Chern-Simons Origin of Superstring integrability

B. Stefański
City, University of London

2005.03064 w/ Kevin Costello

to appear in PRL

4d Chern-Simons

Costello
Costello, Witten, Yamazaki

Long-suspected connection CS theory $\leftrightarrow$ YBE realised for

$$S_{4d} = \int \omega \wedge \left[A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right]$$

gauge group G

$w\ \bar{w}$ $z\ \bar{z}$

ω a holomorphic one-form $\Sigma_{(\ ,\ )} \times C_{(\ ,\ )}$ eg $\omega = dz$

$A = A_w\, dw + A_{\bar{w}}\, d\bar{w} + A_{\bar{z}}\, d\bar{z}\ \cancel{+ A_z\, dz}$

Σ: Riemann surface ($\mathbb{R}^2$ today)

C: complex curve ($\mathbb{C}_\infty$ today)

4d Chern-Simons

$$S_{4d} = \frac{1}{\hbar} \int_{\Sigma_{(w,\bar{w})} \times C_{(z,\bar{z})}} \omega \wedge \left[A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right]$$

Eoms:
$(\omega = dz)$

$F_{w\bar{w}} = 0$ flat bdl on Σ

$F_{wz} = F_{\bar{w}\bar{z}} = 0$ holomorphic on C

Mixed topological / holomorphic theory

All counterterms vanish by Eoms

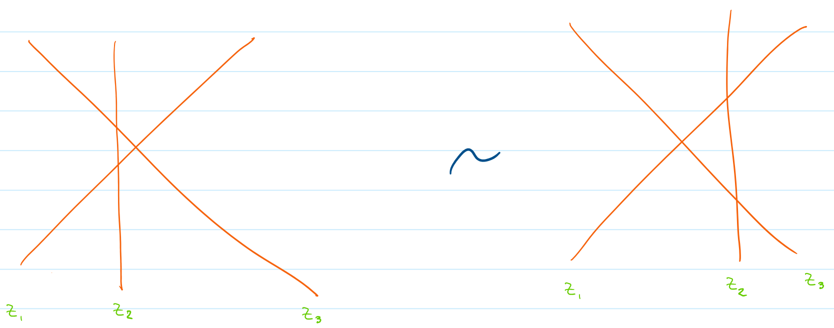
$\rightsquigarrow$ quantization possible

IR-free theory

4d C-S has no local observables (like 3d CS)

Observables are open[*] Wilson lines $\Sigma \times \{z_a\}$

$w, \bar{w}$

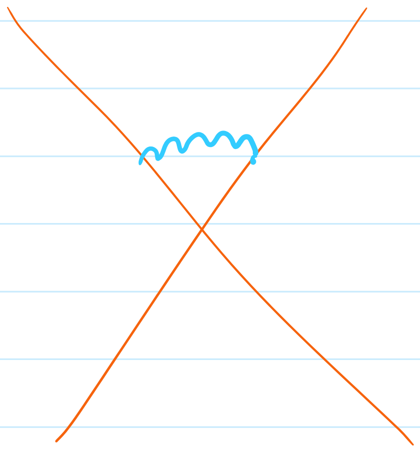


topological in Σ : so $\diagdown \sim |$ YBE ✓

lines don't cross in 4d

(*) IR-free theory gauge fields vanish at infinity so holonomy good observable (no trace)

Interactions in 4d CS



gluon interactions
irrelevant between
uncrossed Wilson lines

$$= 1 + \hbar\, r(z_1 - z_2)$$

r is classical r-matrix
of hom. G spin chain!

Metsaev-Tseytlin σ-model from 4d CS

Costello+Yamazaki '19: A, ω poles + zeros in $\mathbb{C}$

zoo of integrable σ-models from S_{4d} !

Lax: $\mathcal{L}_{w,\bar{w}} = \langle A_{w,\bar{w}} \rangle$

Like CY'19 we set $G = psu(2,2|4)^4$

$$\omega = \frac{\prod_{k=1}^{3} (z - q_k)(z - \tilde{q}_k)}{\prod_{i=1}^{4} (z - p_i)^2} \, dz$$

ation

psu(1

Quantum 4d CS well-defined / for (4)4

$$A_w \sim \frac{1}{z - q_k} \qquad A_{\bar{w}} \sim \frac{1}{z - \tilde{q}_k}$$

$$A_w, A_{\bar{w}}, A_{\bar{z}} \sim (z - p_i)$$

gauge $\mathbb{Z}_4$ sym $\left\{ \begin{array}{l} A^i \longrightarrow \Omega(A^{i+1}) \\ z \longrightarrow e^{\pi i/2} z \end{array} \right.$

get σ-model on 2,2 4) SO)×SO(5)

