Universidad de Granada

High Energy Theory

# On-Shell matching in effective field theories

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with M. Chala, J. López-Miras and J. Santiago [24xx.xxxxx]

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# Why do we need effective field theories?

EFT's are perturbative (Taylor) expansions of a full theory

**In QFT's :**

Operators of mass dimension $d > 4$ → $\mathcal{O}_i^{(d)}$

Useful for finite precision computations

Search for new physics

# Green’s basis and redundant operators

**EFT Lagrangian :** $$\mathcal{L} = \mathcal{L}^{(4)} + \sum_{d>4} \sum_{i} \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}$$

# Green's basis and redundant operators

**EFT Lagrangian :** $$\mathcal{L} = \mathcal{L}^{(4)} + \sum_{d>4} \sum_{i} \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}$$

**Valid operators**

- Local operators
- Preserve the symmetries of the Lagrangian

Finite number of operators → Integration by parts → **Green's basis**

$$\partial_\mu(\phi\partial^\mu\phi) = \partial_\mu\phi\partial^\mu\phi + \phi\partial^2\phi$$

# Green's basis and redundant operators

## Green's basis of the bosonic sector of the SMEFT

| | |
|---|---|
| **$X^3$** | |
| $\mathcal{O}_{3G}$ | $f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$ |
| $\mathcal{O}_{3\widetilde{G}}$ | $f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$ |
| $\mathcal{O}_{3W}$ | $\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$ |
| $\mathcal{O}_{3\widetilde{W}}$ | $\epsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$ |
| **$X^2 D^2$** | |
| $\mathcal{O}_{2G}$ | $-\frac{1}{2}(D_\mu G^{A\mu\nu})(D^\rho G^A_{\rho\nu})$ |
| $\mathcal{O}_{2W}$ | $-\frac{1}{2}(D_\mu W^{I\mu\nu})(D^\rho W^I_{\rho\nu})$ |
| $\mathcal{O}_{2B}$ | $-\frac{1}{2}(\partial_\mu B^{\mu\nu})(\partial^\rho B_{\rho\nu})$ |

| | |
|---|---|
| **$X^2 H^2$** | |
| $\mathcal{O}_{HG}$ | $G^A_{\mu\nu} G^{A\mu\nu}(H^\dagger H)$ |
| $\mathcal{O}_{H\widetilde{G}}$ | $\widetilde{G}^A_{\mu\nu} G^{A\mu\nu}(H^\dagger H)$ |
| $\mathcal{O}_{HW}$ | $W^I_{\mu\nu} W^{I\mu\nu}(H^\dagger H)$ |
| $\mathcal{O}_{H\widetilde{W}}$ | $\widetilde{W}^I_{\mu\nu} W^{I\mu\nu}(H^\dagger H)$ |
| $\mathcal{O}_{HB}$ | $B_{\mu\nu} B^{\mu\nu}(H^\dagger H)$ |
| $\mathcal{O}_{H\widetilde{B}}$ | $\widetilde{B}_{\mu\nu} B^{\mu\nu}(H^\dagger H)$ |
| $\mathcal{O}_{HWB}$ | $W^I_{\mu\nu} B^{\mu\nu}(H^\dagger \sigma^I H)$ |
| $\mathcal{O}_{H\widetilde{W}B}$ | $\widetilde{W}^I_{\mu\nu} B^{\mu\nu}(H^\dagger \sigma^I H)$ |
| **$H^2 X D^2$** | |
| $\mathcal{O}_{WDH}$ | $D_\nu W^{I\mu\nu}(H^\dagger i \overleftrightarrow{D}^I_\mu H)$ |
| $\mathcal{O}_{BDH}$ | $\partial_\nu B^{\mu\nu}(H^\dagger i \overleftrightarrow{D}_\mu H)$ |

