

Hyperbolic compactification of M-theory and de Sitter quantum gravity

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Central problem in theoretical cosmology: understand accelerated expansion of the universe.

Need to formulate quantum gravity in cosmological space-times.

Our approach to this problem, in 2 steps:

1. construct dS/cosmo solutions in string/M theory
2. understand them as deformations of AdS/CFT

By uplifting AdS/CFT from negative to positive c.c., this would give a holographic formulation of cosmology.

Goal of this talk: present a new simple mechanism to obtain de Sitter solutions in M-theory.

This uses crucially hyperbolic manifolds and Casimir energy.

Based on arXiv:2104.13380 w/De Luca and Silverstein

See Baumann/Silverstein, Kachru/Quevedo, ... for other constructions and results

A. Overview of the framework

In string/M-theory we look for sols of the form

$$M_d \times B_n \begin{cases} \rightarrow M_d \text{ max sym } \mathbb{R}^{d-1,1}, (A)dS_d \\ \rightarrow B_n \text{ internal space} \end{cases}$$

obtained by solving $D=d+n$ dim EOMs.

Useful intuition comes from dimensionally reduced effective potential

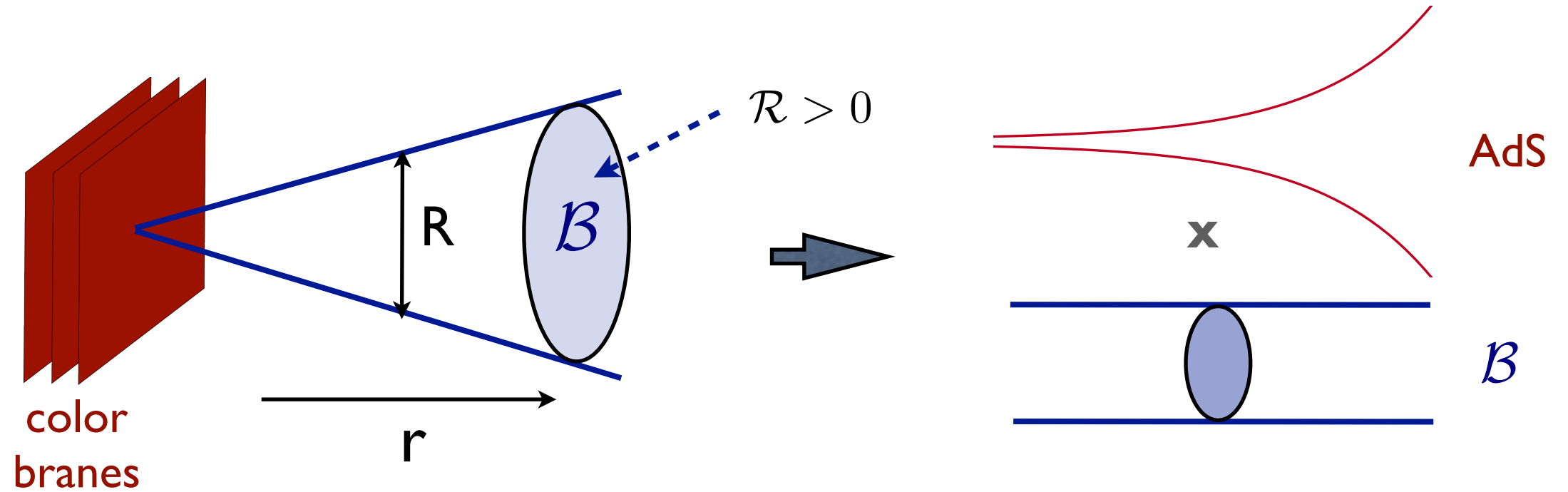
$$V_{naive} \sim \int_B \sqrt{g^{(n)}} \left(-R^{(n)} + |F_p|^2 \right) + \dots$$

d -dim scalar fields descending e.g. from metric modes on B_n should be stabilized.

