Problem Sets

1. Canonical Formalism

1. Find the energy momentum tensor of Maxwell Electrodynamics

$$T_{\mu\nu} = -\frac{2}{\sqrt{g}}\frac{\delta S_M}{\delta g_{\mu\nu}},$$

where

$$S_M = \int d^4x\,\sqrt{g}\,g^{\mu\nu}g^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta}.$$

Derive the expressions of the Hamiltonian and the linear momentum and show that they do commute in canonical quantization.

2. Find the energy momentum tensor of General Relativity without cosmological constant. Derive the expressions of the Hamiltonian and the linear momentum upon quantization on the ADM formalism and show that they do commute.

2. Covariant Formalism

1. Derive the expression of Feynmann propagator in General Relativity in a Minkowski background by linearizing the Einstein action in the harmonic gauge

$$\partial^\mu D_{\mu\nu;\alpha\beta} = 0, \qquad \eta^{\mu\nu}D_{\mu\nu;\alpha\beta} = 0.$$

2. Derive the equation of motion of Stelle's quadratic gravity theory

$$S(g) = -\tfrac{M_P^2}{2}\int\sqrt{g}\,(R-2\Lambda) + \alpha\int\sqrt{g}\,R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} + \beta\int\sqrt{g}\,R_{\alpha\beta}R^{\alpha\beta} + \gamma\int\sqrt{g}\,R^2$$

3. Show that any vacuum solution of Einstein equations is a vacuum solution of the equation of Stelle's quadratic gravity
4. Derive the expression of Feynman propagator in Stelle's quadratic gravity in a Minkowski background by linearizing the Stelle action in the harmonic gauge.

Bibliography

* Claus Kiefer, Quantum Gravity, Oxford Univ. Pub., Oxford (2007)
* Iosif L Buchbinder, Sergei Odintsov, Ilya Shapiro Effective Action in Quantum Gravity, CRC Press (1992)
* Roberto Percacci, An Introduction to Covariant Quantum Gravity and Asymptotic Safety World Sci, Sigapore (2017)
* Steven Weinberg, Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity , Wiley&Sons, Inc. (1972)
* Kip S. Thorne, Charles W. Misner, and John A. Wheeler, Gravitation. Freeman San Francisco, CA (1973).
* Iosif L Buchbinder, Ilya Shapiro, Introduction to Quantum Field Theory with Applications to Quantum Gravity, Oxford University Press (2021)
* Alberto Salvio, Quadratic Gravity, Front. in Phys. 6 (2018) 77
* Chris Isham, Conceptual and geometrical problems in quantum gravity. In: Mitter, H., Gausterer, H. (eds) Recent Aspects of Quantum Fields. Lecture Notes in Physics, vol 396. Springer, Berlin, Heidelberg (1991)