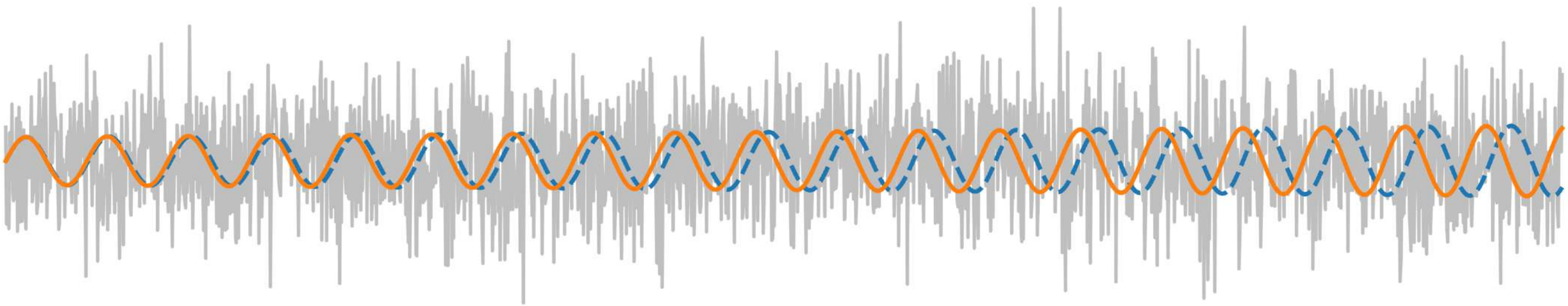
Detecting Dark Matter around Black Holes with Gravitational Waves

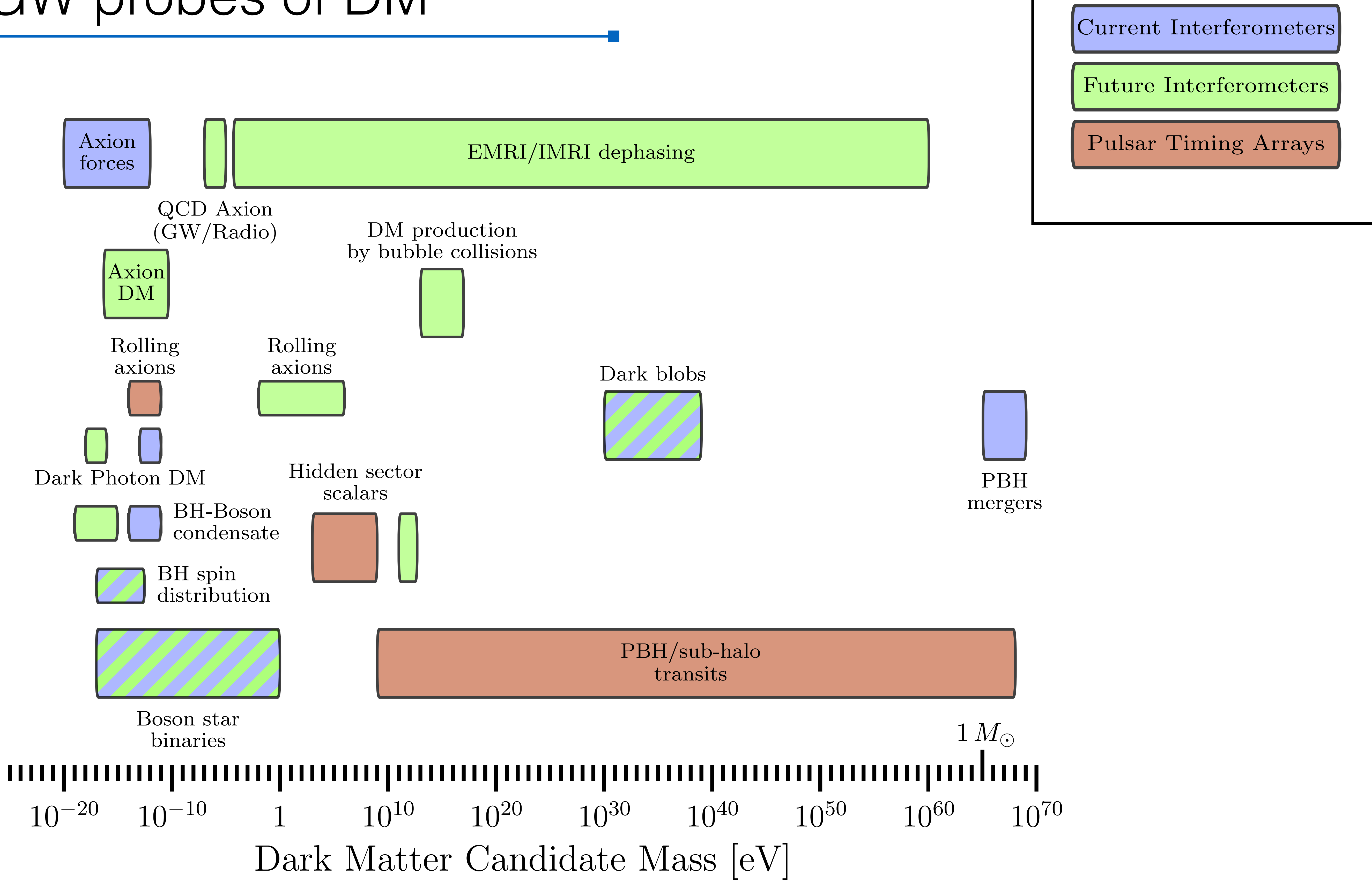


Bradley J Kavanagh
Instituto de Física de Cantabria
(CSIC-Universidad de Cantabria)

#SADM3 - 4th December 2020

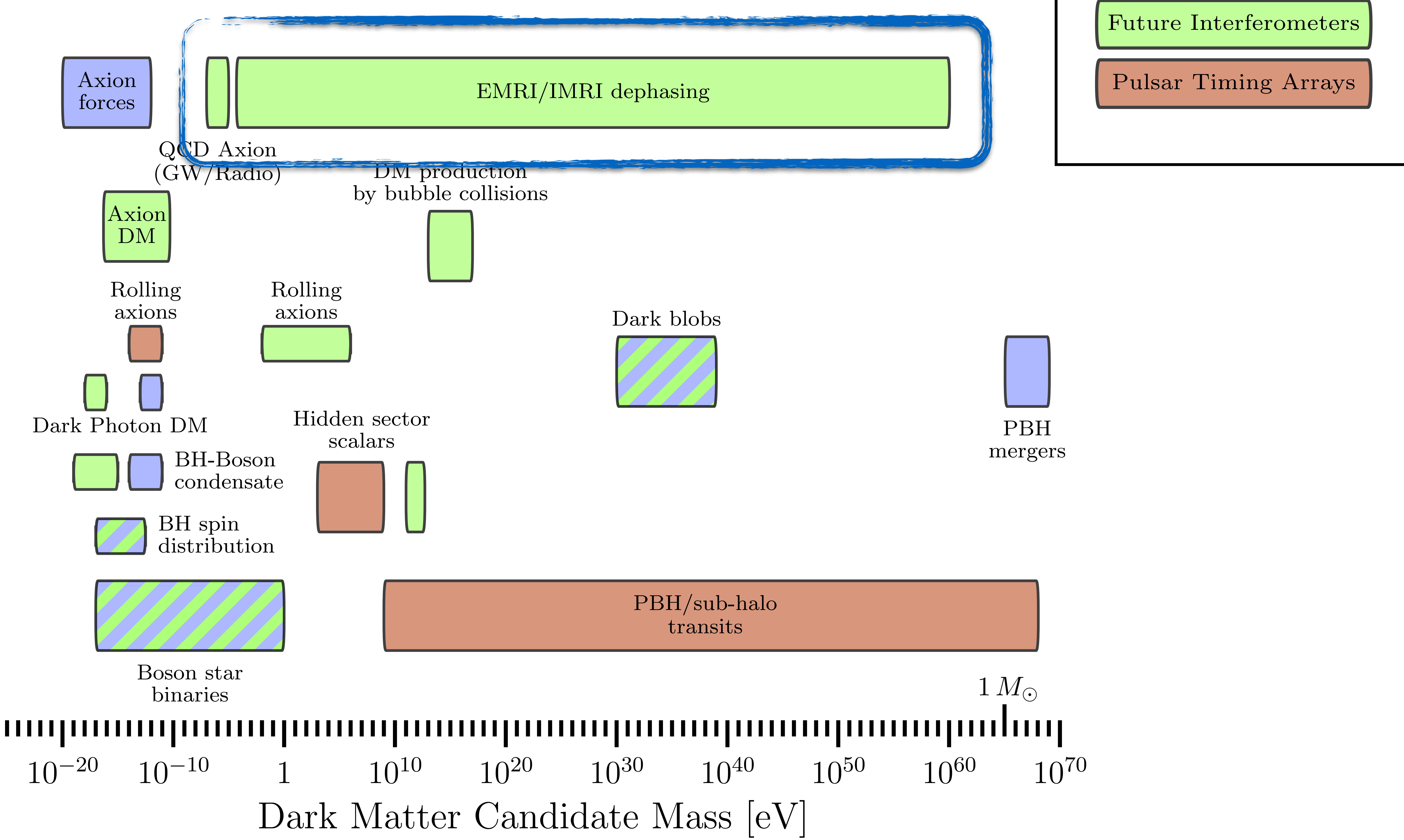
Instituto de Física de Cantabria
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS
UNIVERSIDAD DE CANTABRIA
kavanagh@ifca.unican.es
@BradleyKavanagh

GW probes of DM



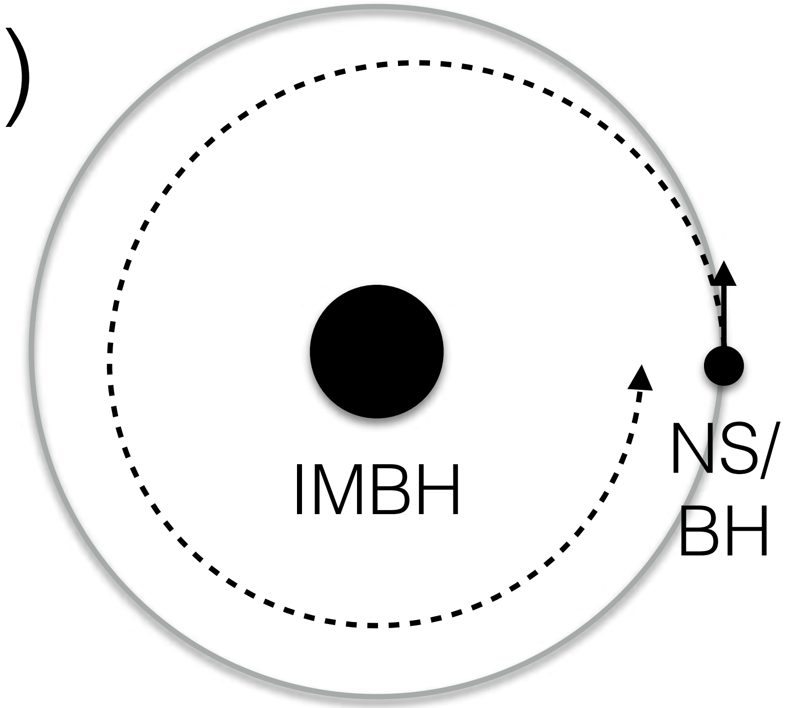
[Bertone, Croon, Amin, Boddy, **BJK**, Mack, Natarajan, Opferkuch, Schutz, Takhistov, Weniger, Yu,
SciPost Phys. Core 3, 007 (2020), 1907.10610]

GW probes of DM



[Bertone, Croon, Amin, Boddy, **BJK**, Mack, Natarajan, Opferkuch, Schutz, Takhistov, Weniger, Yu, SciPost Phys. Core 3, 007 (2020), 1907.10610]

Intermediate Mass Ratio Inspiral (IMRI)

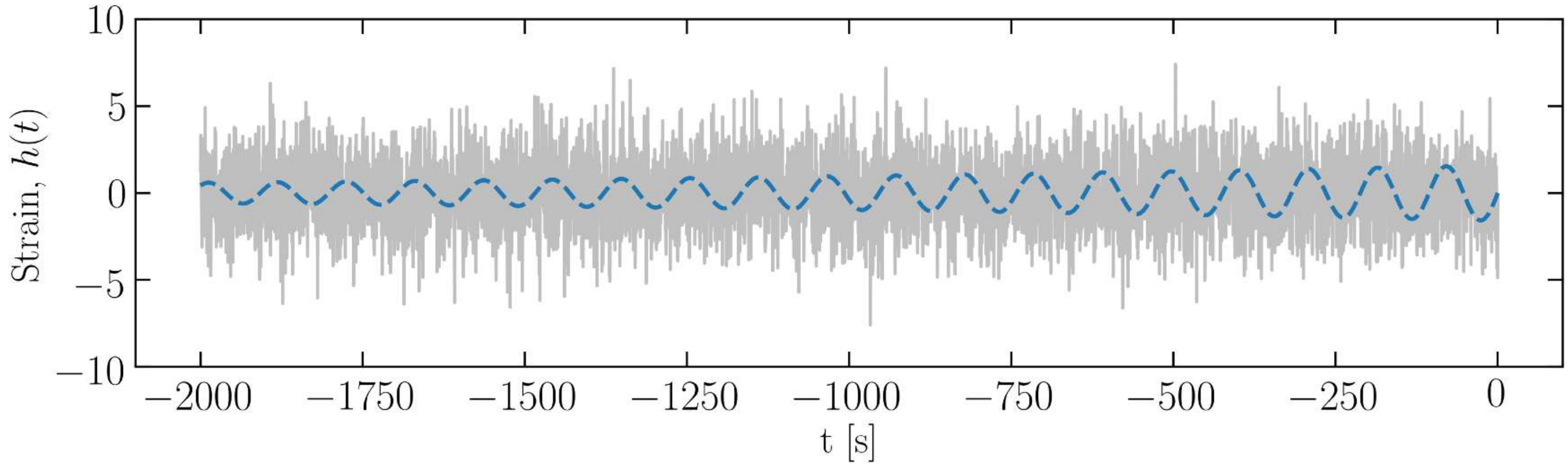


Stellar mass compact object (NS/BH) inspirals towards intermediate mass black hole (IMBH)

$$M_{\mathrm{IMBH}} \sim 10^3 - 10^5\, M_\odot$$

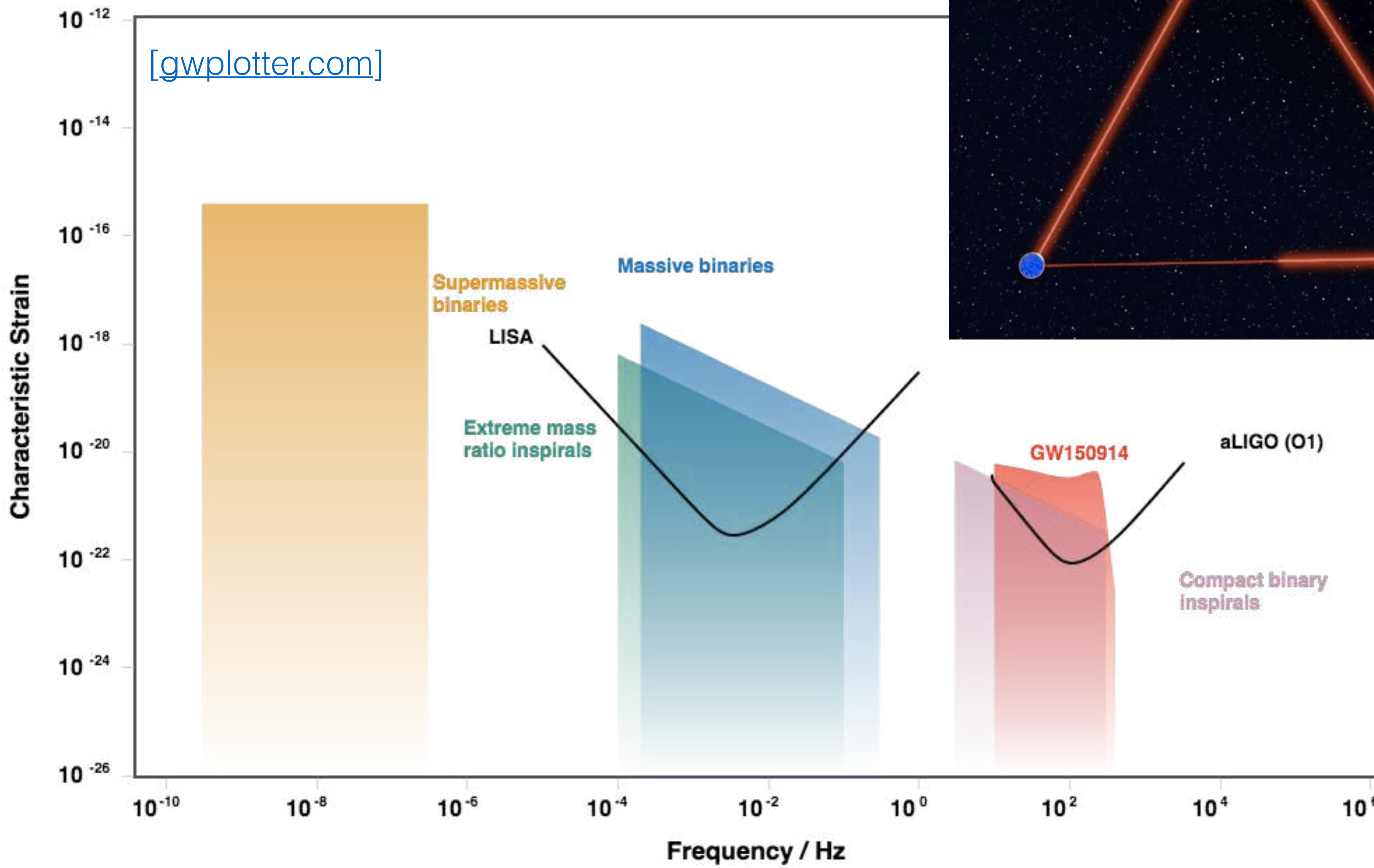
GW emission causes long, slow inspiral:

$$\dot{E}_{\mathrm{GW}} \approx \frac{32G^4}{5c^5}\frac{{M_{\mathrm{IMBH}}}^3 {M_{\mathrm{NS}}}^2}{r^5} \propto (f_{\mathrm{GW}})^{10/3}$$



LISA: GWs in Space

Laser **I**nterferometer **S**pace **A**ntenna
(planned for the 2030s) [1702.00786]



