



Workshop on Dynamical Processes on Complex Networks - May 13 – 17, 2024

# Visibility Graphs for non-equilibrium phase transitions

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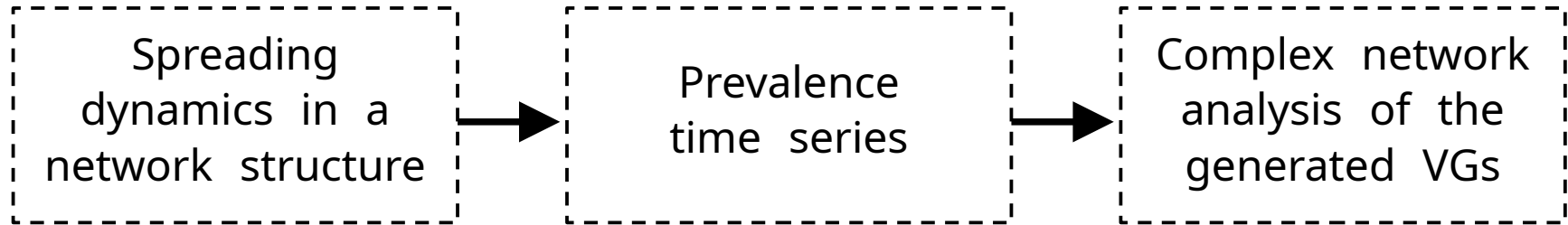
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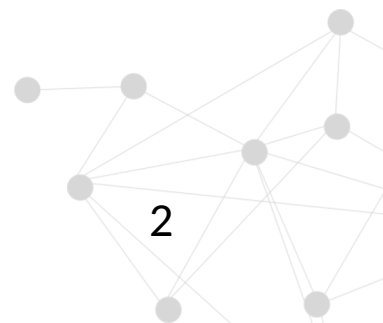




# Working flowchart



Can we infer information on the dynamical process by analyzing some properties of the generated VG?

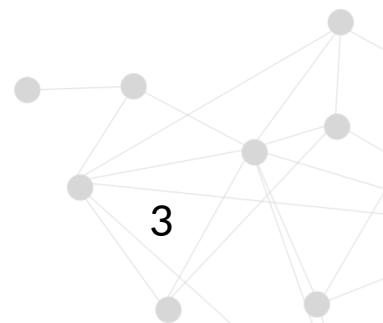




# Spreading dynamics on networks

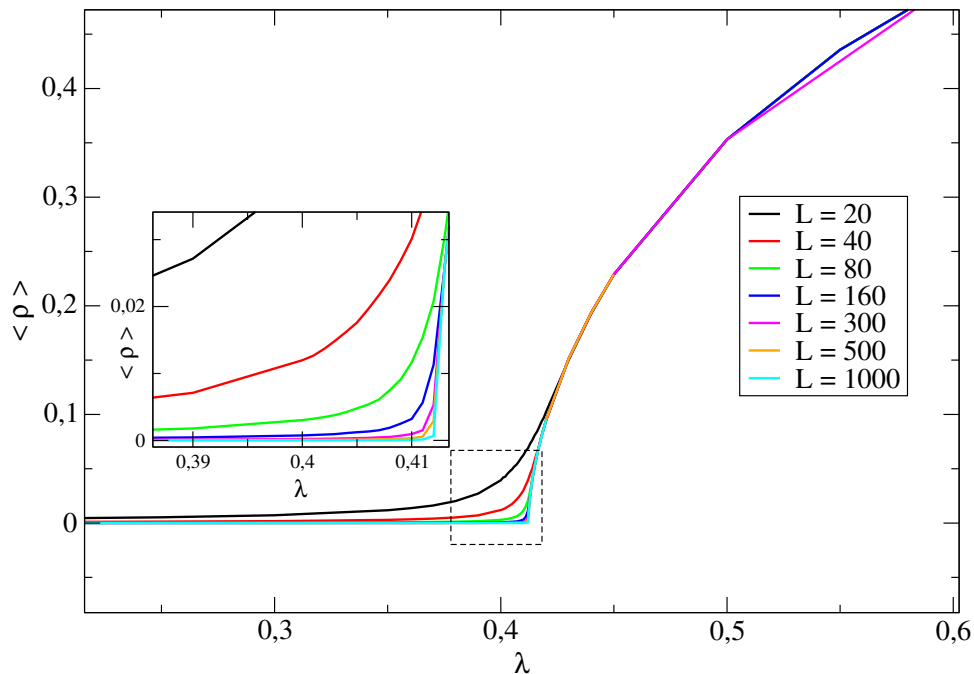
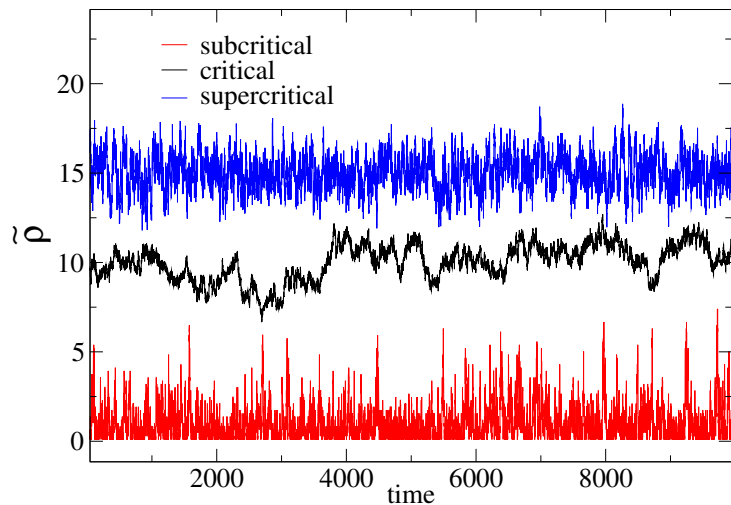
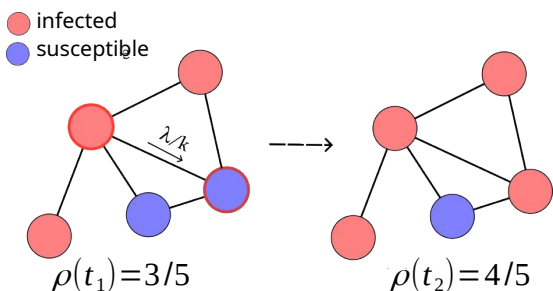
Models:  Network Structures:

- Contact Process (CP);
  - Susceptible-Infected-Susceptible (SIS);
  - Two-Species Contact Process (2SCP);
  - Quenched Contact Process (QCP).
- Annealed (ANN);
  - Lattices (LAT);
  - Random Regular Network (RRN);
  - Random Regular Network with a hub (RRNH);
  - Uncorrelated Configuration Model (UCM);
  - Erdős-Renyi (ER).



# Ex.: Contact Process Model on a 2d LATT ( $N = L^2$ )

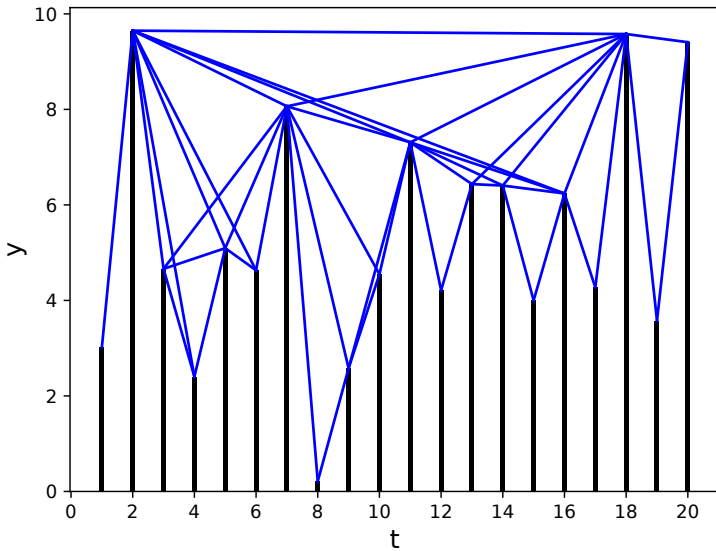
- Absorbing state phase transition.



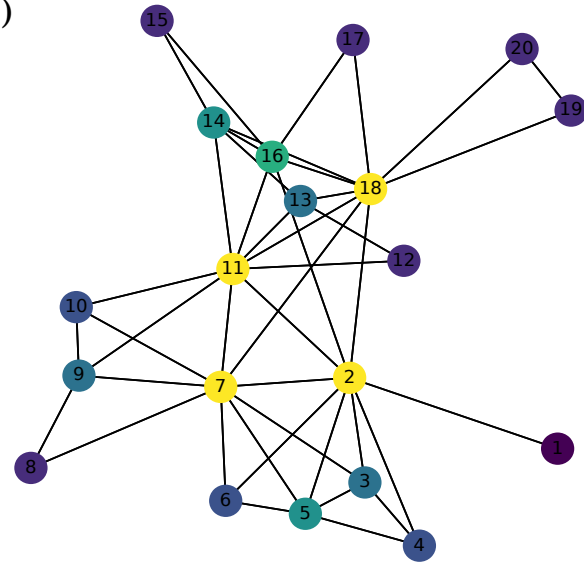
← Epidemic prevalence time series

# Time series to networks: Visibility graphs (VG)

(a)



