Casa abierta al tiempo
UNIVERSIDAD AUTÓNOMA METROPOLITANA
Unidad Iztapalapa

Magnetic screening mass for neutral pions

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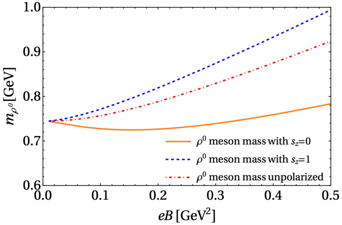
Workshop on Electromagnetic Effects in Strongly Interacting Matter 2022.

In collaboration with: A. Ayala, R. Farias, A. Mizher, C. Villavicencio and R. Zamora

Outline

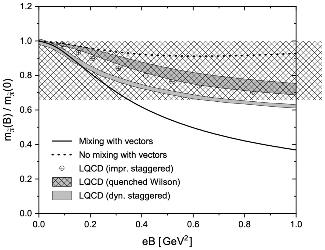
On Tuesday ...

William, Norberto and Ricardo showed results for the magnetic modification to the pole mass for different hadrons



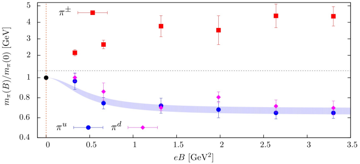
S. Avancini, R. Farias, W. Tavares and V. Timoteo, Nucl. Phys. B981 (2022) 115862

Carlomagno, Gómez Dumm, Noguera and Scoccola, Physics. Rev. D106 (2022), 074002

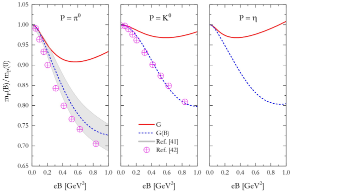


Many other results

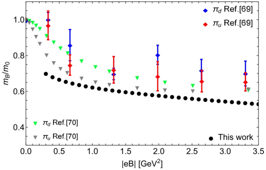
LQCD and effective models results



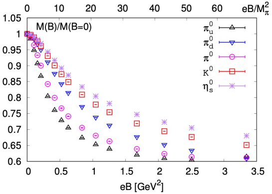
G. S. Bali, B. B. Brandt, G. Endrödi and B. Glässle, Phys. Rev. D97, 034505 (2018)



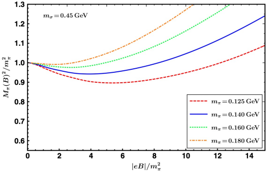
S. Avancini, M. Coppola, N. Scoccola and J. C. Sodré, Physics. Rev. D104 (2021) 9, 094040



A. Ayala, J. L. Hernández, L. A. Hernández, R. Farias and R. Zamora, Phys. Rev. D103 (2021) 5, 054038

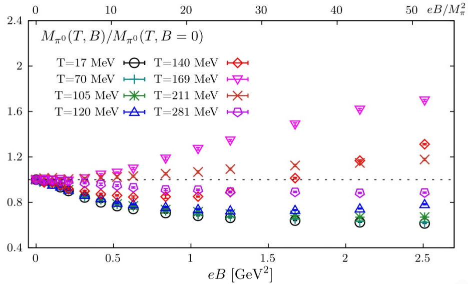


H.-T. Ding, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, Phys. Rev. D104 (2021) 1, 014505



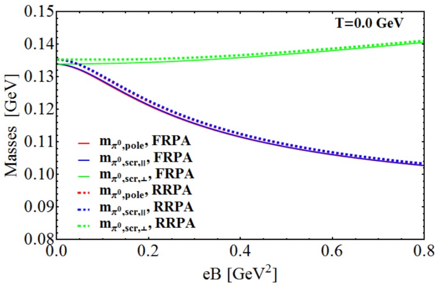
A. Das and N. Haque, Phys. Rev. D101, 074033 (2020)

Magnetic screening mass



H.-T. Ding, S.-T. Li, J.-H. Liu and X.-D. Wang, Phys. Rev. D105, 034514 (2022)