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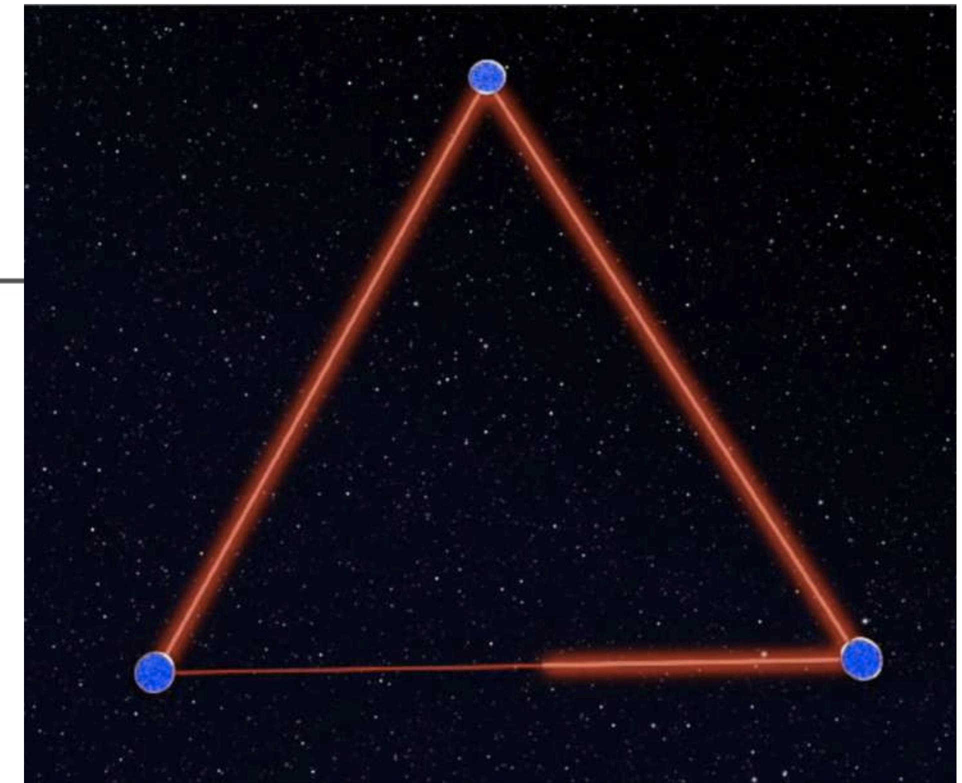


LISA should detect ~ 3 - 10 IMRIs per year [1711.00483]

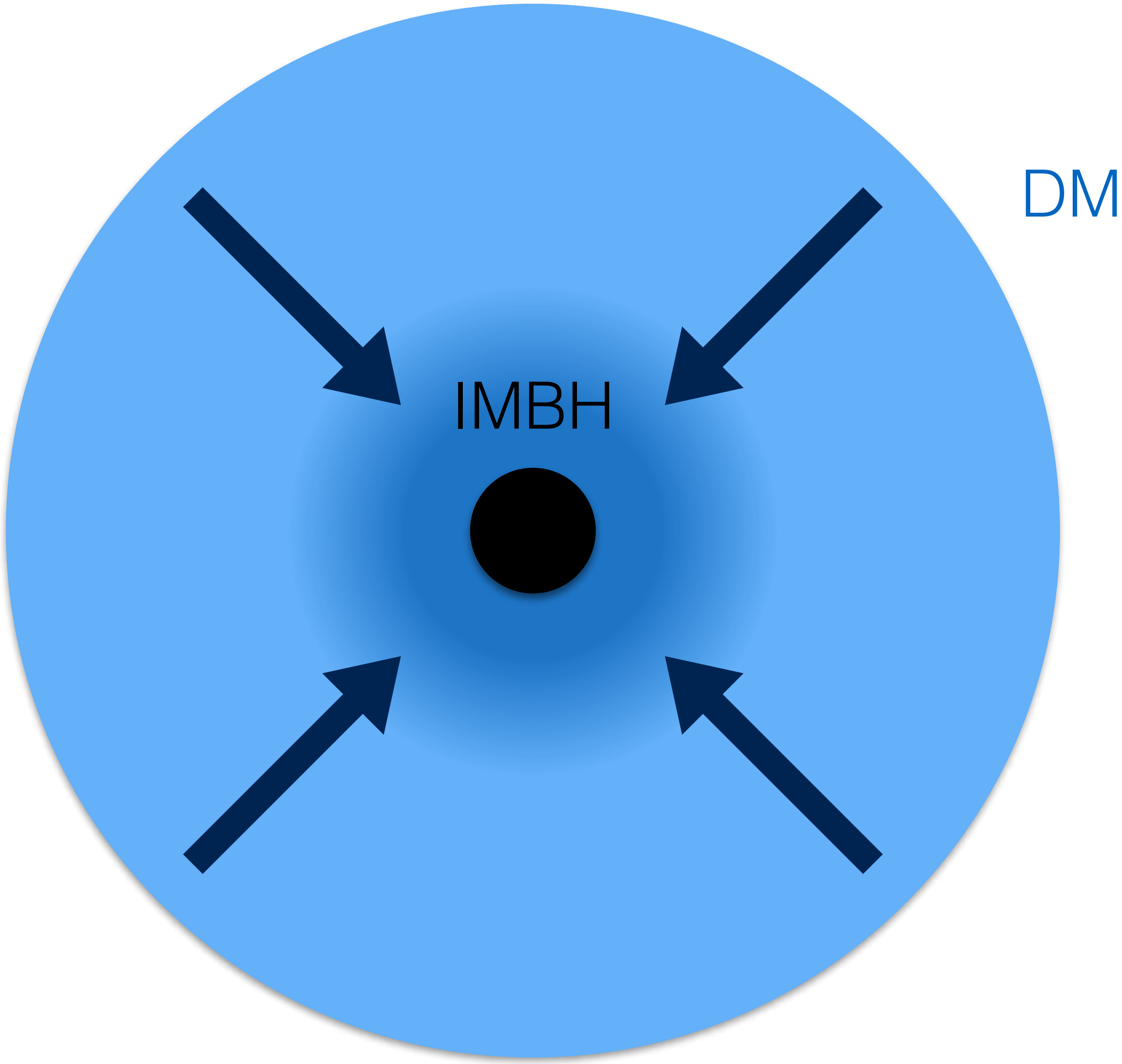
Dark Matter 'spikes'

Depending on the formation mechanism of the IMBH, expect an over-density of DM:

$$\rho_{\mathrm{DM}}(r) = \rho_{\mathrm{sp}} \left(\frac{r_{\mathrm{sp}}}{r}\right)^{\gamma_{\mathrm{sp}}}$$

For BH forming in an NFW halo, from adiabatic growth expect:

$$\gamma_{\mathrm{sp}} = 7/3 \approx 2.333$$



Typical density normalisation:

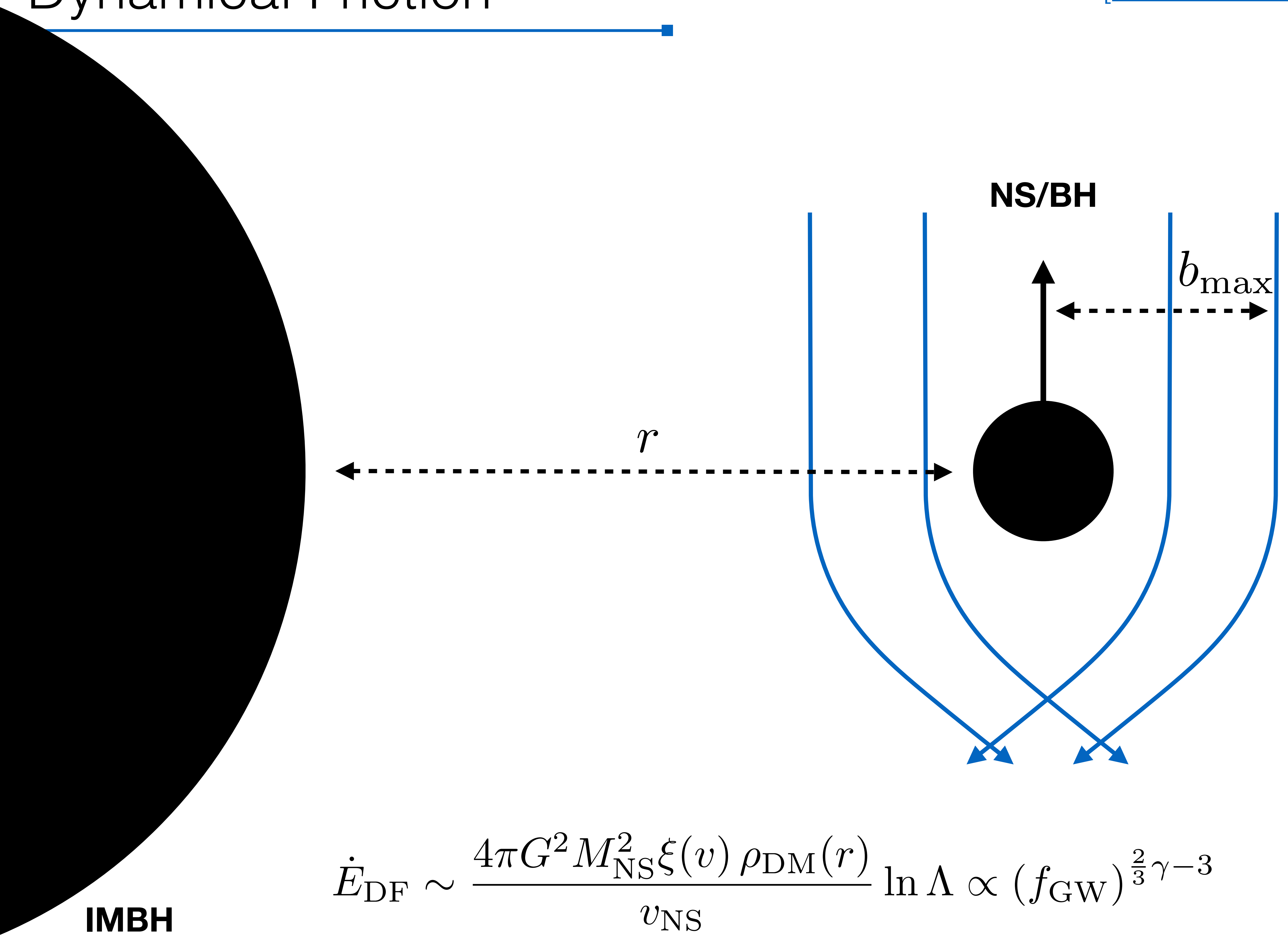
$$\rho_{\mathrm{sp}} \sim 200\, M_\odot\, \mathrm{pc}^{-3}$$

Density can reach $\rho \sim 10^{24}\, M_\odot\, \mathrm{pc}^{-3}$
(~10^{24} times larger than local density)

[astro-ph/9906391, astro-ph/0501555, astro-ph/0501625, astro-ph/0509565, 0902.3665, 1305.2619]

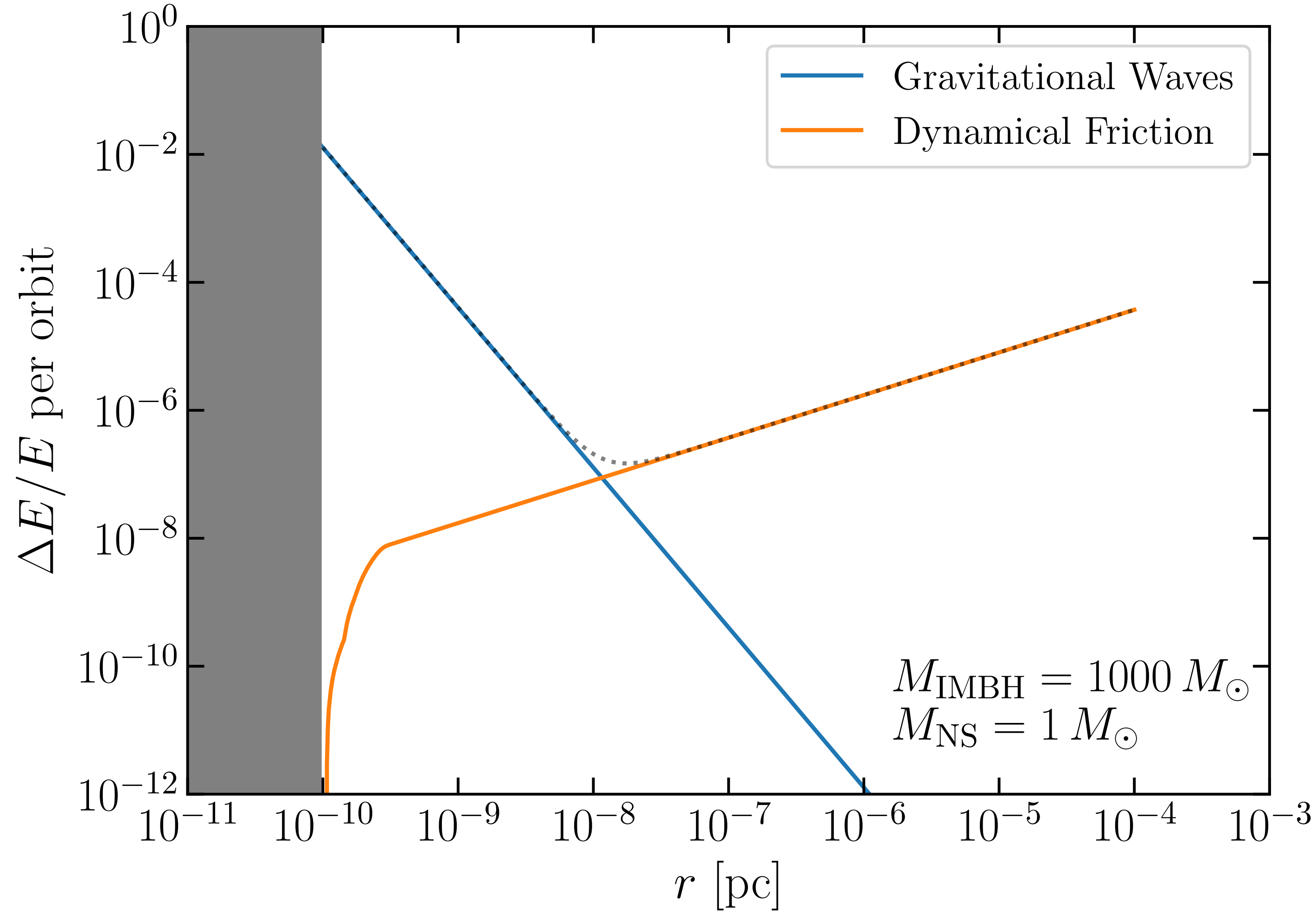
Dynamical Friction

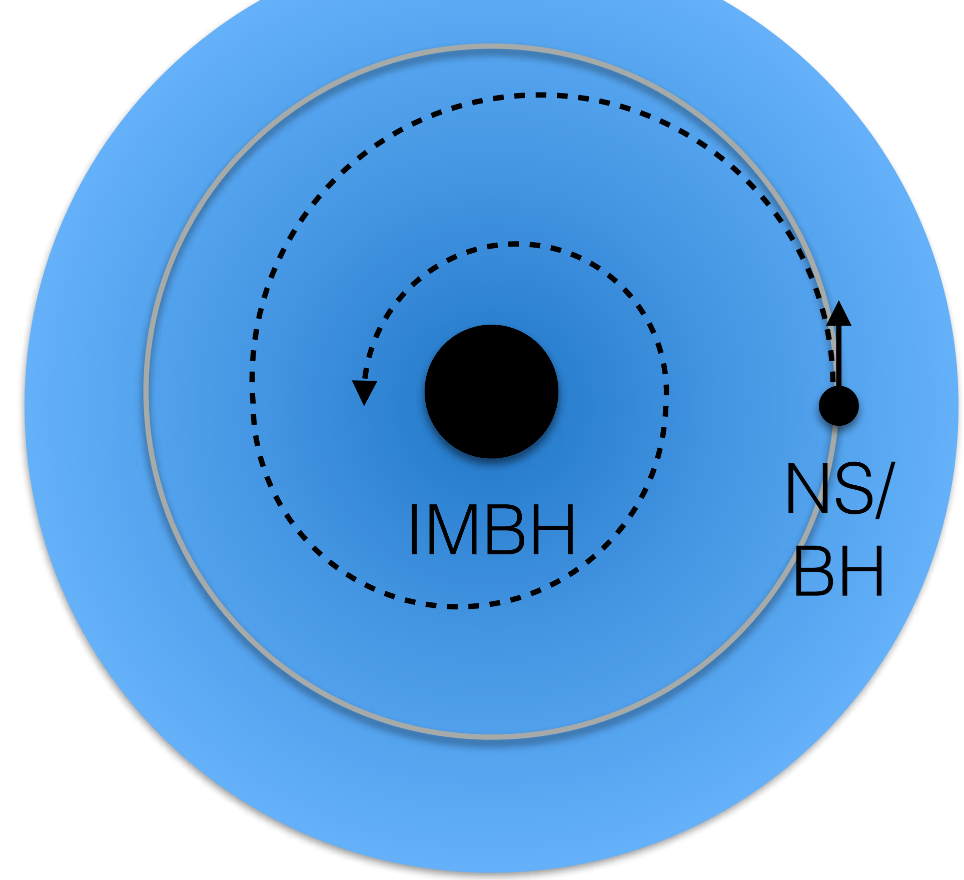
[Chandrasekhar, 1943]



