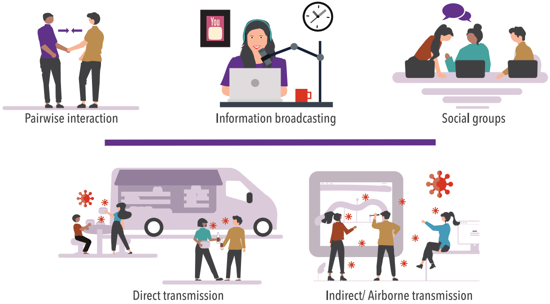
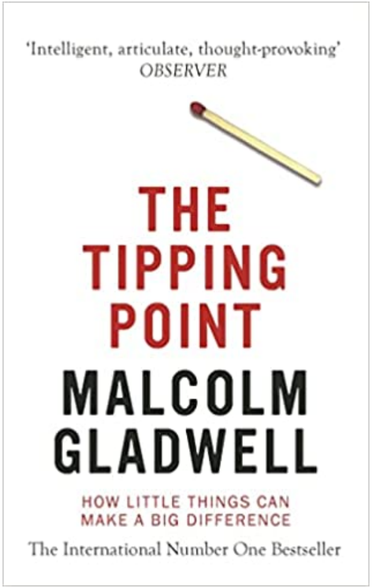
CONTAGION MODELS ON HIGHER-ORDER NETWORKS

Guilherme Ferraz de Arruda

Collaborations with:

Alberto Aleta, Michele Tizzani, Giovanni Petri, Pablo M. Rodriguez, and Yamir Moreno

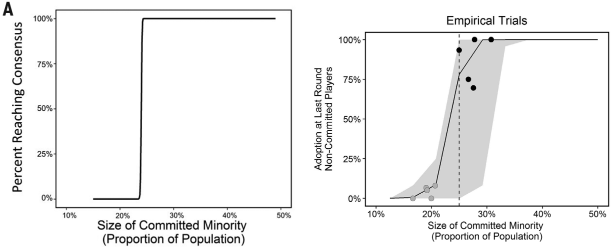




Experimental evidence for tipping points in social convention

Damon Centola[1,2*], Joshua Becker[1], Devon Brackbill[1], Andrea Baronchelli[3]

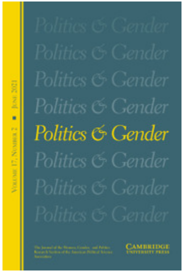
Theoretical models of critical mass have shown how minority groups can initiate social change dynamics in the emergence of new social conventions. Here, we study an artificial system of social conventions in which human subjects interact to establish a new coordination equilibrium. The findings provide direct empirical demonstration of the existence of a tipping point in the dynamics of changing social conventions. When minority groups reached the critical mass—that is, the critical group size for initiating social change—they were consistently able to overturn the established behavior. The size of the required critical mass is expected to vary based on theoretically identifiable features of a social setting. Our results show that the theoretically predicted dynamics of critical mass do in fact emerge as expected within an empirical system of social coordination.



Numbers and Beyond: The Relevance of Critical Mass in Gender Research

Sandra Grey, Victoria University

Political scientists concerned with gender relations have long been interested in the numbers of women in national legislatures. Women make up slightly more than 50% of the world's population, yet average only 16% of the world's elected political posts. This has led to calls for action that would increase the number of women in legislatures based both on arguments of justice and on claims that an increase will substantively change decision-making processes and outcomes. Part of the debate about substantive changes in political decision making has centered on whether women in a legislature must reach a "critical mass" in order to bring about change in the political arena. The term *critical mass* is frequently used by politicians, the media, and academics, but can it offer insights into the influence of gender on political processes and outcomes? In this essay, I argue that critical mass is only useful if we discard the belief that a single proportion holds the key to all representation needs of women and if we discard notions that numbers alone bring about substantive changes in policy processes and outcomes. I use a longitudinal textual analysis of New Zealand parliamentary debates to begin development of a joint-effect model that can better explain the factors that aid (or hinder) the substantive representation of women.

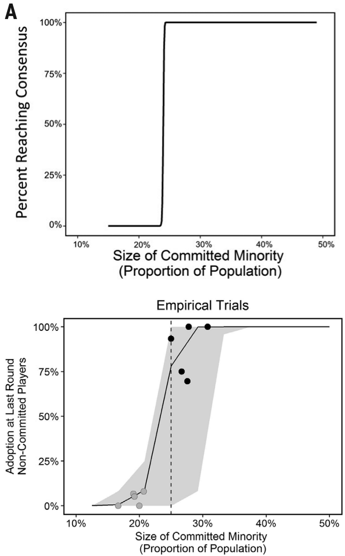


CRITICAL PERSPECTIVES ON GENDER AND POLITICS
Politics & Gender, Vol. 2, Issue 4, Dec. 2006, pp. 492 - 502