B. Sheng, Y. Wang, X. Wang and L. Yu, Phys. Rev. D103 (2021) 9, 094001



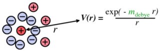
Screening mass $\leftrightarrow$ Debye mass in QED

The Coulomb potential is modified by collective effects as

$$V(r) = Q \int \frac{d^3p}{(2\pi)^3} \frac{e^{i\vec{p}\cdot\vec{r}}}{\vec{p}^2 + \Pi(p_0 = 0, \vec{p})}$$

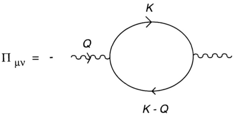
The position of the pole is called the Debye mass or the screening mass. Also the potential can be written as

$$V(r) = e^{-m_D r} \frac{Q}{r},$$



where $m_D = (r_D)^{-1}$.
Then, if we want to compute the screening mass at finite T, we need to solve the equation

$$[p_0^2 - \vec{p}^2 - \Pi(p_0, \vec{p}, T)]|_{p_0=0} = 0$$

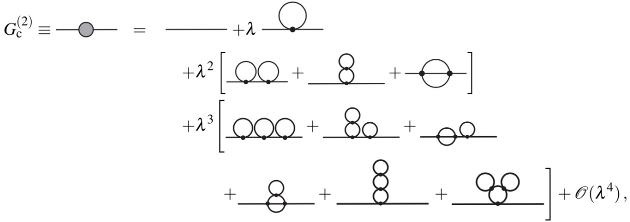


Magnetic screening mass

Now, if we want to compute in general the screening mass at finite $|eB|$, we need to solve the equation

$$[p_0^2 - p_\perp^2 - p_3^2 - m^2 - \Pi(p_0, p_\perp, p_3, |eB|)]|_{p_0=0} = 0,$$

where $\vec{p}^2 \to p_\perp^2 + p_3^2$ and $\Pi(p_0, p_\perp, p_3, |eB|)$ should be computed according the Lagrangian that we use.



Linear Sigma Model with quarks

Renormalizable effective model to describe dynamics at low energies.

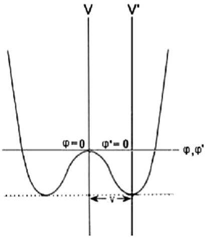
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(D_\mu \vec{\pi})^2 + \frac{a^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2$$
$$+ i\bar{\psi}\gamma^\mu D_\mu \psi - g\bar{\psi}(\sigma + i\gamma_5 \vec{\tau}\cdot\vec{\pi})\psi,$$

where $\vec{\pi} = (\pi_1, \pi_2, \pi_3)$, the model has two species of quarks represented by an $SU(2)$ isospin doublet ψ, and σ meson is a scalar included by means of an isospin singlet.

$$D_\mu = \partial_\mu + iq_{f,b}A_\mu,$$

with

$$A^\mu = \frac{B}{2}(0, -y, x, 0).$$

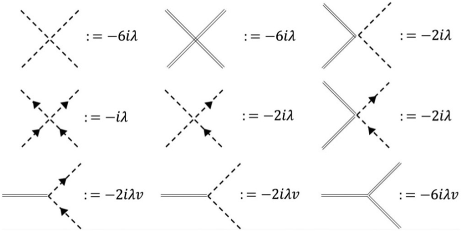


To allow for spontaneous symmetry breaking

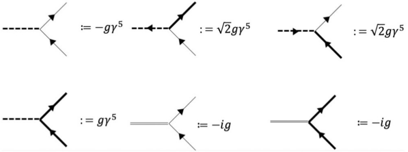
$$\sigma \to \sigma + v.$$

As a consequence of SSB

$$m_\sigma^2 = 3\lambda v^2 - a^2, \quad m_\pi^2 = \lambda v^2 - a^2, \quad m_f = gv.$$



Meson interactions in the LSMq. Dashed lines are used to represent the neutral and charged pions, whereas double lines represent the σ.