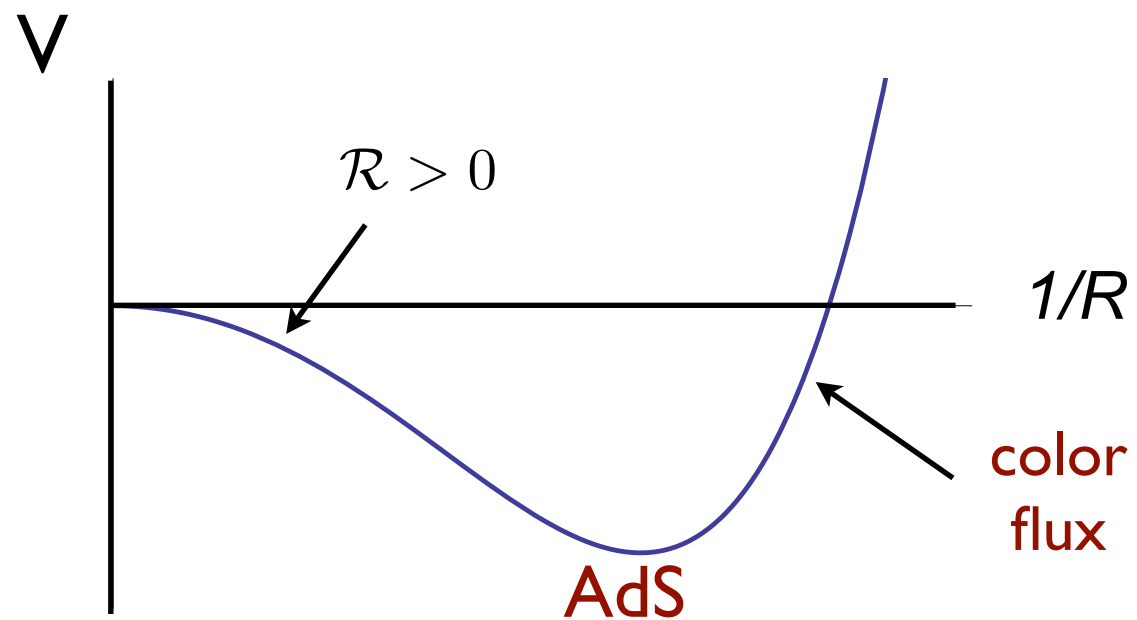


AdS/CFT energetics

Let's summarize how AdS/CFT works from this perspective

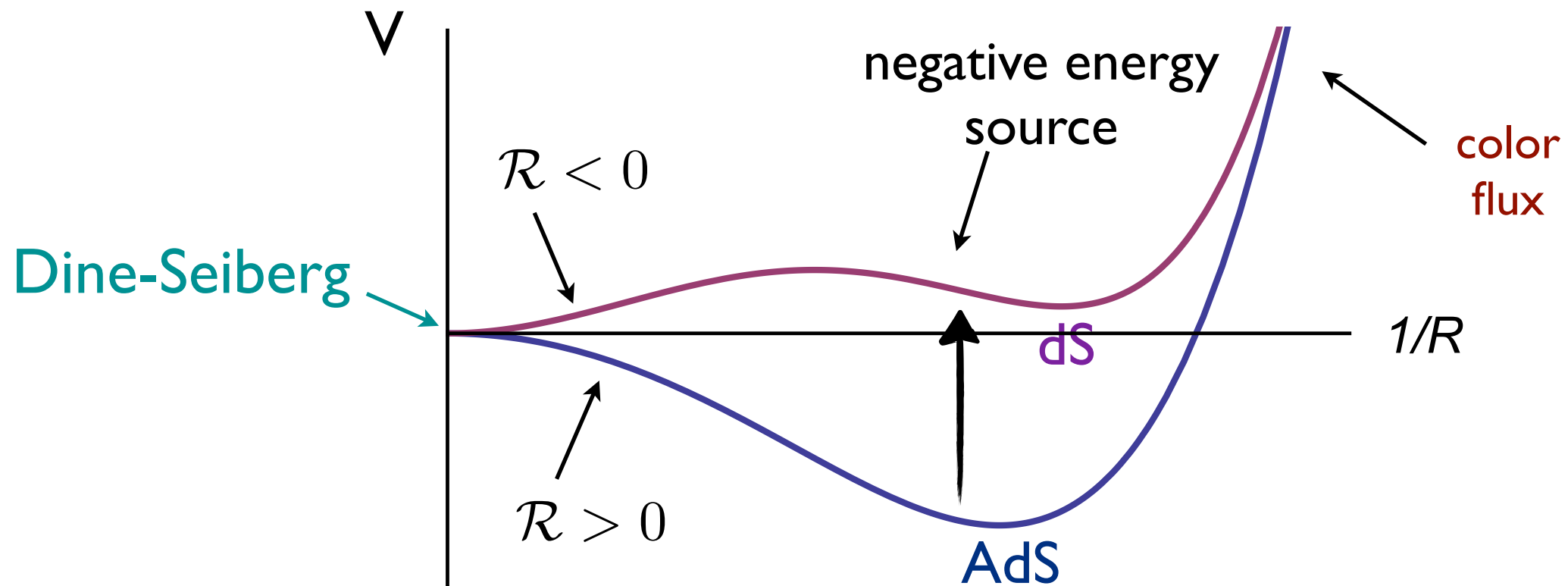


From dimensionally reduced theory:





Uplifting AdS/CFT



- ✓ Internal space of negative curvature
- ✓ Negative energy for the intermediate term in the potential
- ✓ May need other ingredients in place to stabilize all light fields

... can lead to metastable de Sitter solutions

Concrete brane construction for dS_3 given in

Dong, Horn,
Silverstein, GT

arXiv:1005.5403

We will present dS_4 sols with minimal ingredients

B. Setup and stabilization mechanism

We consider 11d SUGRA/M-theory

- ➔ compactified on finite volume hyperbolic manifold \mathbb{H}_7/Γ
- ➔ negative Casimir energy from small circles (antiperiodic boundary conditions for fermions)
- ➔ N units of F7 flux (M2 branes)

Finite volume hyperbolic manifold obtained from \mathbb{H}_7

$$ds_{\mathbb{H}}^2 = \ell^2 \frac{dz^2 + d\vec{y}^2}{z^2}$$

by modding out by freely acting discrete subgroup $\Gamma \subset SO(1, n)$

✓ Mostow rigidity: hyperbolic structure is uniquely fixed by $\pi_1(B)$

All metric fluctuations (except for conformal mode) are gapped!



