S-matrix Bootstrap

Discussion session

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Strings 2021, Sao Paulo, Brazil

1. Introduction

2. State of the art

3. Open problems

(list at the end of the presentation)

Physical theory = a complete list of consistent observables

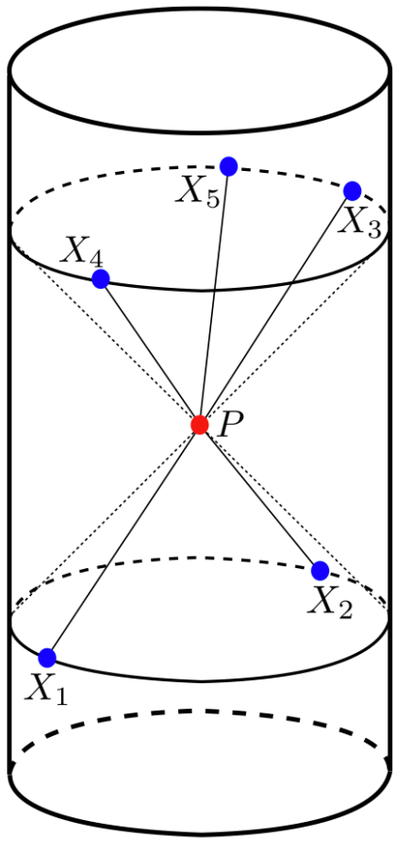
- What are the observables?
- What is a complete list?
- What does *consistent* mean?

What is the space of theories?

(What is the theory?)

Observables are anchored at infinity

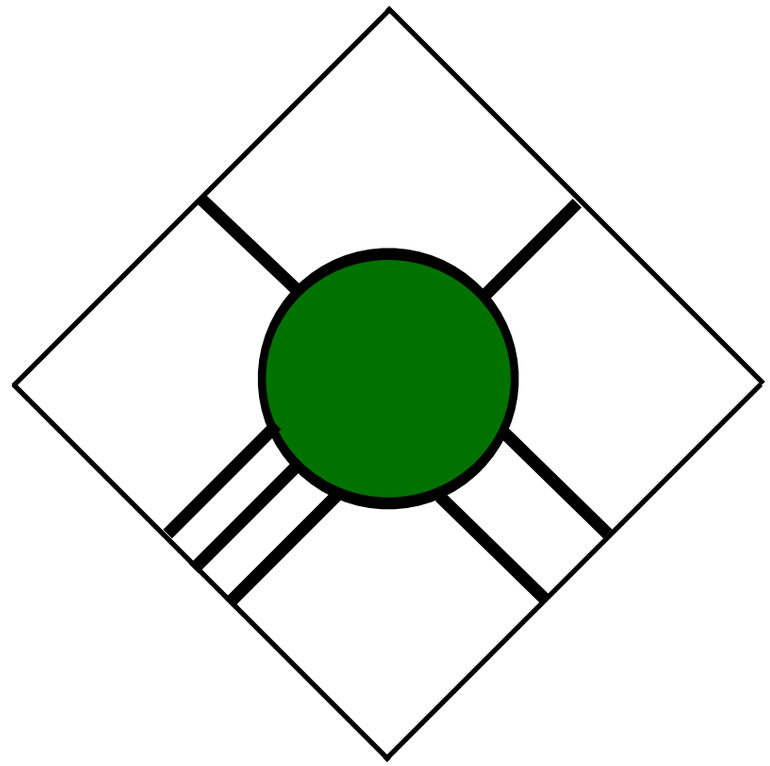
AdS/CFT



correlation functions of local operators

(Euclidean+Lorentzian)

flat space



scattering amplitudes

(Lorentzian)

What is a complete set of observables?

$$\widehat{1} = \sum_i |\mathcal{O}_i\rangle\langle\mathcal{O}_i|$$

$$\langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_k \mathcal{O}_l \rangle$$

(via OPE)

$$\widehat{1} = \sum_n |n\rangle\langle n|$$

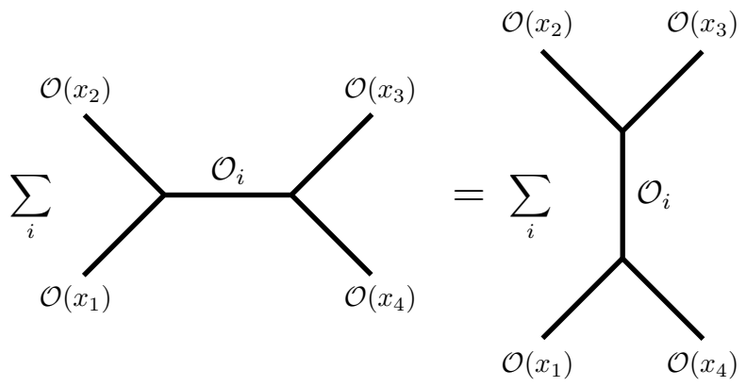
$$\mathcal{A}_{m\to n}$$

(terra incognita beyond 2-2)

What does consistent mean?

(Euclidean) CFT axioms

[Kravchuk, Qiao, Rychkov]

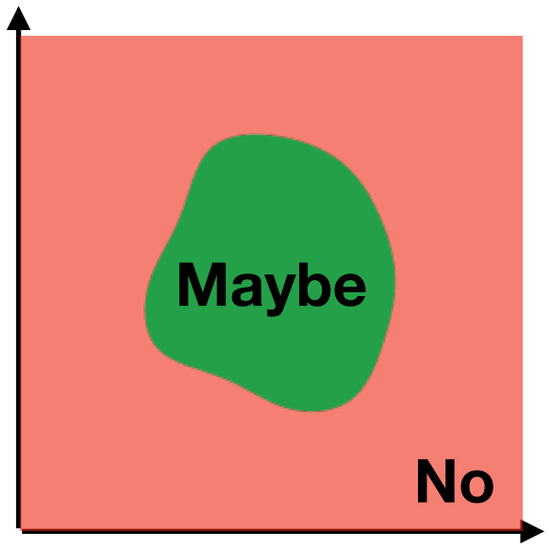


unitarity+symmetries+causality (analyticity, crossing, …)