Quark-meson interactions in the LSMq. Dashed lines represent the neutral and charged pions, whereas the double lines represent the σ. Solid lines represent the quarks. Thin solid lines represent the d quark, and thick solid lines represent the u quark.

Screening mass within the LSMq

Come back with the main topic. In order to obtain the screening mass for the NEUTRAL PION, we need to solve the equation

$$[p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Pi(p_0, p_\perp, p_3, |eB|)]|_{p_0=0} = 0$$

dynamical mass

Screening mass within the LSMq

Come back with the main topic. In order to obtain the screening mass for the NEUTRAL PION, we need to solve the equation

$$[p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Pi(p_0, p_\perp, p_3, |eB|)]|_{p_0=0} = 0$$
$$[p_0^2 - p_\perp^2 - p_3^2 - (\lambda v_0^2 - a^2) - \Pi(p_0, p_\perp, p_3, |eB|)]|_{p_0=0} = 0$$

$v_0 \equiv$ vacuum spectation value (changes as a function of $|eB|$)

?

Screening mass within the LSMq

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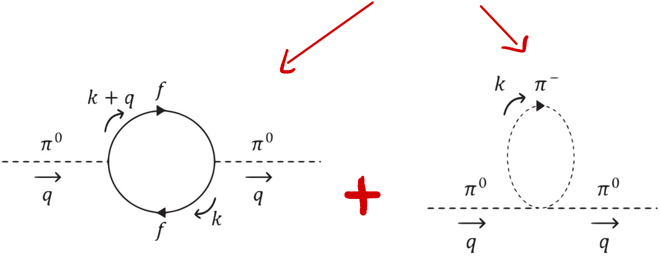
$$[p_0^2 - p_\perp^2 - p_3^2 - (\lambda v_0^2 - a^2) - \Pi(p_0, p_\perp, p_3, |eB|)]|_{p_0=0} = 0$$



Vacuum expectation value and the Magnetic Catalysis

In order to obtain the *vev*, we compute the effective potential up to 1-loop order.

$$V^{\text{eff}} = V^{\text{tree}} + V^1_{\pi^+} + V^1_{\pi^-} + V^1_{\pi^0} + V^1_{\sigma} + \sum_f V^1_f .$$

where

$$V^1_b = -\frac{i}{2}\int \frac{d^4k}{(2\pi)^4} \ln\left[-D_b^{-1}(k)\right], \quad V^1_f = iN_c \int \frac{d^4k}{(2\pi)^4} \text{Tr} \ln\left[S_f^{-1}(k)\right]$$

with the propagators given by

$$S_f(p) = \int_0^\infty \frac{ds}{\cos(|q_f B|s)} e^{is\left(p_\parallel^2 - p_\perp^2 \frac{\tan(|q_f B|s)}{|q_f B|s} - m_f^2 + i\epsilon\right)} \times \left[\left(\cos(|q_f B|s) + \gamma_1\gamma_2 \sin(|q_f B|s)\text{sign}(q_f B)\right) \times \left(m_f + \not{p}_\parallel\right) \frac{\not{p}_\perp}{\cos(|q_f B|s)}\right],$$

$$D_i(p) = \int_0^\infty \frac{ds}{\cos(|q_b B|s)} e^{is\left(p_\parallel^2 - p_\perp^2 \frac{\tan(|q_b B|s)}{|q_b B|s} - m_b^2 + i\epsilon\right)} .$$

In order to obtain the *vev*, we compute the effective potential up to 1-loop order.

$$V^{\rm eff} = V^{\rm tree} + V^1_{\pi^+} + V^1_{\pi^-} + V^1_{\pi^0} + V^1_{\sigma} + \sum_f V^1_f .$$

where vacuum + matter vacuum+matter

$$\underbrace{V^1_b = -\frac{i}{2}\int \frac{d^4k}{(2\pi)^4}\ln\left[-D_b^{-1}(k)\right]}_{\text{vacuum + matter}}, \qquad \underbrace{V^1_f = iN_c\int \frac{d^4k}{(2\pi)^4}{\rm Tr}\,\ln\left[S_f^{-1}(k)\right]}_{\text{vacuum+matter}}$$