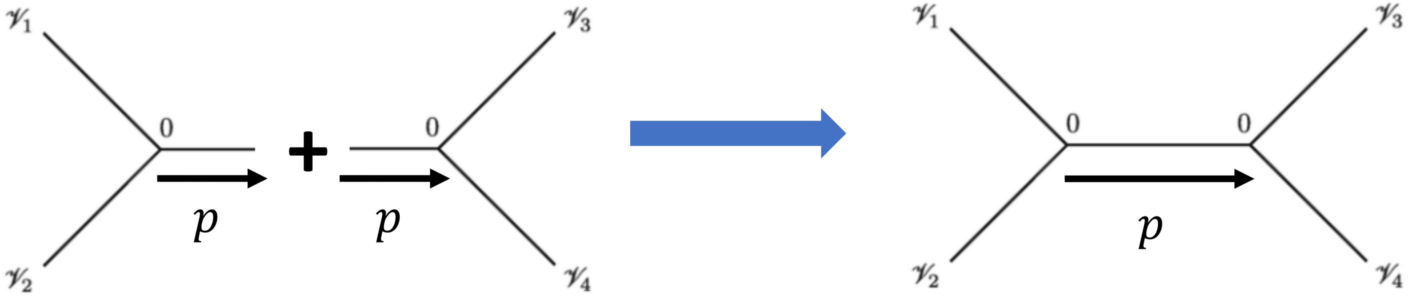
| | |
|---|---|
| **$H^2 D^4$** | |
| $\mathcal{O}_{DH}$ | $(D_\mu D^\mu H)^\dagger (D_\nu D^\nu H)$ |
| **$H^4 D^2$** | |
| $\mathcal{O}_{H\Box}$ | $(H^\dagger H)\Box(H^\dagger H)$ |
| $\mathcal{O}_{HD}$ | $(H^\dagger D^\mu H)^\dagger (H^\dagger D_\mu H)$ |
| $\mathcal{O}'_{HD}$ | $(H^\dagger H)(D_\mu H)^\dagger (D^\mu H)$ |
| $\mathcal{O}''_{HD}$ | $(H^\dagger H) D_\mu (H^\dagger i \overleftrightarrow{D}^\mu H)$ |
| **$H^6$** | |
| $\mathcal{O}_H$ | $(H^\dagger H)^3$ |

V. Gherardi, D. Marzocca y E. Venturini (2021) [2003.12525v5]

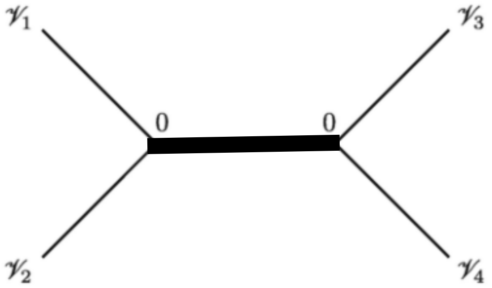
# Matching: Off-Shell vs On-shell

## Off-Shell matching

- Small number of diagrams (1 lPI)




- Heavy bridges contribution directly local




$p^2 \ll M^2$

$$\sim \frac{1}{p^2 - M^2} = -\frac{1}{M^2}\left(1 + \frac{p^2}{M^2} + \frac{p^4}{M^4} + \cdots\right)$$

- But requires the construction and reduction of the Green's basis

# Reduction to the physical basis

**Identification of redundant operators**

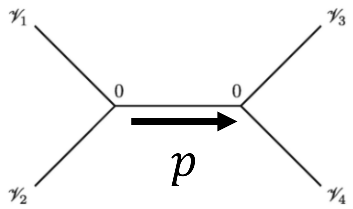
- Field redefinitions $\phi \to f(\phi)$
- EOMs (only valid up to linear order)

Non-trivial process

Hard to program it in a systematic way

**On-Shell matching**

- Huge number of diagrams



- There is delicate cancellation of non-local contributions between between UV and EFT

$$\left.\frac{1}{p^2 - m^2}\right|_{\text{UV}} - \left.\frac{1}{p^2 - m^2}\right|_{\text{EFT}} = Polynomial(p^2)$$

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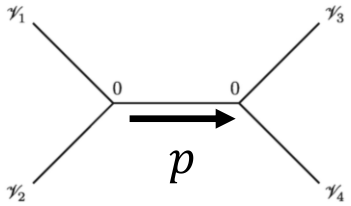


- There is delicate cancellation of non-local contributions between between UV and EFT

**Substitution of randomly generated physical momenta**

**M. Accettulli [2304.01589]**

$$\left.\frac{1}{p^2-m^2}\right|_{\text{UV}} - \left.\frac{1}{p^2-m^2}\right|_{\text{EFT}} = Polynomial(p^2)$$

# On-Shell matching approach

- Find the Green's basis up to dimension $d$ → $\mathcal{L}_{Green}$
- Find the physical basis → $\mathcal{L}_{phys}$