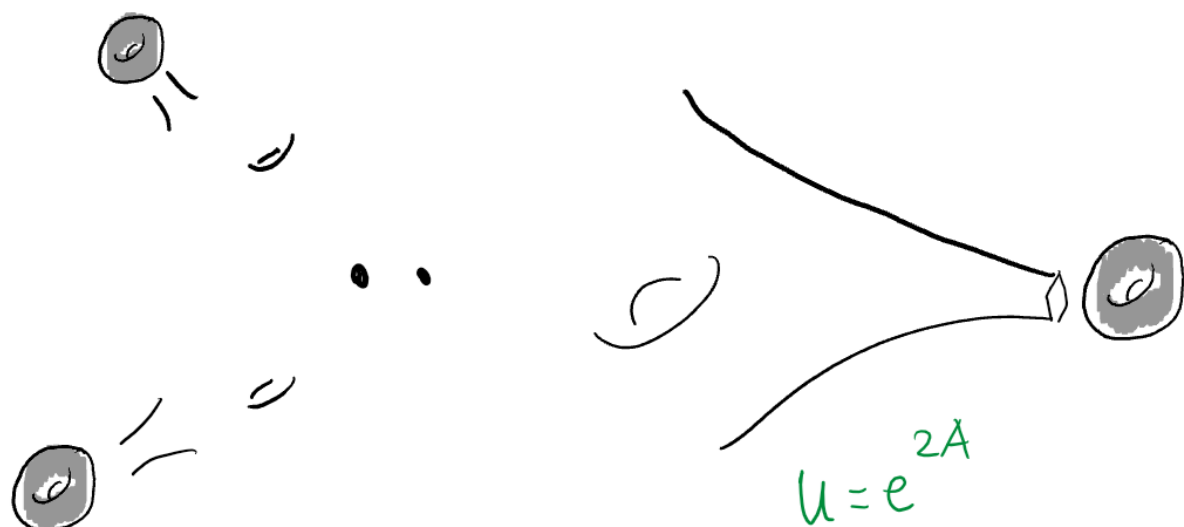
Casimir energy

Source of negative energy: we use hyperbolic manifolds with small cycles. We put antiperiodic boundary conditions for fermions, so Casimir energy is net negative due to bosons.

- Hyperbolic manifolds can contain near-cusps. Described by

$$ds_{cusp}^2 = dy^2 + e^{-2y/\ell} ds_{T^6}^2, \quad y_0 < y < y_c$$

Joined to the bulk part at y_0 . Radial direction should be stopped at finite y_c to get compact space. Achieved via **Anderson/Dehn filling**.



Let λ_c be size of smallest circle in T^6 . Proper size

$$R_c(y) = e^{-y/\ell} \lambda_c$$

shrinks slowly $\frac{dR_c}{dy} \sim \frac{\lambda_c}{\ell} e^{-y/\ell} \ll 1$

- In this situation, the Casimir energy becomes

$$\rho_C(R_c) \sim -\frac{1}{R_c^{11}} \sim -\frac{e^{11y/\ell}}{\lambda_c^{11}}$$

and semi-classical EOMs
$$-\frac{2}{\sqrt{-g^{11}}} \frac{\delta S_{cl}}{\delta g^{MN}} = \langle T_{MN}^{(Cas)} \rangle$$

- Casimir strongly localized near thin regions. Will need to include backreaction.
- This quantum effect will have to be large enough to compete with classical sources. Will use a combination of small cycles and warping.
- At the same time, other quantum corrections are suppressed if

$$\ell \gg R_c \gg \ell_{11}$$



Homogeneous potential

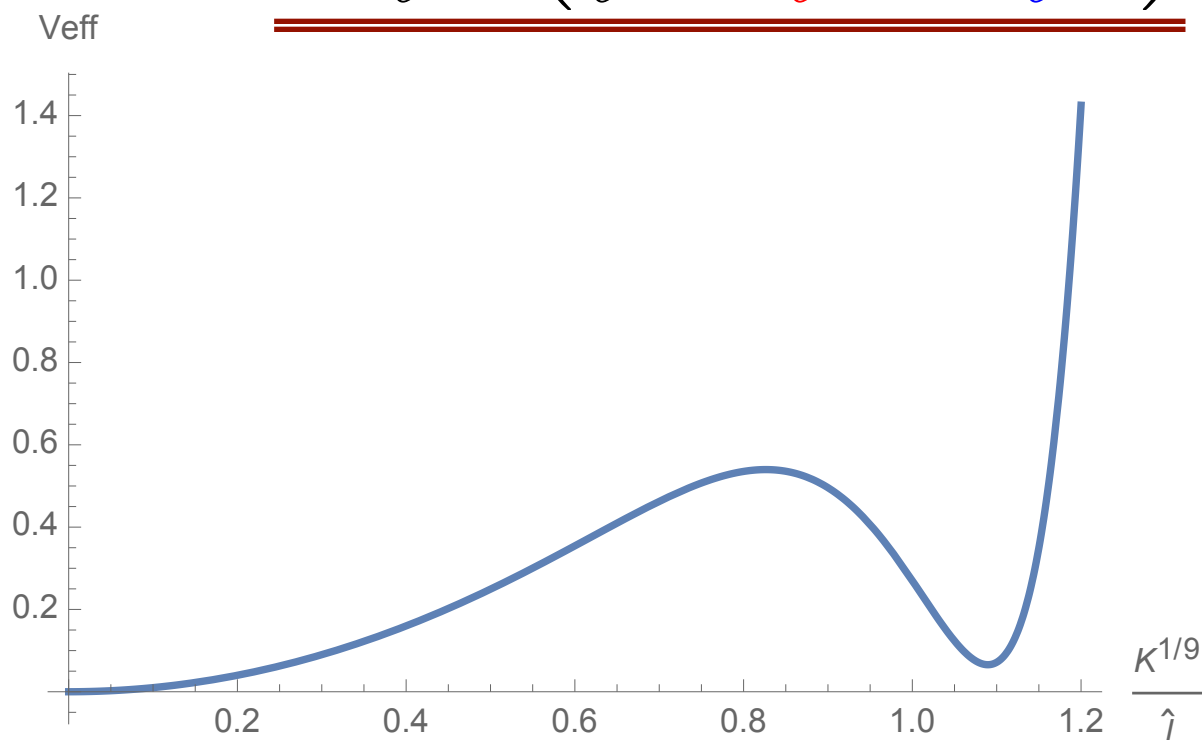
Stabilization mechanism in homogeneous approx.