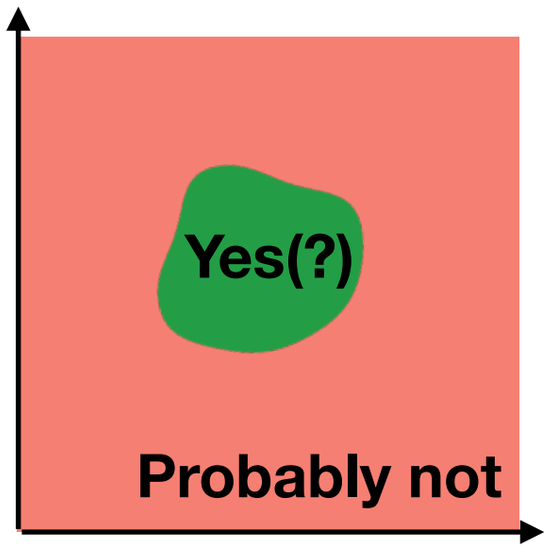
$$SS^\dagger = \widehat{1}$$

What is the space of theories?

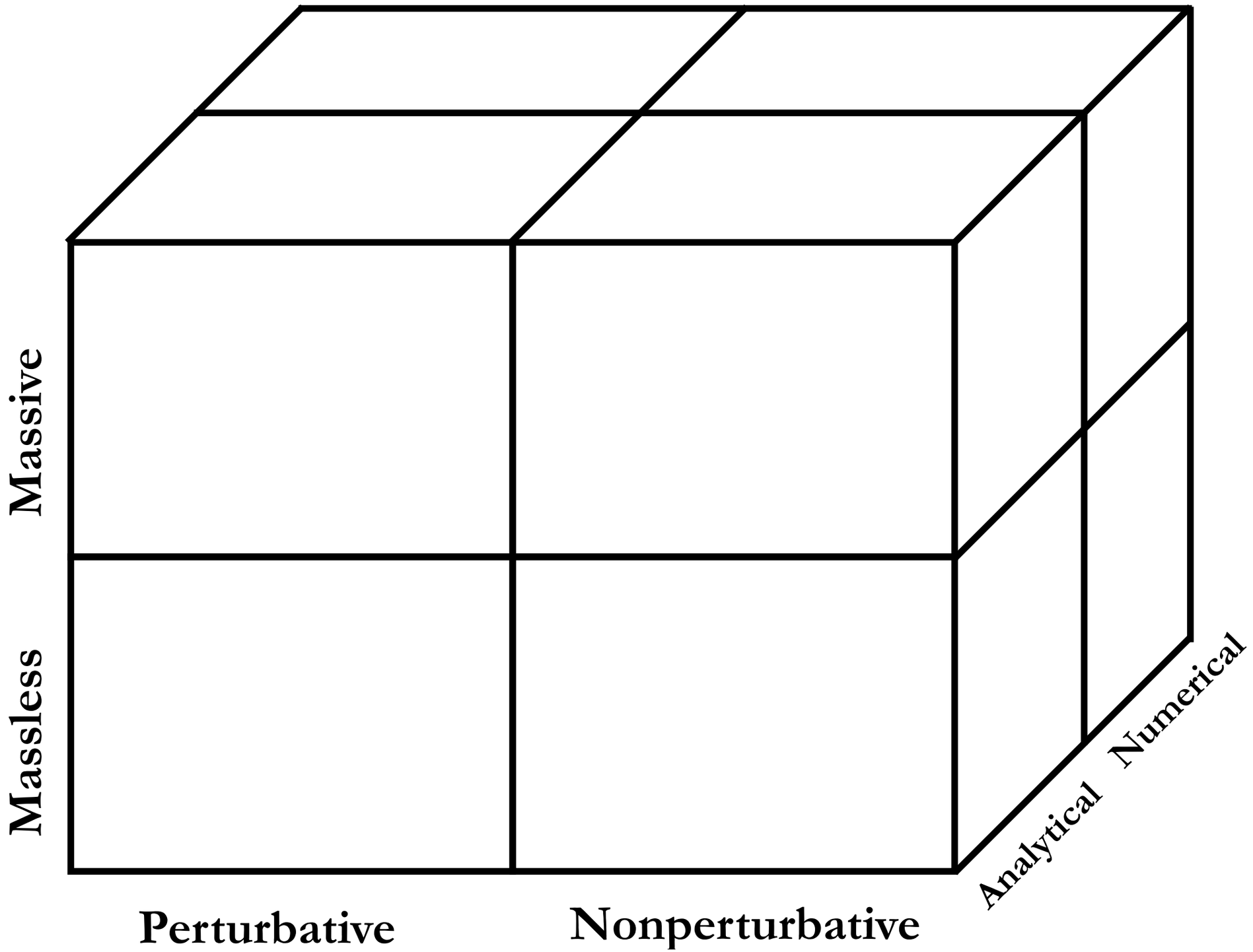
Dual problem: exclude theories (bootstrap bounds)



Primal problem: construct amplitudes (landscape)



The Bootstrap Cube



	Perturbative	Nonperturbative
Massive	Confining gauge theories in the planar limit	Strongly interacting QFTs: • 2D Ising field theory • Confining gauge theories • ...
Massless	Classical UV completions of gravitational theories (tree level string theory) Weakly coupled EFTs (neglecting massless loops)	UV completions of EFTs: • phonons of flux tubes • pions (chiral lagrangian) • photons (Euler-Heisenberg) • (super)gravitons • ...

Interesting theories

Analytic results

	Perturbative	Nonperturbative
Massive	Crossing symmetry (planar) Universality of planar 4-point amplitude at large s,t > 0 On-shell methods [Arkani-Hamed, Caron-Huot, Huang, Huang, Komargodski, Mizera, Sever, AZ]	Analyticity domains Crossing symmetry (2 to 2) Froissart bound Froissart-Gribov formula Dispersion relations Bounds on couplings [Bros, Creutz, Epstein, Froissart, Gell-mann, Glaser, Goldberger, Gribov, Lehmann, Martin, Paulos, JP, Thirring, Toledo, van Rees, Vieira, …]
Massless	On-shell methods […] EFThedron and positivity bounds (neglecting massless loops) String scattering amplitudes from the flat space limit of AdS/CFT [Adams, Agmon, Alberte, Arkani-Hamed, Bellazzini, Bern, Binder, Caron-Huot, Chester, de Rham, Dubovski, Elias-Miro, Gary, Giddings, Gorbenko, Green, Fitzpatrick, Heemskerk, Hijano, Huang, Huang, Jaitly, Kaplan, Komatsu, Kosmopoulos, Li, Maldacena, Mazac, Mirbabayi, Nicolis, Okuda, Paulos, JP, Polchinski, Pufu, Raju, Rastelli, Rattazzi, Riembau, Riva, Simmons-Duffin, Sinha, Sully, Tolley, van Duong, van Rees, Yang, Wang, Wen, Xu, Zahed, Zhao, Zhang, AZ, Zhou …]	Positivity bounds (including massless loops) Bounds on EFT for flux tubes in 3D [Bellazzini, Elias Miro, Guerrieri, Hebbar, JP, Rattazzi, Riembau, Riva, Vieira …]