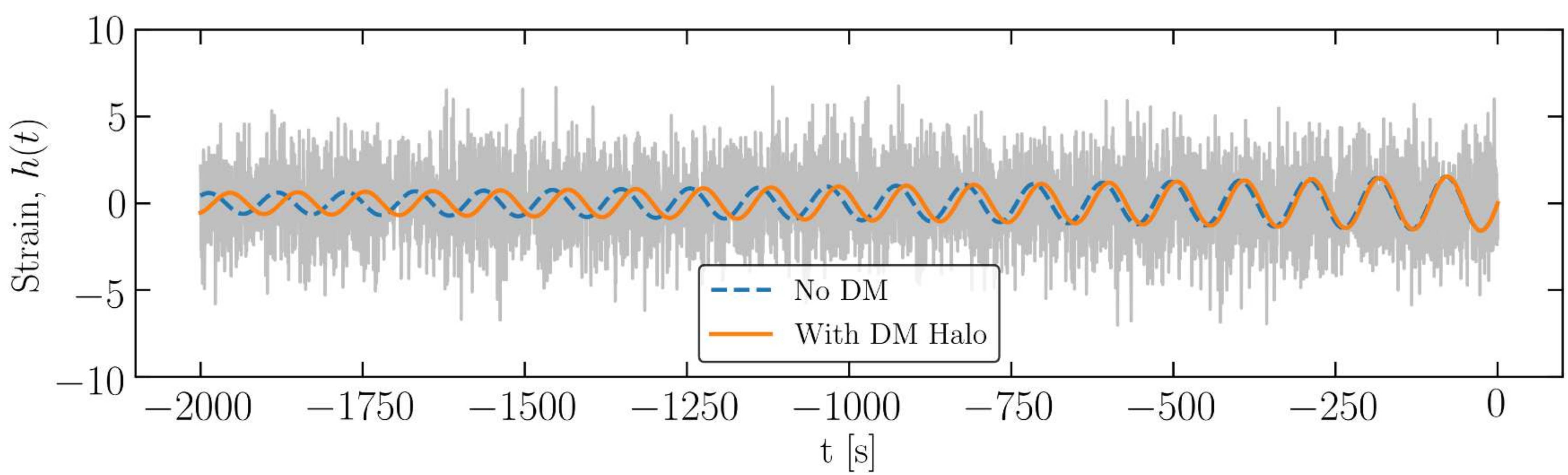
$$\dot{E}_{\rm DF} \sim \frac{4\pi G^2 M_{\rm NS}^2 \xi(v)\, \rho_{\rm DM}(r)}{v_{\rm NS}} \ln\Lambda \propto (f_{\rm GW})^{\frac{2}{3}\gamma - 3}$$

‘Dressed’ IMRI





$$-\dot{E}_{\rm orb} = \dot{E}_{\rm GW} + \dot{E}_{\rm DF}$$



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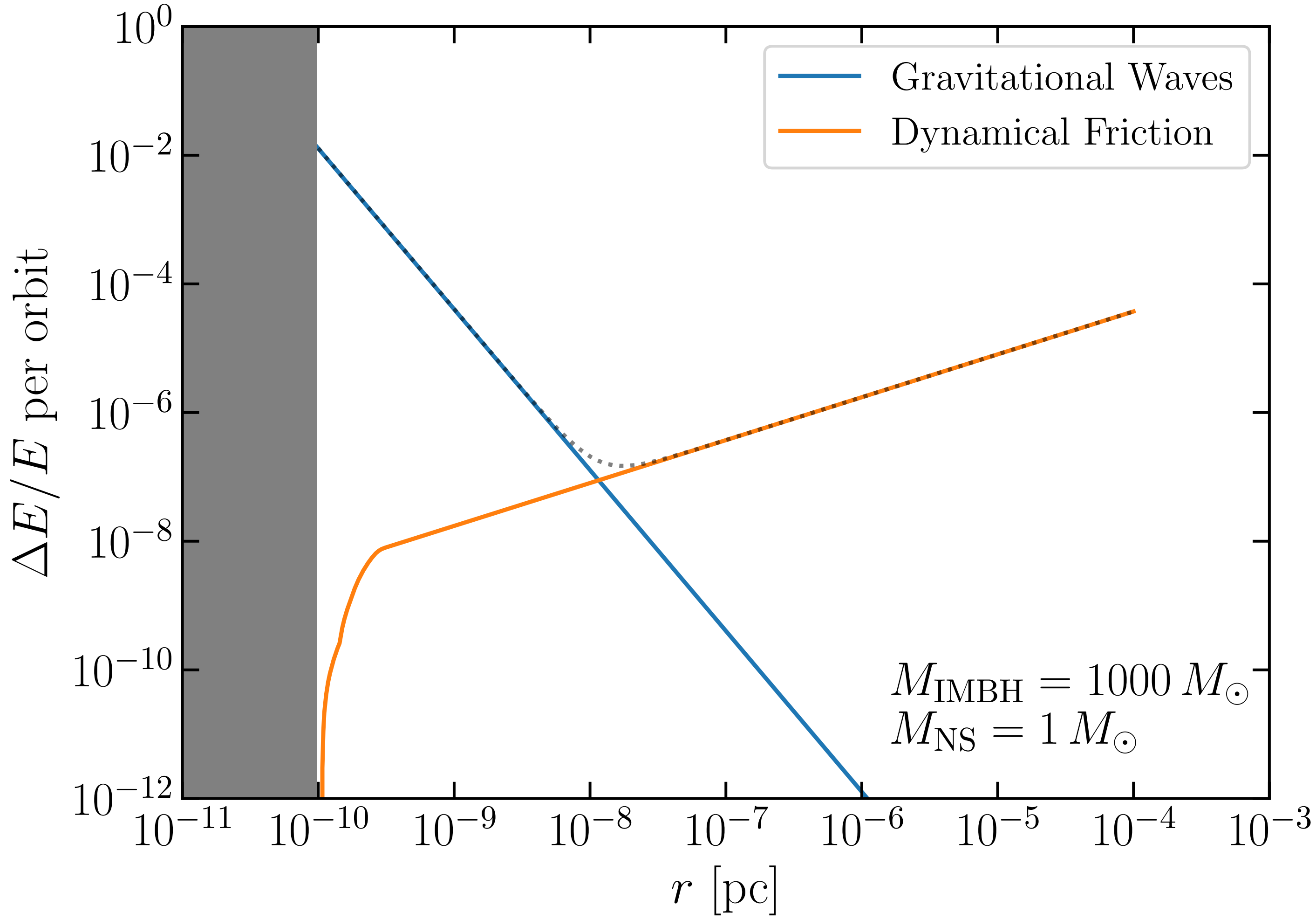

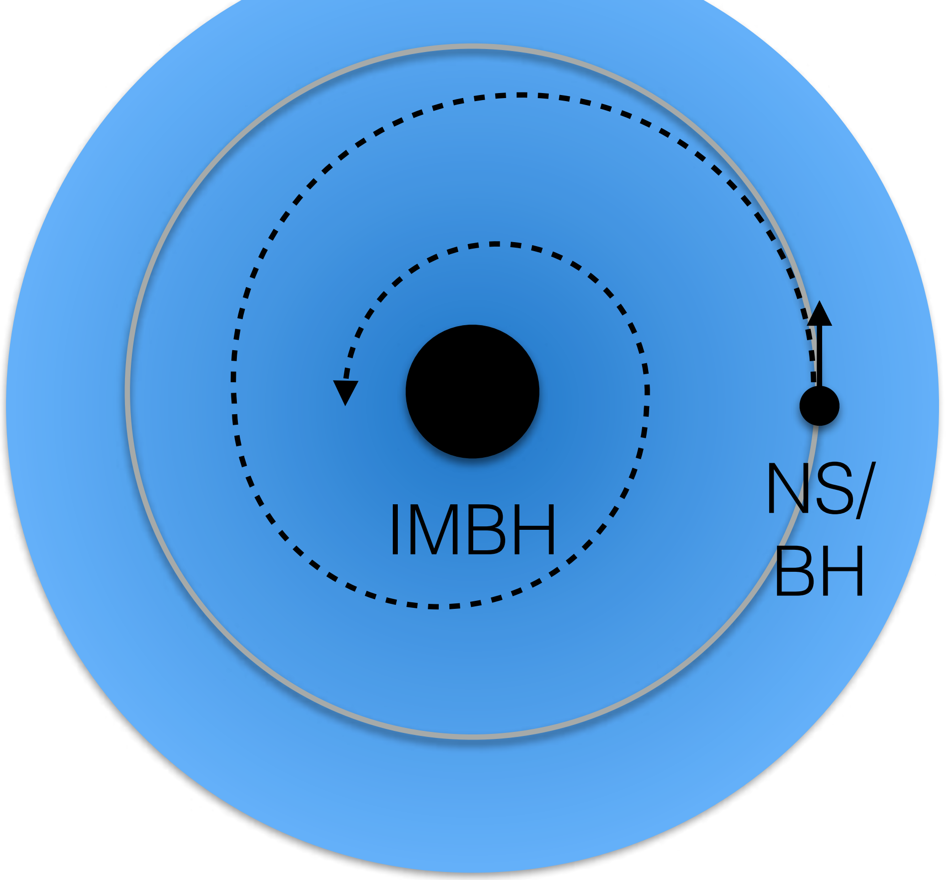

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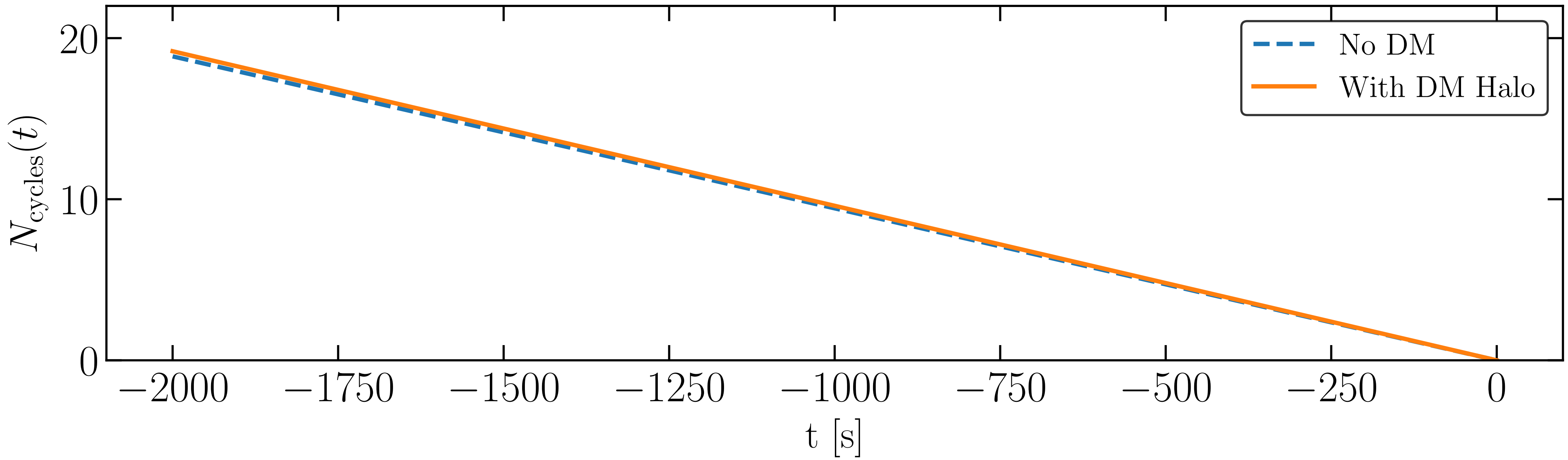


‘De-phasing’ signal

Benchmark:

$$M_{\mathrm{IMBH}} = 10^3\,M_\odot$$

$$M_{\mathrm{NS}} = 1\,M_\odot$$

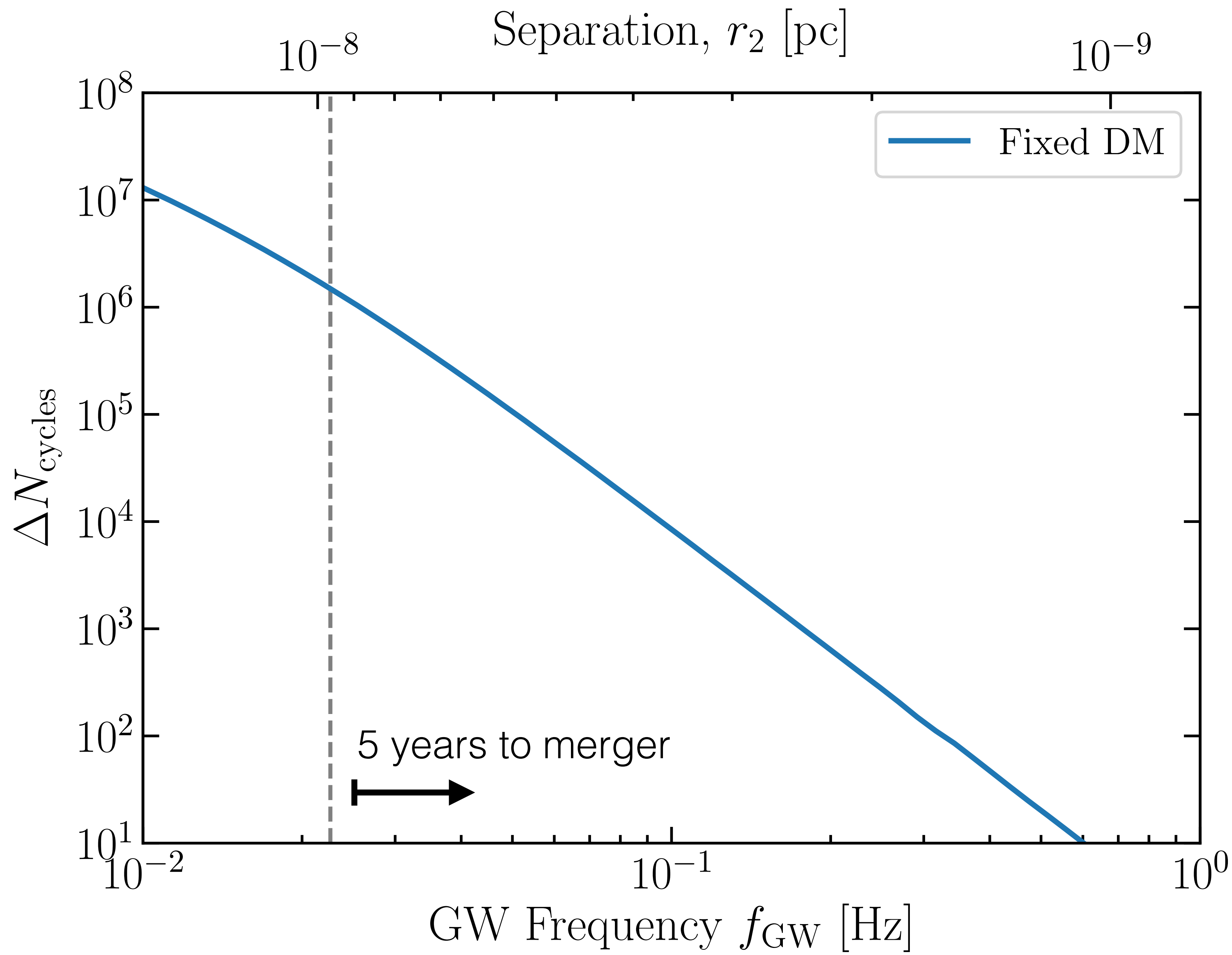
$$r_{\mathrm{ini}} \sim 10^{-8}\,\mathrm{pc}$$

$$t_{\mathrm{merge}}^{\mathrm{vacuum}} \sim 5\,\mathrm{yr}$$

$$N_{\mathrm{cycles}}^{\mathrm{vacuum}} \sim 6 \times 10^6$$

LISA may be sensitive to O(1 cycle) of dephasing!

How does DM affect the number of cycles?

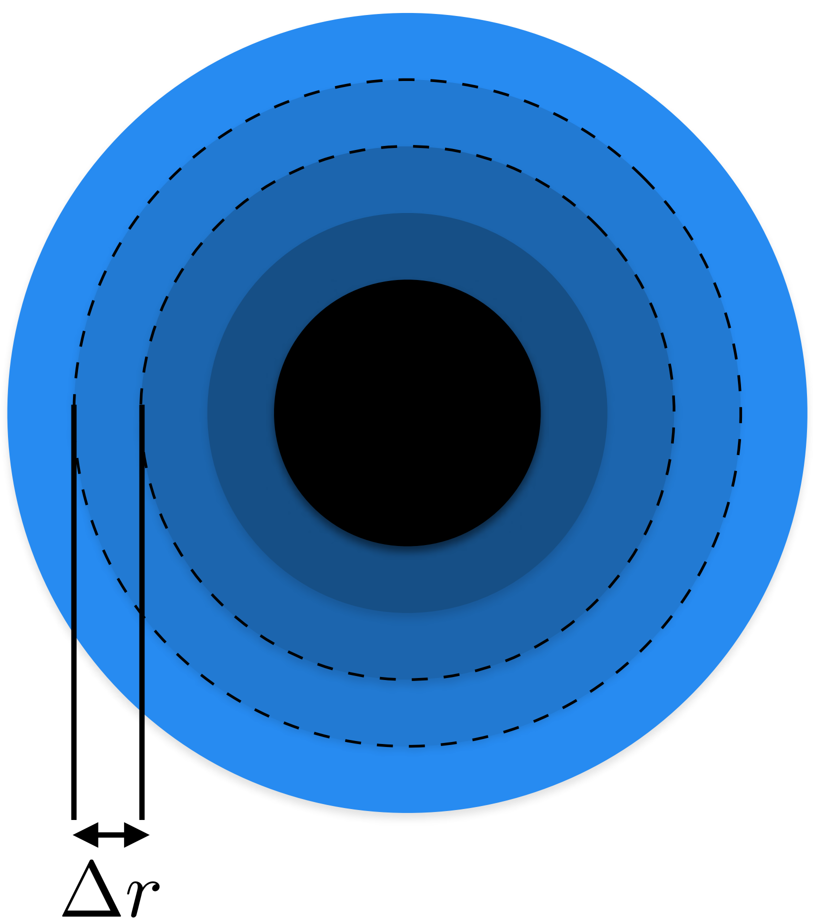


[Eda et al. 1301.5971, 1408.3534 and others]

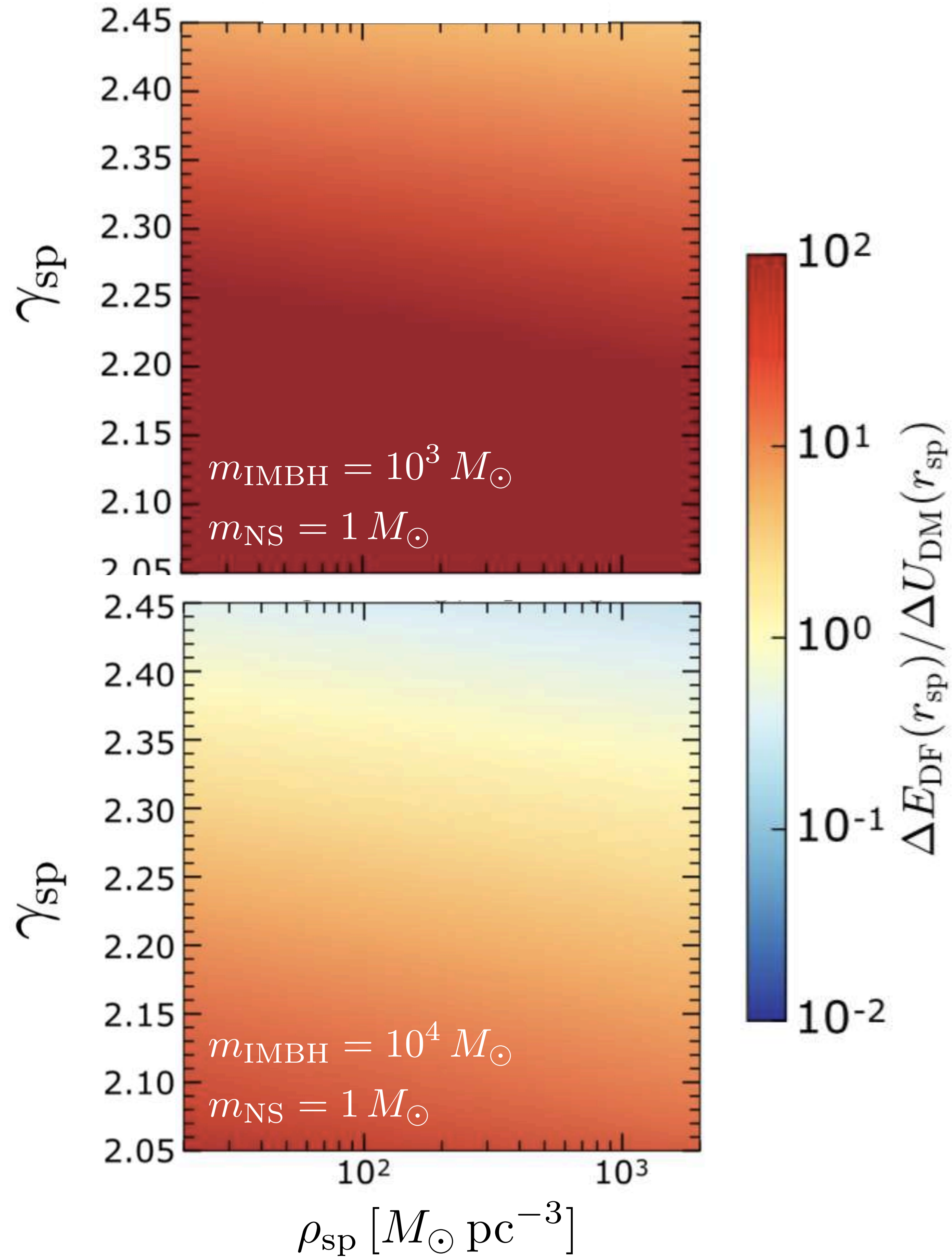
[See talk by Marco Chianese about Edwards, Chianese, **BJK**, Nissanke & Weniger, Phys. Rev. Lett. 124, 161101, 1905.04686]

Energy Budget

Q: How much energy is *available* for dynamical friction?