Pure spinor $q_{1,2} \to 0$ Metsaev-Tseytlin $q_1, q_2, \tilde{q}_2 \to 0$

Pure spinor $q_K \to 0$

$\tilde{q}_K \to \infty$

Metsaev-Tseytlin $q_1, q_2, \tilde{q}_3 \to 0$

$\tilde{q}_1, \tilde{q}_2, q_3 \to \infty$

$$\boxed{\begin{array}{ll} A_w \sim \dfrac{1}{z^2} & A_{\bar{w}} \sim z^2 \\ A_{\bar{w}} \sim \dfrac{1}{z} & A_w \sim z \\ z \sim 0 & z \sim \infty \end{array}}$$

4d CS theory and diffeomorphisms

On manifold without boundary
4d CS is diffeomorphism invariant (no metric)

With bdry/singular fields not necessarily!

$$\delta_v A_a \sim \partial_a v^b A_b \qquad a,b = w,\bar{w},z,\bar{z}$$

Near $z=0$ (v is regular)

$$A_{\bar{w}} \sim \frac{1}{z} \qquad A_w \sim \frac{1}{z^2}$$

δ_v mixes $A_{\bar{w}}$ and A_w so not compatible with

Beltrami-Chern-Simons Theory

Restore diff inv. $\leadsto$ add new field β coupling to de Rham differential

$$d \longrightarrow d + d\bar{w}\,\mathcal{L}_{\beta_w \partial} + d\bar{z}\,\mathcal{L}_{\beta \partial}$$

Lie derivative along $\beta\partial$

$$S_{4d} \longrightarrow S_{4d} + S_\beta$$

Cartan's formula for $\mathcal{L}$

$$\mathcal{L}_V = [d, \iota_V]$$

ι_V interior product

S_β : BRST-exact $\leadsto$ field redefinition removes S_β

$$A_{\bar{w}} \longrightarrow A_{\bar{w}} - \beta_{\bar{w}} A_w$$

$$S_{4d} + S_\beta \longrightarrow S_{4d}$$

$A_w, A_{\bar{w}}$ poles $\Rightarrow$ CANNOT do redefinition β a bona fide field

$$\boxed{S_{BCS} \equiv S_{4d} + S_\beta}$$

Easy check $\delta_v S_{BCS} = 0$ diff ✓

$$\delta_\beta S_{BCS} = 0 \Longrightarrow A^{(2)}_w A^{(2)}_w \Big|_{z=0} = 0$$

$$A^{(2)}_{\bar{w}} A^{(2)}_{\bar{w}} \Big|_{z=\infty} = 0$$

VIRASORO!

Kappa symmetry

S_{BCS} gauge invariant

$\delta_\chi S_{BCS} = 0$ away from boundaries

Usually gauge variation $\chi = 0$ on bdry
to eliminate bdry terms in $\delta_\chi S_{BCS}$

Instead, we allow

$$\chi \sim \frac{\zeta^{(3)}}{z} + \text{regular}$$

$\mathbb{Z}_4$ $(i)^3$ eigenspace

$\because \chi$ is $\mathbb{Z}_4$ inv

$\zeta^{(3)}$ FERMIONIC
as expected for κ-symmetry

So expect $\delta_\chi S_{BCS}\big|_{z=0} \neq 0$

$z = 0$

$A_{\bar w} \sim \frac{A^{(3)}_{\bar w}}{z} + \dots,$ $\quad A \quad \frac{A^{(2)}}{z^2} \quad \frac{A^{(3)}}{z}$, $\quad \omega \quad z^3$

Explicitly, near

$$\delta_\chi S_{4d} = \int \omega \left(A_{\bar w}\, \partial_{\bar z} [A_w, \chi] - A_w\, \partial_{\bar z} [A_{\bar w}, \chi] \right)$$

$$\sim \int z^3 \left(\frac{A^{(3)}_{\bar w}}{z}\, \partial_{\bar z}\left(\frac{1}{z^3}\right) [A^{(2)}_w, \zeta^{(3)}] + \frac{A^{(3)}_{\bar w}}{z}\, \partial_{\bar z}\left(\frac{1}{z^2}\right) [A^{(3)}_w, \zeta^{(3)}] \right.$$
$$\left. + \frac{A^{(2)}_w}{z^2}\, \partial_{\bar z}\left(\frac{1}{z^2}\right) [A^{(3)}_{\bar w}, \zeta^{(3)}] + \frac{A^{(3)}_w}{z}\, \partial_{\bar z}\left(\frac{1}{z^2}\right) [A^{(3)}_w, \zeta^{(3)}] \right) \, dz$$