	Perturbative	Nonperturbative
Massive		"Bounds" on: • quartic couplings (primal and dual) • cubic/Yukawa couplings (primal) (assuming maximal analyticity) Multiple amplitudes bootstrap Fixed point mapping methods [Bercini, Cordova, Doroud, Elias Miro, Fabri, Guerrieri, He, Hebbar, Homrich, Karateev, Kruczenski, Paulos, JP, Sever, Toledo, Tourkine, van Rees, Vieira, AZ …]
Massless	Positivity bounds on EFT couplings: • pions (chiral lagrangian) • photons • Standard Model EFT • (Super)gravitons • … [Arkani-Hamed, Bern, Caron-Huot, Huang, Huang, Kosmopoulos, Mazac, Rastelli, Simmons-Duffin, van Duong, Zhang, Zhou, AZ …]	"Bounds" on EFT couplings: • flux tubes • pions (chiral lagrangian) • (Super)gravitons (assuming maximal analyticity) [Elias Miro, Guerrieri, Hebbar, JP, Vieira]

Numerical results

Solve known theories.
Find new theories

From AdS
to flat space

Multi-particle
amplitudes

New targets
(experiments,
cosmology)

S-matrix Bootstrap
(Open Problems)

IR-finite
observables

High energy
data

Bounds on
the Regge limit

Spinning
particles

Open questions

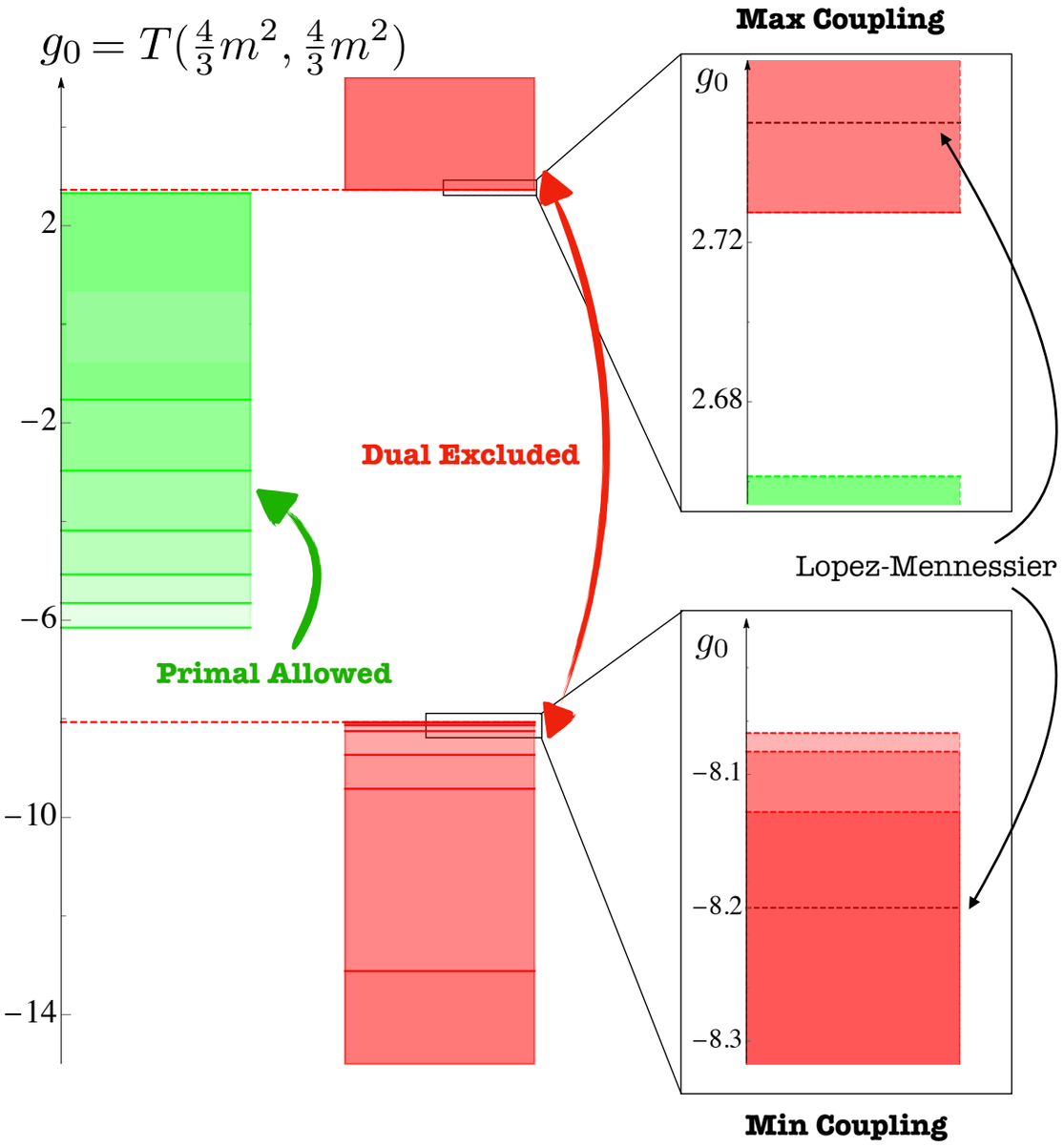
	Perturbative	Nonperturbative
Massive	• Can we constrain the spectrum of planar confining gauge theories?	• Bootstrap n to m amplitudes (inelastic effects) • Prove maximal analyticity • How to input UV information (local operators, hard scattering …)? • Extremal theories
Massless	• Is tree-level string theory unique? • Prove classical Regge bound • Prove low spin dominance • Compare positivity and nonperturbative unitarity • Extremal theories	• Prove quantum Regge bound • Make use of Regge trajectories • How to input BH production at high energies? • S-matrix properties from flat space limit of AdS/CFT • Bootstrap IR safe observables in 4D

Extra slides

Rigorous Upper and Lower bounds on the quartic coupling

[Lopez, Mennessier '77]

[Guerrieri, Sever]



Setup

- We consider the scattering of identical massive scalar particles in four spacetime dimension

- These satisfy double subtracted fixed-t dispersion relations for $-28m^2 < t < 4m^2$.