(b)

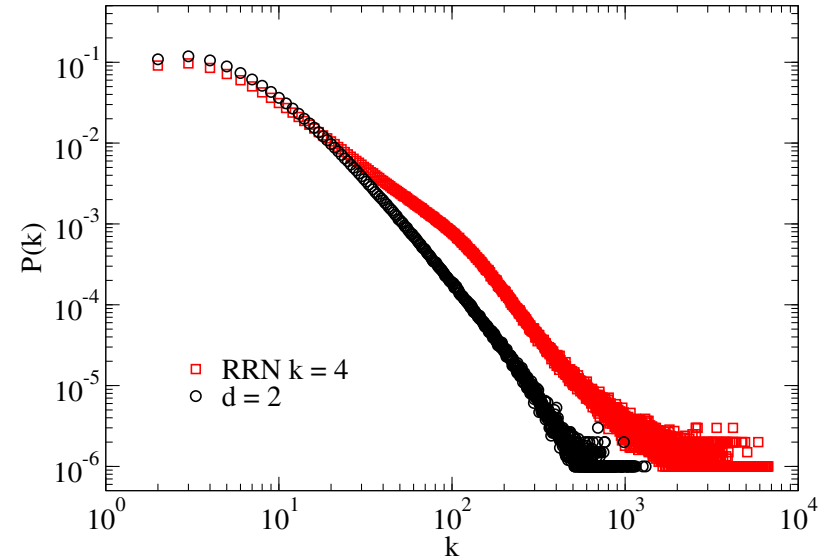
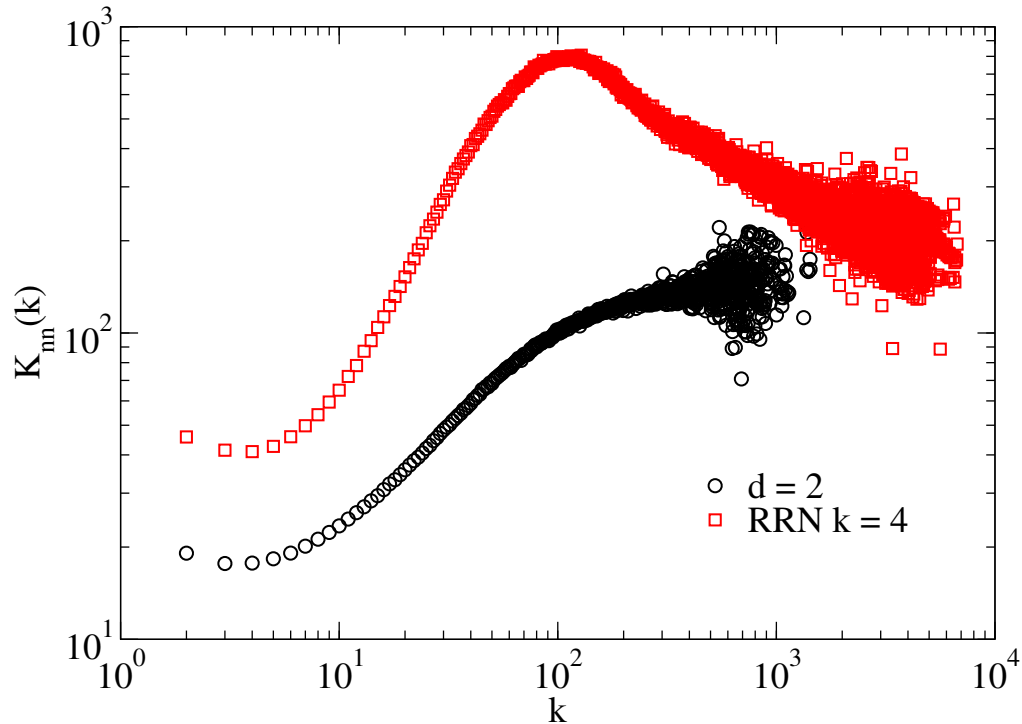


$$y_c < y_b + (y_a - y_b) \frac{(t_b - t_c)}{(t_b - t_a)},$$

$$t_a < t_c < t_b$$

# Degree Correlations of the VG

The average degree of the nearest neighbors  $K_{nn}(k)$  captures much more information than the degree distribution  $P(k)$ .