A: Binding energy of DM $\Delta U_{\rm DM}$ over radius Δr



[**BJK**, Nichols, Gaggero, Bertone, 2002.12811]

Energy Budget

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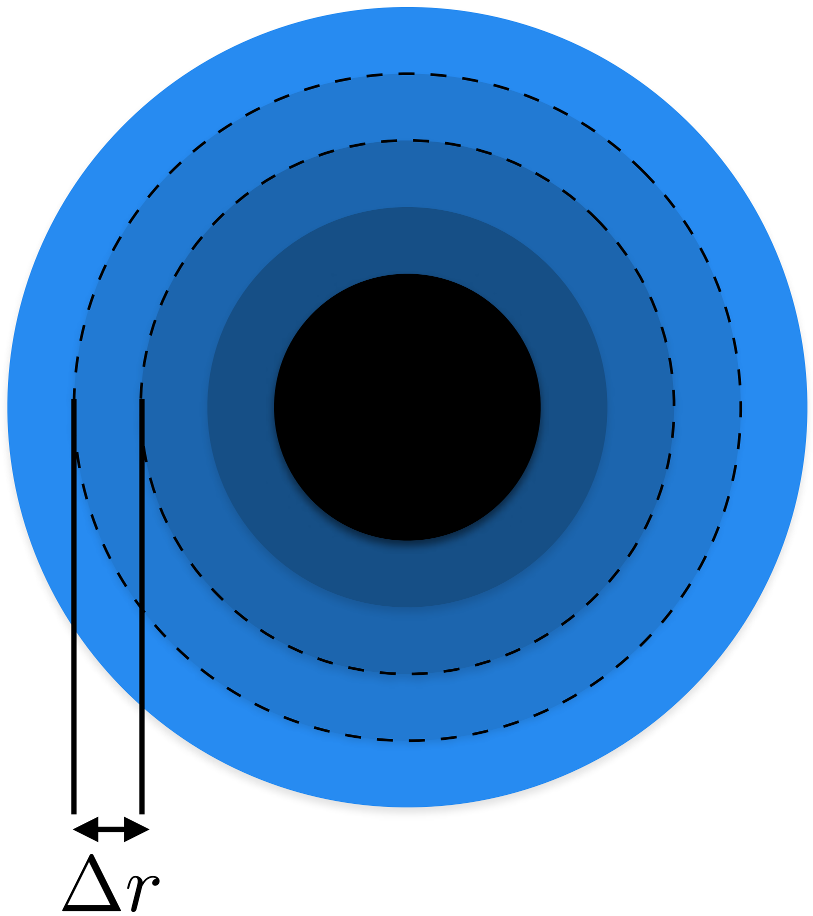
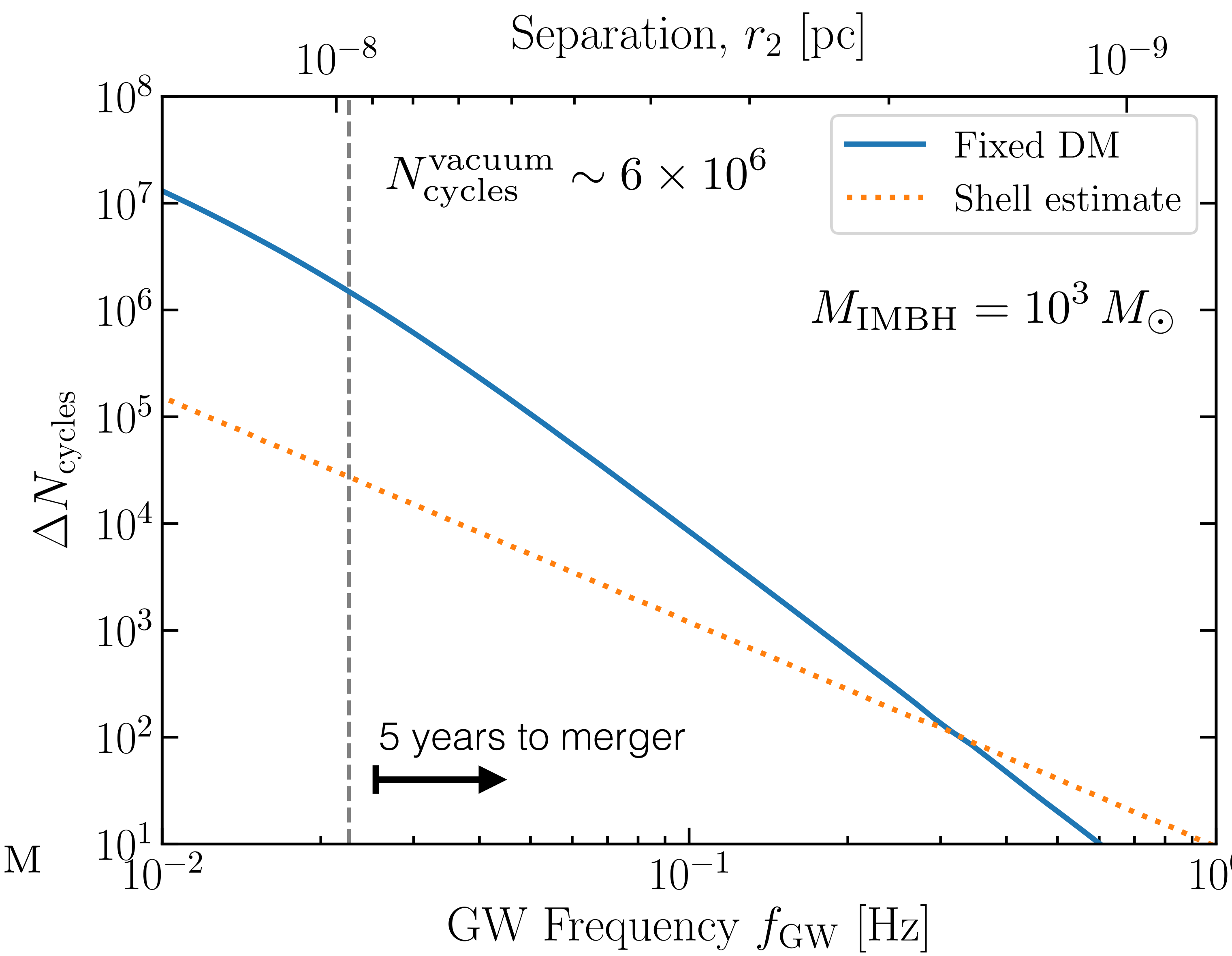


A: Binding energy of DM ΔU_{DM} over radius Δr

Evolve the system by fixing the dynamical friction force to extract *all* binding energy from a shell at a given radius:

$$\dot{E}_{\mathrm{DF}} = \dot{r}\,\frac{\mathrm{d}U_{\mathrm{DM}}}{\mathrm{d}r}$$



[**BJK**, Nichols, Gaggero, Bertone, 2002.12811]

Self-consistent evolution

Follow semi-analytically the phase space distribution of DM:

$$f = \frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{r}\,\mathrm{d}^3\mathbf{v}} \equiv f(\mathcal{E})$$

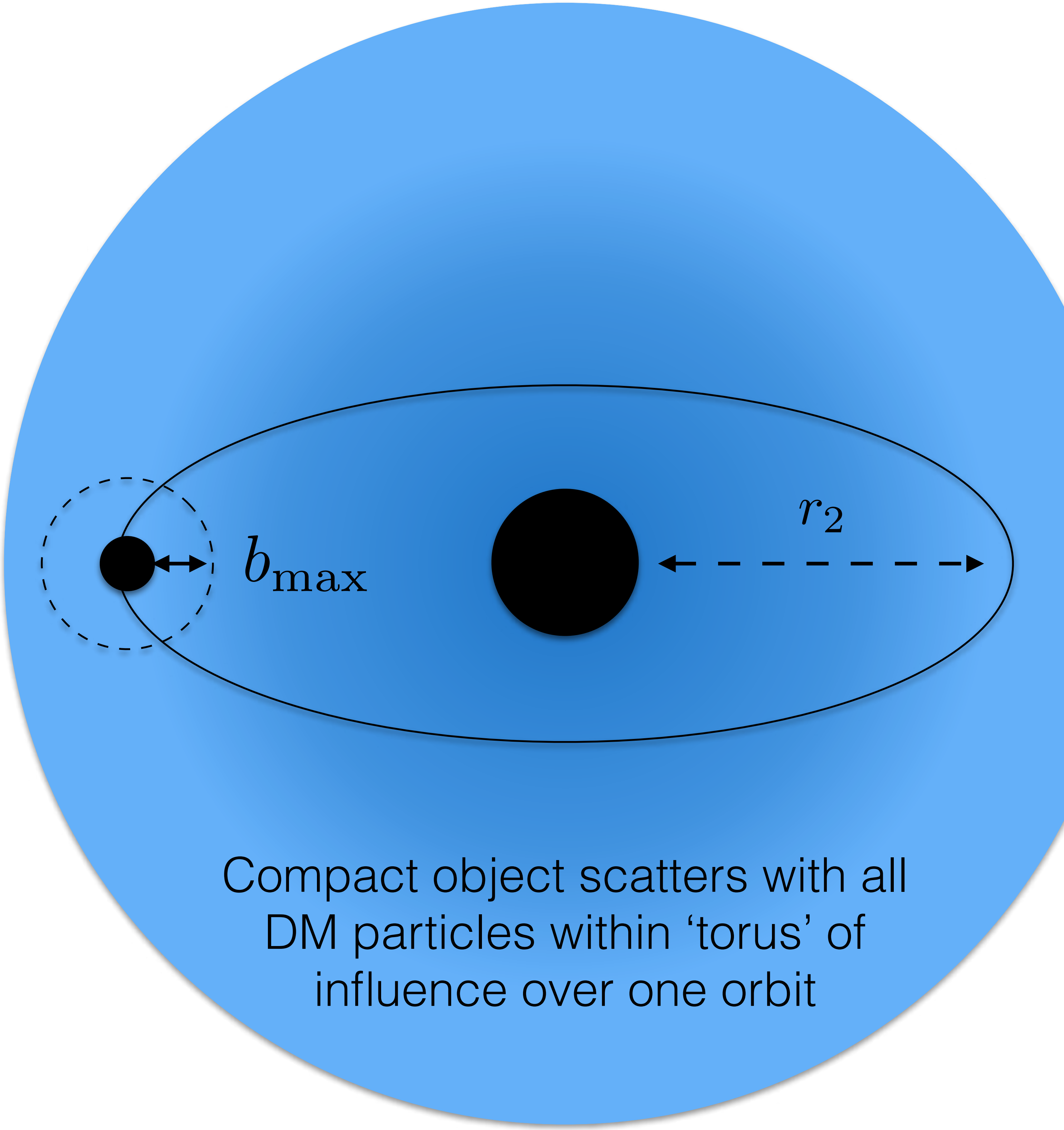
$$\mathcal{E} = \Psi(r) - \frac{1}{2}v^2$$

Each particle receives a 'kick' through gravitational scattering

$$\mathcal{E} \to \mathcal{E} + \Delta\mathcal{E}$$

Reconstruct density from distribution function:

$$\rho(r) = \int \mathrm{d}^3\mathbf{v}\, f(\mathcal{E})$$



Self-consistent evolution

Assuming everything evolves slowly compared to the orbital period:

$$\Delta f(\mathcal{E}) = -p_{\mathcal{E}} f(\mathcal{E}) + \int \left(\frac{\mathcal{E}}{\mathcal{E} - \Delta\mathcal{E}}\right)^{5/2} f(\mathcal{E} - \Delta\mathcal{E}) P_{\mathcal{E}-\Delta\mathcal{E}}(\Delta\mathcal{E}) \,\mathrm{d}\Delta\mathcal{E}$$

$P_{\mathcal{E}}(\Delta\mathcal{E})$ - probability for a particle with energy $\mathcal{E}$ to scatter and receive a 'kick' $\Delta\mathcal{E}$

$$p_{\mathcal{E}} = \int P_{\mathcal{E}}(\Delta\mathcal{E}) \,\mathrm{d}\Delta\mathcal{E}$$

- total probability for a particle with energy $\mathcal{E}$ to scatter

[Code available online: github.com/bradkav/HaloFeedback]

Self-consistent evolution

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Particles scattering from $\mathcal{E} \to \mathcal{E} + \Delta\mathcal{E}$

Particles scattering from $\mathcal{E} - \Delta\mathcal{E} \to \mathcal{E}$

$P_\mathcal{E}(\Delta\mathcal{E})$ - probability for a particle with energy $\mathcal{E}$ to scatter and receive a 'kick' $\Delta\mathcal{E}$

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Self-consistent evolution

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$$T_\mathrm{orb}\frac{\mathrm{d}f(\mathcal{E})}{\mathrm{d}t} = -p_\mathcal{E} f(\mathcal{E}) + \int \left(\frac{\mathcal{E}}{\mathcal{E}-\Delta\mathcal{E}}\right)^{5/2} f(\mathcal{E}-\Delta\mathcal{E}) P_{\mathcal{E}-\Delta\mathcal{E}}(\Delta\mathcal{E})\, \mathrm{d}\Delta\mathcal{E}$$

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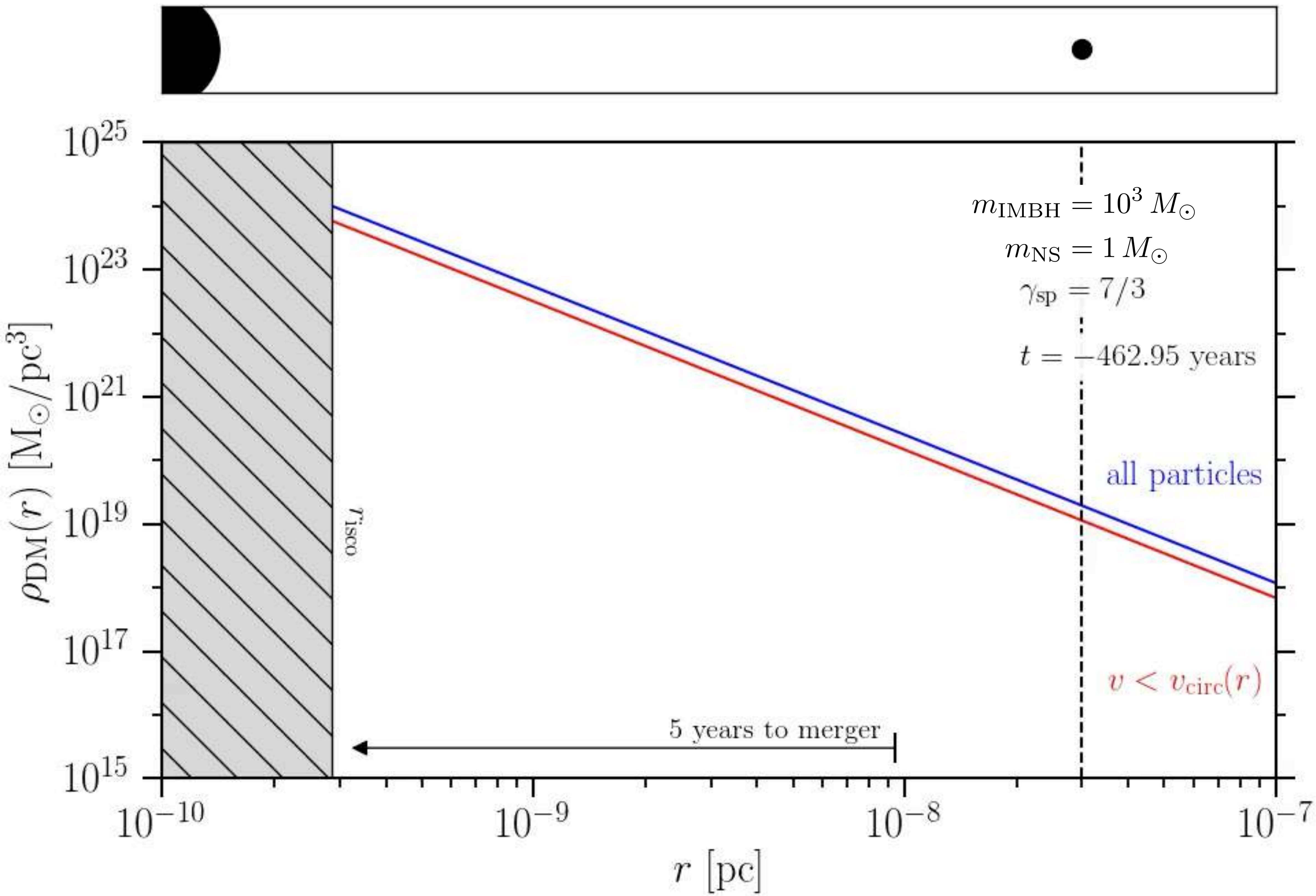
Full evolution of the system

