Entanglement in QM and QFT

5/5 -Holographic entanglement entropy

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It from Qubit
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Quantum Fields, Gravity and Information

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— **II School of Holography and Entanglement Entropy** —
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Outline

1. The holographic principle and AdS/CFT
2. The Ryu-Takayanagi prescription
3. Corrections to the RT formula
4. Some explicit calculations
5. Gravity from entanglement

Some References

- A nice review of the holographic principle is https://arxiv.org/pdf/hep-th/0203101. The original insight is due to 't Hooft.
- The original AdS/CFT paper by Maldacena is https://arxiv.org/pdf/hep-th/9711200.pdf (most cited paper in the history of physics). Seminal papers by Witten: https://arxiv.org/pdf/hep-th/9802150.pdf and Gubser, Klebanov and Polyakov: https://arxiv.org/pdf/hep-th/9802109.pdf.
- The subject of holographic entanglement entropy started with the seminal papers of Ryu and Takayanagi https://arxiv.org/pdf/hep-th/0603001 and https://arxiv.org/pdf/hep-th/0605073, where they presented their prescription. The covariant version, due to Hubeny, Rangamani and Takayanagi was proposed in https://arxiv.org/pdf/0705.0016.
- Other relevant papers on the subject including various generalizations are: https://arxiv.org/pdf/1310.5713.pdf, https://arxiv.org/pdf/1101.5813.pdf, https://arxiv.org/pdf/1307.2892, https://arxiv.org/pdf/1304.4926, https://arxiv.org/pdf/1102.0440. There are many more...
- Various reviews on the subject of holographic entanglement entropy can be found in https://arxiv.org/pdf/0905.0932, https://arxiv.org/pdf/1609.01287.pdf, http://www2.yukawa.kyoto-u.ac.jp/~tadashi.takayanagi/CERNEE.pdf, https://arxiv.org/pdf/1609.00026.pdf
- The derivation of the linearized gravity equations from the entanglement first law is from https://arxiv.org/pdf/1312.7856.pdf.

The holographic principle and AdS/CFT

Black hole entropy and holographic principle

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The idea that all things happening within a volume A should be describable in terms of things happening at its boundary ∂A is called the **holographic principle**.

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We can use the tools of GR to learn quantum things!

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(For today, we mostly restrict our discussion to the vacuum state.)

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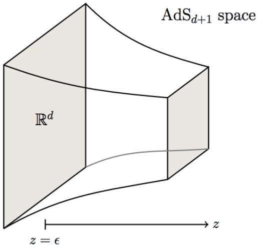
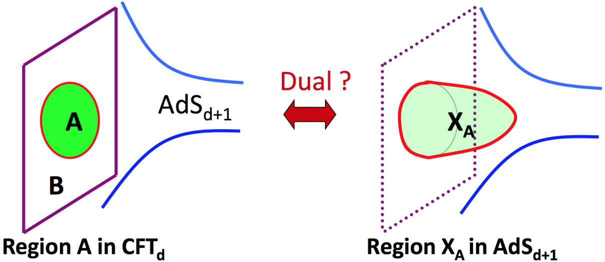
- Greater values of z probe the "bulk region" and corresponds to low energies. Features which distinguish different states become apparent in the geometry (*e.g.*, by the presence of a horizon).



The Ryu-Takayanagi prescription

Holographic entanglement entropy

One may wonder: is there a region in AdS_{d+1} which encodes the information corresponding to a given region A in the CFT_d?



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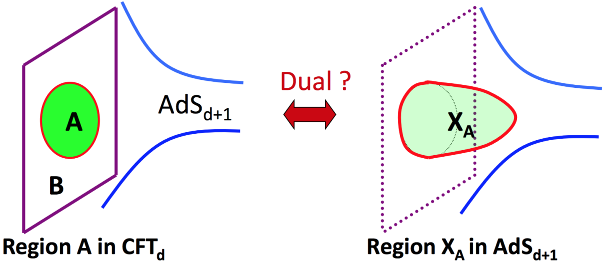


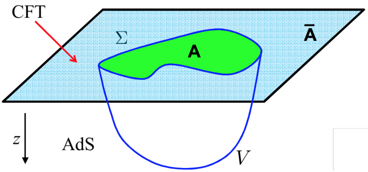
This question was partially answered by Ryu and Takayanagi with their sensational proposal for the computation of entanglement entropy of CFTs with a holographic dual...