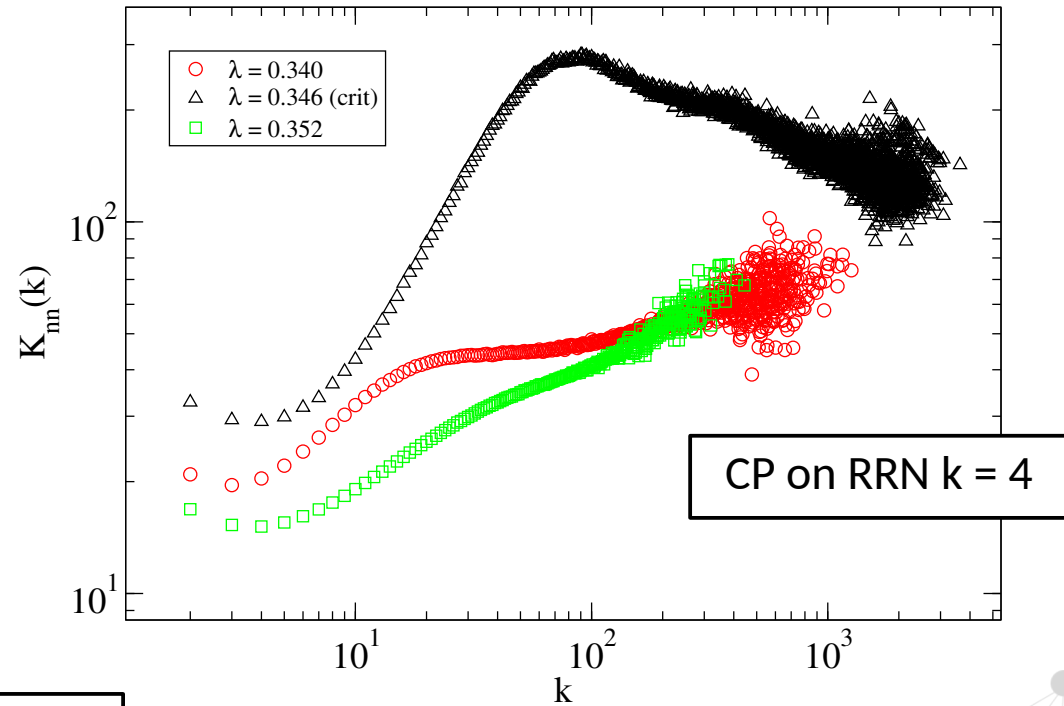
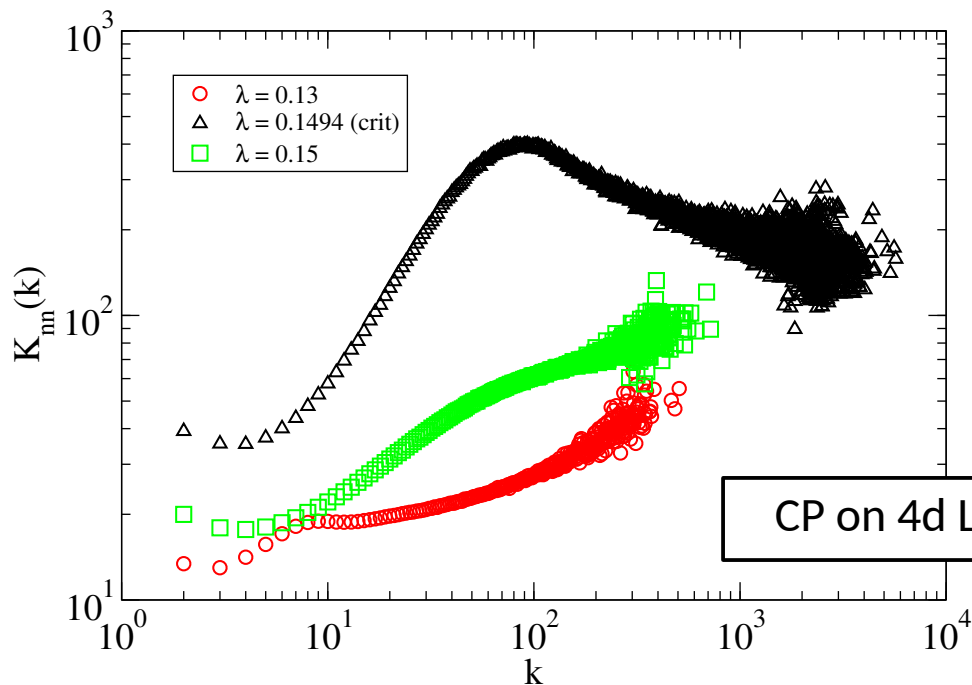


