Decoupling theorem and effective quantum gravity

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Partial support: CNPq, ICTP-SAIFR

Minicourse on perturbative and nonperturbative treatment of quantum gravity problems

ICTP-SAIFR/IFT-UNESP – May 20-24, 2024

Lecture 2.

- **The main features of quantum gravity (QG) based on GR.**
- **Fourth-derivative quantum gravity.**
- **Higher than fourth derivative polynomial QG models.**
- **What we can expect as decoupling in the IR?**
- **Is quantum GR a universal theory of IR QG?**
- **Scalar model with four derivatives. Mixed diagrams.**
- **Renormalization group in quantum GR vs renormalizable or superrenormalizable QG models.**
- **What we can expect from QG in the IR?**

Main references

Pedagogical introduction to quantum gravity (QG).

I.L. Buchbinder, I. Sh., Introduction to Quantum Field Theory with Applications to Quantum Gravity, (Oxford Un. Press, 2021).

Some of the most important references for this lecture.

E.S. Fradkin & A.A. Tseytlin, NPB 201 (1982) 469.

J.F. Donoghue, gr-qc/9310024 (PRL); gr-qc/9405057 (PRD).

Wagno C. e Silva & I.Sh., arXiv:2301.13291, JHEP

Section "Effective Quantum Gravity" edited by C. Burgess and J. Donoghue of the "Handbook of Quantum Gravity" (Editors C. Bambi, L. Modesto and I. Shapiro, Springer Singapore (2023).

Many reviews by Donoghue, Burgess, and others.

Renormalizability in the quantum gravity (QG) models.

Any QG theory starts from a covariant action of gravity,

$$S = \int d^4x \sqrt{-g}\; \mathcal{L}(g_{\mu\nu})\,.$$

$\mathcal{L}(g_{\mu\nu})$ **can be of GR, or another action, including with finite or infinite number of derivatives, local or even nonlocal.**

Gauge transformation $x'^{\mu} = x^{\mu} + \xi^{\mu}$. **The metric transforms as**

$$\delta g_{\mu\nu} = g'_{\mu\nu}(x) - g_{\mu\nu}(x) = -\nabla_{\mu}\xi_{\nu} - \nabla_{\nu}\xi_{\mu} = R_{\mu\nu\,,\,\alpha}\,\xi^{\alpha}.$$

Covariance implies $\quad \dfrac{\delta S}{\delta g_{\mu\nu}}\,R_{\mu\nu\,,\,\alpha}\,\xi^{\alpha} = 0.$

There is a proof *P.M. Lavrov & I.Sh., PRD, 1902.04687* **that the effective action is covariant, independent of the QG model,**

$$\frac{\delta \Gamma(g)}{\delta_{\mu\nu}}\,R_{\mu\nu\,,\,\alpha}\,\xi^{\alpha} = 0,$$

This is called the covariant renormalizability.

Power counting in QG

General definition, where p, d, n are numbers of loops, derivatives acting on external lines, and vertices.

$$D + d \;=\; \sum_{l_{int}} (4 - r_l) \;-\; 4n \;+\; 4 \;+\; \sum_{\nu} K_\nu \quad \textbf{with} \quad l_{int} = p + n - 1.$$

As the first example consider quantum GR.

$$S_{EH} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R, \qquad \textbf{Power counting: } D + d = 2 + 2p.$$

At the 1-loop level we can expect the divergences like

$$\mathcal{O}(R^2_{...}) = R^2_{\mu\nu\alpha\beta}\,,\;\; R^2_{\mu\nu}\,,\;\; R^2\,.$$

t'Hooft and Veltman; Deser and van Nieuwenhuisen, (1974); ...

At 2-loop level we have [*Goroff and Sagnotti, NPB (1986).*]

$$\mathcal{O}(R^3_{...}) = R_{\mu\nu}\Box R^{\mu\nu},\,... \quad R^3, \quad R_{\mu\nu}R^{\mu}_{\alpha}R^{\alpha\nu}, \quad R_{\mu\nu\alpha\beta}R^{\mu\nu}{}_{\rho\sigma}R^{\mu\nu\rho\sigma}.$$

The last structure doesn't vanish on-shell. This demonstrated that the theory of GR-based QG is non-renormalizable.

Another natural choice is four-derivative model, because we need four derivatives anyway for quantum matter field.

$$S_{gravity} = S_{EH} + S_{HD}, \qquad S_{EH} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R + 2\Lambda),$$

and S_{HD} **includes square of the Weyl tensor and** R

$$S_{HD} = \int d^4x\sqrt{-g}\,\Big\{-\frac{1}{2\lambda}C^2 - \frac{\omega}{3\lambda}R^2 + \text{ surface terms}\Big\}\,,$$

$$C^2(4) = R^2_{\mu\nu\alpha\beta} - 2R^2_{\alpha\beta} + R^2/3\,,$$

Propagators of metric and ghosts behave like $\mathcal{O}(k^{-4})$ **and we have** K_4, K_2, K_0 **vertices. The superficial degree of divergence**

$$D + d = 4 - 2K_2 - 4K_0.$$

Dimensions of counterterms are 4, 2, 0.
This theory is renormalizable. *K. Stelle, Phys. Rev. D (1977).*

However there is a price to pay: massive ghosts

$$G_{\text{spin}-2}(k) \sim \frac{1}{m^2}\left(\frac{1}{k^2} - \frac{1}{k^2+m^2}\right), \quad m \propto M_P\,.$$

The tree-level spectrum includes massless graviton and massive spin-2 "ghost" with negative kinetic energy and a huge mass.