Movies: tinyurl.com/GW4DM



Subtlety: dynamical friction only cares about ‘slow-moving’ particles

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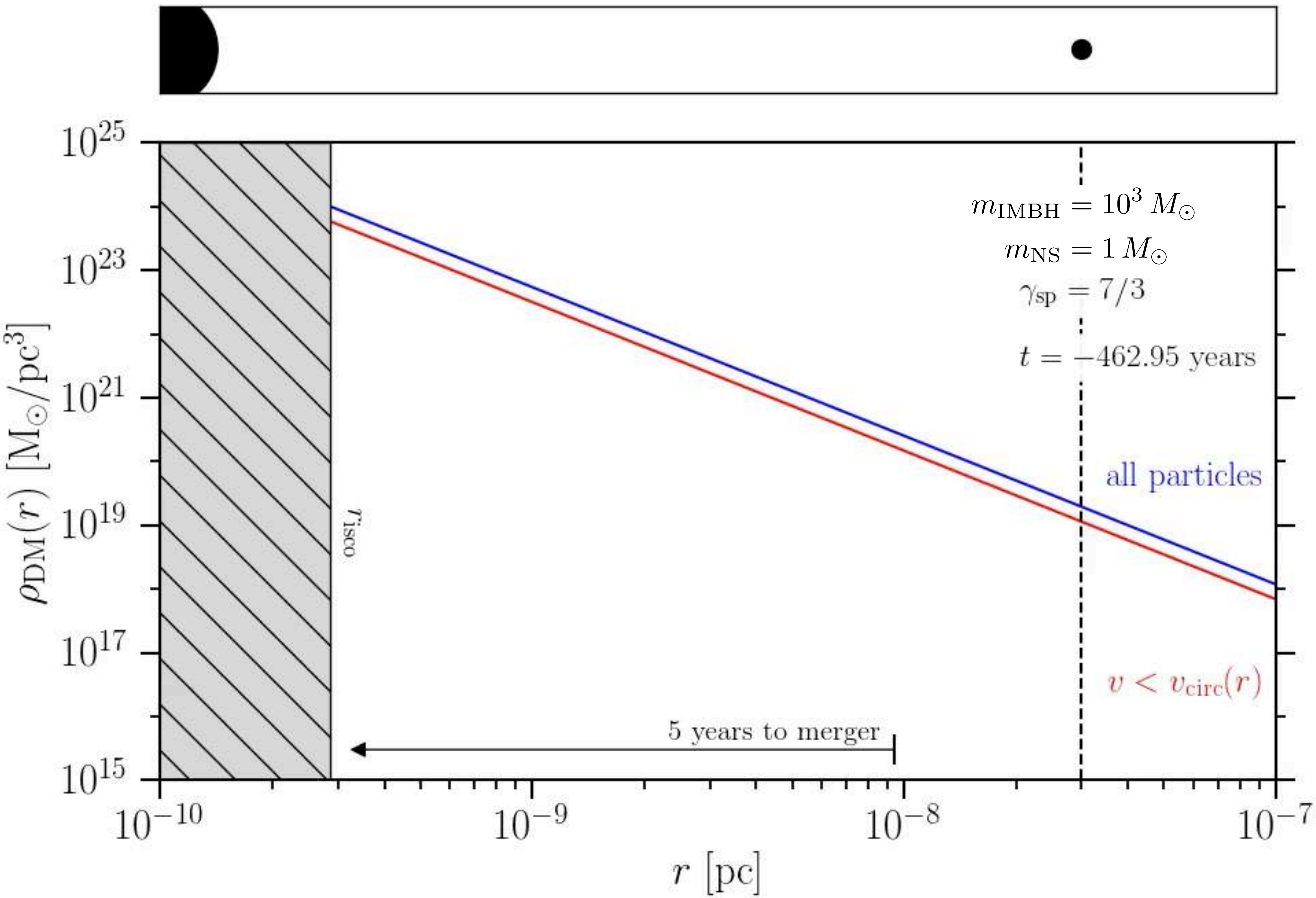

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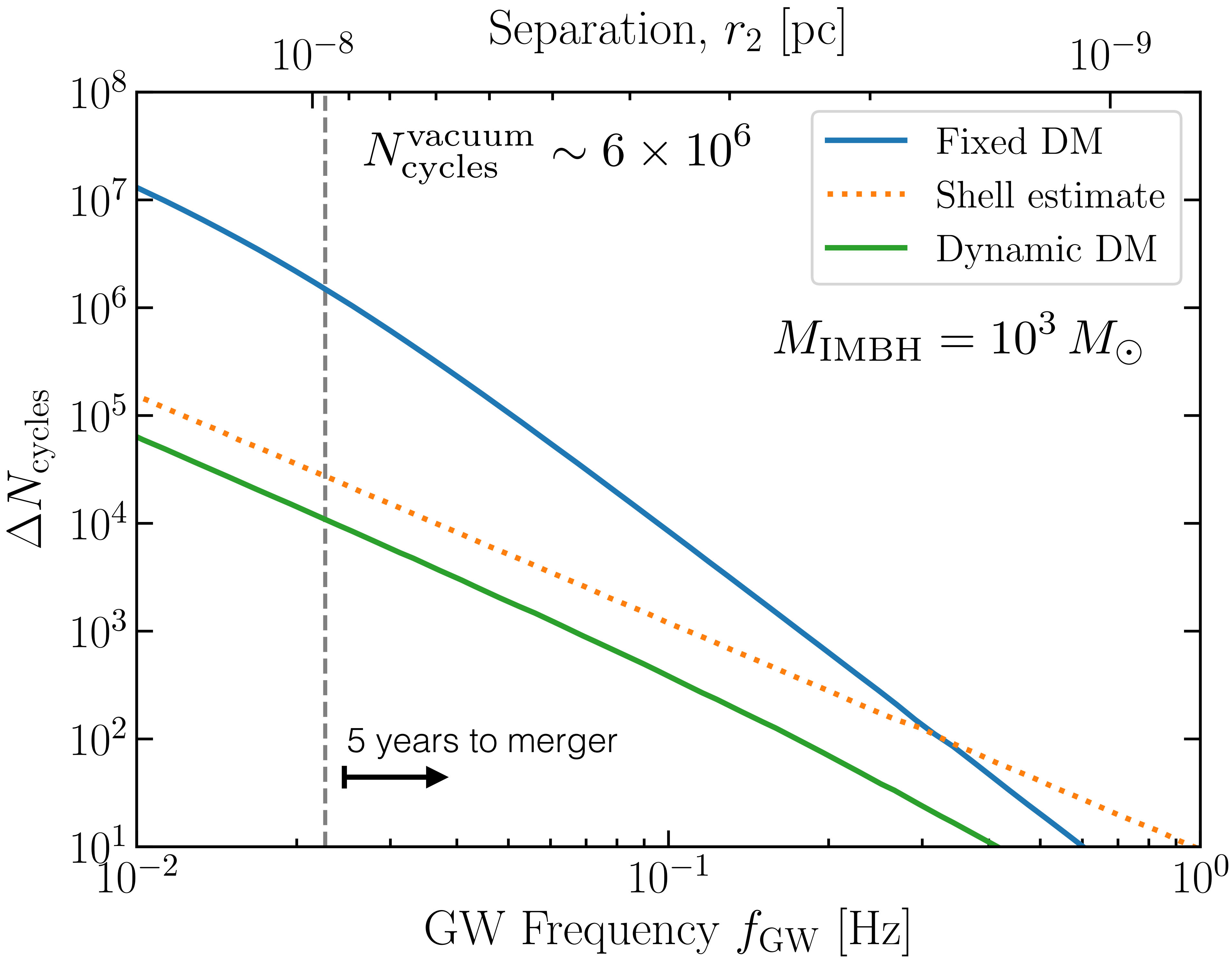
Self-consistent results

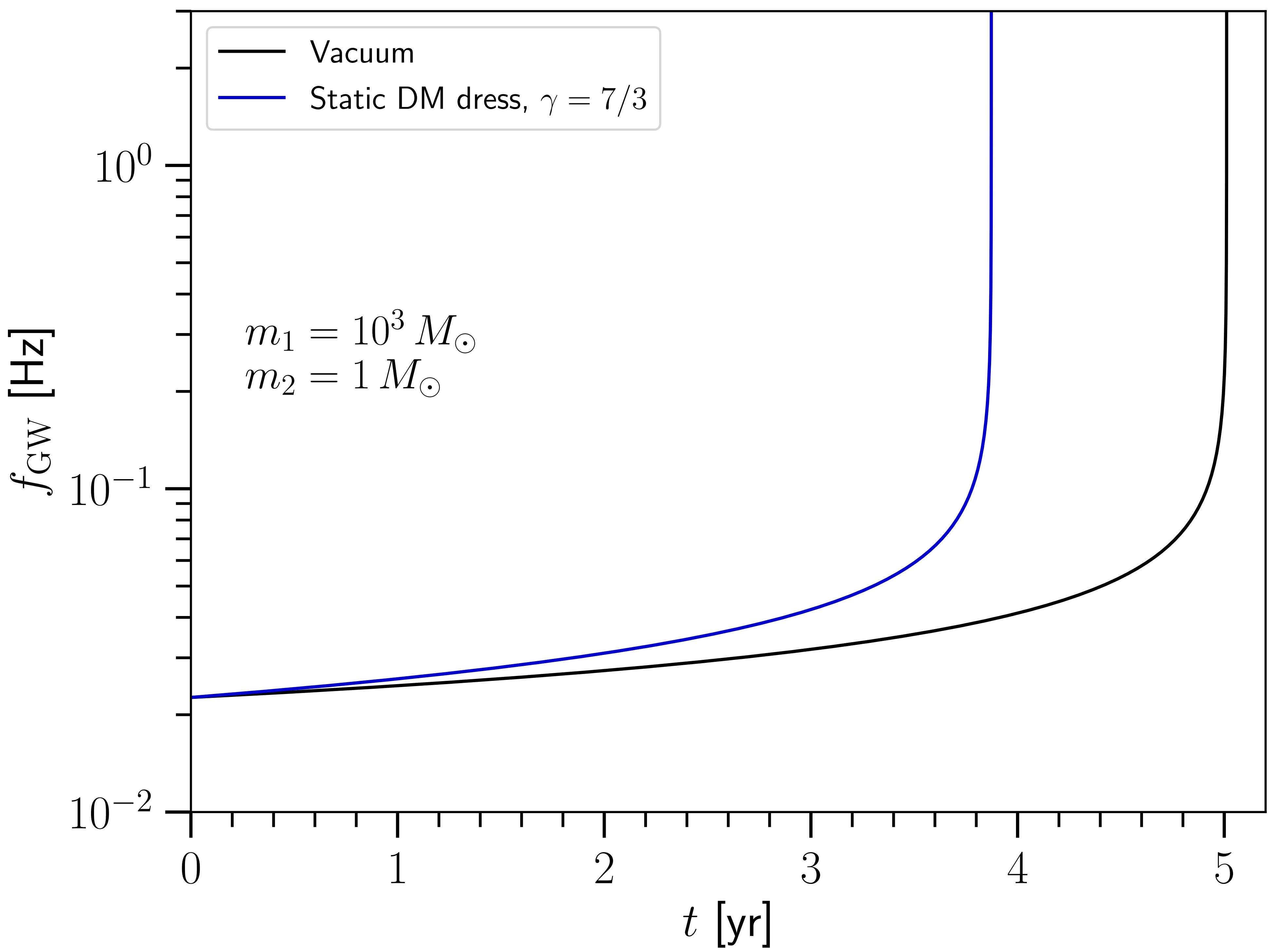
[**BJK**, Nichols, Gaggero, Bertone, 2002.12811]



$$\Delta N_{\rm cycles}(\text{static}) \approx 10^6 \;\rightarrow\; \Delta N_{\rm cycles}(\text{dynamic}) \approx 10^4$$

Spectrograms

[**BJK**, Nichols, Gaggero, Bertone, 2002.12811]



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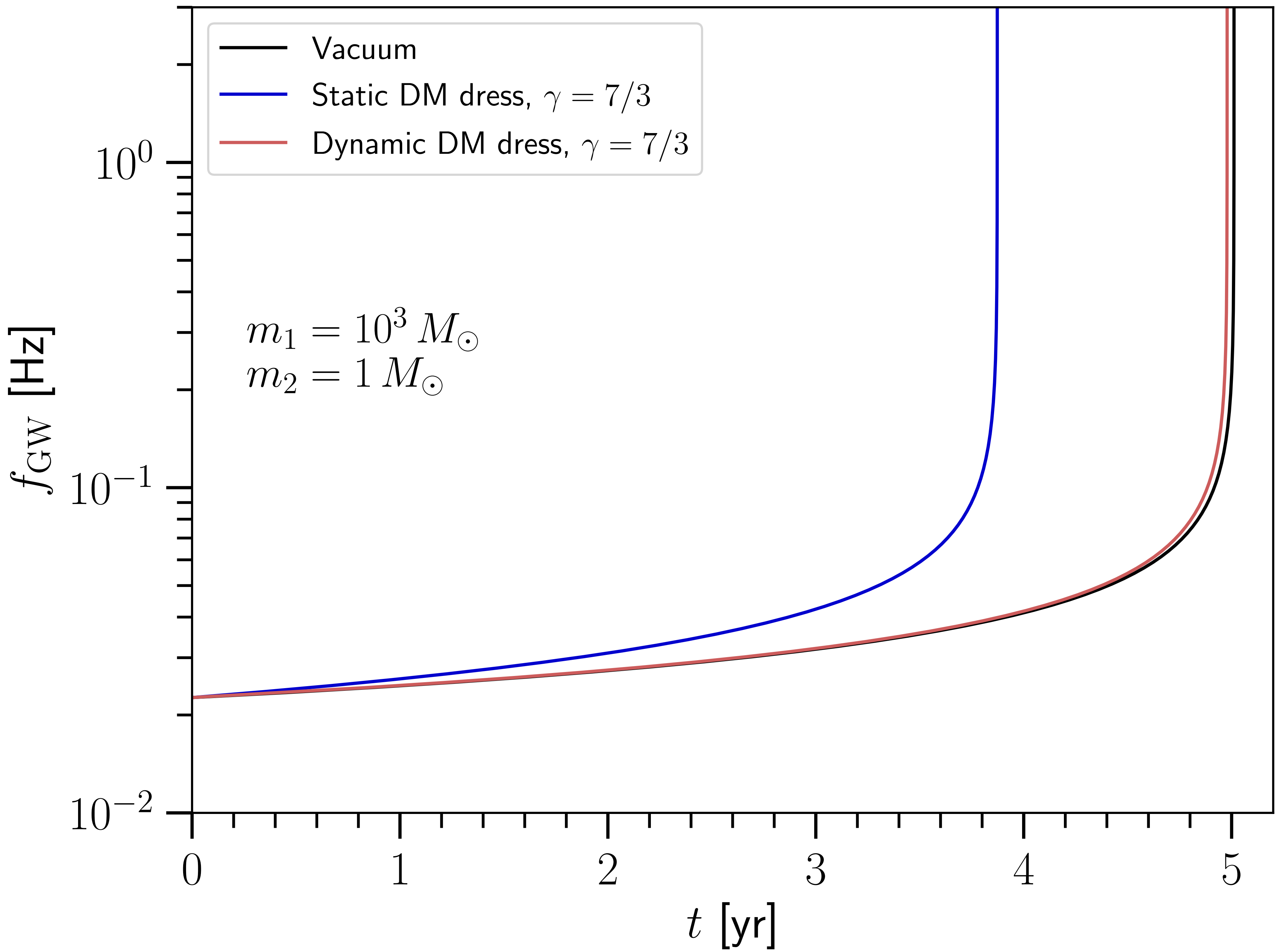

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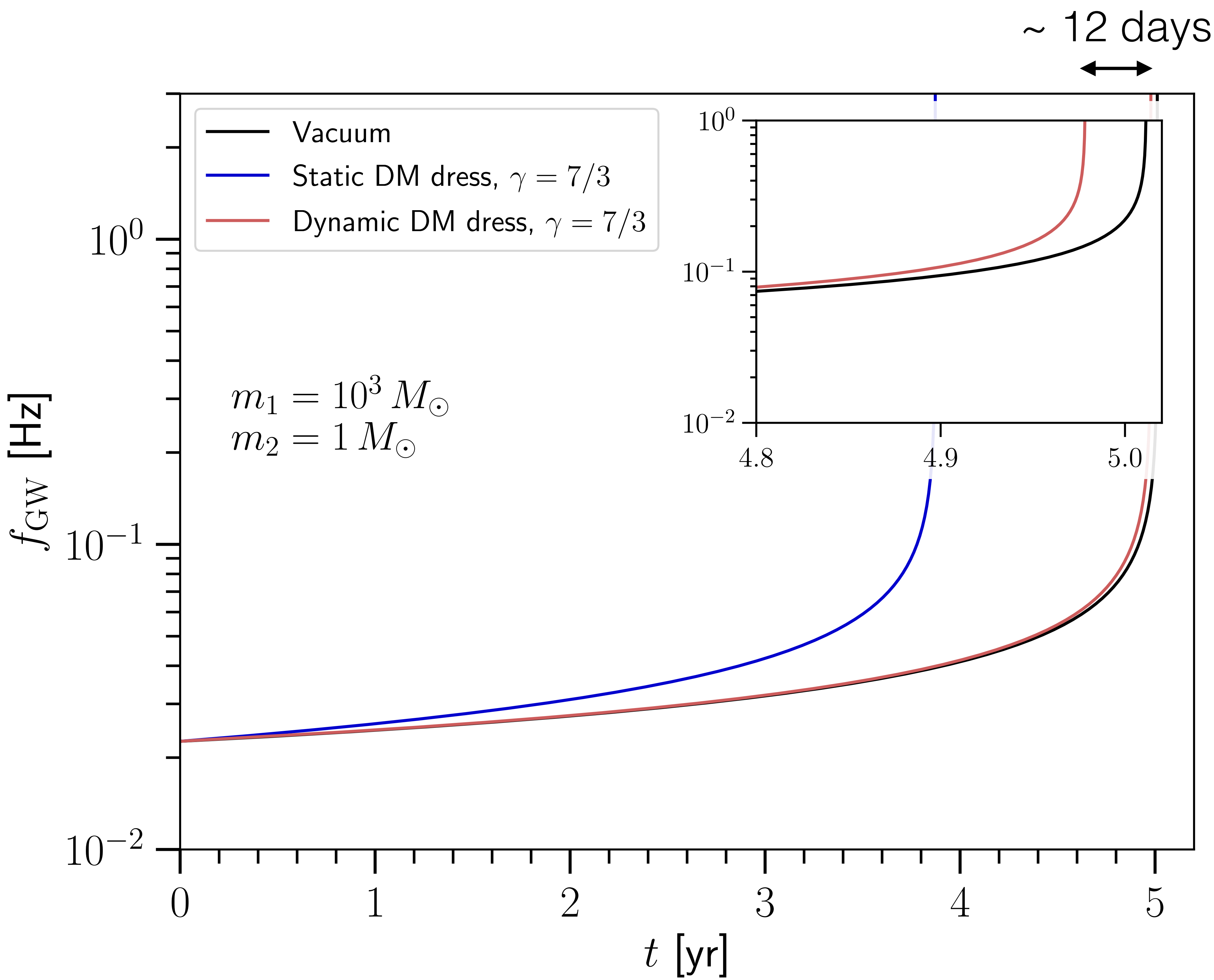


$$\Delta t_{\mathrm{merge}} \approx 1\,\mathrm{yr} \to \Delta t_{\mathrm{merge}} \approx 12\,\mathrm{days}$$

Plans for the future

Improved modelling

- Injection and evolution of angular momentum in the DM halo
- More general orbital parameters (eccentricities etc.)
- Post-Newtonian corrections
- N-body approaches [AMUSE?]