$$K_{nn}(i) = \frac{1}{k_i} \sum_{j=1}^N A_{ij} k_j$$

$$K_{nn}(k) = \frac{1}{N_k} \sum_{i=1}^N K_{nn}(i) \delta_{k,k_i}$$

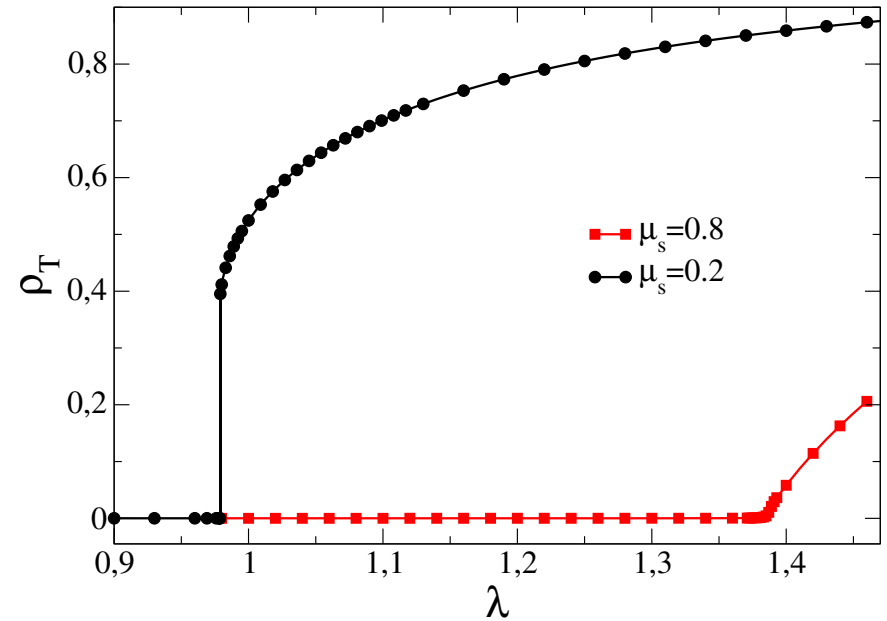
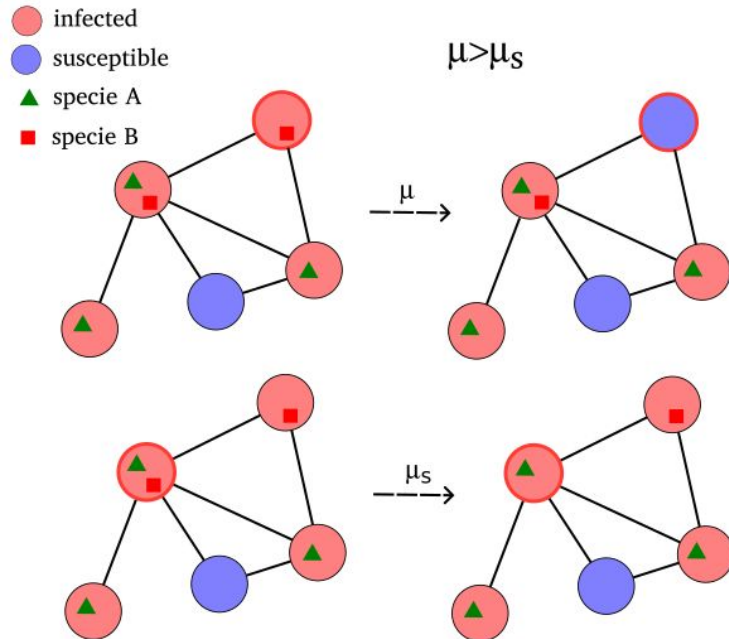
# Criticality through degree correlations of the VG – CP

Distinguishing between critical and off-critical regimes.



# Criticality through degree correlations of the VG – 2SCP

The phase transition in the 2SCP depends on the symbiotic healing rate  $\mu_s$ .