Particle with negative energy means instability of vacuum state.

Even Minkowski space is not protected from spontaneous creation of massive ghost and many gravitons from vacuum.

Different sides of the HDQG problems with massive ghosts:

- **In classical systems higher derivatives generate exploding instabilities at the non-linear level** *(Ostrogradsky, 1850).*

- **Interaction between ghost and gravitons may violate energy conservation in the massless sector** *(Veltman, 1963).*

- **Ghost produce violation of unitarity of the S -matrix.**

The main issue is stability

Certainly, the unitarity of the S- matrix is not the unique condition of consistency of the quantum gravity theory.

The most important feature is the stability of physically relevant solutions of classical general relativity in the presence of higher derivatives and massive ghosts.

The problem is well explored for the cosmological backgrounds.
Gravitational waves on de Sitter space (energy $\ll M_p$):

A. A. Starobinsky, Let. Astr. Journ. (in Russian) (1983).

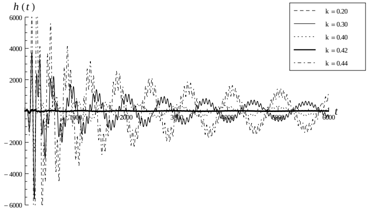
S. Hawking, T. Hertog, and H.S. Real, PRD (2001).
J. Fabris, A. Pelinson and I.Sh., NPB (2001).

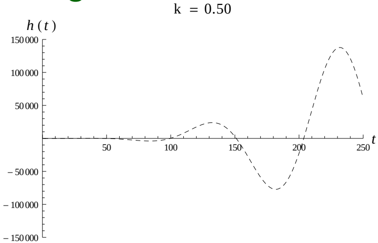
J. Fabris, A. Pelinson, F. Salles and I.Sh., JCAP, arXiv:1112.5202.

More general FRW-backgrounds:

F. Salles and I.Sh., PRD, arXiv:1401.4583.

More general cosmological backgrounds





Example: radiation-dominated Universe. There are no growing modes until the frequency k achieves the value ≈ 0.5 in Planck units. Starting from this value, we observe instability as an effect of massive ghost.

The anomaly-induced quantum correction is $\mathcal{O}(R^3_{....})$. Until the energy is not of the Planck order of magnitude, these corrections can not compete with classical $\mathcal{O}(R^2_{....})$ - terms.

Massive ghosts are present only in the vacuum state. We just do not observe them “alive” until the energy scale M_P .

Behavior at Planck or greater frequencies

The simplest possible equation is for the fourth-derivative gravity without quantum (semiclassical) corrections,

$$\frac{1}{3}\overset{....}{h} + 2H\dddot{h} + \Big(H^2 + \frac{M_P^2}{32\pi a_1}\Big)\ddot{h} + \frac{1}{6}\frac{\nabla^4 h}{a^4} - \frac{2}{3}\frac{\nabla^2 \ddot{h}}{a^2} - \frac{2H}{3}\frac{\nabla^2 \dot{h}}{a^2}$$
$$-\Big(H\dot{H} + \ddot{H} + 6H^3 - \frac{3M_P^2 H}{32\pi a_1}\Big)\dot{h} - \Big[\frac{M_P^2}{32\pi a_1} - \frac{4}{3}\Big(\dot{H} + 2H^2\Big)\Big]\frac{\nabla^2 h}{a^2}$$
$$-\Big[24\dot{H}H^2 + 12\dot{H}^2 + 16H\ddot{H} + \frac{8}{3}\dddot{H} - \frac{M_P^2}{16\pi a_1}\Big(2\dot{H} + 3H^2\Big)\Big]h = 0.$$

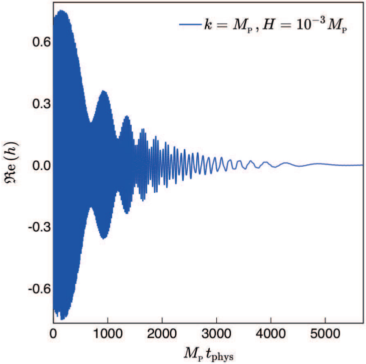
It is easy to note that the space derivatives ∇ and hence the wave vector $\vec{k}$ enter this equation only in the combination

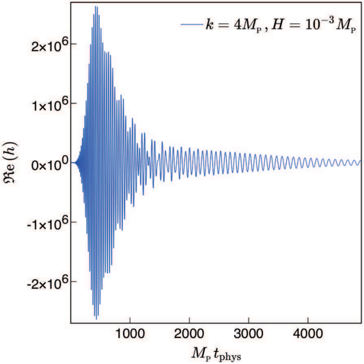
$$\vec{q} = \frac{\vec{k}}{a(t)}.$$

When Universe expands, each frequency becomes smaller!

Filipe de O. Salles, Patrick Peter, I.Sh., On the ghost-induced instability on de Sitter background. PRD (2018), arXiv:1801.00063

The qualitative conclusion is perfectly well supported by numerical analysis, including the case when the semiclassical corrections are taken into account.