$$S_{11} = \frac{1}{\ell_{11}^9} \int d^{11}x \sqrt{-g^{(11)}} \left(R^{(11)} - \frac{1}{2} |F_7|^2 \right) + S_{quantum}$$

$$\Rightarrow V \sim \frac{1}{\ell_{11}^9} \text{Vol}_7 \left(\frac{a}{\ell^2} + \ell_{11}^{12} \frac{N^2}{\text{Vol}_7^2} \right) + \int_{\mathbb{H}_7/\Gamma} d^7y \sqrt{g^{(7)}} \rho_C$$

$$\propto \frac{M_4^4}{\hat{\ell}^7} \left(\frac{a}{\hat{\ell}^2} - \frac{K}{\hat{\ell}^{11}} + \frac{N^2}{\hat{\ell}^{14}} \right)$$

$$\hat{\ell} \equiv \ell/\ell_{11} \quad K \sim \left(\frac{\ell}{R_c} \right)^{11} \frac{\text{Vol}_C}{\text{Vol}_7}$$



Supports a dS minimum

$$\hat{\ell} = \frac{\ell}{\ell_{11}} \sim \left(\frac{K}{a} \right)^{1/9} \gg 1$$

with $\ell_{11} \ll \ell \leq \ell_{dS}$

C. Inhomogeneities and backreaction

Casimir contributions localized in thin regions. Need to determine the effect of these inhomogeneities.

The metric will be deformed to a warped product

$$ds^2 = e^{2A(y)} ds_{dS_4}^2 + e^{2B(y)} (g_{\text{HI} ij} + h_{ij}) dy^i dy^j$$

warp factor $u(y) \equiv e^{2A(y)}$ conformal mode

EOMs will be PDEs in 7 variables; possible numerical approach using NNs described at the end.

Here instead we combine analytic estimates with numerical approximations in the cusp region.



Warped effective potential

[Douglas, 2009]

In the 4d EFT, it is useful to organize 11d EOMs in terms of an off-shell effective potential that includes warping effects:

$$V_{eff} = \frac{1}{2\ell_{11}^9} \int d^7y \sqrt{g^{(7)}} u^2 \left(-R^{(7)} - \frac{1}{4} \ell_{11}^9 T^{(Cas)\mu}_{\mu} + \frac{1}{2} |F_7|^2 - 3 \left(\frac{\nabla u}{u} \right)^2 \right) + \frac{C}{2} \left(\frac{1}{G_N} - \frac{1}{\ell_{11}^9} \int \sqrt{g^{(7)}} u \right)$$

C: Lagrange multiplier $\frac{\delta V_{eff}}{\delta C} = 0 \Rightarrow \frac{1}{G_N} = \frac{1}{\ell_{11}^9} \int \sqrt{g^{(7)}} u \quad C \sim R_{dS}^{(4)}$

GR constraint: $\frac{\delta V_{eff}}{\delta u} = 0 \Rightarrow \left(-\nabla^2 - \frac{1}{3} \left(-R^{(7)} - \frac{1}{4} \ell_{11}^9 T^{(Cas)\mu}_{\mu} + \frac{1}{2} |F_7|^2 \right) \right) u = -\frac{C}{6}$