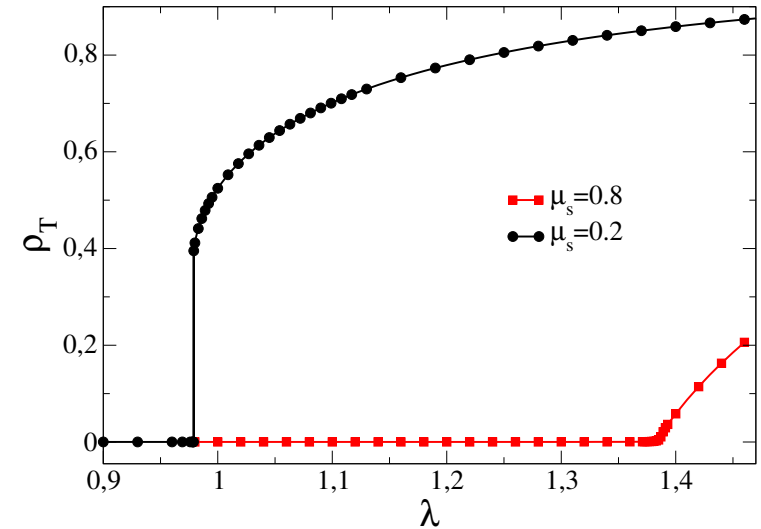
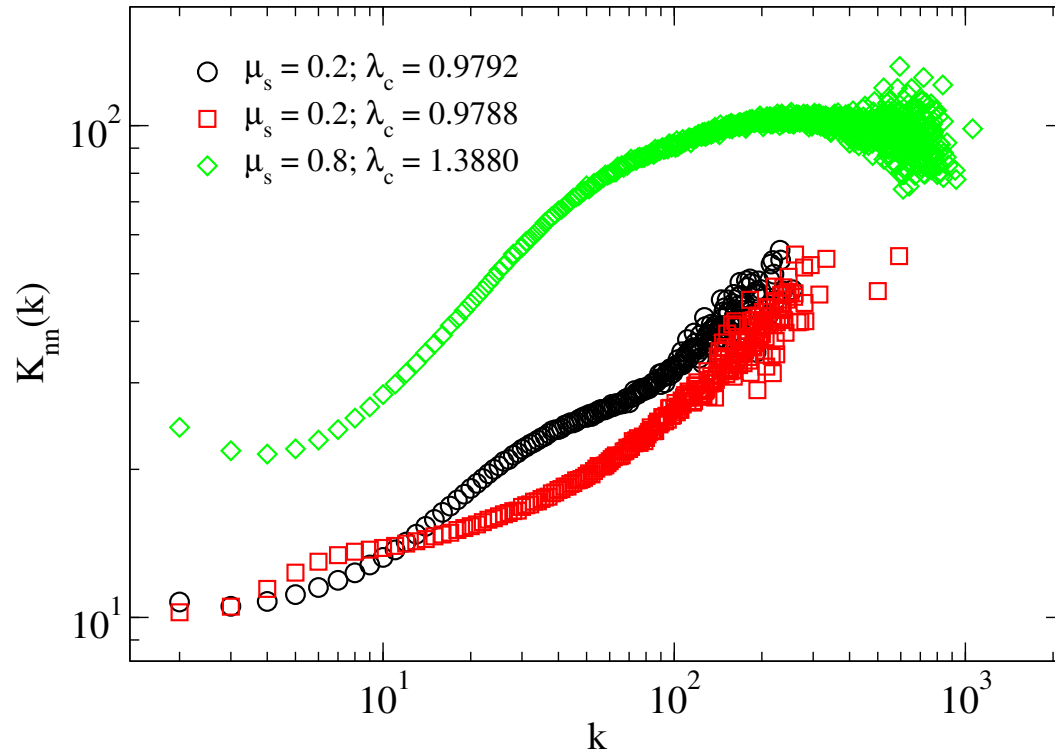