These terms vanish in trace + integration

$$= \int z^2 \delta''(z) A^{(3)}_{\bar w} [A^{(2)}_w, \zeta^{(3)}] \mp \cancel{z\, \delta'(z) A^{(2)}_w [A^{(3)}_{\bar w}, \zeta^{(3)}]} \cdots$$

$$= 2 \int_{z=0} [A^{(3)}_{\bar w}, A^{(2)}_w]\, \zeta^{(3)}$$

$z = \infty$ $\quad \delta_\chi S_{4d} = 2 \int_{z=\infty} [A^{(1)}, A^{(2)}]\, \zeta^{(1)}$

$z=\infty$

$\delta_\chi S_{4d} \neq 0$ localized at $z=0,\infty$

$\delta_\chi S_{4d}$ familiar from Arutyunov + Frolov !

it is χ-variation of matter fields in GS action

Brink, Schwarz, Scherk
Gliozzi, Scherk, Olive

For some G, famous Fierz identities $\Longrightarrow$

can cancel $\delta_\chi S_{4d}$ by δ_χ(metric)

For BCS this is $\xi^{(3)} = A^{(2)}_w \chi^{(3)}$ $\chi^{(3)} A^{(2)}_w$

$$\delta_\chi \beta_{\bar{w}} = \frac{\delta_{|z|<\varepsilon}}{2} \operatorname{tr}\left(\Upsilon \left[\chi^{(3)}_{\bar{w}}, A^{(3)}_{\bar{w}}\right]\right)$$

giving $\boxed{\delta_\chi S_{BCS} = 0}$

$\bar{w}^+$ $\bar{w}$

Quantizing BCS theory

Vacuum config has to reduce to BMN geodesic

$$J_w = J_{\bar{w}} = \kappa\,(T_E - T_J) \qquad T_E, T_J \in psu(2,2|4)$$

with the MT boundary conditions

$$a_{\bar{z}}^{BMN} = (w+\bar{w})\,\partial_{\bar{z}} f\,(T_E - T_J)$$

$$a_w^{BMN} = \left(f + \frac{1}{z^2}\right)(T_E - T_J)$$

$$a_{\bar{w}}^{BMN} = \left(f + z^2\right)(T_E - T_J)$$

where $f(z,\bar{z})$ is a simple fn w/ suitable asymptotics $\left(f, \partial_{\bar{z}} f \sim 0 \quad z = 0,\infty \;,\; f \sim (-1)^i \quad z = p_i\right)$

The BRST operator w/ bkd a^{BMN} is

$$Q_{BRST} = d_B + a^{BMN}$$

This breaks

$$psu(2,2|4) \longrightarrow psu(2|2)^2$$

and is starting point for quantization

Conclusions

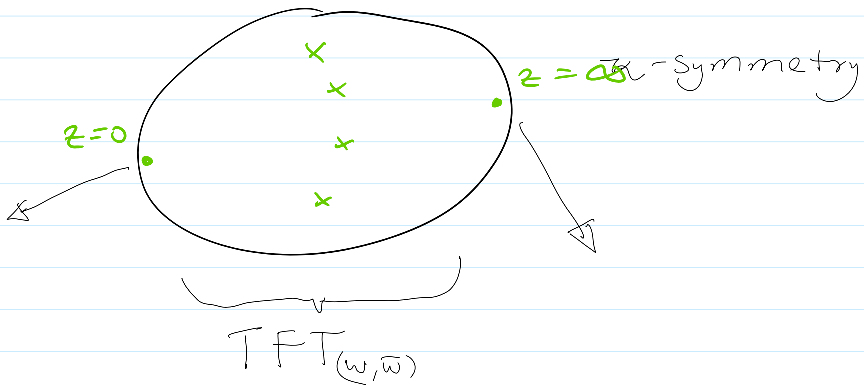
Beltrami-Chern-Simons theory
is a new formulation of
Green-Schwarz superstring
with integrability manifest

works in
plane-wave & flat space Same Fierz

other bkds?

(same Fierz)

QuantizBCS theory ~ Beltramis, ghosts,



chiral b-c ghosts

from BCS

anti-chiral b-c ghosts

Derive holographic integrability