Tune $C \ll 1$, then this is a **Schrodinger problem** $-\nabla^2 u(y) + U_{Schr} u(y) = 0$

Other eoms reproduced by $\frac{\delta V_{eff}}{\delta B} = 0 = \frac{\delta V_{eff}}{\delta h} = \frac{\delta V_{eff}}{\delta C_6}$

General properties can be established using warped V_{eff} :

- $a \ll 1$ tuning, from $\int (-u^2 R^{(7)} - 3(\nabla u)^2)$
- Naive conformal factor instability stabilized by warping [Douglas, 2009]
- Bound on negative energy, since u decays in classically forbidden region
- Positive contribution to mass matrix from warp factor



de Sitter solution in near-cusp region

Casimir energy concentrates inhomogeneously in near-cusp regions. Focus on one of these regions, keeping only radial y -dependence. Angular variations become important when we glue onto the bulk.

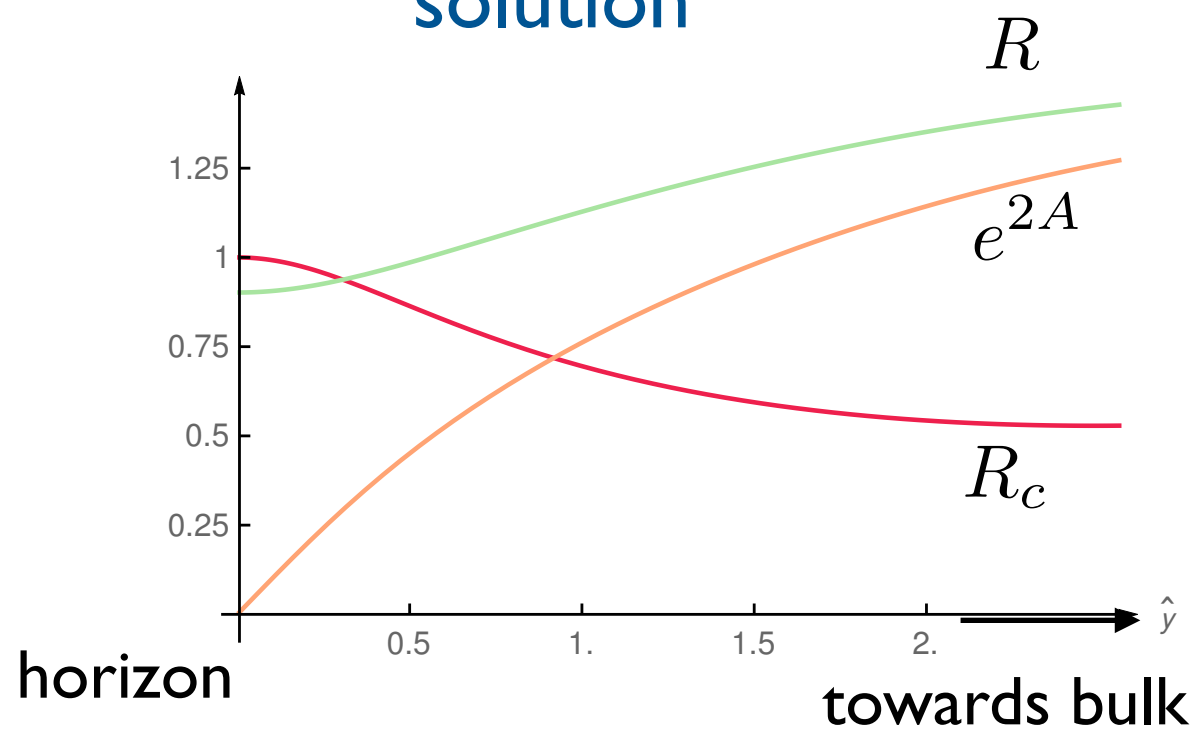
$$ds_{11}^2 = e^{2A(y)} ds_4^2 + dy^2 + R_c(y)^2 ds_{T^5}^2 + R(y)^2 d\theta^2$$