Motivations

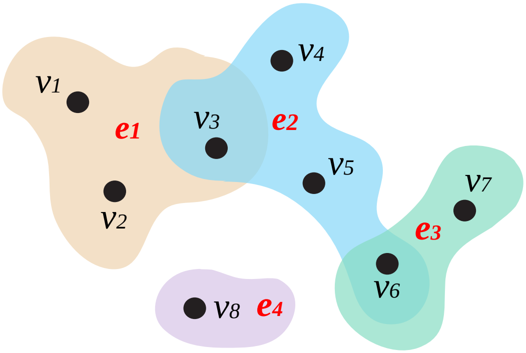
QUESTIONS

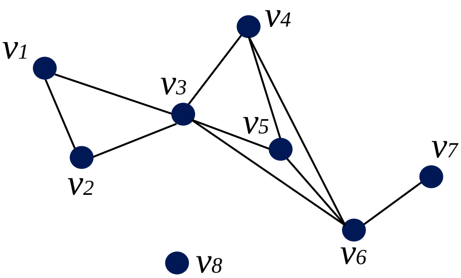
1. How will a collection of groups behave?
2. How might the intersection between these groups change the global dynamics?
3. Can smaller groups have a higher critical-mass threshold than the whole population?



Damon Centola et al. Science 2018;360:1116-1119

Hypergraphs: basic definitions





$\mathcal{V} = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$

$\mathcal{E} = \{e_1, e_2, e_3, e_4\}$

$e_1 = \{v_1, v_2, v_3\}$, $e_2 = \{v_3, v_4, v_5, v_6\}$,

$e_3 = \{v_6, v_7\}$ and $e_4 = \{v_8\}$

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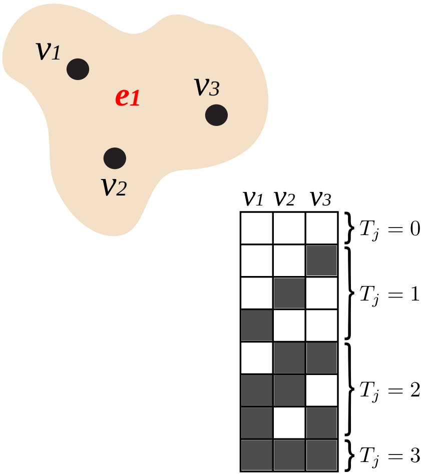
Model definition: Hyperedge process construction



Bernoulli random variable:

$$Y_i = \begin{cases} 1, & \text{if } i \text{ is active} \\ 0, & \text{if } i \text{ is inactive} \end{cases}$$

Critical-mass dynamics:

$\rightarrow T_j = \sum_{k \in e_j} Y_k$

$\rightarrow T_j > \Theta_j$

Poisson binomial distribution:

$$\mathbb{P}(K = k) = \sum_{A \in F_k} \prod_{i \in A} p_i \prod_{j \in A^c} (1 - p_j)$$

DFT [1]:

$$\mathbb{P}_{e_j}(K = k) = \frac{1}{n+1} \sum_{l=0}^{n} C^{-lk} \prod_{m=1}^{n} (1 + (C^l - 1) y_m)$$

$$C = \exp\left(\frac{2i\pi}{n+1}\right)$$

[1] M. Fernandez and S. Williams, IEEE Trans. Aerosp. Electron. Syst. 46, 803 (2010)
Guilherme Ferraz de Arruda, Giovanni Petri, and Yamir Moreno, Phys. Rev. Research 2, 023032

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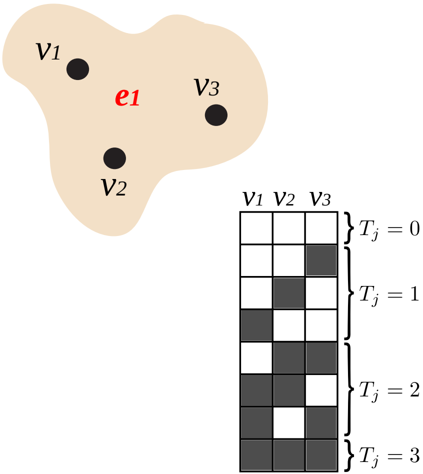

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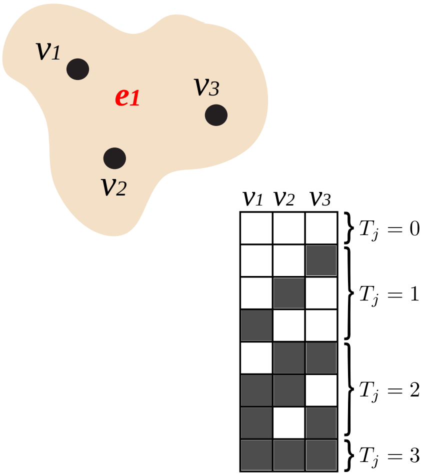

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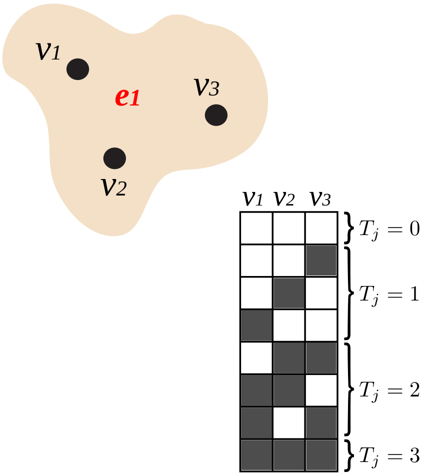