  **R. Fonseca [1907.12584]**
  **J.C. Criado [1901.03501]**

- Compute n-points amplitudes with n $\leq d$ **on-shell** → **By the substitution of randomly generated physical momenta**
- Solve the system $\mathcal{M}_{i,Green} = \mathcal{M}_{i,phys}$

# Some results in the SMEFT

**HIGG'S SECTOR**

$$\mathcal{L}_{Green} = \mathcal{L}^{(4)} + \mathcal{L}^{(6)}_{Green} \qquad \mathcal{L}_{phys} = \mathcal{L}^{(4)} + \mathcal{L}^{(6)}_{phys} + \mathcal{L}^{(8)}_{phys}$$

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$$\mathcal{L}_{Green} = \mathcal{L}^{(4)} + \mathcal{L}^{(6)}_{Green}$$

$$\mathcal{L}_{phys} = \mathcal{L}^{(4)} + \mathcal{L}^{(6)}_{phys} + \mathcal{L}^{(8)}_{phys}$$

$$\mathcal{L}^{(4)} = \frac{1}{2}\left(D_\mu H\right)^\dagger \left(D^\mu H\right) - m_0^2 H^\dagger H - \lambda \left(H^\dagger H\right)^2$$

$$\mathcal{L}^{(6)}_{Green} = c_H \left(H^\dagger H\right)^3 + c_{H\Box}\left(H^\dagger H\right)\Box\left(H^\dagger H\right) + c_{HD}\left(H^\dagger D^\mu H\right)^\dagger \left(H^\dagger D_\mu H\right) + \\ r'_{HD}\left(H^\dagger H\right)\left(D_\mu H\right)^\dagger\left(D^\mu H\right) + r''_{HD}\left(H^\dagger H\right) D_\mu \left(H^\dagger i \overleftrightarrow{D}^\mu H\right) + r_{DH}\left(D^2 H\right)^\dagger \left(D^2 H\right)$$

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$$\mathcal{L}^{(6)}_{phys}$$

$$\mathcal{L}^{(6)}_{Green} = c_H \left(H^\dagger H\right)^3 + c_{H\Box}\left(H^\dagger H\right)\Box\left(H^\dagger H\right) + c_{HD}\left(H^\dagger D^\mu H\right)^\dagger \left(H^\dagger D_\mu H\right) +$$
$$r'_{HD}\left(H^\dagger H\right)\left(D_\mu H\right)^\dagger\left(D^\mu H\right) + r''_{HD}\left(H^\dagger H\right) D_\mu \left(H^\dagger i \overleftrightarrow{D}^\mu H\right) + r_{DH}\left(D^2 H\right)^\dagger \left(D^2 H\right)$$

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$$\mathcal{L}^{(6)}_{Green} = c_H \left(H^\dagger H\right)^3 + c_{H\Box} \left(H^\dagger H\right) \Box \left(H^\dagger H\right) + c_{HD} \left(H^\dagger D^\mu H\right)^\dagger \left(H^\dagger D_\mu H\right) + r'_{HD} \left(H^\dagger H\right) \left(D_\mu H\right)^\dagger \left(D^\mu H\right) + r''_{HD} \left(H^\dagger H\right) D_\mu \left(H^\dagger i \overleftrightarrow{D}^\mu H\right) + r_{DH} \left(D^2 H\right)^\dagger \left(D^2 H\right)$$

