left(\tau;\Delta\right) &= \kappa_N \int_{\mathcal{C}} \prod_{\mu=1}^{N-1} du_\mu \frac{\prod_{a=1}^{3}\prod_{i\neq j}\widetilde{\Gamma}\left(u_{ij}+\Delta_a;\tau\right)}{\prod_{i\neq j}\widetilde{\Gamma}\left(u_{ij};\tau\right)}, \\ \kappa_N &= \frac{1}{N!}\left((q;q)_\infty^2 \prod_{a=1}^{3}\widetilde{\Gamma}(\Delta_a;\tau)\right)^{N-1} .\end{aligned}$$

- Saddle point approach.

Saddle-point approach and $SU(N)$ CS Theory

$$N^2 S_{\text{eff}}(\hat{u};\Delta,\tau) = \sum_{i\neq j}\left(\sum_{a=1}^{3}\log\widetilde{\Gamma}(u_{ij}+\Delta_a;\tau)+\log\theta_0(u_{ij};\tau)\right)$$
$$+(N-1)\sum_{a=1}^{3}\log\widetilde{\Gamma}(\Delta_a;\tau)+2(N-1)\log(q;q)_\infty,$$

- Saddle-point equation up to $\mathcal{O}(e^{-1/|\tau|})$

$$i\eta\, v_j = \frac{1}{N}\sum_{k=1\,(\neq j)}^{N}\cot\pi v_{jk}\quad (i=1,\cdots,N)$$

- Exact in N.

$$\mathcal{I}(\tau;\Delta)\sim N\tau^{N-1}e^{-\frac{\pi i(N^2-1)}{2}}\mathcal{A}\,Z^{CS}_{\mathsf{SU}(N)_{k=-\eta N}}$$
$$+\text{(contribution from other saddles)}.$$

BA presentation

$$\mathcal{I}(\tau;\Delta)=\kappa_N\sum_{\hat{u}\in\mathrm{BA}}\mathcal{Z}(\hat{u};\Delta,\tau)H(\hat{u};\Delta,\tau)^{-1},$$

$H\left(\hat{u};\Delta,\tau\right)$ is the Jacobian $\left(u_i\to Q_i\right)$

$$\mathcal{Z}\left(\hat{u};\Delta,\tau\right)=\prod_{i\neq j}^{N}\frac{\prod_{a=1}^{3}\widetilde{\Gamma}(u_{ij}+\Delta_a;\tau)}{\widetilde{\Gamma}\left(u_{ij};\tau\right)}$$

$$Q_i(\hat{u};\Delta,\tau)\equiv e^{2\pi i\lambda}\prod_{j=1}^{N}\frac{\theta_1(u_{ji}+\Delta_1;\tau)\theta_1(u_{ji}+\Delta_2;\tau)\theta_1(u_{ji}-\Delta_1-\Delta_2;\tau)}{\theta_1(u_{ij}+\Delta_1;\tau)\theta_1(u_{ij}+\Delta_2;\tau)\theta_1(u_{ij}-\Delta_1-\Delta_2;\tau)},$$

$$Q_i(\hat{u};\Delta,\tau)=1\qquad \mathrm{Bethe-Ansatz\ \ Equation}$$

- Basic BA solutions: $u_{ij}=\frac{\tau}{N}\left(i-j\right)$

Evaluation beyond large N [González-Hong-Liu-PZ]

- Exact in N in terms of SU(N) Chern-Simons matrix model. Different saddles contributing at N^2.

$$\mathcal{I}(\tau;\Delta) = \mathcal{I}(\tau;\Delta)\Big|_{\text{Main Saddle Point}} + (\text{ other saddles})$$
$$\log \mathcal{I}(\tau;\Delta)\Big|_{\text{Main Saddle Point}} = -\frac{\pi i(N^2-1)}{\tau^2}\prod_{a=1}^{3}[\Delta_a]_\tau + \log N + \mathcal{O}(e^{-1/|\tau|}).$$

- Valid for all τ.

$$\mathcal{I}(\tau;\Delta) = \mathcal{I}(\tau;\Delta)\Big|_{\text{Basic BA}} + (\text{other BA solutions})$$
$$\log \mathcal{I}(\tau;\Delta)\Big|_{\text{Basic BA}} = -\frac{\pi i(N^2-1)}{\tau^2}\prod_{a=1}^{3}[\Delta_a]_\tau + \log N + \mathcal{O}(N^0),$$

Similar Results for toric 4d $\mathcal{N}=1$

- The SCI of toric quiver gauge theories is:

$$\mathcal{I}^{\Gamma_e} = \kappa_G \oint_{\mathbb{T}^{\mathsf{rk}(G)}} \frac{\prod_{\Phi_{ab}} \prod_{i_a \neq j_b} \Gamma_e\left(u_{i_a} - u_{j_b} + \Delta_{ab}; \tau, \sigma\right)}{\prod_{\alpha \in \Delta} \Gamma_e\left(\alpha\left(u\right); \tau, \sigma\right)} \prod_{i=1}^{\mathsf{rk}(G)} \frac{dz_i}{2\pi i z_i}$$

$$\kappa_G = (q;q)_\infty^{2\mathsf{rk}(G)} / |\mathcal{W}_G|, \;\; z_i = e^{2\pi i u_i}$$

- Evaluating the integral [Benini-Milan, Closset-Kim-Willet]:

$$\mathcal{I}^{BA} = \kappa_G \sum_{\hat{u} \in \mathfrak{M}_{BAE}} \mathcal{Z}\left(\hat{u};\tau\right) H\left(\hat{u};\tau\right)^{-1}; \; \mathfrak{M}_{BAE} = \{u : \mathcal{Q}_{i_a}\left(u;\tau\right) = 1\}$$

where $H = \det\left(\left[\frac{1}{2\pi i}\frac{\partial Q_{i_a}}{\partial u_{j_b}}\right]_{i_a j_b}\right)$ and $\mathcal{Q}_{i_a}$ is the BA operator.

- Logarithmic correction: $+\log(n_v N)$.

Open Problems

- The IIB gravity computation of $\log N$, in progress [with Marina David]
- Higher curvature corrections:
  - ABJM: $R^2_{string} \sim \sqrt{N/k} \sim N^{1/2}$. MSW precedent, Wald entropy [with Jewel Ghosh]
  - $\mathcal{N}=4$ SYM: The SCI should be independent of couplings, $\sqrt{\lambda} \sim R^2_{string} \Longrightarrow$ No higher derivative corrections!
- Univesal picture of AdS$_{4,5,6,7}$ black hole entropy from near-horizon (Kerr-AdS/CFT) [2005.10251, David-Nian-PZ].
- Integrabily Connection:
  - Insights and technical arguments to tackle the BA equations for the topologically twisted index and the superconformal index.
  - Understanding of the space of BA solutions as function of fugacities equals understanding of phases in quantum supergravity.