Constraints imposed

- Crossing: $T(s,t) = T(t,s) = T(s, 4-s-t)$

- Unitarity:

$$2\,\mathrm{Im} f_l(s) \geq \sqrt{\frac{s-4m^2}{s}}\,|f_l(s)|^2 \quad \text{for} \quad 4m^2 < s < 12m^2\,,\ l \leq L$$

$$0 \leq \mathrm{Im} f_l(s) \leq \sqrt{\frac{4s}{s-4m^2}} \quad \text{otherwise}$$

Dual constraints $\Rightarrow$ Rigorous bounds

Still to improve

- Extending the region where unitarity is imposed

- Impose the Froissart high energy behavior

Maximal Coupling in Gapped QFT

(massive+nonperturbative+numerical+primal)

[Paulos, Penedones, Toledo, van Rees, Vieira]

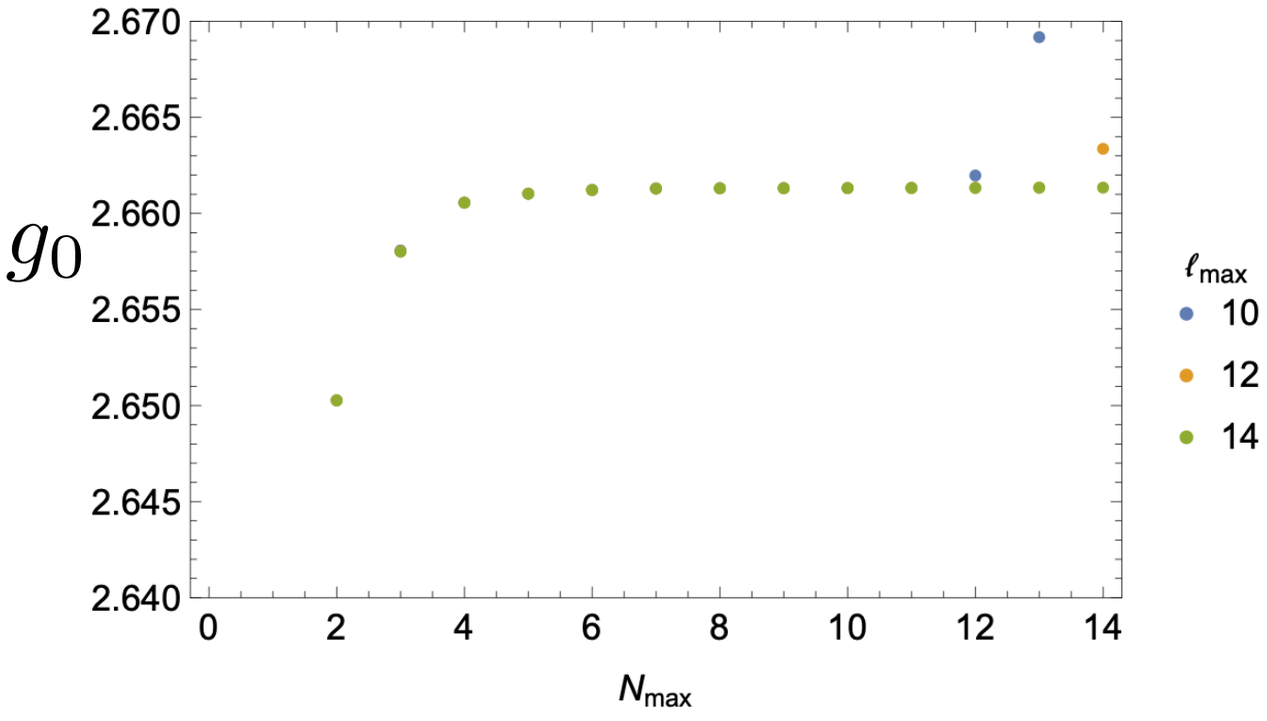
$$T(s,t) = \sum_{a,b,c=0}^{N_{\max}} \alpha_{abc}\rho_s^a\rho_t^b\rho_u^c + \text{extra}|_{u=4m^2-s-t}$$

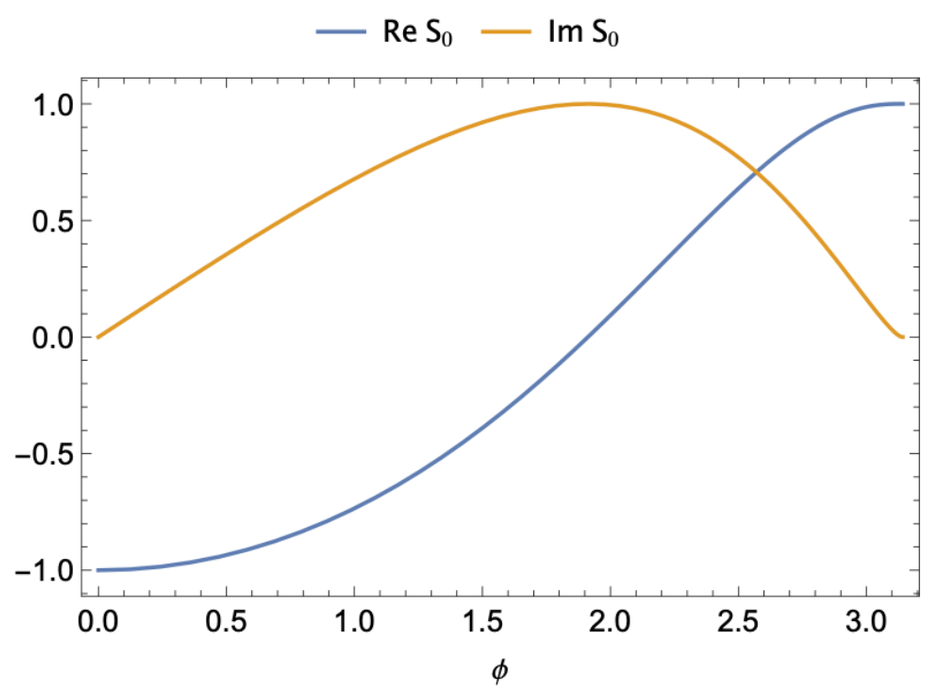
assuming maximal analyticity

$$g_0 \equiv \frac{1}{32\pi}T\left(\frac{4m^2}{3},\frac{4m^2}{3}\right)$$

$$\boxed{-8.2 \leq g_0 \leq 2.75}$$

[Lopez, Mennessier '77]





+higher partial waves are very small!