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Model definition: Arbitrary hypergraphs

→ Bernoulli random variable Y_i

→ Set of independent Poisson Processes:

$$\{N_i^{\delta}, N_j^{\lambda_j}\}$$

Deactivation (spontaneous)

Critical-mass dynamics:
$\rightarrow T_j > \Theta_j$
$\rightarrow T_j = \sum_{k \in e_j} Y_k$

→ Exact Model:

$$\frac{d\mathbb{E}(Y_i)}{dt} = \mathbb{E}\left(-\delta Y_i + [1 - Y_i] \sum_{e_j \cap \{v_i\} \neq \emptyset} \lambda_j \sum_{B} \mathbb{1}_{\{Y_i = 0, T_j \geqslant \Theta_j\}}\right)$$

→ First-order approximation

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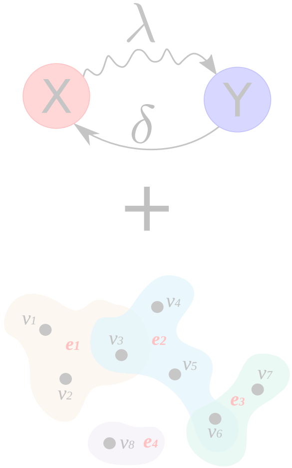

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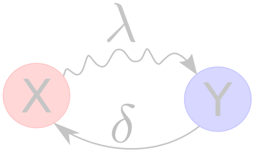

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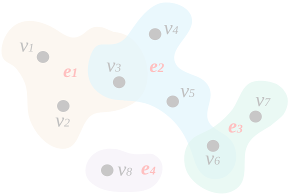

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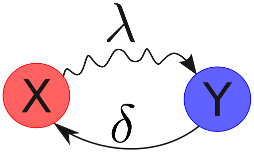

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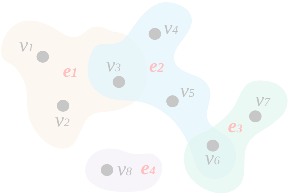

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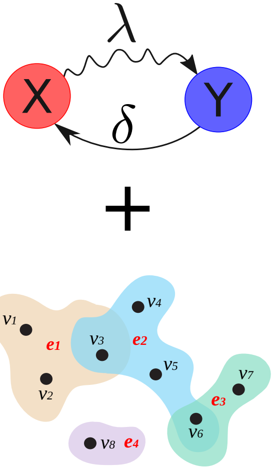

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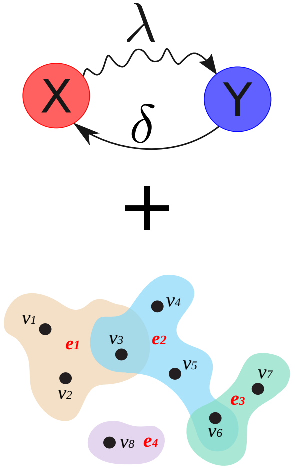

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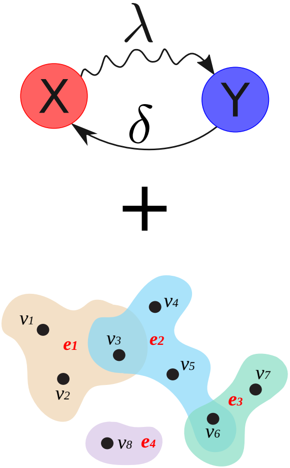

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Monte Carlo Simulations: Definition

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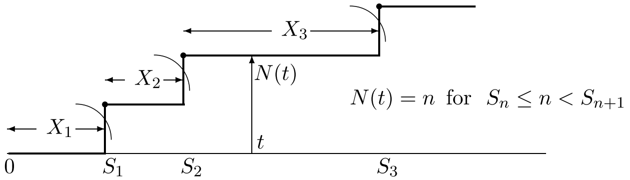


→ Gillespie-like algorithm:
Continuous-time Simulations

Poisson Process:

Let $\lambda > 0$ be fixed. The counting process $\{N(t), t \in [0, \infty)\}$ is called a Poisson Process with rate λ if all the following conditions hold:

1. $N(0) = 0$
2. $N(t)$ has independent and stationary increments
3. We have

$$\begin{aligned} \mathbb{P}\,(N(\Delta) = 0) &= 1 - \lambda\Delta + o(\Delta), \\ \mathbb{P}\,(N(\Delta) = 1) &= \lambda\Delta + o(\Delta), \\ \mathbb{P}\,(N(\Delta) = 2) &= o(\Delta). \end{aligned}$$

Interarrival Times for Poisson Processes :

If $N(t)$ is a Poisson process with rate λ, then the interarrival times $X_1, X_2, \ldots$ are independent and

$$X_i \sim \text{Exp}(\lambda) \qquad i = 1, 2, \ldots$$

Gillespie, Daniel T. (1977). The Journal of Physical Chemistry. 81 (25): 2340–2361.
Guilherme Ferraz de Arruda, Giovanni Petri, and Yamir Moreno, Phys. Rev. Research 2, 023032 - SM

Monte Carlo Simulations: Finite size effects and the QS algorithm

Finite size effects:

Due to finite-size effects we will always reach the absorbing state!