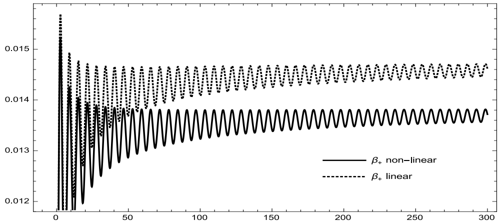
The growth of the waves really stops at some point. At least in the cosmological setting this may be a solution of the problem.

The verification of the general logic, that the first order stability is sufficient for a higher order stability, can be done for the Bianchi-I metric.

A. Salvio, PRD (2019), arXiv:1902.09557.
Simpliciano dos Reis, Gr. Chapiro, I.Sh. PRD (2019), 1903.01044.

$$ds^2 = dt^2 - a_1^2(t)\,dx^2 - a_2^2(t)\,dy^2 - a_3^2(t)\,dz^2,$$

where $a_{1/2}(t) = e^{\sigma(t)}\,e^{\beta_+(t)\pm\sqrt{3}\beta_-(t)}, \quad a_3(t) = e^{\sigma(t)}\,e^{-2\beta_+(t)}.$



For small initial amplitudes and frequencies: a very good correspondence between linear and exact numerical solutions.

This is a convincing, still very phenomenological approach. What about further modifications of a fundamental theory?

One can include more than four derivatives,

$$S = S_{EH} + \int d^4x \sqrt{-g} \sum_{n=0}^{N} \Big\{ \omega_n^C C_{\mu\nu\alpha\beta} \Box^n C_{\mu\nu\alpha\beta} + \omega_n^R R \Box^n R \Big\} + \mathcal{O}\big(R^3_{\dots}\big).$$

Simple analysis shows that this theory is superrenormalizable, but the massive ghost-like states are still present.

For the real poles case:

$$G_2(k) = \frac{A_0}{k^2} + \frac{A_1}{k^2 + m_1^2} + \frac{A_2}{k^2 + m_2^2} + \cdots + \frac{A_{N+1}}{k^2 + m_{N+1}^2}\,.$$

For any sequence $0 < m_1^2 < m_2^2 < m_3^2 < \cdots < m_{N+1}^2$,
the signs of the corresponding terms alternate: $A_j \cdot A_{j+1} < 0$.

M. Asorey, J.-L. Lopez & I. Sh., IJMPhA (1997), hep-th/9610006.

The general feature of (almost) all QG models

Particle contents is defined by the structure of propagator and the last depends on up to the second-order in curvature terms,

$$S_{gen} = \int d^4x \sqrt{-g}\,\Big\{-\frac{1}{\kappa^2}\,(R+2\Lambda) + \frac{1}{2}\,C_{\mu\nu\alpha\beta}\,\Phi(\Box)\,C_{\mu\nu\alpha\beta} + \frac{1}{2}\,R\,\Psi(\Box)\,R + \mathcal{O}(R^3_{\dots})\Big\}.$$

At least for the polynomial functions $\Phi(\Box)$ and $\Psi(\Box)$ there is always a massless graviton and also massive degrees of freedom, which can be ghosts, tachyons or normal particles.

The question of our interest is what happens with the quantum loops with these modes in the IR? Is there a decoupling?

The main hypothesis is that the IR limit in the QG models is always the quantum version of GR *(J. Donoghue, 1994).*

However, there may be other options for the QG remains in the IR, which can be ruled our only by a direct verification.

There may be, in principle, other options for the QG "remains" in the IR, which can be ruled our only by a direct verification.

Do we need to make these calculations in such a complex theory are higher derivative QG? In the QG case, we need to work with scalar and tensor modes, gauge fixing, Faddeev-Popov ghosts, third (Nakanishi-Laudrup) ghost, etc.

A simplified version of the theory with qualitatively similar structure is the Antoniadis and Mottola model.

I. Antoniadis & E. Mottola, PRD 45 (1992) 2013.

This theory has the following relevant properties:

- **Fourth derivative and renormalizable.**
- **Non-polynomial interactions.**
- **Admits effective theory in the IR limit.**
- **All this makes it a good toy model for QG.**

S.D. Odintsov & I.Sh., Class. Quant. Grav. 8 (1991) L57.

Classical action of the model results from the integration of trace anomaly, owing to quantum effects of conformal matter fields,

$$S_{\rm cf} = \int d^4x \Big\{ \gamma e^{2\sigma}(\partial\sigma)^2 - \lambda e^{4\sigma} - \theta^2(\Box\sigma)^2 - \zeta\,\big[2(\partial\sigma)^2\Box\sigma + (\partial\sigma)^4\big]\Big\},$$

where

$$\theta^2 = (2w+3c), \quad \zeta = \big(2w+2b+3c\big), \quad \gamma = \frac{3}{8\pi G}, \quad \text{and} \quad \lambda = \frac{\Lambda}{\kappa}.$$

The coefficients w, b, c depend on the particle contents of the original theory of conformal matter fields.

$$w = \frac{1}{120(4\pi)^2}(N_s + 6N_f + 12N_v),$$

$$b = -\frac{1}{360(4\pi)^2}(N_s + 11N_f + 62N_v),$$

$$c = \frac{1}{180(4\pi)^2}(N_s + 6N_f - 18N_v),$$

The IR sector includes Einstein-Hilbert and cosmological terms,

$$S_{\rm IR} = \int d^4x \Big\{ \gamma e^{2\sigma}(\partial\sigma)^2 - \lambda e^{4\sigma}\Big\}.$$