Introducing the *vacuum stability conditions*

$$\frac{1}{2v}\frac{dV^{\rm vac}}{dv}\bigg|_{v=v_0} = 0, \qquad \frac{d^2V^{\rm vac}}{dv^2}\bigg|_{v=v_0} = 2a^2 + 2m_0^2.$$

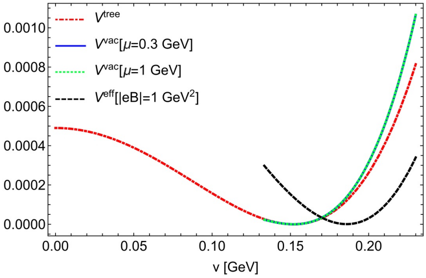
$$V^{\rm vac} = -\frac{(a^2+m_0^2+\delta a^2)}{2}v^2 + \frac{(\lambda+\delta\lambda)}{4}v^4 - 3\frac{m_0^4}{64\pi^2}\left[\frac{3}{2}+\ln\left(\frac{\mu^2}{m_0^2}\right)\right]$$
$$-\frac{m_\sigma^4}{64\pi^2}\left[\frac{3}{2}+\ln\left(\frac{\mu^2}{m_\sigma^2}\right)\right] + 2N_c\frac{m_f^4}{16\pi^2}\left[\frac{3}{2}+\ln\left(\frac{\mu^2}{m_f^2}\right)\right].$$

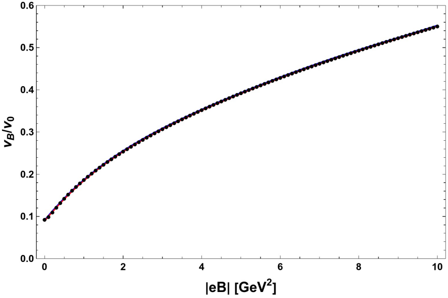
Effective potential

Then, the effective potential is

$$
\begin{aligned}
V^{eff}(B) = &-\frac{(a^2+m_0^2)}{2}v^2 - \frac{\delta a^2}{2}v_0^2 + \frac{\lambda}{4}v^4 + \frac{\delta\lambda}{4}v_0^4 - 3\frac{m_0^4(v_0)}{64\pi^2}\left[\frac{3}{2} + \ln\left(\frac{\mu^2}{m_0^2(v_0)}\right)\right] \\
&- \frac{m_\sigma^4(v_0)}{64\pi^2}\left[\frac{3}{2} + \ln\left(\frac{\mu^2}{m_\sigma^2(v_0)}\right)\right] + 2N_c\sum_f \frac{m_f^4(v_0)}{16\pi^2}\left[\frac{3}{2} + \ln\left(\frac{\mu^2}{m_f^2(v_0)}\right)\right] \\
&+ \frac{2}{16\pi^2}\left[2|eB|^2\psi^{-2}\left(\frac{1}{2} + \frac{m_0^2(v)}{2|eB|}\right) + \frac{3m_0^4(v)}{8} - \frac{1}{2}|eB|\, m_0^2(v)\ln(2\pi)\right. \\
&\left.- \frac{m_0^4(v)}{4}\ln\left(\frac{m_0^2(v)}{2|eB|}\right)\right] - \frac{N_c}{8\pi^2}\sum_f\left[4|q_fB|^2\psi^{-2}\left(\frac{m_f^2(v)}{2|q_fB|}\right) + \frac{3}{4}m_f^4(v)\right. \\
&\left.- \frac{m_f^4(v)}{2}\ln\left(\frac{m_f^2(v)}{2|q_fB|}\right) - m_f^2(v)|q_fB| + m_f^2(v)|q_fB|\ln\left(\frac{m_f^2(v)}{4\pi|q_fB|}\right)\right].
\end{aligned}
$$