Implementation details:

- C/C++
- GNU Scientific Library (GSL)
- GNU Parallel (a shell tool)

QS method:

Restricts the dynamics to non-absorbing states

Order parameter:

$$\rho = \langle n_a \rangle$$

Susceptibility:

$$\chi = \frac{\langle n_a^2 \rangle - \langle n_a \rangle^2}{\langle n_a \rangle} = N \left(\frac{\langle \rho^2 \rangle - \langle \rho \rangle^2}{\langle \rho \rangle} \right)$$

Guilherme Ferraz de Arruda, Giovanni Petri, and Yamir Moreno, Phys. Rev. Research 2, 023032
S. C. Ferreira, C. Castellano, and R. Pastor-Satorras, Phys. Rev. E 86, 041125 (2012)
A. S. Mata and S. C. Ferreira, Phys. Rev. E 91, 012816 (2015)
M. e. a. Galassi, Gnu scientific library reference manual (2018), https://www.gnu.org/software/gsl/
O. Tange, ;login: The USENIX Magazine 36, 42 (2011).

Results: Blues reviews hypergraph

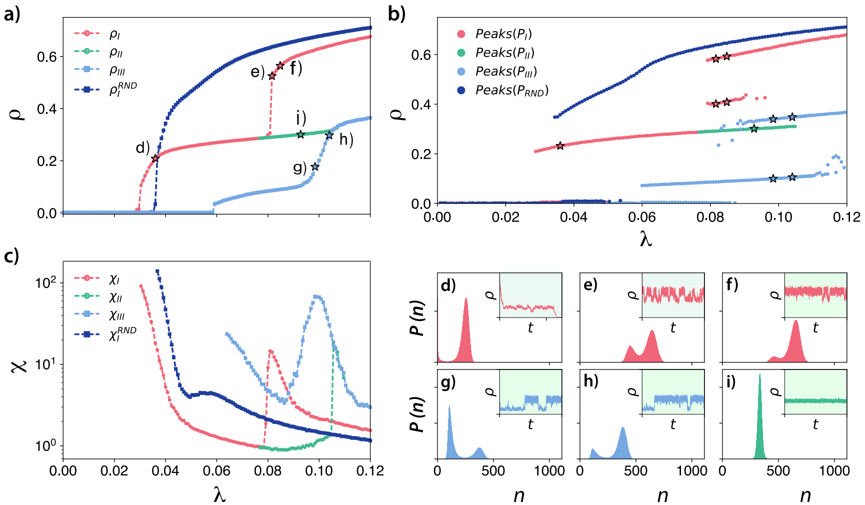
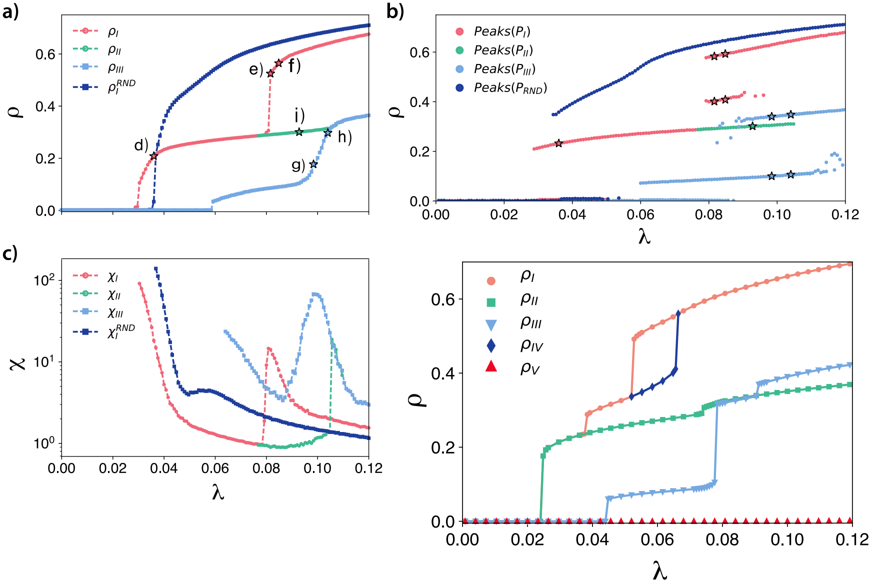


FIG. 2. Monte Carlo simulations for the social contagion model in the blues reviews hypergraph showing multistability and intermittency. In (a), (b) and (c), we present the order parameter, susceptibility, and the peaks of the state distributions respectively. In (d) – (i) we show the state distributions marked in (a). In the insets of these plots we show a small example of temporal behavior that generate these distributions. We performed the simulations using the real blues review hypergraph and a random version using $\delta = 1.0$, $\Theta^* = 0.5$, and $\lambda^*(|e|) = \log_2(|e_j|)$.