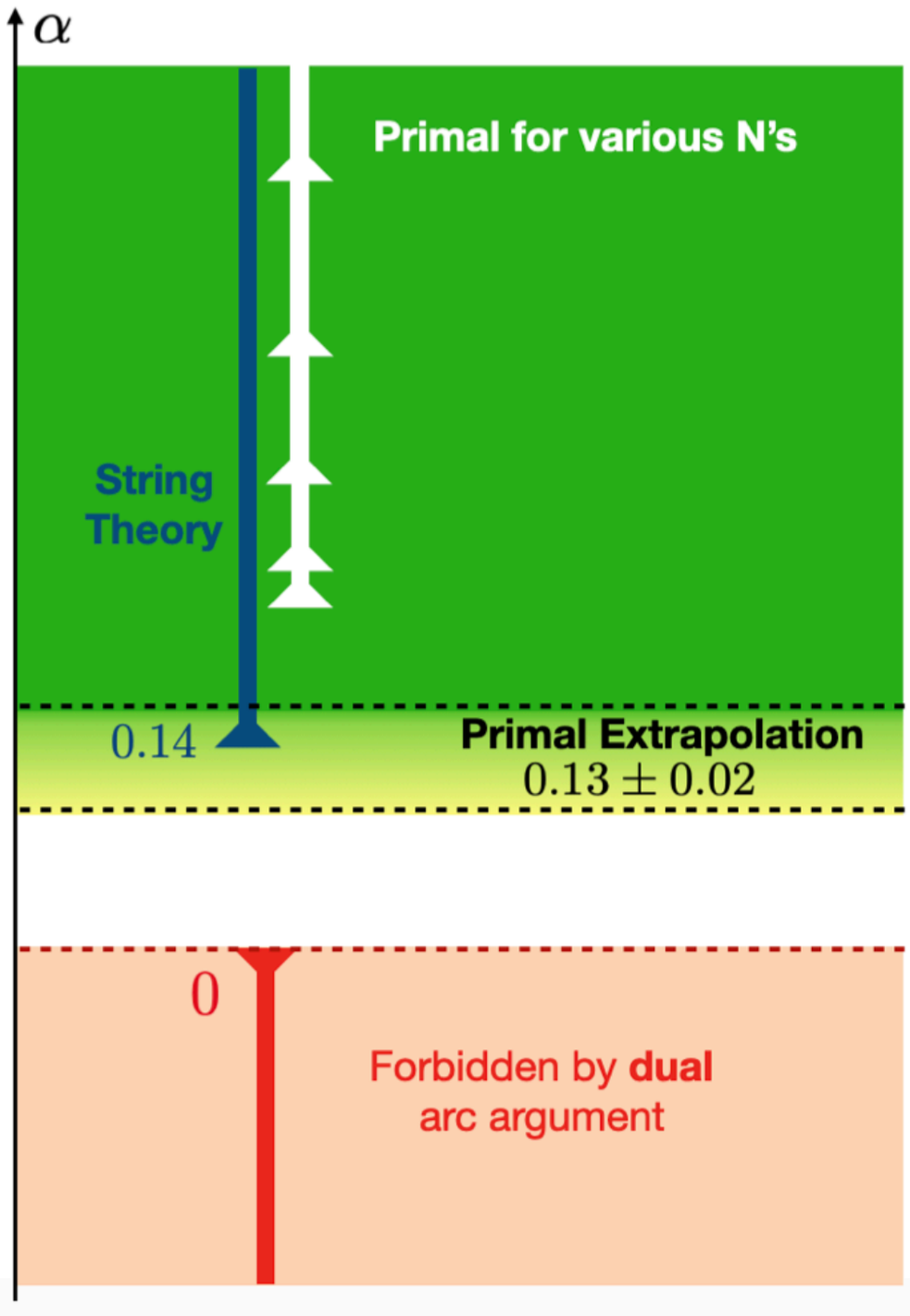
- make it rigorous?
- solvable 4d S-matrix?
- can production be made arbitrarily small?

  (Aks theorem: in d>2 scattering implies production)
- impose elastic unitarity + multi-particle unitarity?
- Regge limit (Froissart bound)/fixed angle scattering?