Using the standard algorithm for the fourth-order operators, we get the divergences in the "fundamental" four-derivative model

$$\bar{\Gamma}^{(1)}_{\rm div} = -\frac{1}{\varepsilon}\int d^4x\Big\{\frac{5\zeta^2}{\theta^4}\left[\Box\sigma + (\partial\sigma)^2\right]^2 + \frac{\gamma}{\theta^2}\Big(\frac{3\zeta}{\theta^2}+2\Big)(\partial\sigma)^2 e^{2\sigma} - \Big(\frac{8\lambda}{\theta^2} - \frac{\gamma^2}{2\theta^4}\Big)e^{4\sigma}\Big\}, \quad \textbf{where} \quad \varepsilon = (4\pi)^2(n-4). \qquad (1)$$

This is supposed to fit the UV limit of the nonlocal form factors, providing correspondence between MS (Minimal Subtraction) and (physical) Momentum Subtraction renormalization schemes.

The divergences of the effective theory have the form

$$\bar{\Gamma}^{(1)}_{\rm div,\,IR} = -\frac{1}{\varepsilon}\int d^4x\,\Big\{\frac{1}{2}\left[\Box\sigma + (\partial\sigma)^2\right]^2 - \frac{8}{3}\Lambda\, e^{2\sigma}(\partial\sigma)^2 + \frac{32}{9}\Lambda^2 e^{4\sigma}\Big\}. \qquad (2)$$

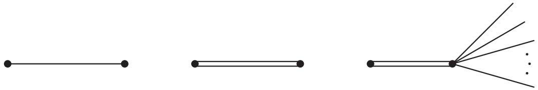
In agreement with the power counting, the fourth-derivative counterterms emerge, as the theory is non-renormalizable.

Taking only lower-derivative terms, we arrive at the expression to compare with the IR limit of the full theory form factors.

Like the four-derivative QG, the Antoniadis and Mottola model has a light mode and heavy mode propagating. The masses are

$$m^2 = \frac{8\Lambda}{3}, \qquad M^2 = \frac{\gamma}{\theta^2}\,.$$

Without the cosmological constant, the light mode is massless.

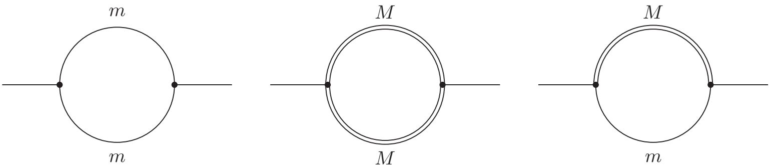


Propagators of light and heavy modes, and an example of the mixed vertex. Since the theory is non-polynomial, the number of lines of each type in a vertex is not restricted.

At the tree level, the heavy mode collapses in the IR and there is only the light field propagating. The question is what happens in the IR with the loop corrections.

For the one-loop contributions to the propagators and lowest-order vertices, we need only a few vertices.

In the self-energy part, there are three types of the diagrams, but only the bubble type contribute to the nonlocal form factors.



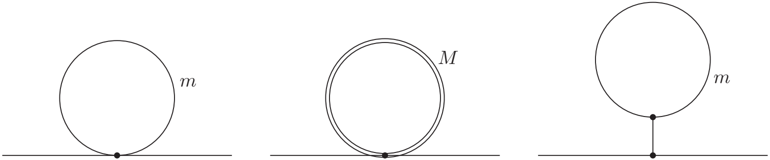
Three types of diagrams of the bubble type.

We already know that the first type of loop produce the form factors which are pure logarithms. This is also expected in QG, for both quantum metric and the Faddeev-Popov ghost sectors.

The diagrams of the second type produce more complicated form factors, which demonstrate usual quadratic decoupling, according to the Appelquist and Carazzone theorem.

The remaining question is what happens, in the IR, with the contributions of the third type of diagrams.

Other relevant diagrams.



Examples of other types of diagrams: snail and tadpole.

These diagrams are important in the sense they produce logarithmic divergences, the same as bubble diagrams.

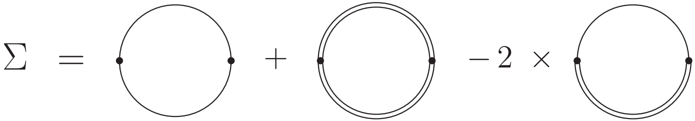
The total sum of these divergences is expected to fit the UV limit, i.e., the logarithmic part of the form factors, produced exclusively by the bubble diagrams.

In the IR, snail and tadpole type diagrams do not make relevant contributions to compare with the low-energy effective theory.

All this is our expectation.

Do things really work in the described way?