Results: Blues reviews hypergraph



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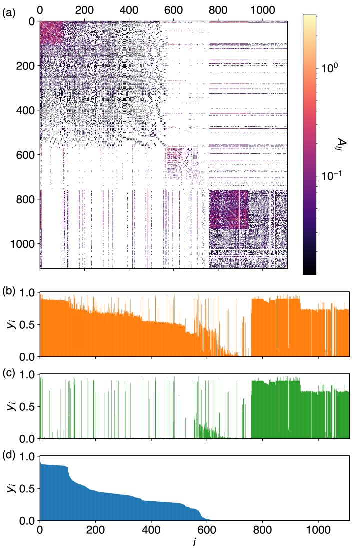

G. F. de Arruda, G. Petri, P. M. Rodriguez and Y. Moreno, Multistability, intermittency and hybrid transitions in social contagion models on hypergraphs, Nat Commun 14, 1375 (2023)

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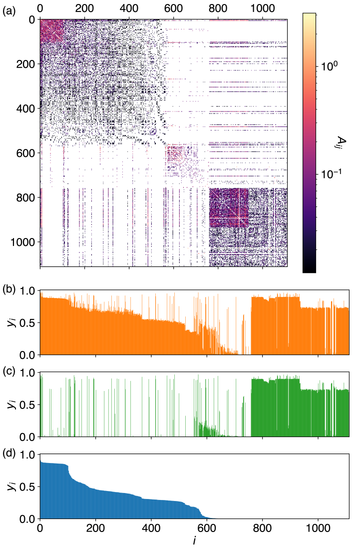

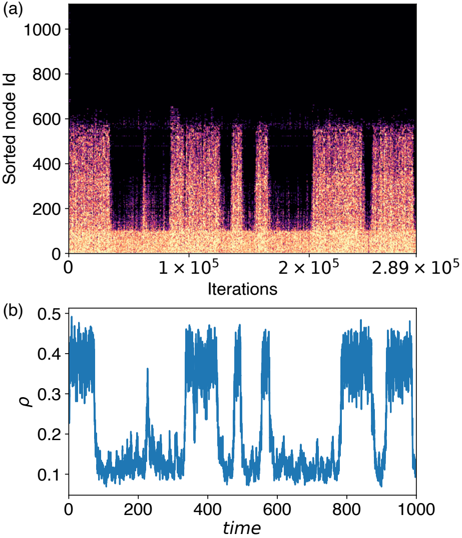

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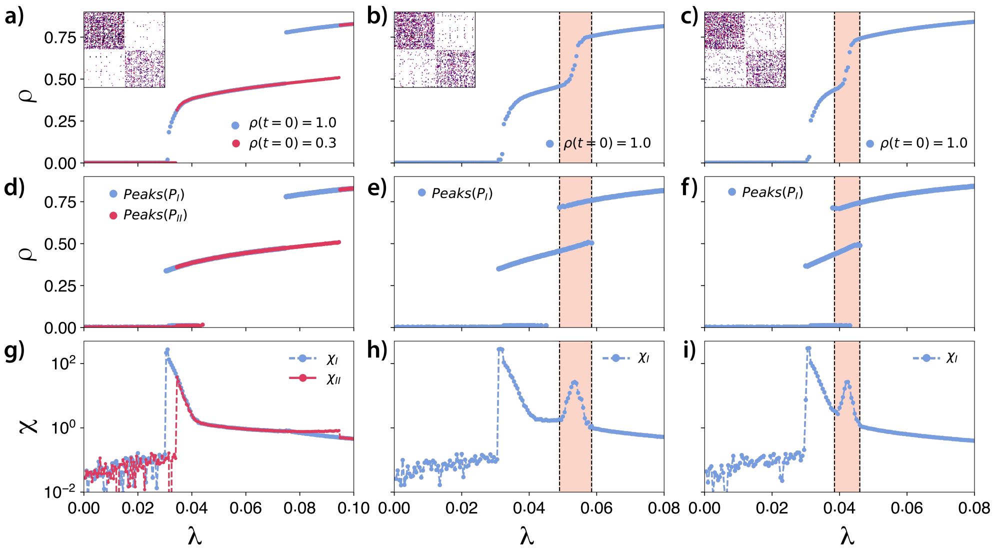
Results: Explaining Multistability and Intermittency



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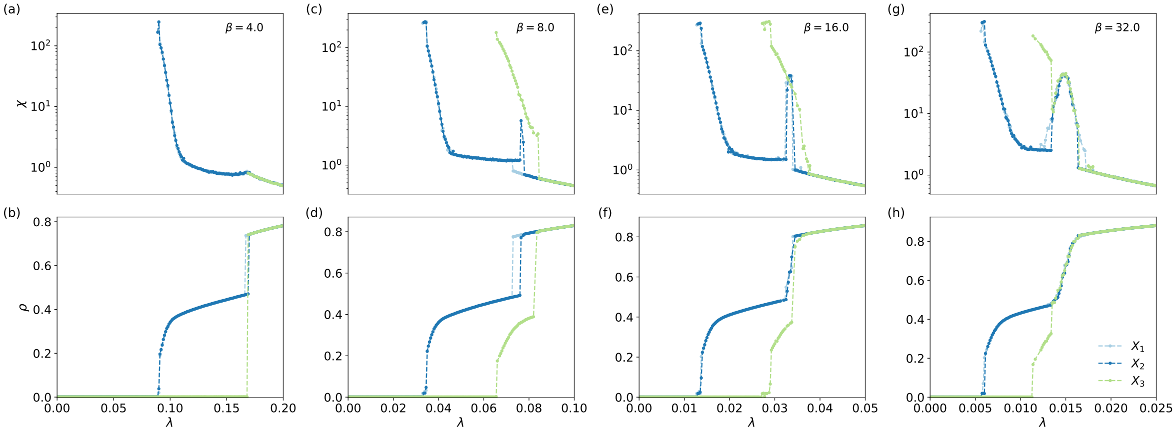