Supergraviton Scattering

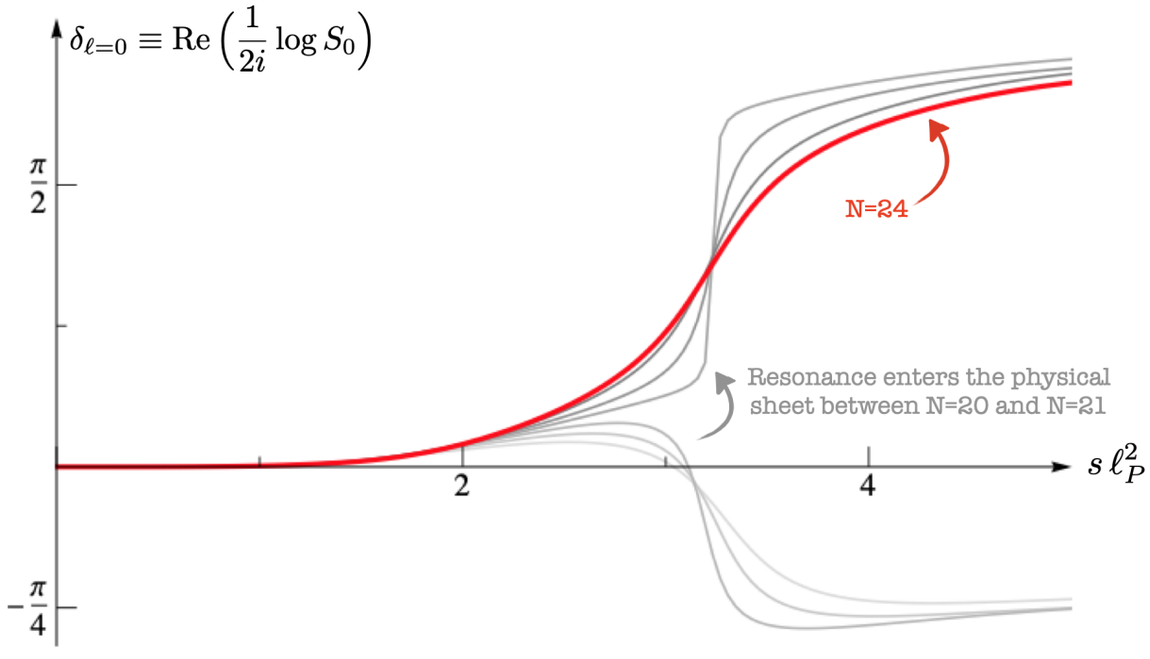
(massless+nonperturbative+numerical+primal)

[Guerrieri, JP, Vieira]



$$\frac{T(s,t,u)}{8\pi G_N = 64\pi^7 \ell_P^8} = s^4 \left(\frac{1}{stu} + \alpha\, \ell_P^6 + O(s) \right)$$

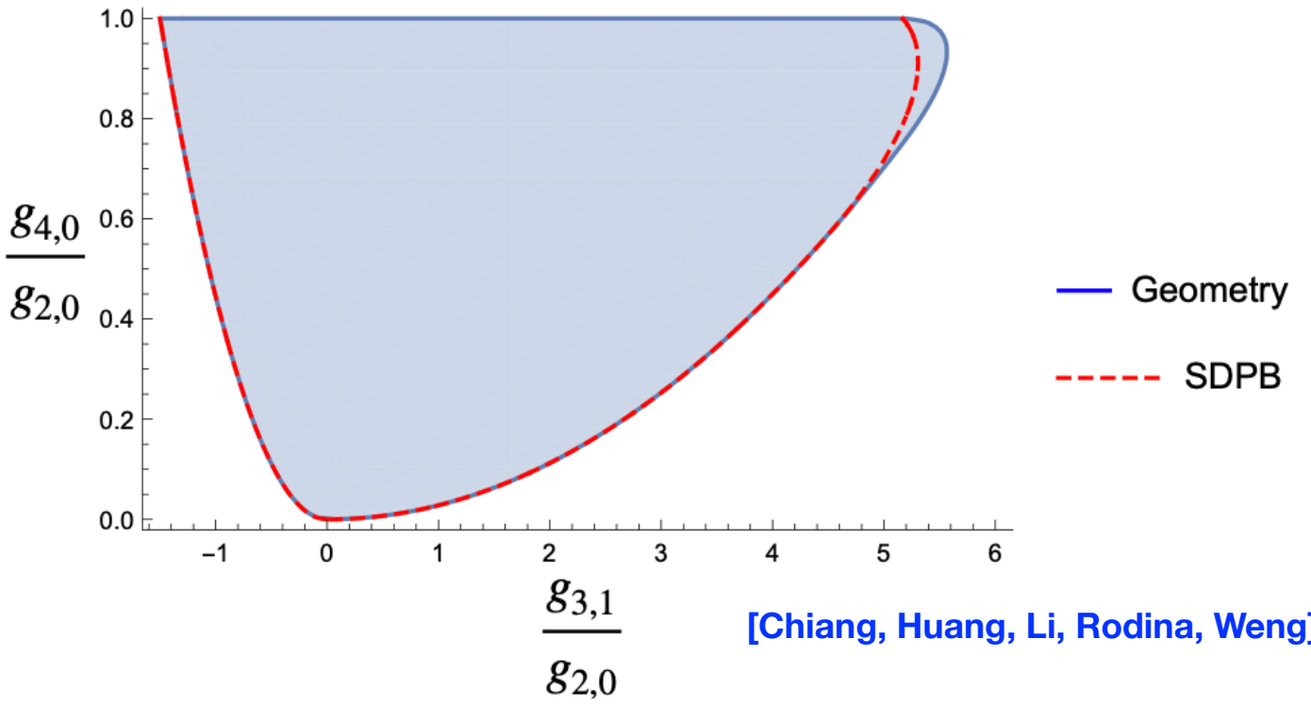
R⁴ correction



Graviball resonance

Graviton Scattering

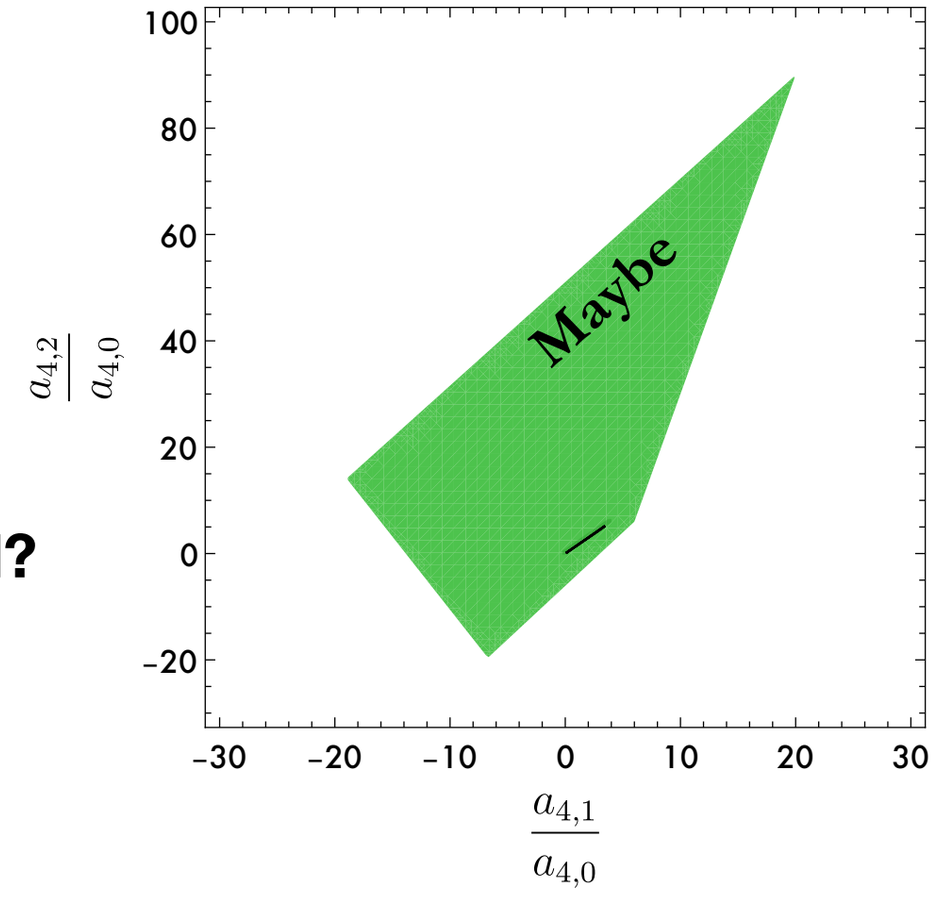
(massless+perturbative+analytical/numerical+dual)