The contributions to the nonlocal form factors can be presented in the form of the self-energy graphs



where

$$G^{(2)}_{\text{1-loop}}(p,-p) \propto \frac{i}{p^2-m^2}\Big(\bar{\bar{\Sigma}}_\gamma + \bar{\bar{\Sigma}}_\lambda + \widetilde{\widetilde{\Sigma}}_{\gamma\lambda} + \widetilde{\widetilde{\Sigma}}_{\lambda^2} + \Sigma_{\zeta^2} + \Sigma_{\gamma\zeta} + \Sigma_{\gamma^2} + \dots\Big)\frac{i}{p^2-m^2},$$

The form factors in this case are relatively complicated.
The basic elements are

$$\frac{1}{\epsilon} \equiv \frac{1}{2-\omega} - \gamma_E + \ln(4\pi), \qquad a = \frac{4m^2}{p^2}, \qquad b = \frac{M^2-m^2}{4m^2}$$

$$A = \sqrt{(1+ab)^2+a}, \qquad c^2 = \frac{p^2}{p^2+4m^2}, \qquad d^2 = \frac{p^2}{p^2+4M^2}.$$

$\gamma_E \approx 0.577$ **is the Euler-Mascheroni constant.**

The explicit expressions for the form factors are rather cumbersome compared to the single-mass loops.

As an example, one of the two shortest elements is

$$
\begin{aligned}
\Sigma_{\zeta^2}(p) \;=\; & \frac{i\zeta^2 p^4}{(4\pi)^2\theta^4}\Bigg\{5\left[\frac{1}{\epsilon}+\ln\left(\frac{\mu^2}{m^2}\right)\right]-\frac{1}{4}\left[9A^2-5(ab)^2-37\right] \\
& -\frac{1}{2(ab)^2c^5}\ln\left(\frac{1+c}{1-c}\right)-\frac{1}{2(ab)^2d^5}\ln\left(\frac{1+d}{1-d}\right) \\
& -\Bigg[\frac{1}{2}(ab)^3+\frac{5}{2}ab\left(ab+\frac{a}{2}+2\right)+\frac{15a}{4}\left(1+\frac{1}{4b}\right) \\
& +5\left(2+\frac{3}{4b}+\frac{1}{2ab}\right)\Bigg]\ln(1+4b) \\
& +\frac{A^5}{2(ab)^2}\ln\left[\frac{(A+1)^2-(ab)^2}{(A-1)^2-(ab)^2}\right]\Bigg\}.
\end{aligned}
$$

Things change if we consider the UV limit, $p^2 \gg M^2 \gg m^2$. In this case, the formulas are much simpler, as shown on the next page.

$$\Sigma^{\rm UV}_{\zeta^2}(p^2 \to \infty) = \frac{i\zeta^2 p^4}{(4\pi)^2\theta^4}\Bigg\{5\left[\frac{1}{\epsilon} - \ln\left(\frac{p^2}{\mu^2}\right)\right] + 3 - \frac{15(M^2+m^2)}{p^2} + \frac{10(m^4+m^2M^2+M^4)}{p^4}\ln\left(\frac{p^2}{M^2}\right) + \frac{35(M^2+m^2)}{6p^4} + \frac{40M^2m^2}{3p^4} + \frac{10m^6}{p^4M^2}\ln\left(\frac{m^2}{M^2}\right) + \mathcal{O}\left(\frac{M^6}{p^6}\right)\Bigg\},$$

$$\Sigma^{\rm UV}_{\gamma\zeta}(p^2 \to \infty) = -\frac{i\gamma\zeta p^2}{(4\pi)^2\theta^4}\Bigg\{3\left[\frac{1}{\epsilon} - \ln\left(\frac{p^2}{\mu^2}\right)\right] + 7 - \frac{9(M^2+m^2)}{p^2} - \frac{6(M^2+m^2)}{p^2}\ln\left(\frac{p^2}{M^2}\right) - \frac{6m^4}{p^2M^2}\ln\left(\frac{m^2}{M^2}\right) + \mathcal{O}\left(\frac{M^4}{p^4}\right)\Bigg\},$$

$$\Sigma^{\rm UV}_{\gamma^2}(p^2 \to \infty) = \frac{i\gamma^2}{(4\pi)^2\theta^4}\Bigg\{2\left[\frac{1}{\epsilon} + \ln\left(\frac{p^2}{\mu^2}\right) + 2\ln\left(\frac{\mu^2}{M^2}\right)\right] + 5\,\frac{4m^2}{M^2}\ln\left(\frac{m^2}{M^2}\right) + \mathcal{O}\left(\frac{M^2}{p^2}\right)\Bigg\}.$$

What should be expected:

In the UV regime, the leading logarithmic terms in the form factor, $\ln\left(p^2/\mu^2\right)$, **are proportional to the corresponding divergences in the fundamental fourth-derivative theory.**

The logarithmic dependencies of the Euclidean momentum p **and of the parameter of renormalization** μ **are exactly the same in the UV regime, where the masses are irrelevant.**

In the UV limit, there is a perfect correspondence between Minimal Subtraction and Momentum Subtraction schemes.

What about the IR limit? In this case, $p^2 \ll M^2$.
For the sake of simplicity, we can set $m^2 = 0$.

Our expectation is a fit between UV limit in the form factors of effective model and the remnants of the logarithmic form factors in the IR limit of the fundamental model.