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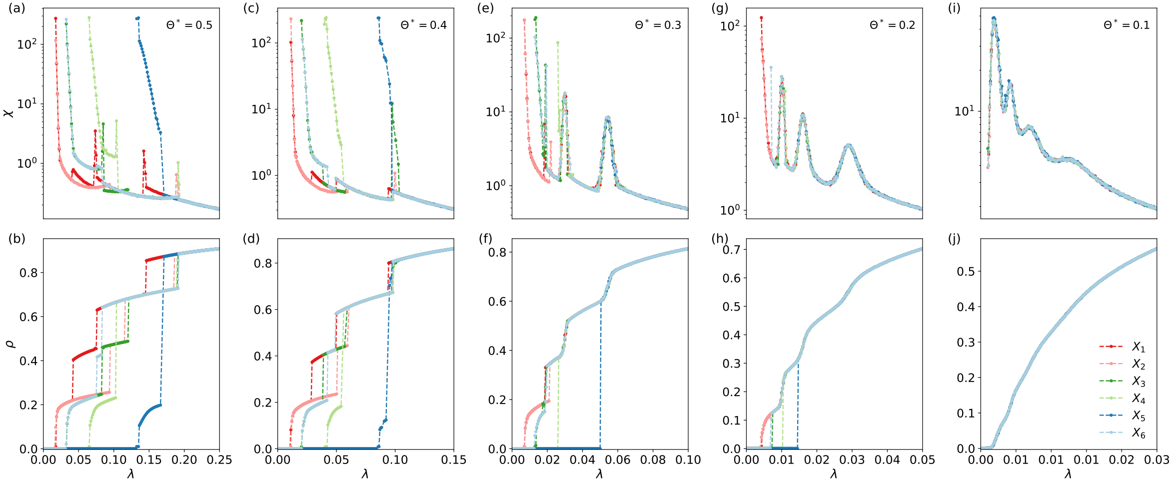

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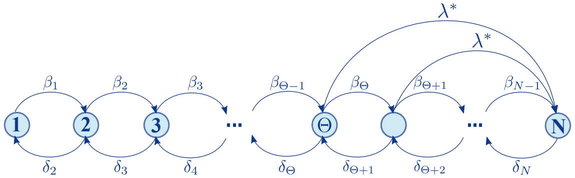


Transition rates:

$$\begin{cases} \mathbf{Q}_{n,n-1} = \delta n = \delta_n \\ \mathbf{Q}_{n,n+1} = \lambda \langle k \rangle n \frac{(N-n)}{N} = \beta_n \\ \mathbf{Q}_{n,N} = \lambda^* U(n-\Theta), \end{cases}$$

where $U(n-\Theta)$ is the Heaviside step function.

Dynamics:

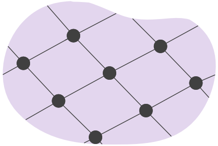
$$\frac{d\boldsymbol{P}}{dt} = \mathbf{Q}^T P$$

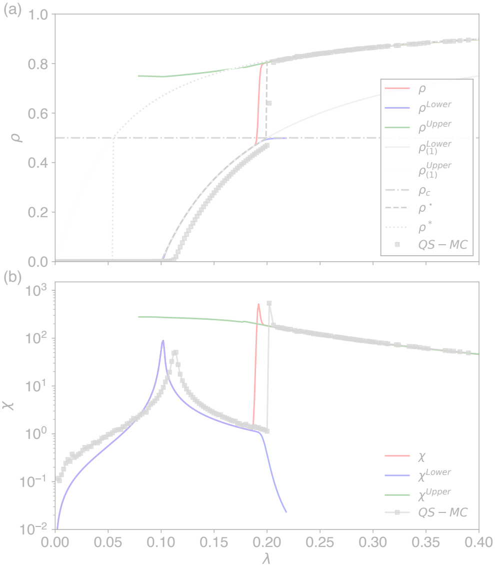
QS constraint:

$$\mathbf{Q}_{1,0} = 0$$

Steady-state solution:

$$\pi = \boldsymbol{P}\pi$$





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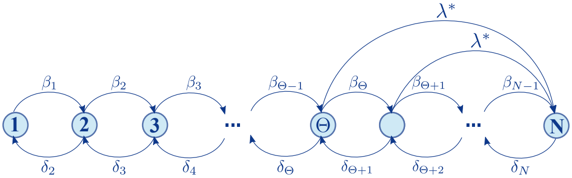

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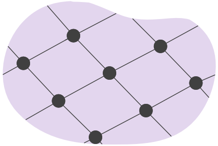