Casimir circles

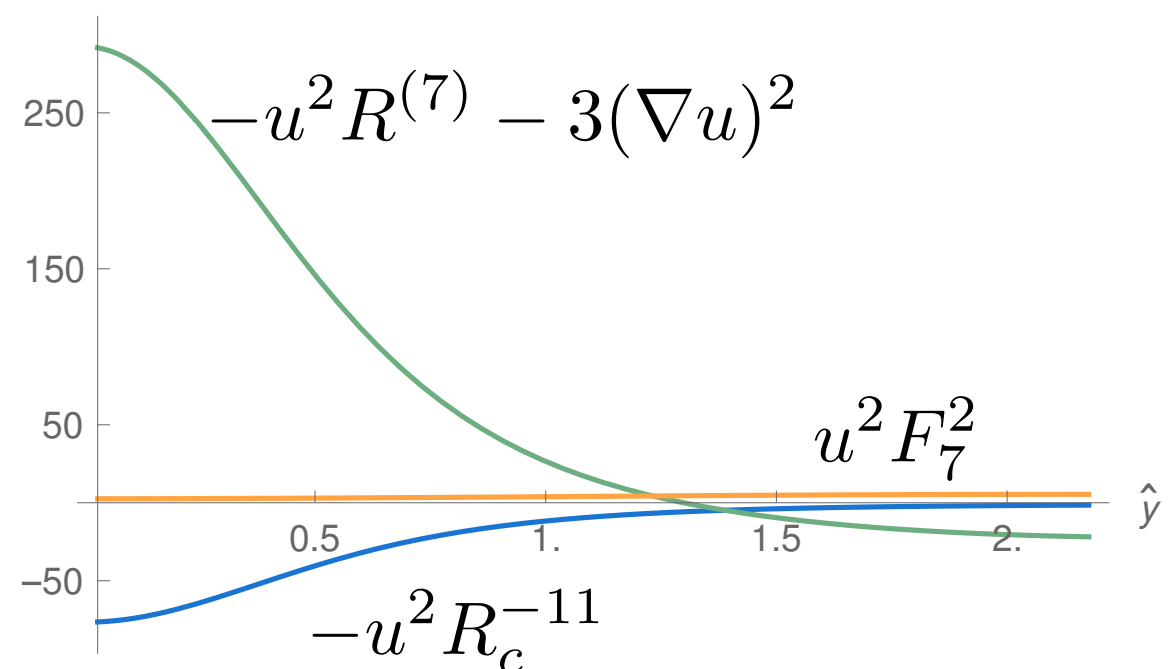
Anderson/Dehn-filling

We find numerical smooth solutions with dS4:

solution



energy sources

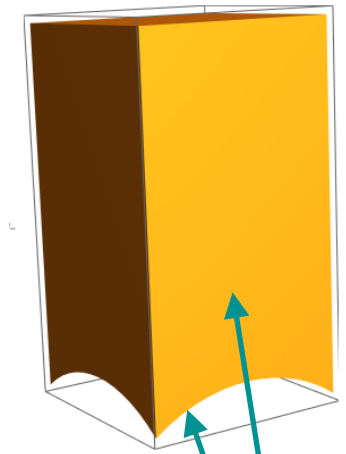




Matching onto the bulk

Previous approx. cusp joined to the bulk of the hyperbolic manifold at a totally geodesic face of polygon. **Boundary conditions involve angular variables and more intense numerics.**