Detection methods

- Producing template banks for LISA searches
- Incoherent searches for continuous GWs
- ‘General’ de-phased waveform templates [2004.06729]

Detection prospects

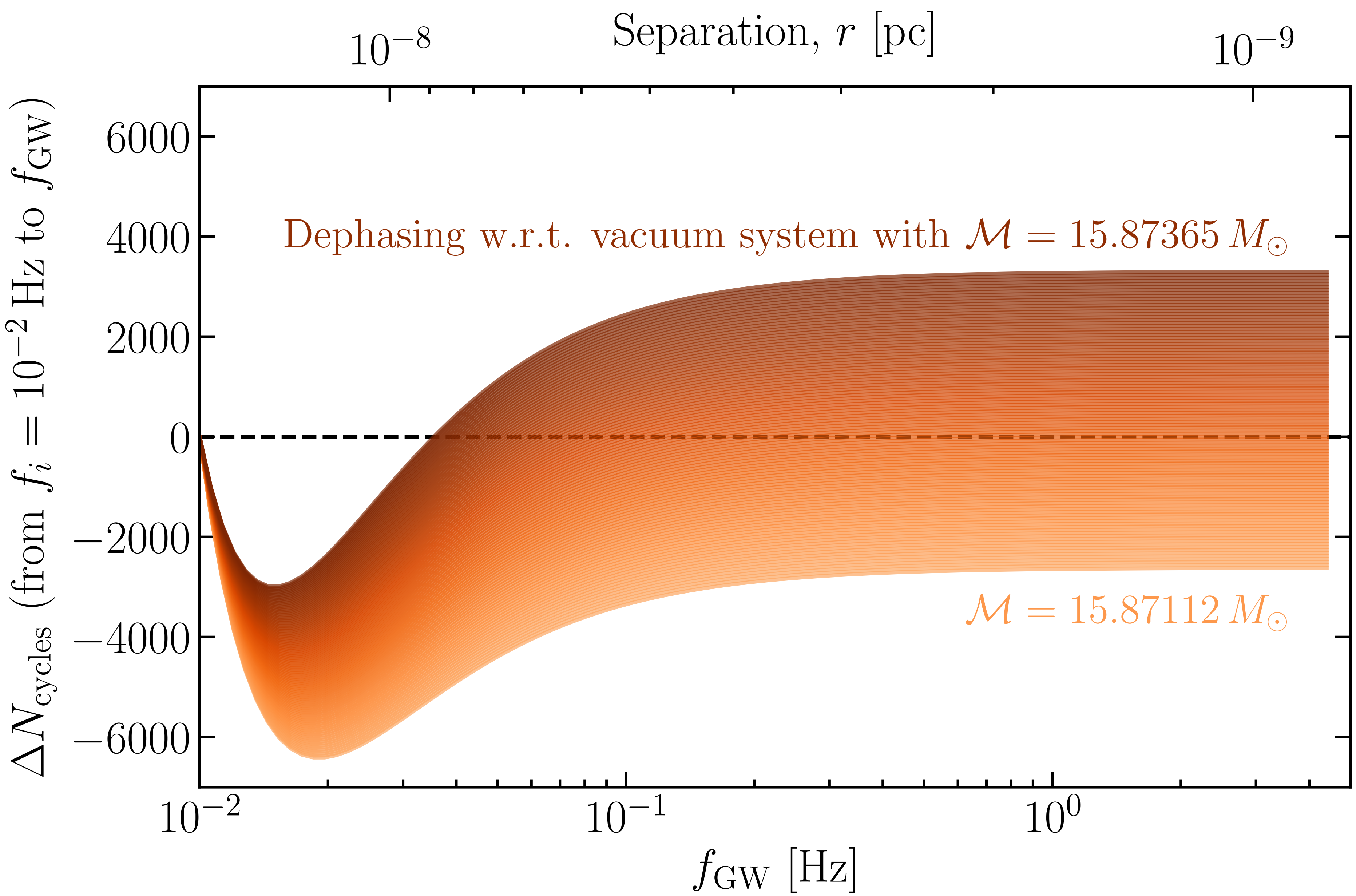
- How many IMRI systems form?
- How many systems have a (surviving) spike?
- *Prospects for detection and parameter reconstruction (DM density)*

Detectability

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

Assume we observe a GW signal from our benchmark 'dressed' IMRI, which has $\mathcal{M} = 15.846\, M_\odot$.

Compare phase evolution with different vacuum binaries:

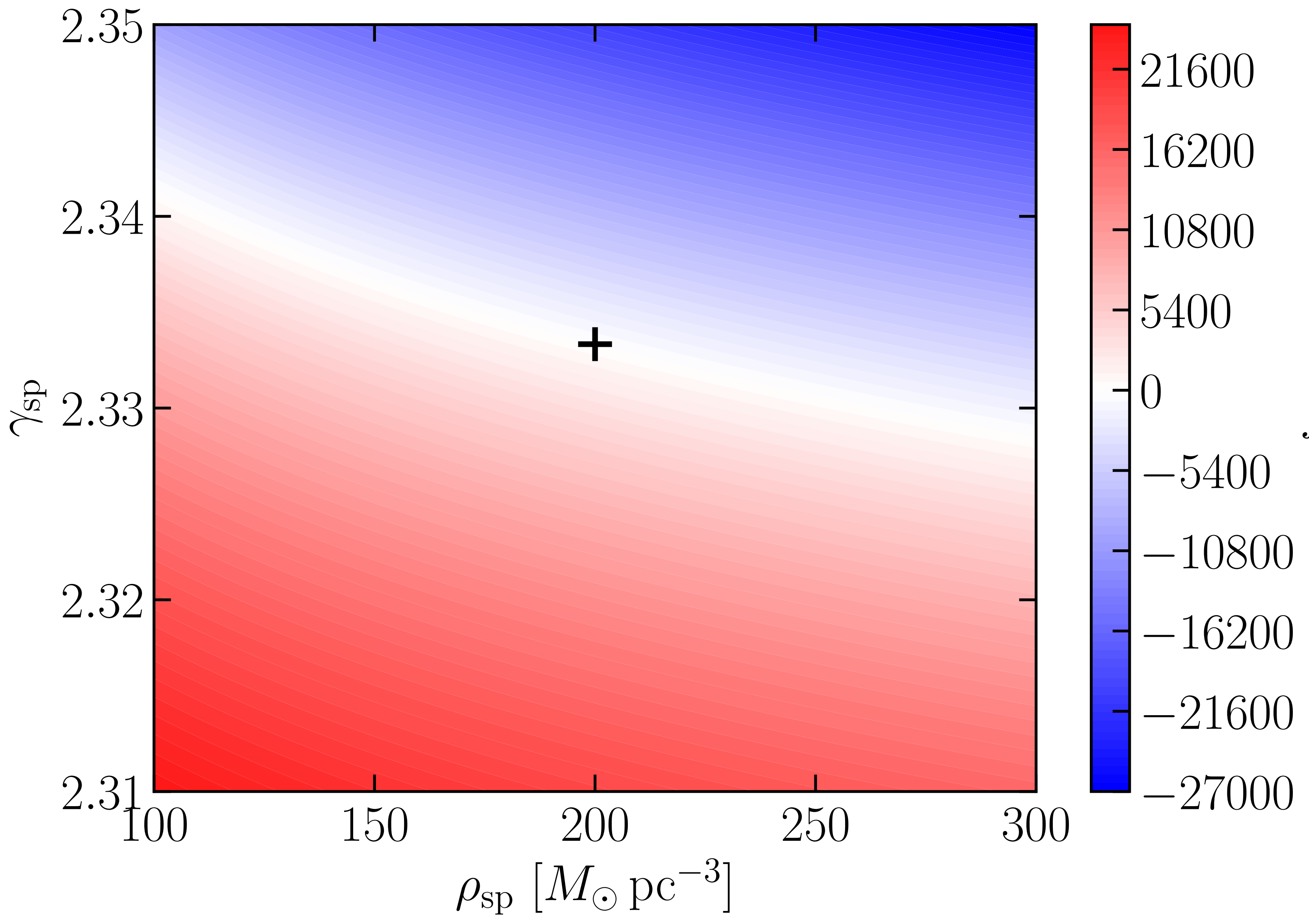


Parameter sensitivity

$$\rho_{\rm DM}(r) = \rho_{\rm sp}\left(\frac{r_{\rm sp}}{r}\right)^{\gamma_{\rm sp}}$$

Assume we observe a GW signal from our benchmark 'dressed' IMRI, which has $\rho_{\rm sp} = 200\, M_\odot\, {\rm pc}^3$ and $\gamma_{\rm sp} = 7/3 \approx 2.333$.

Compare phase evolution with different dressed binaries:



Conclusions

Exciting prospects for detecting Dark Matter through GW ‘de-phasing’

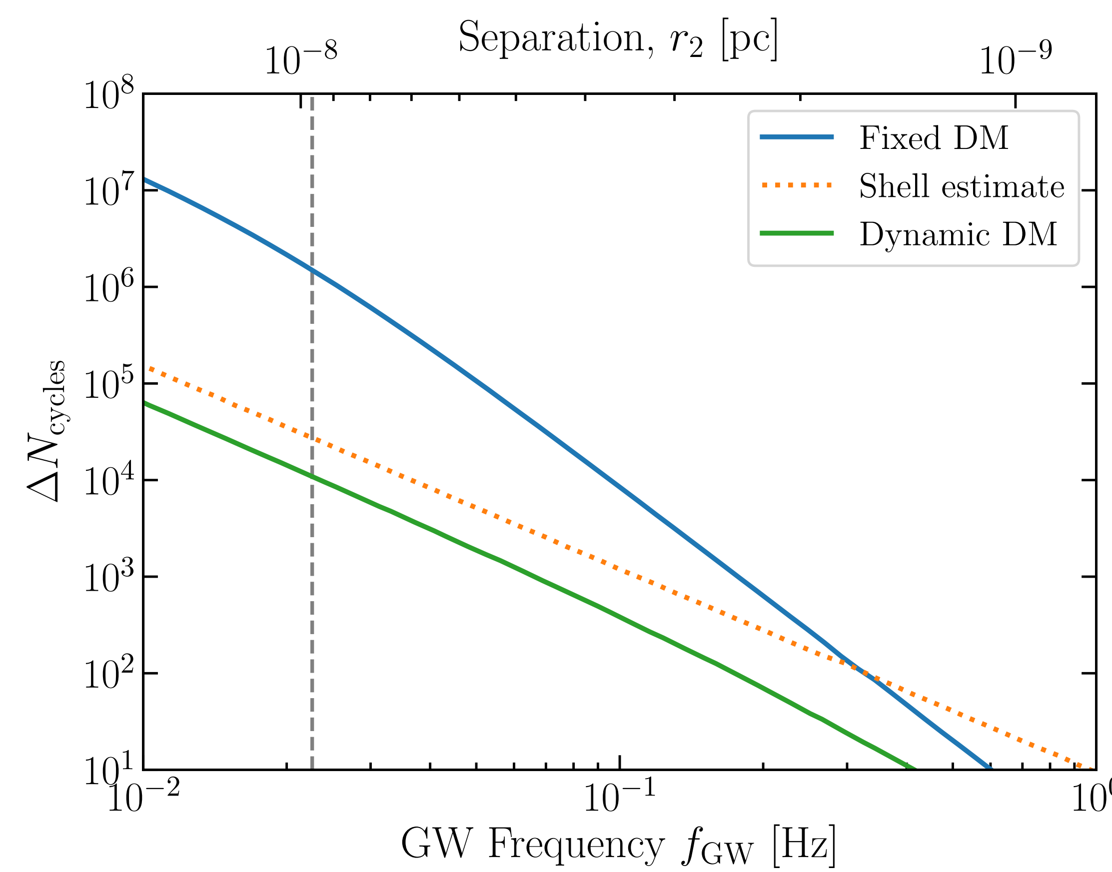
[Edwards, Chianese, **BJK**, Nissanke & Weniger, Phys. Rev. Lett. 124, 161101, 1905.04686]

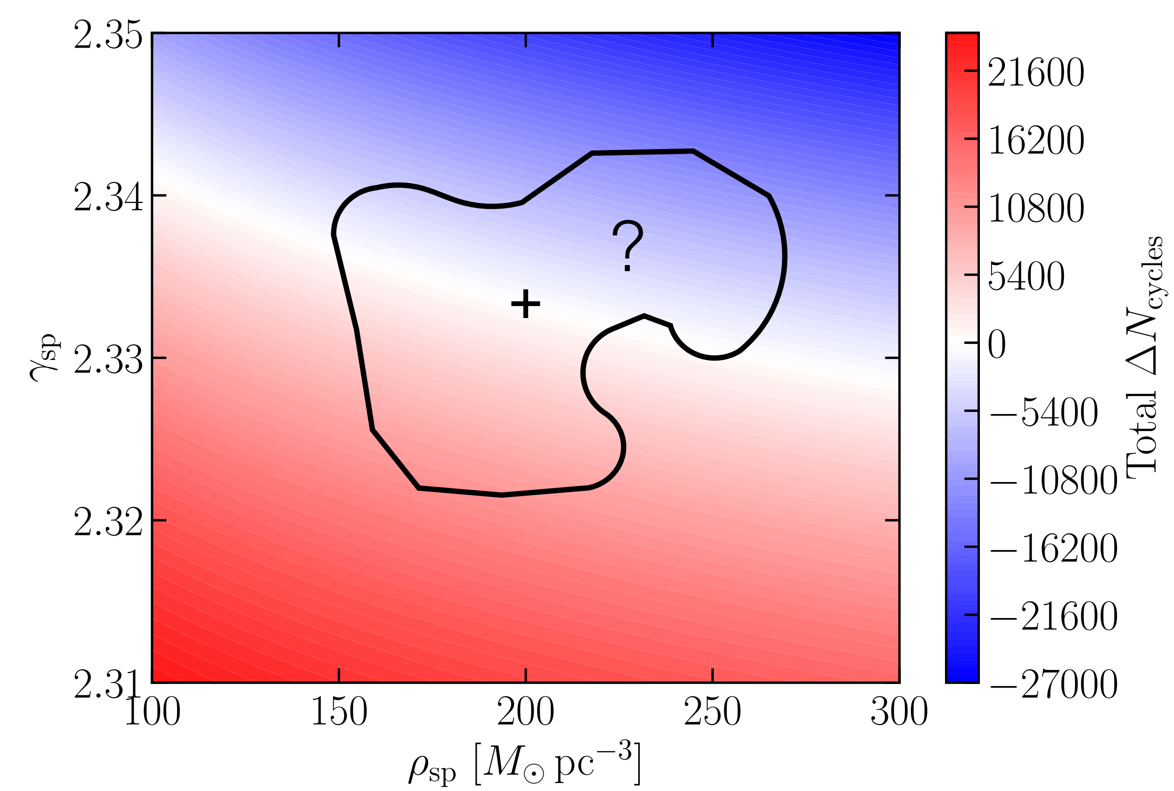
‘Dressed' IMRI systems need to be modelled carefully

[**BJK**, Nichols, Gaggero, Bertone, 2002.12811]

Next: develop search strategies and look at parameter estimation

[Ongoing work with all of the above, and *especially* Adam Coogan]