Hyperblob:
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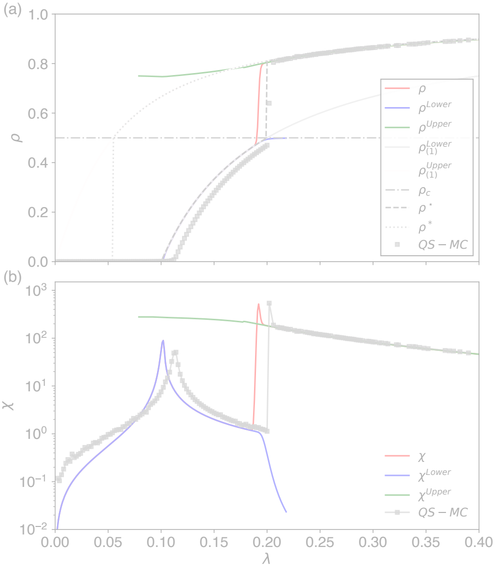

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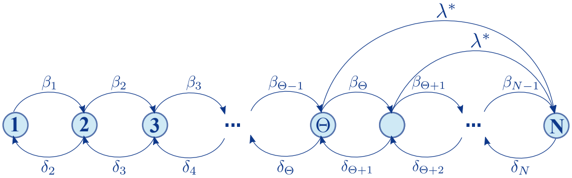

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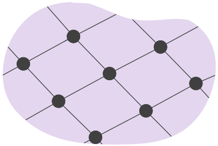

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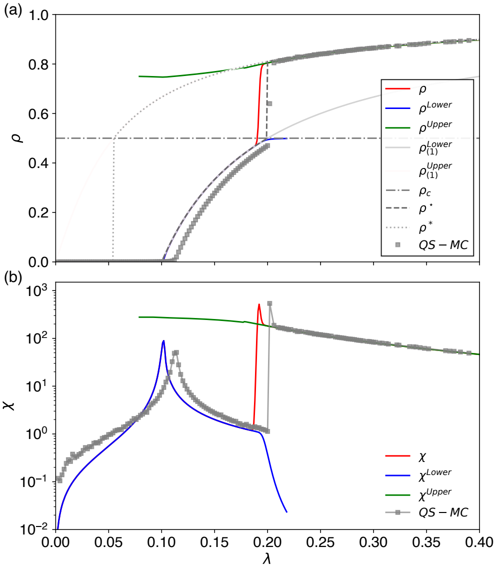

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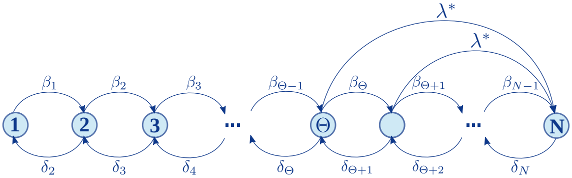

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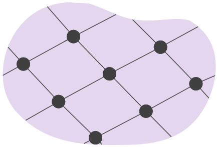

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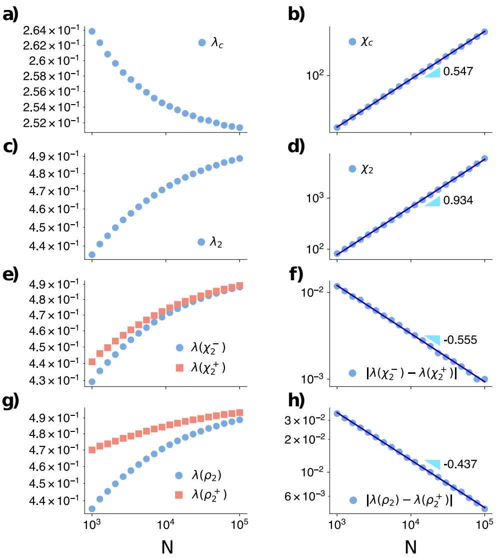

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“Conclusions”

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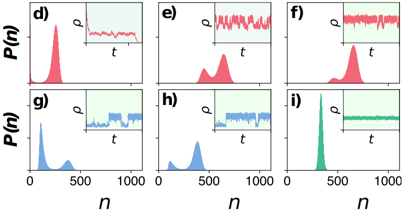

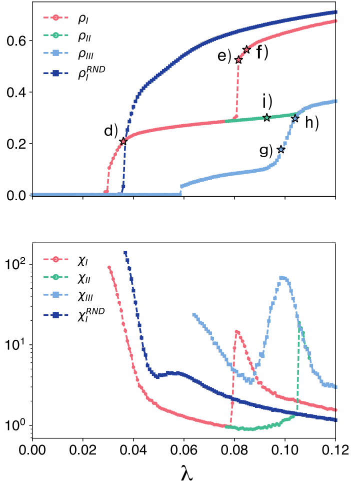

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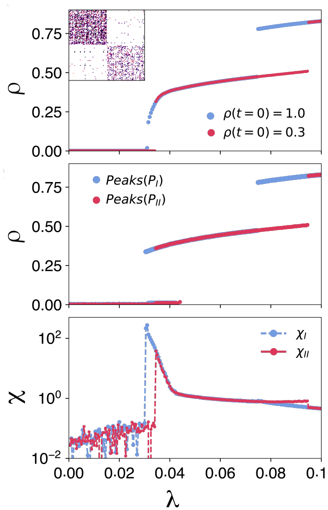