$$\mathcal{L}^{(8)}_{phys} = c_{H^8} \left(H^\dagger H\right)^4 + c^{(1)}_{H^6 D^2} \left(H^\dagger H\right)^2 \left(D_\mu H^\dagger D^\mu H\right) + c^{(2)}_{H^6 D^2} \left(H^\dagger H\right) \left(H^\dagger \sigma^I H\right) \left(D_\mu H^\dagger \sigma^I D^\mu H\right) + c^{(1)}_{H^4 D^4} \left(D_\mu H^\dagger D_\nu H\right) \left(D^\nu H^\dagger D^\mu H\right) + c^{(2)}_{H^4 D^4} \left(D_\mu H^\dagger D_\nu H\right) \left(D^\mu H^\dagger D^\nu H\right) + c^{(3)}_{H^4 D^4} \left(D^\mu H^\dagger D_\mu H\right) \left(D^\nu H^\dagger D_\nu H\right)$$

```
rules12 = Rules[4, 0, 10] /. {rules`k → k, rules`Pair → Pair} // Simplify;
equations = {};
For[j = 1, j ≤ Length[amp1], j ++,
|para cada     |longitud

    For[i = 1, i ≤ Length[rules12], i++,
    |para cada      |longitud

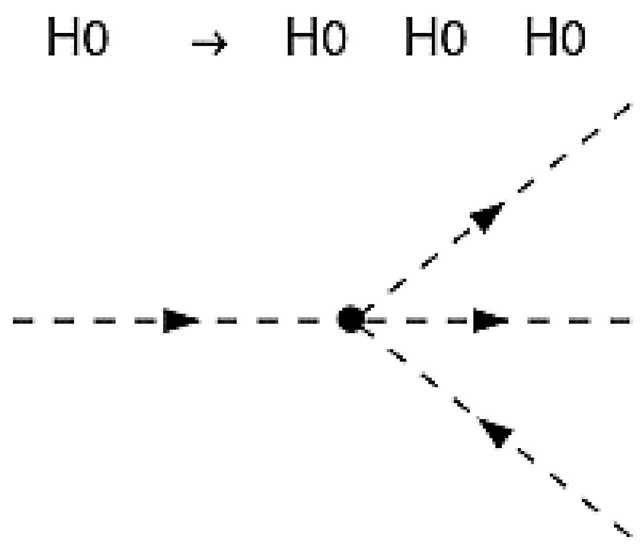
     final = amp1[[j]] /. Flatten[rules12[[i]]] // TermCollect;
                          |aplana

     final = I Sum[final[[aa]], {aa, 1, Length[final]}] // Expand;
              |··|suma                 |longitud           |expande factores

     final = final /. Sust;
     final = final /. {x^3 → 0, x^4 → 0,  x^5 → 0, x^6 → 0} /. {x → 1};
     ampIR = final /. propEFT /. limitIR;
     ampUV = Z^2 final /. propEFT /. limitUV;
     ampsUV[[i]] = ampsUV[[i]] + ampUV;
     ampsIR[[i]] = ampsIR[[i]] + ampIR;

  ];
  AppendTo[equations, Table[ampsUV[[i]] == ampsIR[[i]], {i, 1, Length[rules12]}]];
  |añade al final       |tabla                              |longitud
 ];

solution1 = Solve[Flatten[equations], coefsol] /. massPhys // Simplify;
            |resuelve |aplana                               |simplifica
```



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      final = final /. {x^3 → 0, x^4 → 0,  x^5 → 0, x^6 → 0} /. {x → 1};
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      ampsUV[[i]] = ampsUV[[i]] + ampUV;
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 ];

solution1 = Solve[Flatten[equations], coefsol] /. massPhys // Simplify;
```

→ Generate momenta by randomly generated values

para cada, longitud, aplana, suma, longitud, expande factores, añade al final, tabla, resuelve, simplifica

```
rules12 = Rules[4, 0, 10] /. {rules`k → k, rules`Pair → Pair} // Simplify;
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```

→ Generate momenta by randomly generated values

→ Running through every amplitude in the process

```
rules12 = Rules[4, 0, 10] /. {rules`k → k, rules`Pair → Pair} // Simplify;
equations = {};
For[j = 1, j ≤ Length[amp1], j ++,
|para cada    |longitud

    For[i = 1, i ≤ Length[rules12], i ++,
    |para cada    |longitud

     final = amp1[[j]] /. Flatten[rules12[[i]]] // TermCollect;
                          |aplana

     final = I Sum[final[[aa]], {aa, 1, Length[final]}] // Expand;
               |··|suma              |longitud           |expande factores

     final = final /. Sust;
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  ];
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  |añade al final      |tabla                          |longitud
 ];

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            |resuelve |aplana                            |simplifica
```

→ Generate momenta by randomly generated values

→ Running through every amplitude in the process

→ Replace the randomly generated kinematics

```
rules12 = Rules[4, 0, 10] /. {rules`k → k, rules`Pair → Pair} // Simplify;
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  añade al final      tabla                                  longitud
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solution1 = Solve[Flatten[equations], coefsol] /. massPhys // Simplify;
            resuelve aplana                                   simplifica
```

→ Generate momenta by randomly generated values

→ Running through every amplitude in the process

→ Replace the randomly generated kinematics

→ Setting both theories amplitudes with their propagators and wavefunction renormalizations

```
rules12 = Rules[4, 0, 10] /. {rules`k → k, rules`Pair → Pair} // Simplify;
equations = {};
For[j = 1, j ≤ Length[amp1], j ++,
 (para cada)   (longitud)
        For[i = 1, i ≤ Length[rules12], i++,
         (para cada)   (longitud)
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                              (aplana)
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                  (suma)              (longitud)          (expande factores)
         final = final /. Sust;
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  (añade al final)     (tabla)                                (longitud)
 ];

solution1 = Solve[Flatten[equations], coefsol] /. massPhys // Simplify;
            (resuelve)(aplana)                                (simplifica)
```