[Chiang, Huang, Li, Rodina, Weng]

$$\mathcal{M}_4(1^+,2^-,3^-,4^+) = (\langle 23\rangle[14])^4\, f(s,u)$$

$$f(s,u) = \sum_{k,i} a_{k,i} s^k u^i + \ldots$$



[Arkani-Hamed, Huang, Huang]
[Bern, Kosmopoulos, AZ]

- **EFT geometry?**
- **Analyticity+Regge bound?**
- **Graviton loops (logarithms)**

[Bellazzini, Miro, Rattazzi, Riembau, Riva]

- **d=4: what are the (dispersive sum) rules for IR finite observables?**

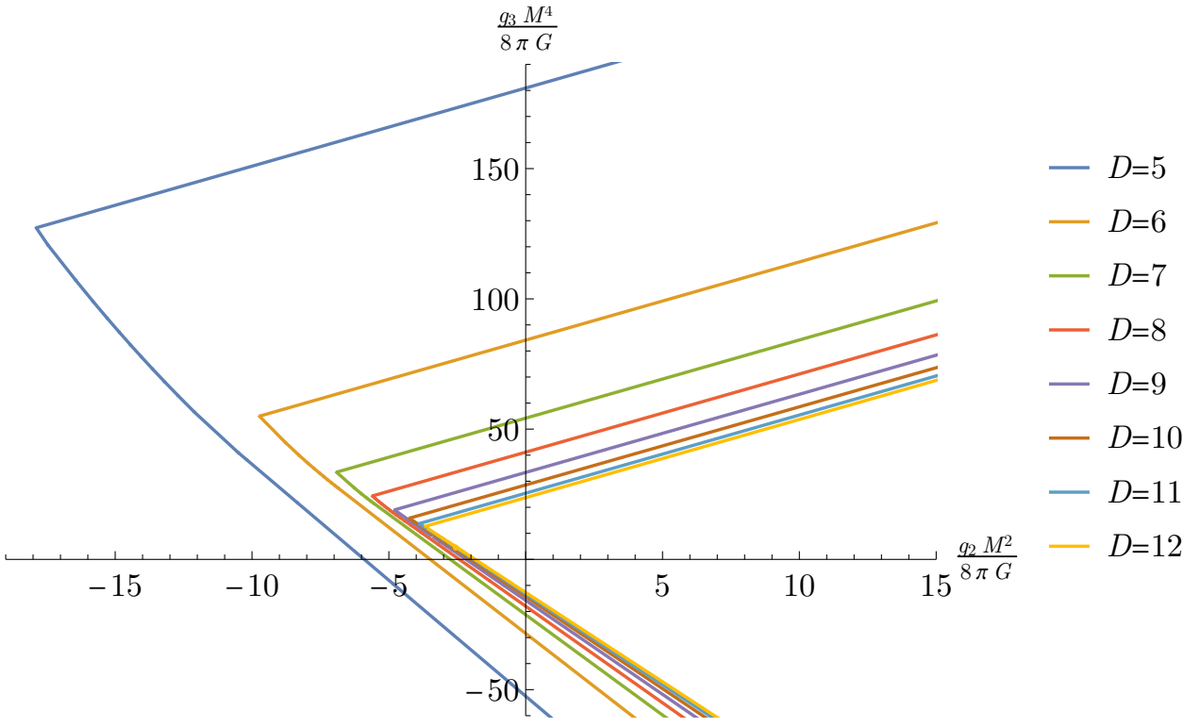
[Arkani-Hamed, Pate, Raclariu, Strominger]

- **Extremal gravitational theories?**
- **Higher-point amplitudes/sum rules?**
  (low spin dominance)

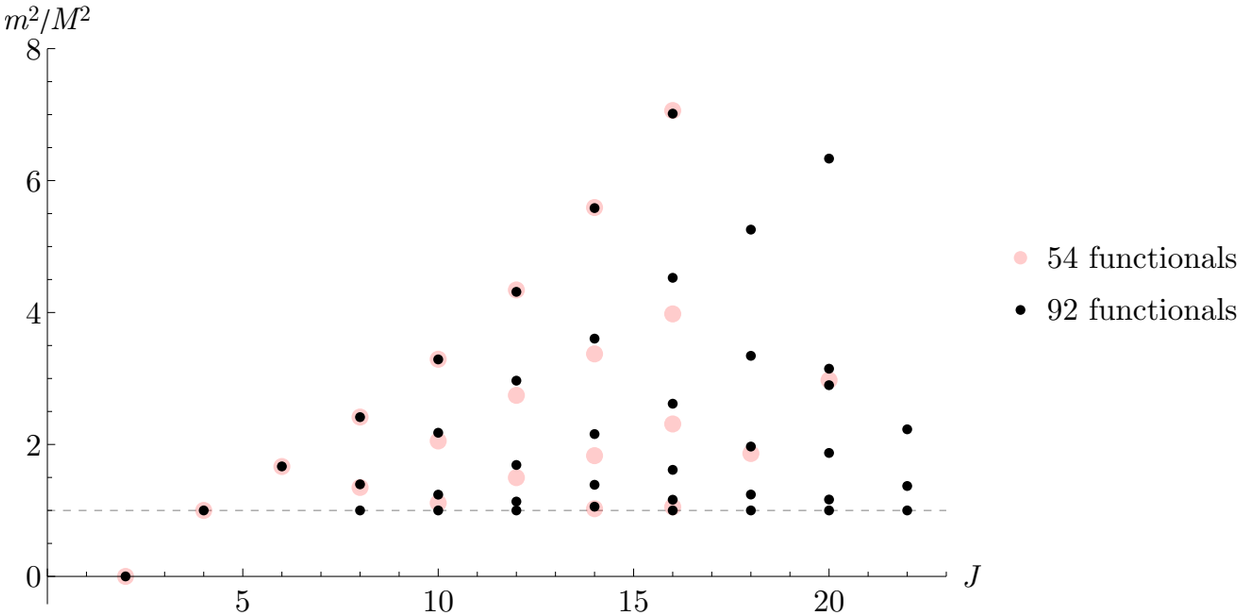
$$f(s,u) = \frac{1}{(s-m^2)(u-m^2)(t-\tilde{m}^2)}$$