Holographic entanglement entropy

The **Ryu-Takayanagi prescription** tells us that the EE for a region A in the CFT [dual to Einstein (super)gravity in the bulk] can be computed as

$$S_{\text{HEE}}(A) = \min_{V \sim A} \left[\frac{\text{Area}(V)}{4G} \right]$$

We need to minimize amongst all bulk surfaces V whose boundary coincides with the one of A



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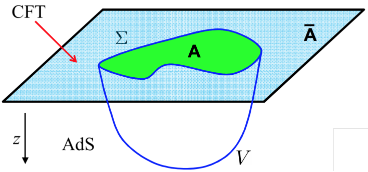

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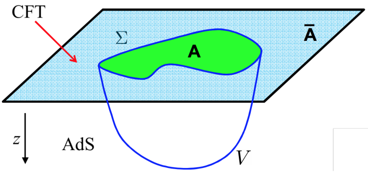

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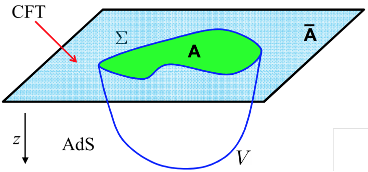

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- However, the formula passed numerous tests, including reproducing all expected properties of EE such as SSA, structure of universal terms, etc. in various dimensions.
- Using the AdS/CFT dictionary, the replica trick and the interpretation of EE as the $n \to 1$ limit of Rényi entropies, Lewkowycz and Maldacena finally rigorously proved it.

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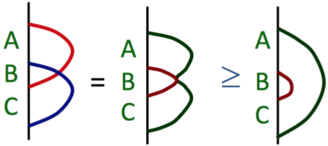

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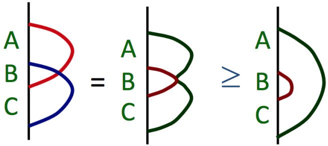

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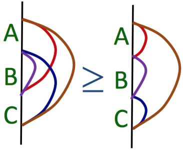
Holographic entanglement entropy satisfies an additional property called "monogamy", not satisfied by general CFTs: $I_3(A, B, C) \equiv I(A, B) + I(A, C) - I(A, BC) \leq 0$



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Corrections to the RT formula

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$$S_{\mathrm{EE}}^{\mathrm{Riem}^2} = \frac{\mathcal{A}(V)}{4G} + \frac{L^2}{4G}\int_V \mathrm{d}^{d-1}y\sqrt{h}\left[2\alpha_1 R + \alpha_2(R_a^a - \frac{1}{2}K^aK_a) + 2\alpha_3\left(R_{ab}^{ab} - K_{aij}K^{aij}\right)\right]$$

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General formula exists for arbitrary theories, but there are some obstructions to its application beyond the present cases.

Quantum corrections

The RT expression is a "tree-level" result, but there are quantum corrections, which are subleading in $1/G$. The expression including the leading one reads:

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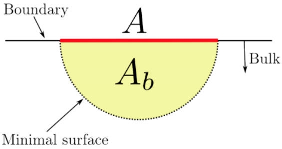

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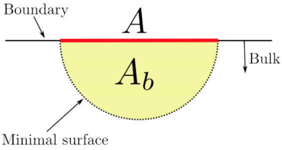

The leading correction (order zero in $1/G$) comes from the entanglement entropy of bulk modes subject to the bipartitioning across the RT surface.

Some explicit calculations

Interval in a CFT$_2$

In Poincaré coordinates, $\mathrm{d}s^2_{\mathrm{AdS}_3} = L_\star^2/z^2[-\mathrm{d}t^2 + \mathrm{d}z^2 + \mathrm{d}x^2]$, we consider the EE of an interval $x \in [-\ell/2, \ell/2]$ in a time slice.

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where we introduced a UV cutoff at $Z = \delta$. This is exactly the result expected for a CFT_2 with central charge $c = 3L_\star/(2G)$. The value of c for holographic Einstein gravity can be computed via alternative methods and perfectly matches this result!

Spheres in a CFT$_d$

We write now the AdS$_{d+1}$ metric as: $\mathrm{d}s^2_{\mathrm{AdS_{d+1}}} = \frac{L_\star^2}{z^2}[-\mathrm{d}t^2 + \mathrm{d}z^2 + \mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2_{d-2}]$.

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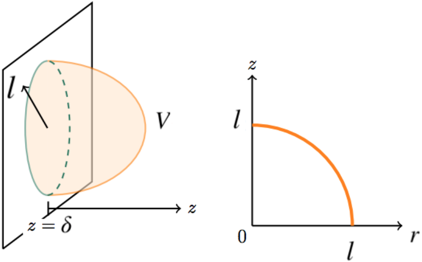
$$rZZ'' + (d-2)ZZ'(1 + Z'^2) + (d-1)r(1 + Z'^2) = 0$$

Spheres in a CFT$_d$

The solution for this differential equation satisfying the boundary condition $Z(\ell) = 0$ is $Z = \sqrt{\ell^2 - r^2}$.