→ Generate momenta by randomly generated values

→ Running through every amplitude in the process

→ Replace the randomly generated kinematics

→ Setting both theories amplitudes with their propagators and wavefunction renormalizations

→ Matching both theories

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Generate momenta by randomly generated values

Running through every amplitude in the process

Replace the randomly generated kinematics

Setting both theories amplitudes with their propagators and wavefunction renormalizations

Matching both theories

Solving the system

# Some results in the SMEFT

## Final solution: redefinition of coefficients

$$\text{cH} \to \text{aH} - \text{lmbd}^2\,\text{rDH} + \text{lmbd}\,\text{rHDp} + \text{m0}^2\left(6\,\text{aH}\,\text{rDH} + \text{aHD}\,\text{lmbd}\,\text{rDH} - 8\,\text{aHDD}\,\text{lmbd}\,\text{rDH} - 11\,\text{lmbd}^2\,\text{rDH}^2 - \frac{\text{aHD}\,\text{rHDp}}{2} + 4\,\text{aHDD}\,\text{rHDp} + 9\,\text{lmbd}\,\text{rDH}\,\text{rHDp} - \frac{\text{rHDp}^2}{4} - \text{rHDpp}^2\right)$$

$$\text{cH41} \to 0$$

$$\text{cH42} \to 0$$

$$\text{cH43} \to 0$$

$$\text{cH61} \to \text{aHD}\,\text{lmbd}\,\text{rDH} + 8\,\text{aHDD}\,\text{lmbd}\,\text{rDH} + \text{lmbd}^2\,\text{rDH}^2 - \frac{\text{aHD}\,\text{rHDp}}{2} - 4\,\text{aHDD}\,\text{rHDp} + 3\,\text{lmbd}\,\text{rDH}\,\text{rHDp} - \frac{7\,\text{rHDp}^2}{4} + \text{rHDpp}^2$$

$$\text{cH62} \to 2\,\text{aHD}\,\text{lmbd}\,\text{rDH} - \text{aHD}\,\text{rHDp}$$

$$\text{cHD} \to \text{aHD} + 4\,\text{aHD}\,\text{m0}^2\,\text{rDH}$$

$$\text{cHDD} \to \frac{1}{2}\left(2\,\text{aHDD} + \text{rHDp}\right) + \text{m0}^2\left(4\,\text{aHDD}\,\text{rDH} + 2\,\text{rDH}\,\text{rHDp}\right)$$

$$\text{mH}^2 \to \text{m0}^2 + \text{m0}^4\,\text{rDH} + 2\,\text{m0}^6\,\text{rDH}^2$$

$$\lambda \to \text{lmbd} + \text{m0}^2\left(4\,\text{lmbd}\,\text{rDH} - 2\,\text{rHDp}\right) + \text{m0}^4\left(16\,\text{lmbd}\,\text{rDH}^2 - 10\,\text{rDH}\,\text{rHDp}\right)$$

J. Aebischer, M. Fael and J. Fuentes-Martín|2023
[2307.08745v1]

V. Gherardi, D. Marzocca and E. Venturini |2021
[2003.12525v5]

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## Final solution: redefinition of coefficients

$$\text{cH} \to \text{aH} - \text{lmbd}^2\,\text{rDH} + \text{lmbd}\,\text{rHDp} + \text{m0}^2\left(6\,\text{aH}\,\text{rDH} + \text{aHD}\,\text{lmbd}\,\text{rDH} - 8\,\text{aHDD}\,\text{lmbd}\,\text{rDH} - 11\,\text{lmbd}^2\,\text{rDH}^2 - \frac{\text{aHD}\,\text{rHDp}}{2} + 4\,\text{aHDD}\,\text{rHDp} + 9\,\text{lmbd}\,\text{rDH}\,\text{rHDp} - \frac{\text{rHDp}^2}{4} - \text{rHDpp}^2\right)$$