Magnetic catalysis

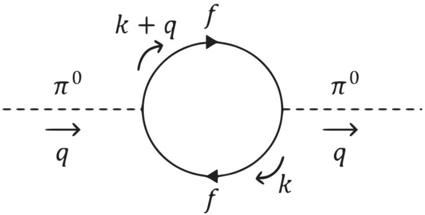


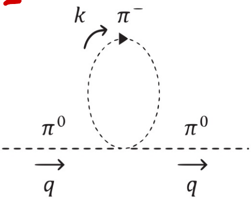


Neutral pion self-energy

$$\Pi(B,q) = \sum_f \Pi_{f\bar{f}}(B,q) + \Pi_{\pi^-}(B) + \Pi_{\pi^+}(B) + \Pi_{\pi^0} + \Pi_\sigma .$$

with





Neutral pion self-energy

$$-i\Pi_{f\bar{f}}(B,q) = -g^2 \int \frac{d^4k}{(2\pi)^4} \text{Tr}[\gamma_5 i S_f(k)\gamma_5 i S_f(k+q)] + \text{CC},$$

$$-i\Pi_{\pi^\pm} = \int \frac{d^4k}{(2\pi)^4}(-2i\lambda) i D_{\pi^\pm}(k).$$

where the propagators are

$$S_f(p) = \int_0^\infty \frac{ds}{\cos(|q_f B|s)} e^{is\left(p_\parallel^2 - p_\perp^2 \frac{\tan(|q_f B|s)}{|q_f B|s} - m_f^2 + i\epsilon\right)}$$
$$\times \left[\left(\cos(|q_f B|s) + \gamma_1\gamma_2 \sin(|q_f B|s)\text{sign}(q_f B)\right) \times \left(m_f + \not{p}_\parallel\right) \frac{\not{p}_\perp}{\cos(|q_f B|s)}\right],$$

$$D_i(p) = \int_0^\infty \frac{ds}{\cos(|q_b B|s)} e^{is\left(p_\parallel^2 - p_\perp^2 \frac{\tan(|q_b B|s)}{|q_b B|s} - m_b^2 + i\epsilon\right)}.$$

Neutral pion self-energy

Boson contribution

$$\begin{aligned}\Pi_{\pi^\pm} &= \Pi^{vac}_{\pi^\pm} + \Pi^{B}_{\pi^\pm} \\ &= \frac{\lambda}{4\pi^2}\left[\frac{m_\pi^2}{2}\ln\left(\frac{\mu^2}{m_\pi^2}\right) + \frac{m_\pi^2}{2}\ln\left(\frac{m_\pi^2}{2|q_bB|}\right)\right. \\ &\left. - |q_bB|\left(\ln\left(\Gamma\left(\frac{1}{2}+\frac{m_\pi^2}{2|q_bB|}\right)\right) + \ln(\sqrt{2\pi})\right) - \frac{m_\pi^2}{2}\right]\end{aligned}$$

Fermion contribution

$$\Pi_{f\bar{f}} = \Pi^{vac}_{f\bar{f}} + \Pi^{B}_{f\bar{f}}$$

Computed without any approximation

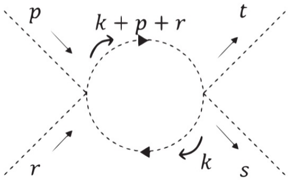
$\Rightarrow$ numerically.

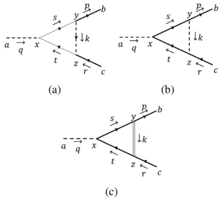
We are ready to find the magnetic screening mass for the neutral pion by joining all the results showed

$$[p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Pi(p_0, p_\perp, p_3, |eB|)]|_{p_0=0} = 0$$

... Hold your horses ! We can include one more ingredient in the recipe. $\rightarrow$ Effective coupling constants.

$$\lambda_{eff} = \lambda(1 + \Gamma_\lambda^B), \qquad g_{eff} = g(1 + \Gamma_g^B).$$