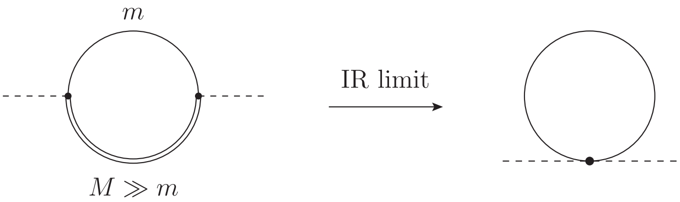
Assuming $m^2 = 0,\ M^2 \gg p^2$, we get

$$\Sigma^{\rm IR}_{\zeta^2}(M^2 \gg p^2)\Big|_{m^2=0} = \frac{i\zeta^2 p^4}{(4\pi)^2\theta^4}\left\{5\left[\frac{1}{\epsilon} + \ln\left(\frac{\mu^2}{M^2}\right)\right] - \frac{1}{6}\left(7 + \frac{35p^2}{2M^2} - \frac{9p^4}{2M^4}\right) + \frac{p^4}{2M^4}\ln\left(\frac{M^2}{p^2}\right) + \mathcal{O}\left(\frac{p^6}{M^6}\right)\right\},$$

$$\Sigma^{\rm IR}_{\gamma\zeta}(M^2 \gg p^2)\Big|_{m^2=0} = -\frac{i\gamma\zeta p^2}{(4\pi)^2\theta^4}\left\{3\left[\frac{1}{\epsilon} + \ln\left(\frac{\mu^2}{M^2}\right)\right] - \frac{1}{2} + \frac{2p^2}{3M^2} - \frac{p^4}{M^4}\left[\frac{7}{20} - \frac{1}{2}\ln\left(\frac{p^2}{M^2}\right)\right] + \mathcal{O}\left(\frac{p^6}{M^6}\right)\right\},$$

$$\Sigma^{\rm IR}_{\gamma^2}(M^2 \gg p^2)\Big|_{m^2=0} = \frac{i\gamma^2}{(4\pi)^2\theta^4}\left\{2\left[\frac{1}{\epsilon} + \ln\left(\frac{\mu^2}{M^2}\right)\right] + \frac{13}{6}\frac{p^2}{M^2} - \frac{p^4}{2M^4}\left[\frac{8}{5} + \ln\left(\frac{p^2}{M^2}\right)\right] + \mathcal{O}\left(\frac{p^6}{M^6}\right)\right\}.$$

Qualitatively, the result can be summarized by a single figure:



Wagno C. e Silva and I.Sh., Effective approach to the Antoniadis-Mottola model: quantum decoupling of the higher derivative terms, arXiv: 2301.13291, JHEP.

In this paper, it was also confirmed that the same "collapse" of massive lines and the consequent decoupling in the form factors coming from mixed diagrams, occurs also for the three-point and four-point vertices.

This is a serious argument in favor of the universal IR limit, something contested, e.g., in the polemic paper in 2008:

I.Sh., arXiv:0812.3521 (IJMPA).

In the effective low-energy model, we meet, e.g.,

$$\Sigma^{\rm eff}_{\gamma^2}(p) \;=\; \frac{ip^4}{(4\pi)^2}\Big\{\Big(\frac{1}{2}-\frac{5}{4}\,a+\frac{3}{8}\,a^2\Big)\,\Big[\frac{1}{\epsilon}-\ln\Big(\frac{\mu^2}{m^2}\Big)\Big] - \frac{1}{2c}\Big(\frac{1}{4}\,a^2-\frac{1}{c^2}+2\Big)\ln\Big(\frac{1+c}{1-c}\Big)+\Big(1-\frac{7}{4}\,a+\frac{1}{2}\,a^2\Big)\Big\}.$$

To make the things simpler, consider the limit $\Lambda = 0, \;\; p^2 \to \infty$.

Then the expression reduces to

$$\Sigma^{\rm eff}_{\gamma^2}(p)\Big|_{\Lambda=0} \;=\; \frac{ip^4}{2(4\pi)^2}\Big[\frac{1}{\epsilon}-\ln\Big(\frac{p^2}{\mu^2}\Big)+2\Big].$$

This UV-leading nonlocal contribution with $\ln\big(p^2/M^2\big)$ fits the γ^2 term in the IR regime of the “fundamental” theory.

All in all, our main expectations in the scalar version of higher derivative quantum gravity are successful. It is natural to ask which kind of result should we expect in the models of QG.

Gauge and parametrization in quantum gravity

The UV "running" depends on divergences and the last may be not universal. Indeed, there maybe dependence on the gauge-fixing and parametrization. Typically,

$$S_t \;=\; S_{QG} + S_{gf} + S_{ghost}\;,$$

where $S_t = S_t(\alpha_i)$ **and** $\alpha_i = (\beta_k,\;\gamma_j)$**. For instance,**

$$S_{gf} = \int_x \chi_\mu\, Y^{\mu\nu}(\beta_2,\,\beta_3,\,...)\,\chi_\nu, \qquad \chi_\mu = \nabla_\lambda\,\phi^\lambda_\mu - \beta_1\,\nabla_\mu\phi^\lambda_\lambda\,.$$

The most general one-loop parametrization is close to

J. Gonçalves, T. de Paula Netto & I.Sh., arXiv:1712.03338, PRD.

$$g_{\mu\nu} \quad\longrightarrow\quad g'_{\mu\nu} \;=\; g_{\mu\nu} + \kappa\big(\gamma_1\,\phi_{\mu\nu} + \gamma_2\,\phi\,g_{\mu\nu}\big) + \kappa^2\big(\gamma_3\,\phi_{\mu\rho}\phi^\rho_\nu + \gamma_4\,\phi_{\rho\omega}\phi^{\rho\omega}\,g_{\mu\nu} + \gamma_5\,\phi\,\phi_{\mu\nu} + \gamma_6\,\phi^2\,g_{\mu\nu}\big),$$

where $g_{\mu\nu}$ **is the background metric and** $\phi_{\mu\nu}$ **are quantum fields.**

At one-loop, this is all possible ambiguity in the divergences.