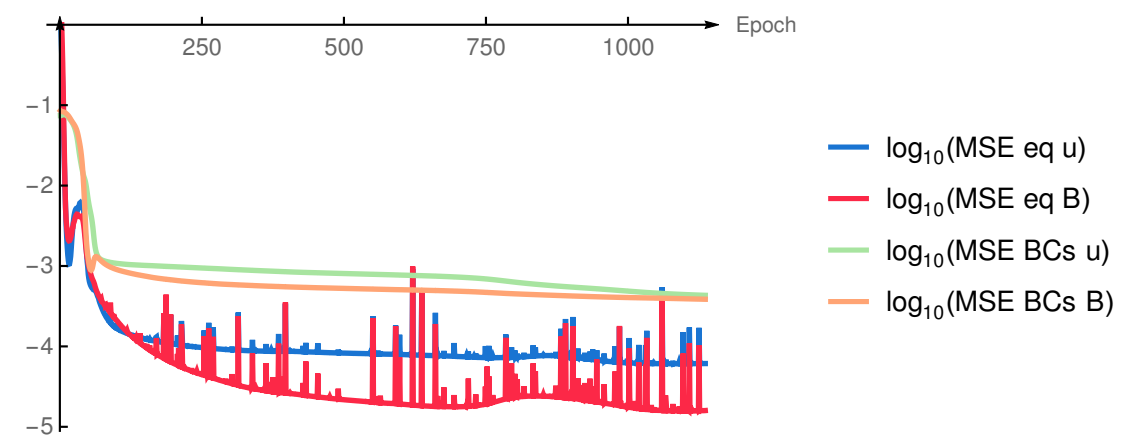
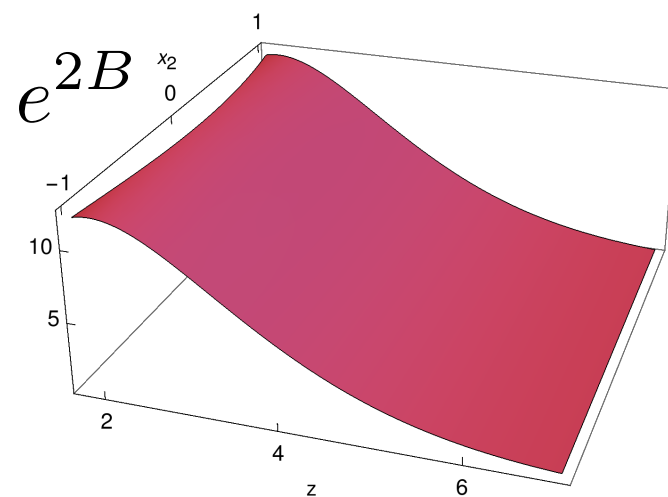
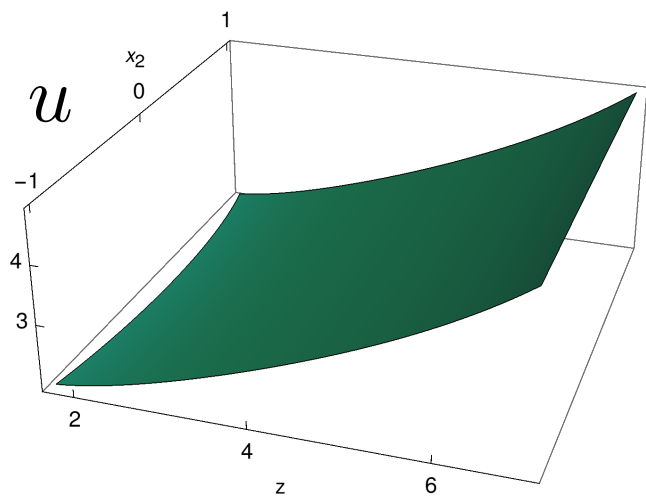
Anderson filling



vanishing extrinsic curvature

Promising direction: use neural network methods to approximate PDE sol, flowing down a loss function $\text{Loss} = \sum (\text{eqs})^2$

Preliminary results in \mathbb{H}_3/Γ



- Current analytic estimates and general constraints suggest a positive mass matrix

D. Discussion & other directions

We have found a simple mechanism for obtaining dS sols in M-theory: uplift of the large N M2 brane theory using negative curvature and automatically generated Casimir energy.

- We gave arguments for the existence of sol's beyond the homogeneous level. Explicit sol's will require solving PDEs but seems doable.
- Holography: the uplifts provide a microscopic realization of dS/dS correspondence. Combines with recent progress on \overline{TT} -type deformations in field theory. Add matter fields & uplift.
- Can also look for more general cosmological solutions w/accelerated expansion (slow-roll functionals in the landscape)

Thanks!