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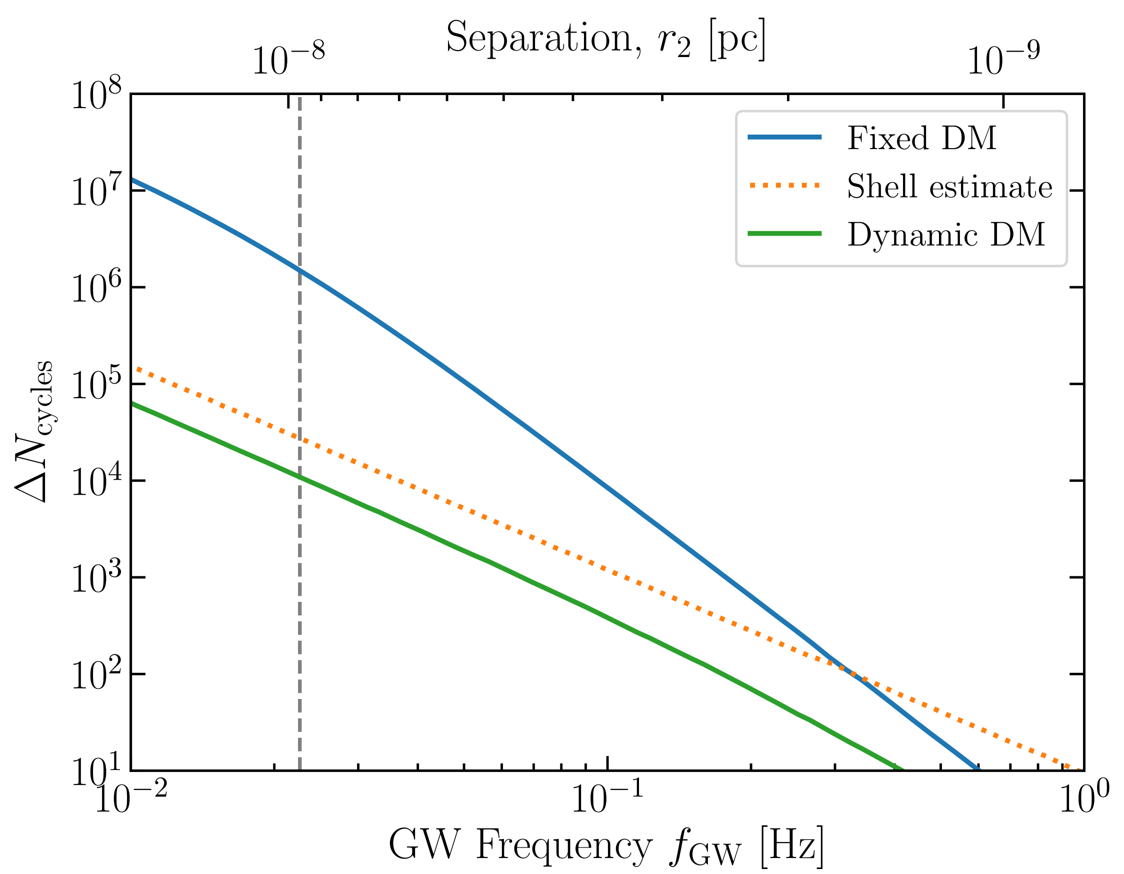
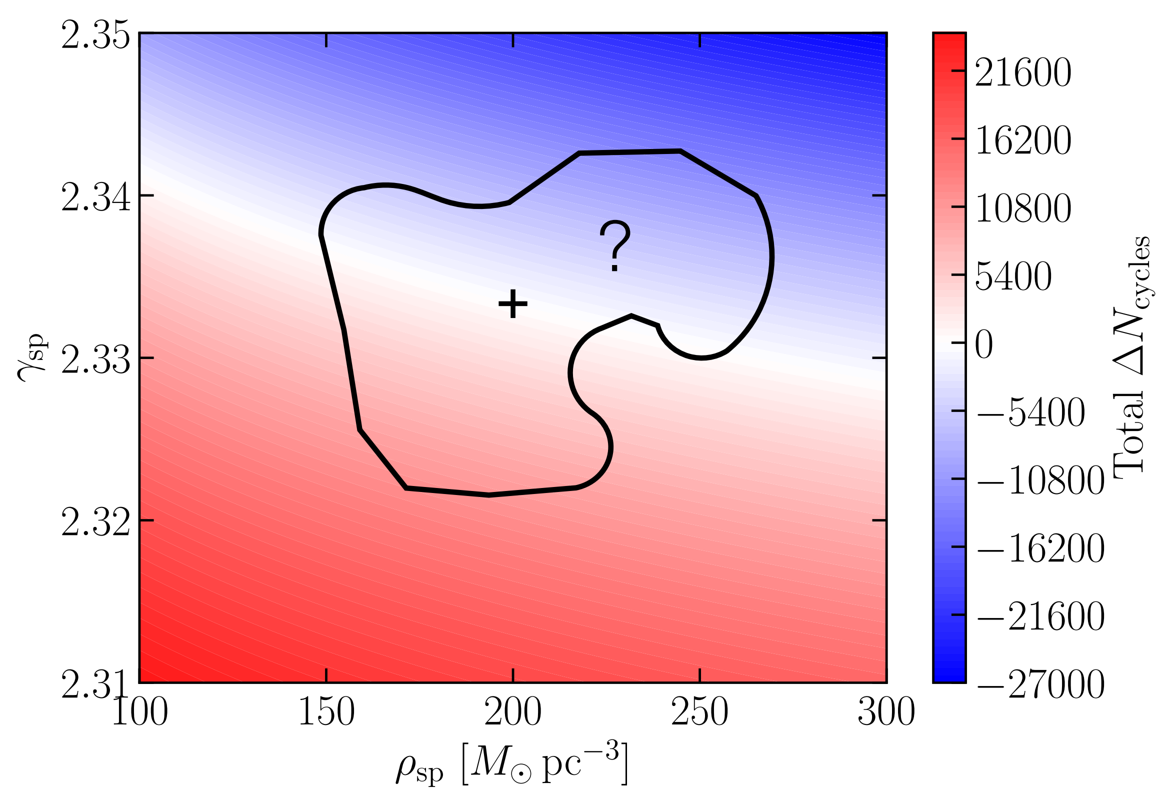




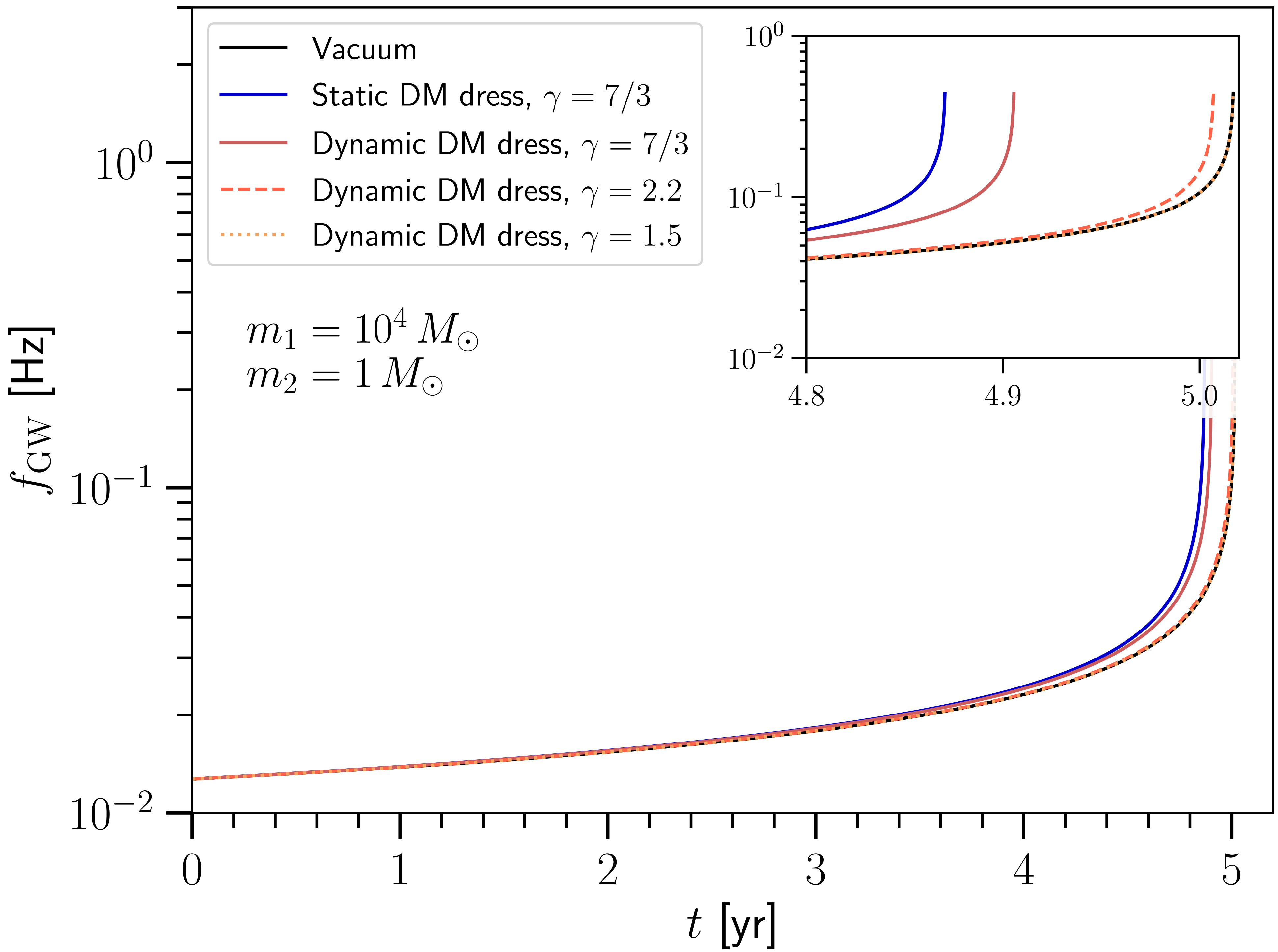
Thank you!

Backup Slides

Spectrograms: $m_{\mathrm{IMBH}} = 10^4\,M_\odot$

$$\rho_{\mathrm{DM}}(r) = \rho_{\mathrm{sp}} \left(\frac{r_{\mathrm{sp}}}{r}\right)^{\gamma_{\mathrm{sp}}}$$

As we increase the IMBH mass, the correction from having a dynamic DM halo decreases (but can still be very relevant)

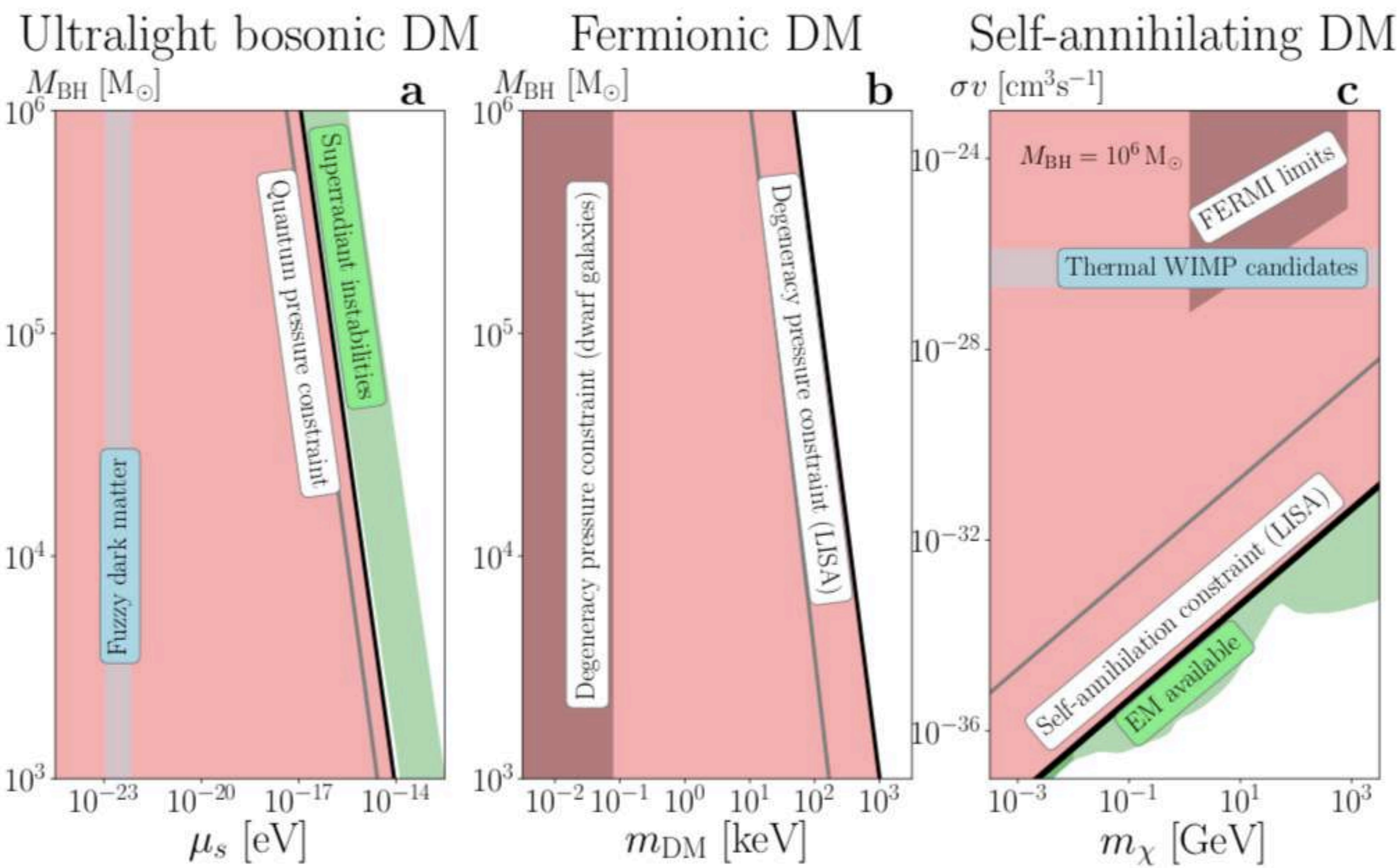


NB: $7/3 \approx 2.333$

Nature of Dark Matter

Red regions would be ruled out by observation of a DM spike!

[1906.11845]



[See also Bertone, Coogan, Gaggero, **BJK** & Weniger, 1905.01238]

Parameter Reconstruction

Prospects for parameter reconstruction in the *static DM* case:

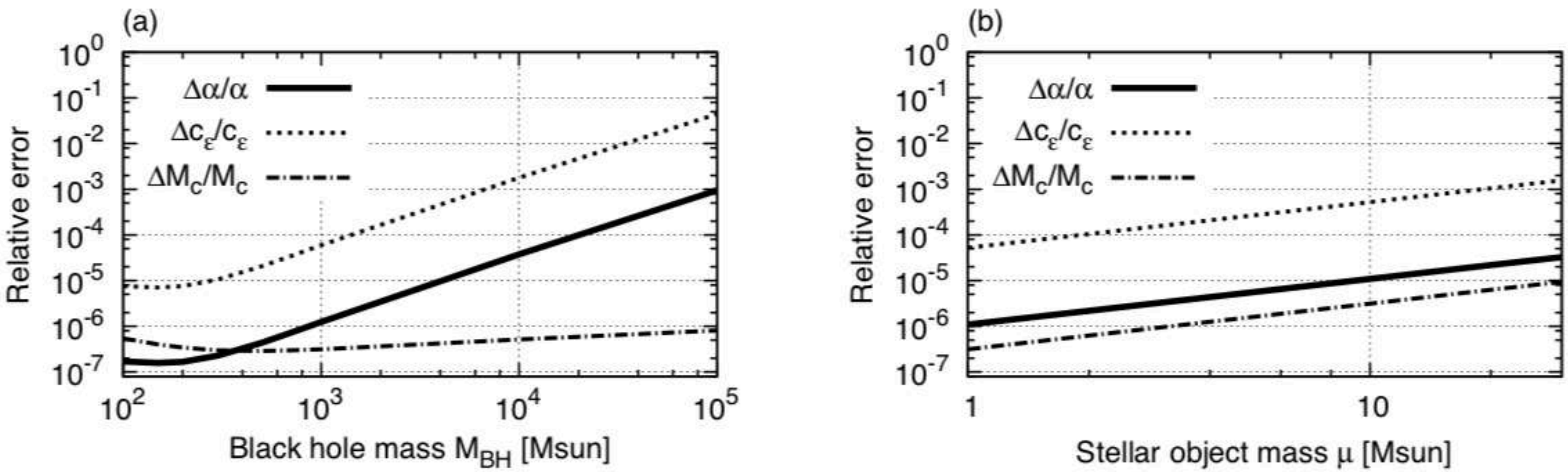
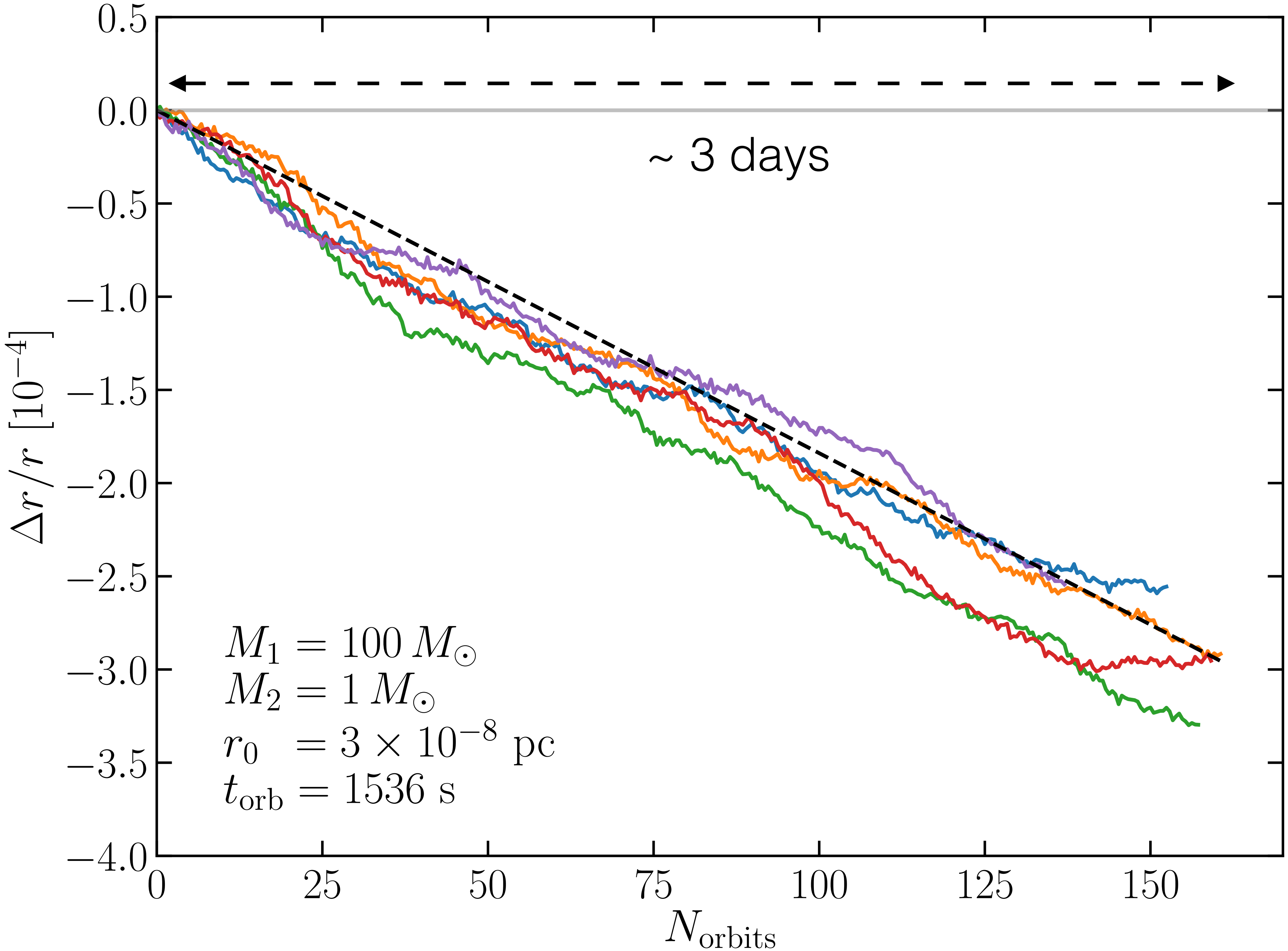


FIG. 4: The relative errors of the parameters in the phase $\tilde{\Phi}(f)$ versus (a) the central BH mass $M_{\rm BH}$ and (b) the stellar mass object mass μ for $S/N = 10$ and $\alpha = 7/3$. For this plot, $\rho_{\rm sp}$ and $r_{\rm sp}$ are taken from the table I. The other parameter is fixed to be $\mu = 1M_\odot$ in the left and $M_{\rm BH} = 10^3 M_\odot$ in the right, respectively. Note that the both axes are in the logarithmic scales. The solid line, the dashed line, the dashed-dotted line correspond to $\Delta\alpha/\alpha, \Delta c_\varepsilon/c_\varepsilon, \Delta M_c/M_c$ respectively.