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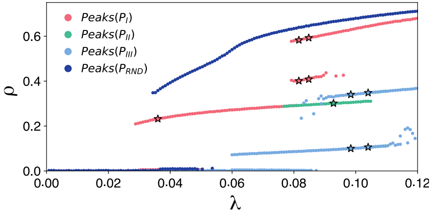

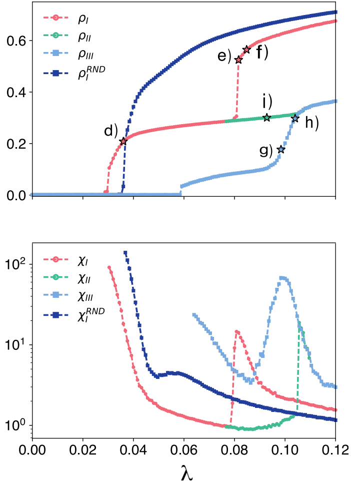

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The general problem

a)

Pairwise interaction

b)

Information broadcasting

Social groups

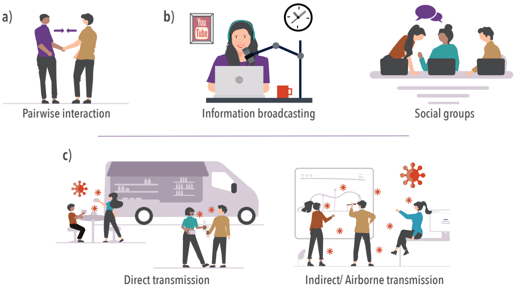
c)

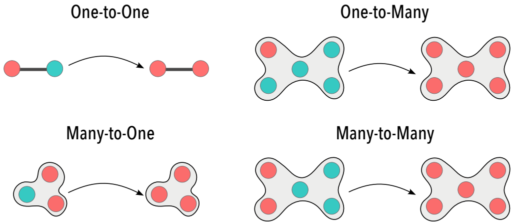
Direct transmission

Indirect/ Airborne transmission

Contagion dynamics on higher-order networks, *arXiv:2402.14938* (accepted on Nature Reviews Physics)

General models





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$$\frac{d\mathbb{E}(Y_i)}{dt} = \mathbb{E}\left(-\delta Y_i + \lambda \sum_{j:v_i\in e_j} \lambda^*(|e_j|)\, X_i f_j^i(\{Y\})\right)$$

- Pairwise SIS:

  $f_j^i(\{Y\}) = Y_k$

- SIS on hypergraphs:

  $$f_j^i(\{Y\}) = \begin{cases} m & \text{if } m < c \\ c & \text{else} \end{cases}$$

- Simplicial contagion model:

  $f_j^i(\{Y\}) = \prod_{k:v_k\in e_j; v_k\neq v_i} Y_k$

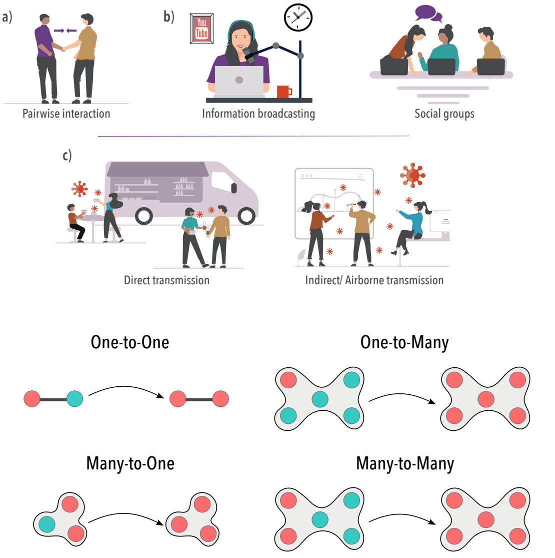
- Power-law infection kernel:

  $f_j^i(\{Y\}) = \left(\sum_{k:v_k\in e_j; v_k\neq v_i} Y_k,\right)^{\nu}$

- **Critical-mass model**:

  $f_j^i(\{Y\}) = H\left(\sum_{k:v_k\in e_j; v_k\neq v_i} Y_k - \Theta_j\right)$

General models: Perspectives



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Perspectives

- Generalities and particularities
  - What are general results and mechanisms?
    - How general?
    - We can translate/reinterpret these results to the different models!
  - What are particular results?
    - Why?
  - Opportunities for new experiments!
    - Data-oriented
    - Controlled experiments
  - Opportunities for new applications!
    - Social tipping points
    - Disease modeling
    - Wireless communications

Acknowledgments

Thank you!

CENTAI

- References:

  - Social contagion models on hypergraphs *Phys. Rev. Research 2, 023032*
  - Social contagion on higher-order structures, *arXiv:2103.03709*
  - **Multistability, intermittency, and hybrid transitions in social contagion models on hypergraphs. *Nat Commun 14, 1375 (2023).***
  - Phase transitions and stability of dynamical processes on hypergraphs, *Communications Physics, v. 4, n. 24 (2021)*
  - The physics of higher-order interactions in complex systems, *Nature Physics v. 17, p. 1093–1098 (2021)*
  - **Contagion dynamics on higher-order networks, *arXiv:2402.14938* (accepted on Nature Reviews Physics)**

- *http://guifarruda.gitlab.io/*