$$\rho_T = \rho_A + \rho_B + 2\rho_{AB}$$



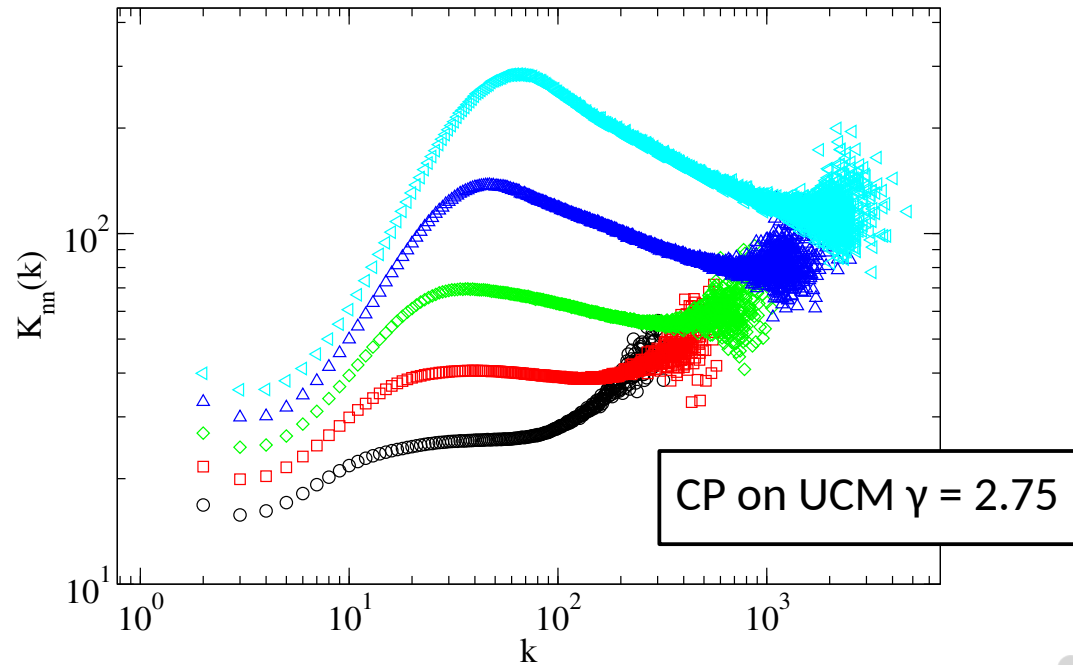
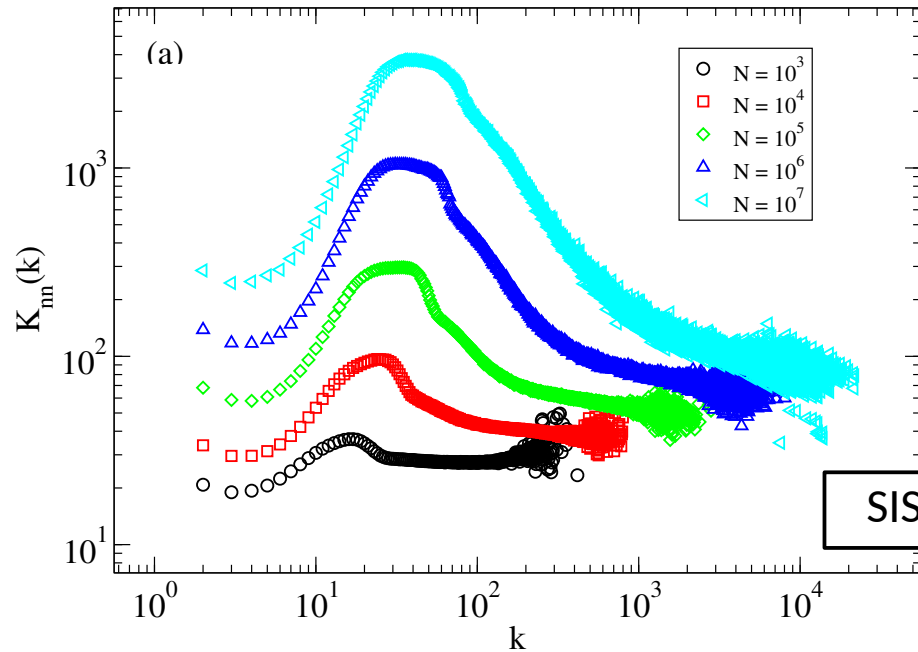
# Criticality through degree correlations of the VG – 2SCP

Degree correlations of VG distinguish between continuous ( $\mu_s = 0.8$ ) and discontinuous ( $\mu_s = 0.2$ ) transitions of 2SCP on RRNs.