[Eda et al. 1301.5971, 1408.3534]

N-body simulations



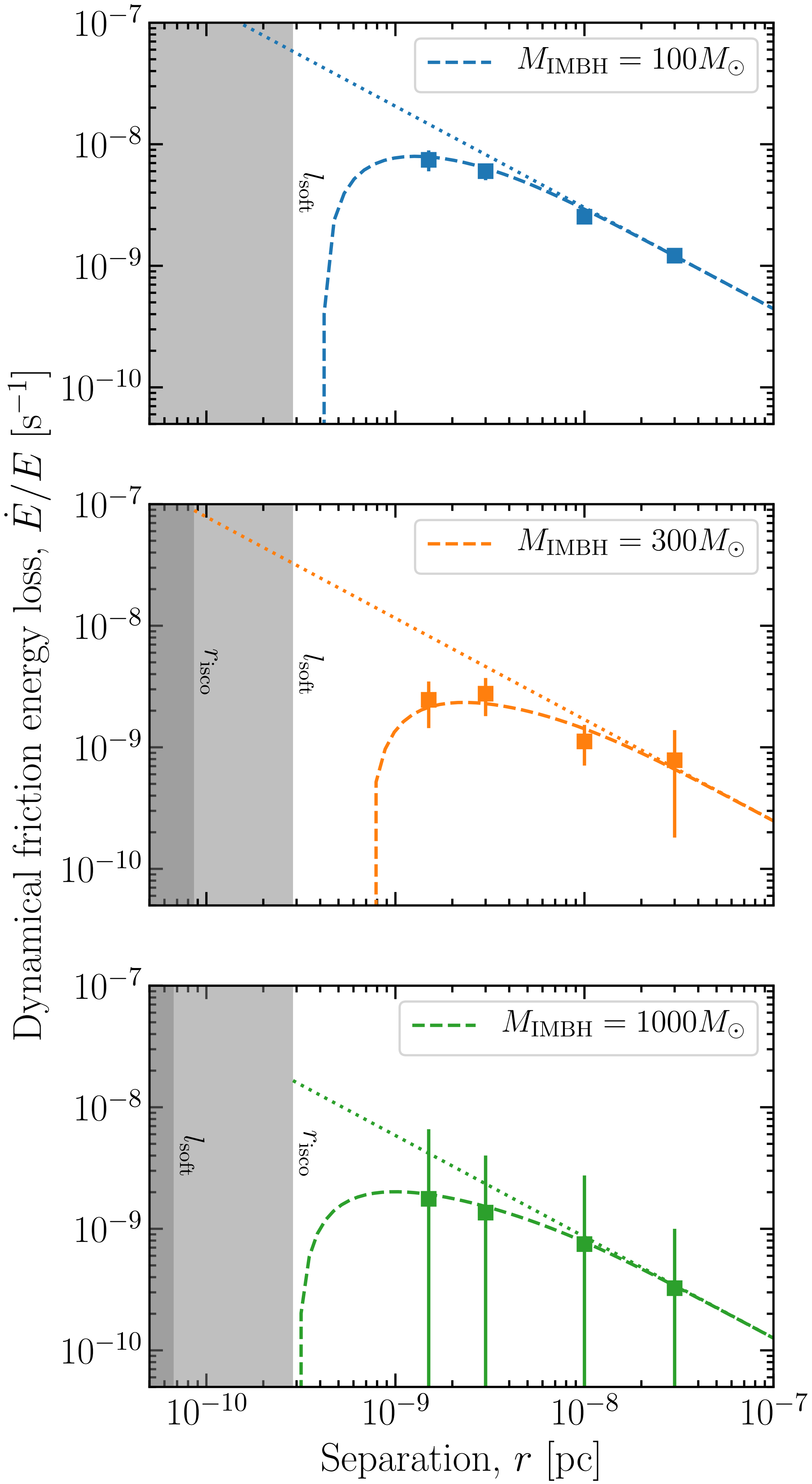
Allows us to check assumptions and fix normalisation of DF force (lnΛ), but can't simulate the whole 5 year inspiral!

N-body results

Dependence of dynamical friction force on mass and separation matches expectations

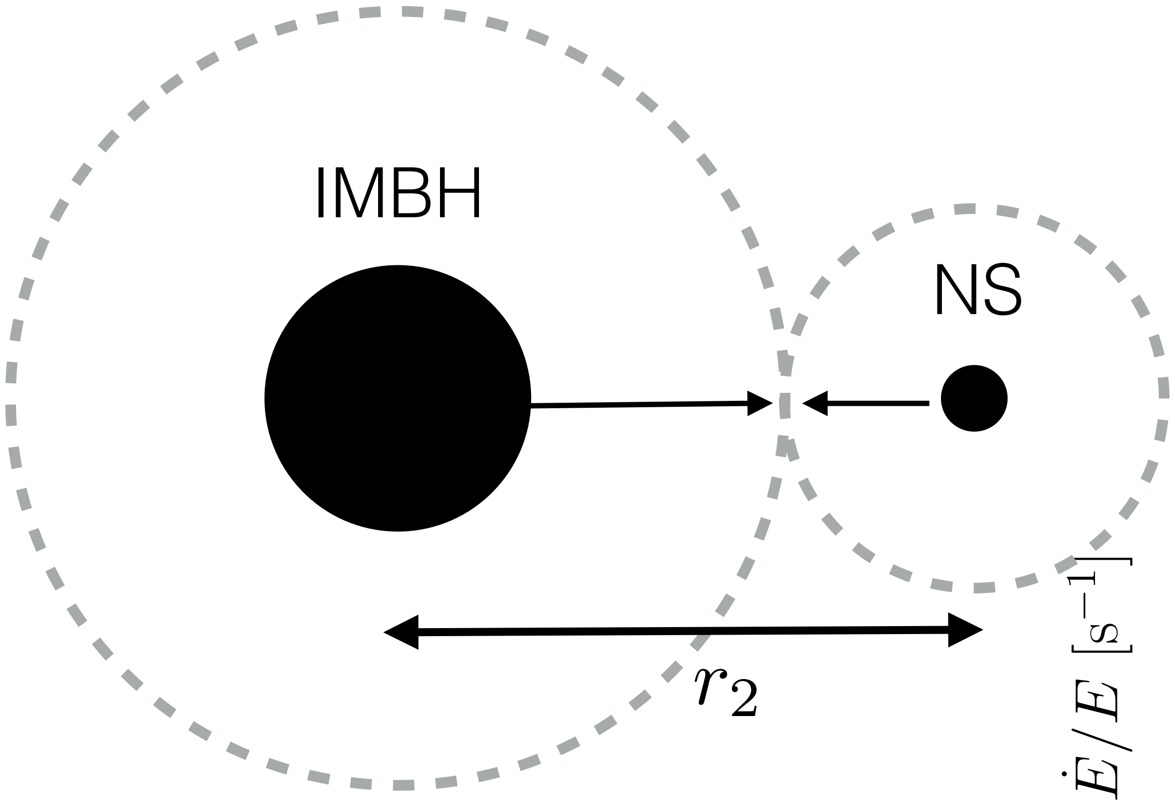
Dynamical friction traces local DM density (to better than 1%)

Drop off in DF force at small separations due to softening of simulations



N-body results

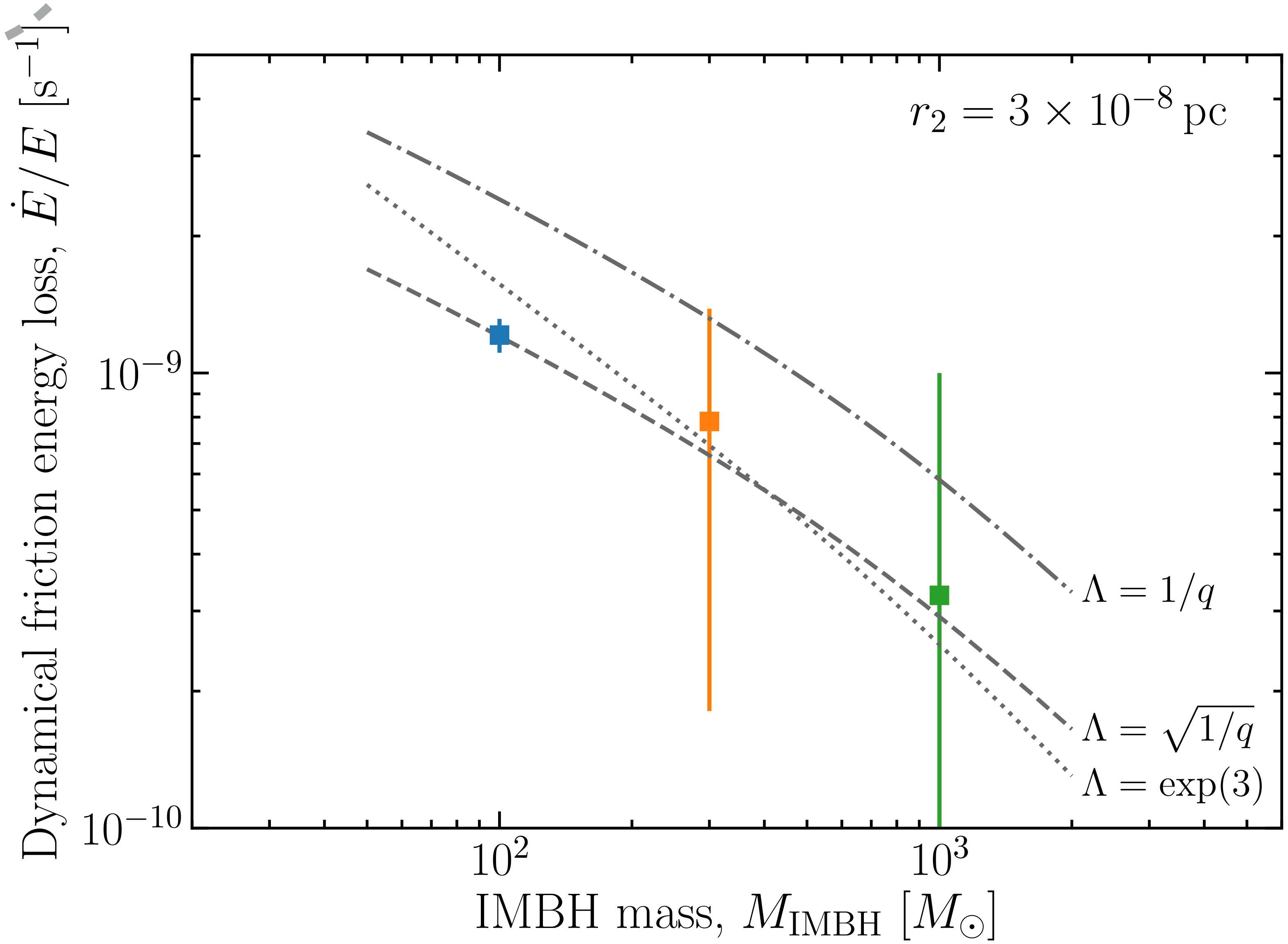
$$q \equiv m_{\rm NS}/m_{\rm IMBH} \ll 1$$



Allows us to calibrate the maximum impact parameter; tells us which particles scatter with the NS.

$$b_{\rm max} = \sqrt{q}\, r_2 \sim 3\%\, r_2$$

$$\Lambda = b_{\rm max} \frac{v_0^2}{G m_{\rm NS}} = \frac{b_{\rm max}}{q\, r_2} = 1/\sqrt{q}$$



Assumptions

- Spherical symmetry and isotropy of the DM halo

- DM particles only scatter within an impact parameter

$$b < b_{\max} = \Lambda \times G_N M_{\rm NS}/v_{\rm NS}^2$$

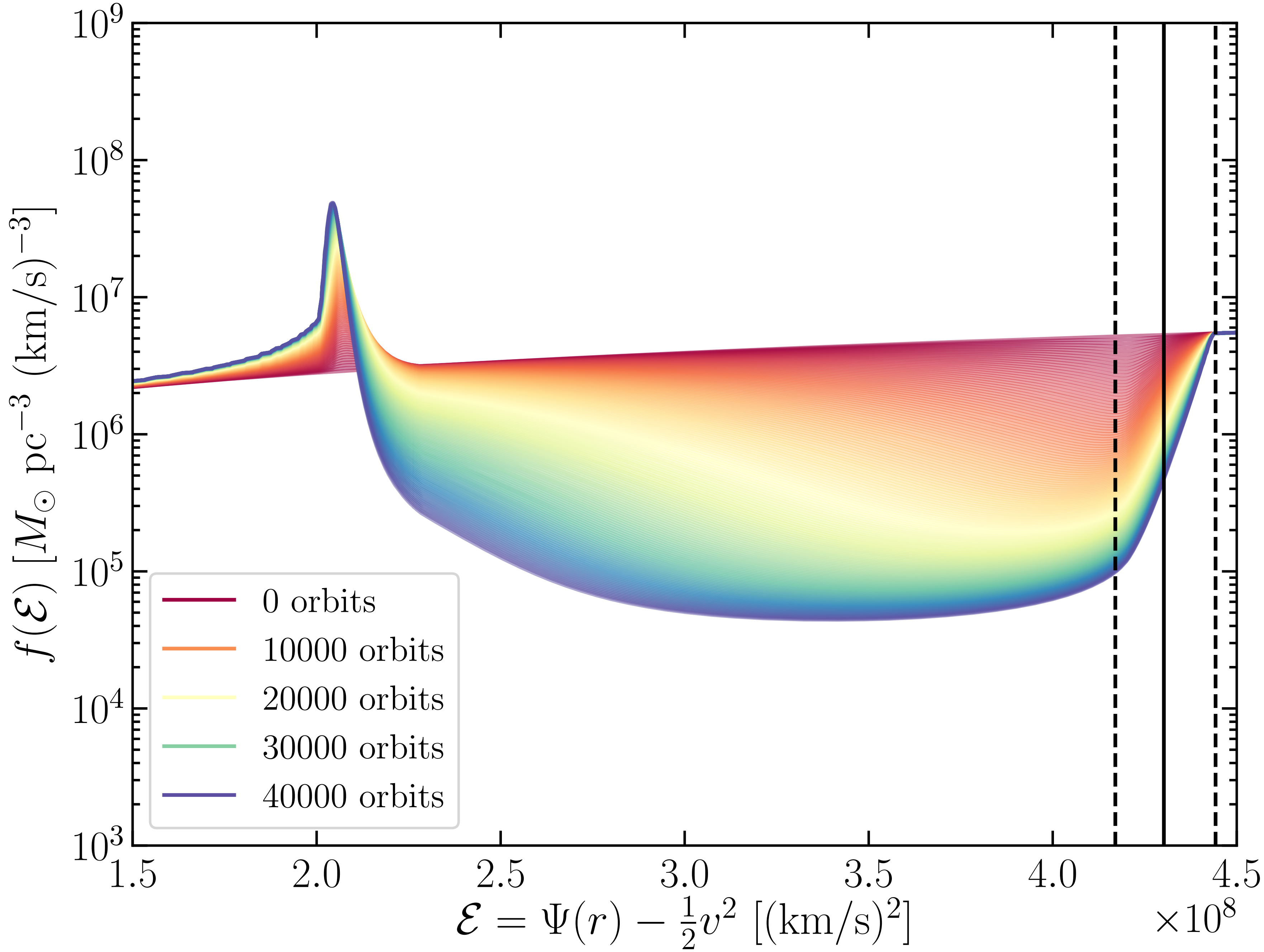
- DM distribution is 'locally' uniform

$$b_{\max} \ll r_0$$

- Halo 'relaxation' is instantaneous

- Orbital properties evolve slowly compared to the orbital period

Distribution function



Self-consistently reconstruct density from distribution function:

$$\rho(r) = 4\pi \int_0^{v_{\mathrm{max}}(r)} v^2 f\left(\mathcal{E}\right) \mathrm{d}v$$

Numbers of cycles

$m_1 = 10^3\, M_\odot$, $N_{\rm cycles} = 5.71 \times 10^6$ in vacuum

	$\gamma_{\rm sp} = 1.5$	$\gamma_{\rm sp} = 2.2$	$\gamma_{\rm sp} = 2.3$	$\gamma_{\rm sp} = 2.\bar{3}$
Static	< 1	2.4×10^4	1.6×10^5	2.9×10^5
Dynamic	< 1	2.7×10^2	1.9×10^3	3.5×10^3

$m_1 = 10^4\, M_\odot$, $N_{\rm cycles} = 3.20 \times 10^6$ in vacuum

	$\gamma_{\rm sp} = 1.5$	$\gamma_{\rm sp} = 2.2$	$\gamma_{\rm sp} = 2.3$	$\gamma_{\rm sp} = 2.\bar{3}$
Static	< 1	1.4×10^3	8.7×10^3	1.6×10^4
Dynamic	< 1	6.2×10^2	4.0×10^3	7.4×10^3

TABLE I. **Change in the number of cycles $\Delta N_{\rm cycles}$ during the inspiral.** Change in the total number of GW cycles due to dynamical friction, starting 5 years from the merger.