Gauge invariance in quantum gravity (QG)

To understand what we can expect in QG, one has to account for the differences between Antoniadis and Mottola model and the fourth- and higher-derivative models of quantum metric. The main difference is that in QG there are parametrization and gauge dependencies in both fundamental and effective cases.

We use the general statement about the gauge-fixing and parametrization independence of the on-shell effective action.

The difference between the divergences of two versions of the one-loop effective action, evaluated using different gauge and parametrization parameters α_i and α_0 is proportional to the classical equations of motion

$$\delta\bar{\Gamma}^{(1)}_{div} = \bar{\Gamma}^{(1)}_{div}(\alpha_i) - \bar{\Gamma}^{(1)}_{div}(\alpha_0) = \frac{1}{\epsilon}\int_x \varepsilon^{\mu\nu} f_{\mu\nu},$$

where, in pure GR, $\varepsilon^{\mu\nu} = R^{\mu\nu} - \dfrac{1}{2} g^{\mu\nu}(R + 2\Lambda).$

In QG/GR one meets the divergences (can be proved local)

$$\Gamma^{(1)}_{div} = \frac{1}{\epsilon} \int_x \Big\{ c_1 R^2_{\mu\nu\alpha\beta} + c_2 R^2_{\alpha\beta} + c_3 R^2 + c_4 \Box R + c_5 R + c_6 \Big\}.$$

and $$\varepsilon^{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}} = R^{\mu\nu} - \frac{1}{2} \big(R + 2\Lambda \big) g^{\mu\nu}\,.$$

Therefore, in this case

$$f_{\mu\nu} = b_1 R_{\mu\nu} + b_2 R g_{\mu\nu} + b_3 \Lambda g_{\mu\nu} + b_4 g_{\mu\nu} \Box + b_5 \nabla_\mu \nabla_\nu$$

and we arrive at

$$\delta\Gamma^{(1)}_{div} = \Gamma^{(1)}_{div}(\alpha_i) - \Gamma^{(1)}_{div}(\alpha_i^0)$$
$$= \frac{1}{\epsilon} \int_x \big(b_1 R_{\mu\nu} + b_2 R g_{\mu\nu} + b_3 \Lambda g_{\mu\nu} + b_4 g_{\mu\nu} \Box + b_5 \nabla_\mu \nabla_\nu \big)\, \varepsilon^{\mu\nu},$$

where the parameters $b_{1,2,..,5}$ depend on the full set of parametrization and gauge parameters α_i.

Are there universal (invariant) beta functions? Obviously, one of those is the coefficient of the Gauss-Bonnet term.

To derive another combination, we note that the coefficients in the divergences vary according to

$$c_2 \to c_2 + b_1, \qquad c_3 \to c_3 - \big(b_2 + \tfrac{1}{2}\,b_1\big), \qquad c_4 \to c_4 - b_4,$$
$$c_5 \to c_5 - \big(b_1 + 4b_2 + b_3\big)\Lambda, \quad c_6 \to c_6 - 4b_3\Lambda^2, \quad c_1 \to c_1.$$

The two gauge-fixing and parametrization invariants are

$$c_1 \qquad \text{and} \qquad c_{inv} = c_6 - 4\Lambda c_5 + 4\Lambda^2 c_2 + 16\Lambda^2 c_3\,.$$

Let us stress that, in the “usual” quantum GR, there are no reasonably defined beta functions for the C^2 and R^2 terms.

Does this conclusion apply to other models of QG?

In the fourth-derivative QG, local divergences have the form

$$\Gamma^{(1)}_{div} = \frac{1}{\epsilon}\int_x \big\{ c_1 R^2_{\mu\nu\alpha\beta} + c_2 R^2_{\alpha\beta} + c_3 R^2 + c_4 \Box R + c_5 R + c_6 \big\}.$$

However, in this case, the classical equations of motion are also four-derivative. Therefore, in this case,

$$\delta\bar{\Gamma}^{(1)}_{div} \,=\, \bar{\Gamma}^{(1)}_{div}(\alpha_i) - \bar{\Gamma}^{(1)}_{div}(\alpha_0) \,=\, -\frac{1}{\epsilon}\int_x \varepsilon^{\mu\nu} f_{\mu\nu},$$

with $f_{\mu\nu}(\alpha_i) \,=\, g_{\mu\nu}\, f(\alpha_i).$

Here $f(\alpha_i)$ is an arbitrary dimensionless function of the parameters of gauge fixing and metric parametrization.

The gauge/parametrization dependence of the divergent part is controlled by the "conformal shift" of the classical action

$$\Gamma^{(1)}_{div}(\alpha_i) \,-\, \Gamma^{(1)}_{div}(\alpha_i^0) \,=\, f(\alpha_i)\int d^4x\; g_{\mu\nu}\,\frac{\delta S}{\delta g_{\mu\nu}}.$$

In the conformal model of QG, the *r.h.s.* vanishes owing to Noether identity for this symmetry. Generally, this isn't the case.