Effective coupling constants

Magnetic corrections to the boson self-coupling

$$-i6\lambda\Gamma^B_\lambda = \int \frac{d^4k}{(2\pi)^4}(-2i\lambda)iD_{\pi^-}(k)(-2i\lambda)\times iD_{\pi^-}(k+p+r)+\text{CC},$$

$$\Gamma^B_\lambda = -\frac{\lambda}{12\pi^2}\left[\ln\left(\frac{\mu^2}{2|q_bB|}\right)-\psi^0\left(\frac{|q_bB|+m_\pi^2}{2|q_bB|}\right)\right].$$

Magnetic corrections to the boson-fermion coupling

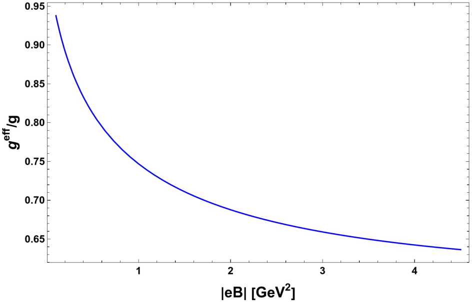
$$\Gamma^{LLL}_g = \Gamma^B_{1,g}+\Gamma^B_{2,g}+\Gamma^B_{3,g}.$$

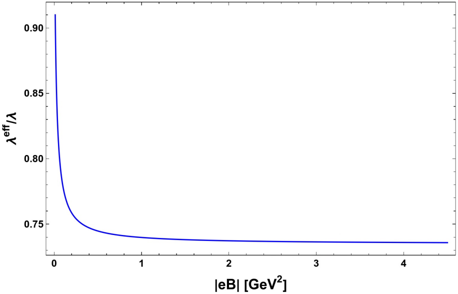
$$g\gamma^5\Gamma^B_{1,g} = \int \frac{d^2s_\perp d^2t_\perp}{\pi^2|eB|^2}\frac{d^4k}{(2\pi)^4}\left(\sqrt{2}g\gamma^5\right)iS_d(k_\parallel+p_\parallel,s_\perp)\left(-g\gamma^5\right)iS_d(k_\parallel+r_\parallel,t_\perp)$$
$$\times\left(\sqrt{2}g\gamma^5\right)iD_{\pi^-}(k_\parallel,k_\perp)e^{i\frac{2}{|eB|}\varepsilon_{ij}(s-q-t)_i(s-p-k)_j}+\text{CC},$$

$$g\gamma^5\Gamma^B_{2,g} = \int \frac{d^4k}{(2\pi)^4}\left(g\gamma^5\right)iS_u(k+p)\left(g\gamma^5\right)iS_u(k+r)\left(g\gamma^5\right)iD_{\pi^0}(k)+\text{CC},$$

$$g\gamma^5\Gamma^B_{3,g} = \int \frac{d^4k}{(2\pi)^4}(-ig)\,iS_u(k+p)\left(g\gamma^5\right)iS_u(k+r)(-ig)\,iD_\sigma(k)+\text{CC}.$$

Effective coupling constants behaviour





LSMq

Where only the quark-antiquark pair fluctuation is considered, we have

$$[-p_\perp^2 - p_3^2 - m_\pi^2 - \Pi(0, p_\perp, p_3, |eB|)] = 0$$

which can be rewritten as follows

$$(-p_\perp^2 - p_3^2 - m_\pi^2)\left(1 - \frac{\Pi(0, p_\perp, p_3, |eB|)}{-p_\perp^2 - p_3^2 - m_\pi^2}\right) = 0$$