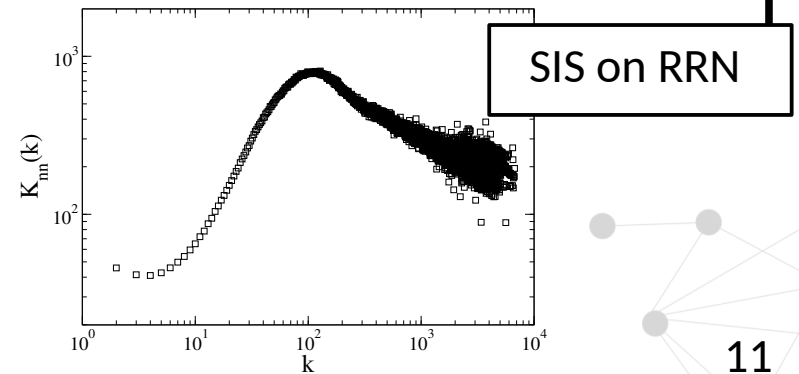
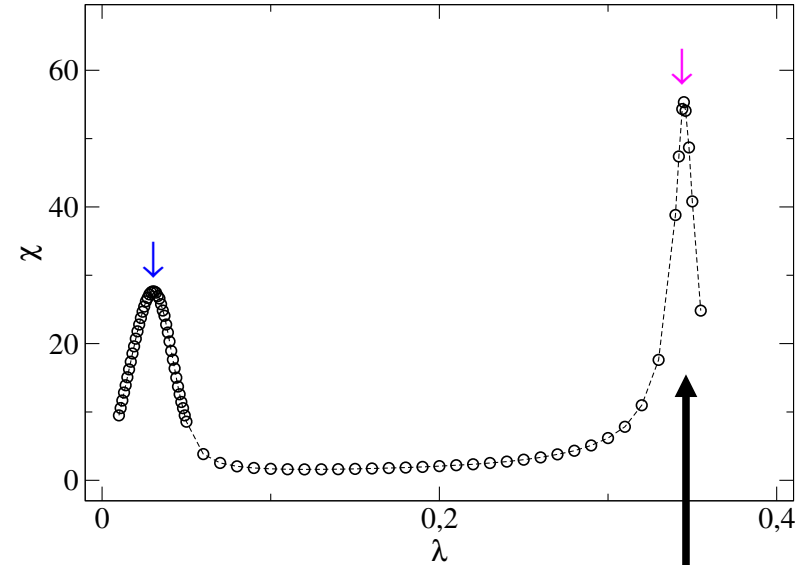
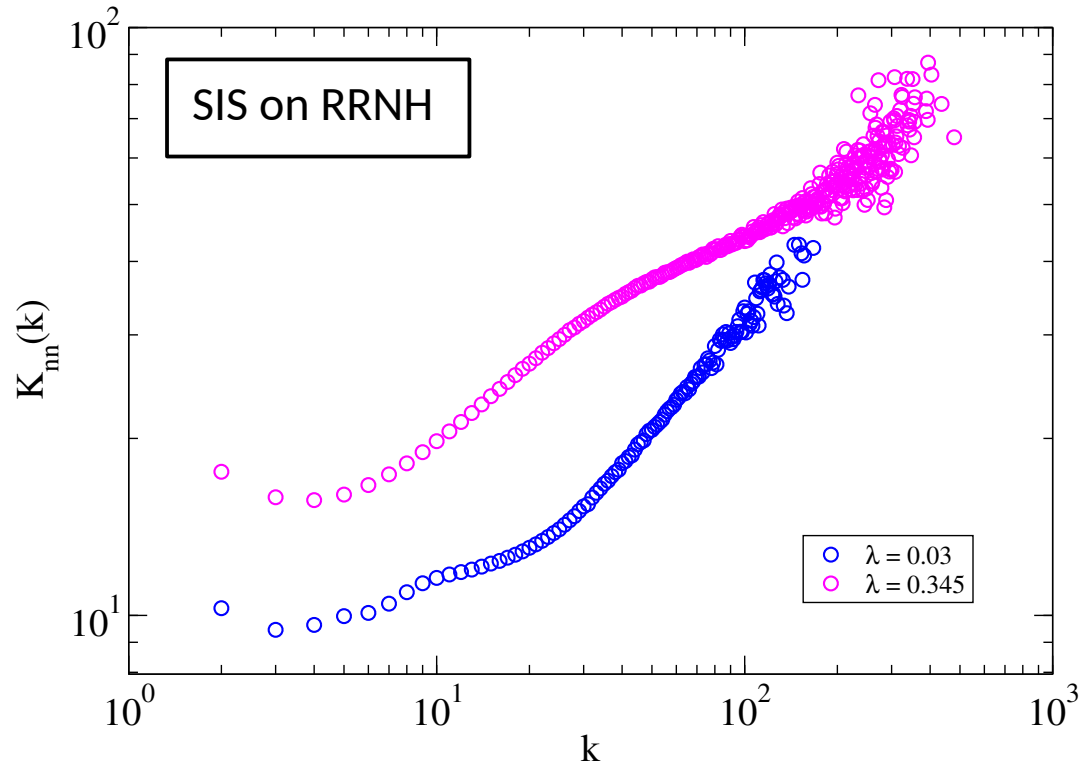
# Localization through degree correlations of the VG

Delocalized processes have VG with disassortative degree correlations.



# Localization through degree correlations of the VG

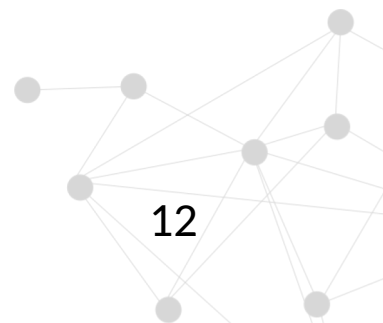
The hub in the RRN generates a rare region, captured by the degree correlations of the VG.





## Concluding remarks

- The degree correlations of the VG are an effective hallmark to distinguish between critical and off-critical dynamics;
- Only critical dynamics feature large degree disassortative correlations in VG;
- Localization in the original network destroys the criticality of the system. This is signaled by assortative degree correlations of the VG.






Visibility graphs of critical and off-critical time series for absorbing state phase transitions

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Thanks!

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# Localization through degree correlations of the VG

Smearred transitions caused by rare regions generate VG with assortative degree correlations.