In this way, we arrive at

$$\Gamma^{(1)}_{div}(\alpha_i) - \Gamma^{(1)}_{div}(\alpha_i^0) = f(\alpha_i) \int_x \Big\{ \frac{2\omega}{\lambda} \Box R - \frac{1}{\kappa^2}(R + 4\Lambda) \Big\}.$$

These three divergent coefficients depend on the the gauge fixing and parametrization, while other three are invariant.

Furthermore, there are two gauge-invariant combinations of the coefficients of Einstein-Hilbert and cosmological terms.

An important consequence is that, in the fourth-derivative model, there is a well-defined UV (in fact, for this particular theory, this means trans-Planckian energy scale) running of the coefficients of the Gauss-Bonnet, C^2 and R^2 terms.

On top of this, there is a single parametrization- and gauge-invariant combination of the beta functions for the Newton constant and cosmological constant.

This combination is the beta function of the dimensionless ratio $G\Lambda$. This parameter and λ, ω, ρ have well-defined runnings.

Consider UV running in quantum GR and fourth-derivative QG.

- **In quantum GR there is only one well-defined on shell renormalization group equation.**

$$\mu \frac{d\gamma}{d\mu} = -\frac{29}{5(4\pi)^2}\gamma^2, \qquad \gamma = 16\pi G\Lambda.$$

This can be seen as an indication of asymptotic freedom, but this is a kind of exaggeration, as it is on shell.

E.S. Fradkin, A.A. Tseytlin, Nucl. Phys. B201 (1982) 469.

The invariant running can be met only using the Vilkovisky–DeWitt (VdW) scheme in QG.

*G.A. Vilkovisky, Nucl. Phys. B***234** *(1984) 125.*

B.S. DeWitt, The effective action, (1987).

Indeed, there are invariant equations. But not in a usual QFT.

T. Taylor and G. Veneziano, Nucl. Phys. B ***345*** *(1990).*

B. Giacchini, T. de Paula Netto, I.Sh., JHEP (2020), 2009.04122.

- **On the contrary, in the fourth-derivative QG**

$$S = -\int_x \left\{ \frac{1}{2\lambda} C^2 - \frac{1}{\rho} E_4 + \frac{1}{\xi} R^2 - \frac{1}{\kappa^2} (R - 2\Lambda) \right\}$$

there are well-defined renormalization group equations.

$$(4\pi)^2 \frac{d\rho}{dt} = -\frac{196}{45} \rho^2 , \quad (4\pi)^2 \frac{d\lambda}{dt} = -\frac{133}{10} \lambda^2 ,$$

$$(4\pi)^2 \frac{d\xi}{dt} = -10 \lambda^2 \xi^2 + 5 \lambda \xi - \frac{5}{36} .$$

In this case, we certainly have asymptotic freedom for the effective charge λ and the UV stable fixed point for the ratio ξ/λ.

I.G. Avramidi and A.O. Barvinsky, PLB 159 (1985) 269
(with several subsequent verifications).

There is one more difference between the running in the two models of QG. The energy scales where the two kinds of running can be applied are very much different.

Thus, the ambiguities in in the fourth-derivative QG model and in quantum GR are qualitatively different. The question is what we can expect from the decoupling in this situation?

The answer can be provided only by the explicit and well-checked calculations, including with (at least) some of the metric parametrization and/or gauge ambiguities.

However, what do we know before doing these, extremely difficult and complicated, calculations?

The two main working assumptions are:

- **The GR is a universal limit of QG in the IR. For a while, this was checked only for a four-derivative scalar Antoniadis and Mottola model, and it was found correct.**

- **The decoupling of the mixed diagrams follows the same pattern in the scalar model and in "real" QG.**

If both things are true:

- **In the UV, the masses of the extra degrees of freedom are irrelevant. We expect a perfect fit between Minimal Subtraction and Momentum Subtraction renormalization schemes.**

- **In the IR, there will be a "mess", in the sense that those beta functions which can be calculated, are expected to be ambiguous and have no physical meaning.**

The unique well-defined beta function in the IR corresponds to the dimensionless ratio of the Newton constant G **and** Λ**.**

Unfortunately, this is exactly the part which cannot be explored using Feynman diagrams. Those are certainly "bad news", but the positive aspect is that we can clearly see what we have to calculate and how we can check the general statements.

E.S. Fradkin & A.A. Tseytlin, NPB 201 (1982) 469.

I.Sh. & A. Jacksenaev, PLB 324 (1994) 284. ...

Conclusions

- **Effective approach is our simplest option for QG.**

Assuming effective QG means we are forced to give up from the main target of QG, i.e., from the describing QG in the deep UV, explanation (or removal) of singularities and alike.

- **Still, it is very important for us to have “correct” and understandable results, as this would mean we can make solid statements, maybe one day being compared with some kind of observational data.**

- **The decoupling is a fruitful concept, with the solid basis formed by the Appelquist and Carazzone theorem. This theorem was extended to curved space, to mixed diagrams and, for a while, the results were always confirming the general concepts.**

- **Using general theorems about renormalization, we can see what is the expected fit between fundamental and effective QG theories in the UV and in the IR. As usual, general arguments in QG cannot replace real calculations, still to be done.**