Example of an extremal theory

[Caron-Huot, Mazac, Rastelli, Simmons-Duffin]



spectrum minimizing g_2 (D=6)



List of questions and hints

We are grateful to everyone that suggested questions for this list.

Questions and hints

1. Can we solve d>2 S-matrices in the same way we can solve CFTs? Can we find new theories this way?

It is definitely possible in d=2 for integrable theories with no particle production, in d>2 particle production is unavoidable which makes amplitudes much more complicated. Approximate solutions?

2. What can we learn about nonperturbative S-matrices using the flat space limit of AdS/CFT?

Perturbatively the mapping is quite direct, however nonperturbative unitarity and mapping to the flat space Hilbert space is not clear.
Is it just a technical difficulty or there is some conceptual problem?

Questions and hints

3. Can we constrain the theory spectrum?

Current bounds are systematic but not obviously very useful. Is it possible to put constraints on the spectrum of the theory? Is it a technical problem or there is some fundamental difficulty?

4. Analyticity for 2-2 amplitudes

Is it possible to improve the old arguments? Minimal set of singularities dictated by unitarity is complete = maximal analyticity. For example, scattering of lightest particles in the theory. Is there still something to be learned from perturbative analysis?

Questions and hints

5. Constraints from m-n amplitudes

Essentially all the nonperturbative work so far is limited to 2-2 amplitudes. Can we go beyond that (possibly with some not fully justified assumptions)? Amazing progress in understanding perturbative amplitudes could guide us.

6. Nonperturbative Regge Theory

Is there some semi-universal theory of nonperturbative Regge limit (similarly to universal late-time physics in chaotic systems; Regge Field Theory)? Can we put in the Regge trajectories information into our bootstrap schemes?

Questions and hints

7. Bounds from multiparticle scattering

What is wrong with $\frac{1}{(s-m^2)(t-m^2)(u-m^2)}$? This amplitude has an accumulation point in the spectrum, can we turn it into a full-fledged m->n S-matrix?

8. How to use fixed angle/high energy scattering data in the bootstrap analysis?

In most of the current methods fixed angle scattering data is not used. One famous exception is scattering of boost eigenstates (called celestial amplitudes). There existence of the Mellin transform is guaranteed by the soft fixed angle behavior in gravity.

Questions and hints

9. What are the properties of IR-finite amplitudes with gravitons and photons in d=4?

It is not hard to define IR-finite dressed scattering amplitudes, but it is less clear what are their properties and how to bootstrap them. Eg are there celestial dispersion relations?

10. What can be said about scattering of weakly coupled higher spin particles?

Can we constrain the mass spectrum of planar confining gauge theories studying the 2 to 2 amplitude?

Questions and hints

11. Lower bound on inelasticity?

We know that in d>2 scattering implies production. What is the minimal bound on inelasticity? Recent work suggests that multi-particle unitarity should be important to derive such a bound. At the level of the 2-2 scattering amplitude it seems that inelasticity can be made arbitrarily small.

12. Interplay with local operators?

It is possible to enlarge the S-matrix Bootstrap to include form factors and spectral densities of local operators. Can this be used to input information about the UV CFT?

Questions and hints

13. Nonperturbative unitarity and positivity

In the most of the current work about weakly coupled gravitational theory only perturbative version of unitarity is imposed. What can we learn from nonperturbative unitarity?

14. Entanglement and scattering?

What can we learn from studying the quantum entanglement produced in a scattering process?

Questions and hints

15. Scattering in different dimensions?

Higher spacetime dimensions can be useful (e.g. to remove IR divergences). 2D is simpler because there are no scattering angles. The Ising field theory is probably the best laboratory to test new ideas to solve non-integrable QFTs.

16. Polynomial boundedness in gravity?

Can we use causality to derive polynomial boundedness of gravitational amplitudes (signal model suggests that it might be possible)? What is the optimal power in the Regge limit?

Questions and hints

17. Is any of this relevant for cosmology?

Perturbatively there are many points of contact. Nonperturbatively it is not clear due to the usual problem of defining observables in cosmological spacetimes.

18. Is mixed signature nonperturbatively relevant?

In many situations it is very convenient to work in mixed signature. Usually this is understood as analytic continuation to unphysical kinematics. Is there a way to give it some independent interpretation?

Questions and hints

19. New targets in the S-matrix bootstrap?

At the moment we are used to asking a particular class of question about the theory (e.g. minimizing or maximizing the coupling). Are there any missed targets in the S-matrix bootstrap which can lead to interesting results? For example, is there a sense in which we can maximize particle production?

20. Can we bootstrap anyon scattering?

The crossing properties of anyon scattering amplitudes are different (because of the Wilson lines attached to the charged particles). Can we establish these rules and implement the corresponding bootstrap?

Questions and hints

21. Can we bootstrap scattering amplitudes of resonances?

Firstly, one would need to define nonperturbative scattering amplitudes of resonances (possibly by factorizing higher particle amplitudes on complex poles). Secondly, what are the properties of these amplitudes? This strategy could help extend the reach of the S-matrix bootstrap using 2 to 2 amplitudes.