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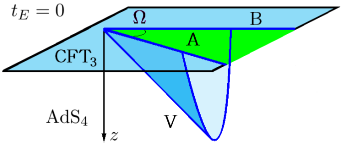
This is the expression expected for a spherical entangling surface in a CFT$_d$. The coefficient $a^\star$ can also be computed using alternative methods and it exactly matches the one obtained here.

Corners in a CFT_3

What about a corner region?



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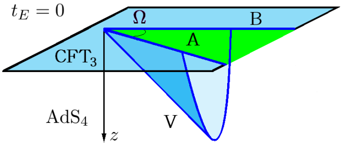

The holographic result reads

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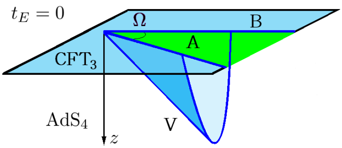

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where the holographic corner function is given by

$$a_{\mathrm{E}}(\Omega) = \frac{L_\star^2}{2G}\int_0^\infty \mathrm{d}y\left[1 - \sqrt{\frac{1+h_0(\Omega)^2(1+y^2)}{2+h_0(\Omega)^2(1+y^2)}}\right]\,,$$

$$\Omega(h_0) = \int_0^{h_0}\mathrm{d}h\,\frac{2h^2\sqrt{1+h_0^2}}{\sqrt{1+h^2}\sqrt{(h_0^2-h^2)(h_0^2+(1+h_0^2)h^2)}}\,.$$

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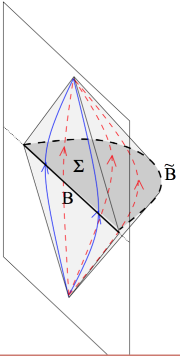
- The modular Hamiltonian is a complicated and non-local object in general. However, for a general CFT$_d$ with stress tensor T_{ab}, for a spherical ball $B(R, \vec{x}_0)$ of radius R centered at $\vec{x}_0$, it has the following useful representation

$$\langle H_{\text{ball}} \rangle = 2\pi \int_{B(R,\vec{x}_0)} \mathrm{d}^{d-1}x \left[\frac{R^2 - |\vec{x} - \vec{x}_0|^2}{2R} \right] \langle T_{tt} \rangle$$

Gravity from entanglement

- Holographically, entanglement entropy is computed via the RT prescription, and the surface corresponding to a ball-shaped region B is given by $\tilde{B} = \{t = t_0, |x^i - x_0^i|^2 + z^2 = R^2\}$. We have then for holographic theories

$$\delta S_{\text{EE}}(B) = \delta \langle H_B \rangle \overset{\text{AdS/CFT}}{\Longrightarrow} \frac{\delta \mathcal{A}(\tilde{B})}{4G} = \delta \langle H_B^{\text{grav.}} \rangle$$



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- On the other hand, using the CHM map, we know that the EE for a ball-shaped region can be obtained as the thermal entropy of the CFT in hyperbolic space, $S_{\text{EE}}(B) = S_{\text{therm.}}(\mathbb{H}^{d-1})$.

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- Holographically, this means that the EE can be obtained from the Bekenstein-Hawking entropy of a hyperbolic black hole.

$$\frac{\delta \mathcal{A}(\tilde{B})}{4G} = \delta \left\langle H_B^{\text{grav.}} \right\rangle \overset{\text{CHM}}{\Longleftrightarrow} \delta S_{\text{BH}} = \delta \left\langle H_B^{\text{grav.}} \right\rangle$$

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$$\delta\left[R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - 8\pi G T_{\mu\nu}\right] = 0 \quad \overset{\text{IW}}{\Longrightarrow} \quad \delta S_{\text{BH}} = \delta\left\langle H_B^{\text{grav.}}\right\rangle$$

$$\overset{\text{CHM+AdS/CFT}}{\Longleftrightarrow} \delta S_{\text{EE}}(B) = \delta\left\langle H_B\right\rangle$$

Namely, the linearized Einstein equations imply the quantum first law of entanglement entropy for ball-shaped regions.

Gravity from entanglement

- Interestingly, the opposite implication also holds!

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- **Classical (linearized) gravitational dynamics in AdS spaces follows from a fundamental quantum principle!**

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- Linearized gravitational dynamics in AdS spaces follows from the first law of EE through the Ryu-Takayanagi formula. This is an explicit illustration of the "It from qubit" idea...

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