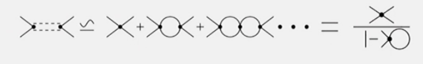
$$(-p_\perp^2 - p_3^2 - m_\pi^2)\left(\boxed{1 - \frac{g^2\tilde{\Pi}(0, p_\perp, p_3, |eB|)}{-p_\perp^2 - p_3^2 - m_\pi^2}}\right) = 0$$

$$\frac{g^2}{-p_\perp^2 - p_3^2 - m_\pi^2} \longleftrightarrow 2G$$

$g^2 \rightarrow g_{\pi qq}$

NJL

Using random phase approximation

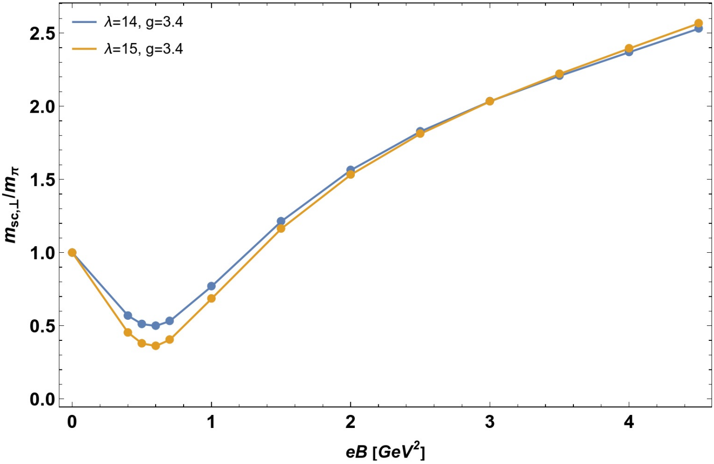


$$\frac{2iG}{1 - 2G\tilde{\Pi}(p_0, p_\perp, p_3, |eB|)}$$

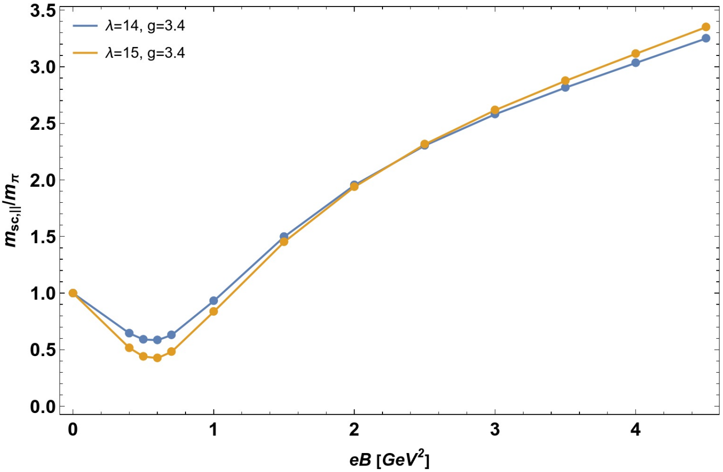
It is interpreted as an effective meson propagator where the pole mass is obtained when $p_\perp$ and p_3 go to zero, and the screening mass is obtained when p_0 goes to zero and $p_\perp$ or p_3 is finite. Then, the equation to solve is

$$\boxed{1 - 2G\tilde{\Pi}(0, p_\perp, p_3, |eB|)} = 0$$

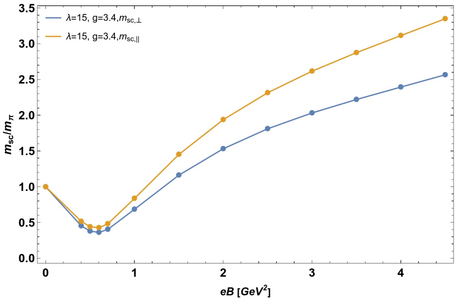
magnetic screening mass for $p_3 = 0$

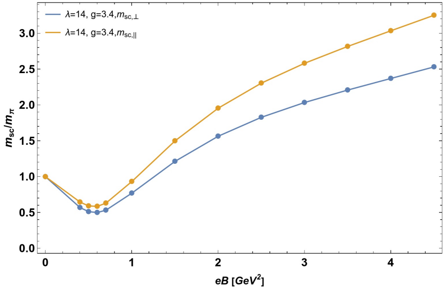


magnetic screening mass for $p_\perp = 0$



Both magnetic screening masses





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¡Gracias!

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AP-CP