$$\text{cH41} \to 0$$

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$$\text{cH61} \to \text{aHD}\,\text{lmbd}\,\text{rDH} + 8\,\text{aHDD}\,\text{lmbd}\,\text{rDH} + \text{lmbd}^2\,\text{rDH}^2 - \frac{\text{aHD}\,\text{rHDp}}{2} - 4\,\text{aHDD}\,\text{rHDp} + 3\,\text{lmbd}\,\text{rDH}\,\text{rHDp} - \frac{7\,\text{rHDp}^2}{4} + \text{rHDpp}^2$$

$$\text{cH62} \to 2\,\text{aHD}\,\text{lmbd}\,\text{rDH} - \text{aHD}\,\text{rHDp}$$

$$\text{cHD} \to \text{aHD} + 4\,\text{aHD}\,\text{m0}^2\,\text{rDH}$$

$$\text{cHDD} \to \frac{1}{2}\left(2\,\text{aHDD} + \text{rHDp}\right) + \text{m0}^2\left(4\,\text{aHDD}\,\text{rDH} + 2\,\text{rDH}\,\text{rHDp}\right)$$

$$\text{mH}^2 \to \text{m0}^2 + \text{m0}^4\,\text{rDH} + 2\,\text{m0}^6\,\text{rDH}^2$$

$$\lambda \to \text{lmbd} + \text{m0}^2\left(4\,\text{lmbd}\,\text{rDH} - 2\,\text{rHDp}\right) + \text{m0}^4\left(16\,\text{lmbd}\,\text{rDH}^2 - 10\,\text{rDH}\,\text{rHDp}\right)$$

J. Aebischer, M. Fael and J. Fuentes-Martín|2023
[2307.08745v1]

V. Gherardi, D. Marzocca and E. Venturini |2021
[2003.12525v5]

**BOSONIC SECTOR**

| $X^3$ | | $X^2H^2$ | | $H^2D^4$ | |
|---|---|---|---|---|---|
| $\mathcal{O}_{3G}$ | $f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$ | $\mathcal{O}_{HG}$ | $G^A_{\mu\nu}G^{A\mu\nu}(H^\dagger H)$ | $\mathcal{O}_{DH}$ | $(D_\mu D^\mu H)^\dagger(D_\nu D^\nu H)$ |
| $\mathcal{O}_{3\widetilde{G}}$ | $f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$ | $\mathcal{O}_{H\widetilde{G}}$ | $\widetilde{G}^A_{\mu\nu}G^{A\mu\nu}(H^\dagger H)$ | **$H^4D^2$** | |
| $\mathcal{O}_{3W}$ | $\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$ | $\mathcal{O}_{HW}$ | $W^I_{\mu\nu}W^{I\mu\nu}(H^\dagger H)$ | $\mathcal{O}_{H\Box}$ | $(H^\dagger H)\Box(H^\dagger H)$ |
| $\mathcal{O}_{3\widetilde{W}}$ | $\epsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$ | $\mathcal{O}_{H\widetilde{W}}$ | $\widetilde{W}^I_{\mu\nu}W^{I\mu\nu}(H^\dagger H)$ | $\mathcal{O}_{HD}$ | $(H^\dagger D^\mu H)^\dagger(H^\dagger D_\mu H)$ |
| **$X^2D^2$** | | $\mathcal{O}_{HB}$ | $B_{\mu\nu}B^{\mu\nu}(H^\dagger H)$ | $\mathcal{O}'_{HD}$ | $(H^\dagger H)(D_\mu H)^\dagger(D^\mu H)$ |
| $\mathcal{O}_{2G}$ | $-\frac{1}{2}(D_\mu G^{A\mu\nu})(D^\rho G^A_{\rho\nu})$ | $\mathcal{O}_{H\widetilde{B}}$ | $\widetilde{B}_{\mu\nu}B^{\mu\nu}(H^\dagger H)$ | $\mathcal{O}''_{HD}$ | $(H^\dagger H)D_\mu(H^\dagger i\overleftrightarrow{D}^\mu H)$ |
| $\mathcal{O}_{2W}$ | $-\frac{1}{2}(D_\mu W^{I\mu\nu})(D^\rho W^I_{\rho\nu})$ | $\mathcal{O}_{HWB}$ | $W^I_{\mu\nu}B^{\mu\nu}(H^\dagger\sigma^I H)$ | **$H^6$** | |
| $\mathcal{O}_{2B}$ | $-\frac{1}{2}(\partial_\mu B^{\mu\nu})(\partial^\rho B_{\rho\nu})$ | $\mathcal{O}_{H\widetilde{W}B}$ | $\widetilde{W}^I_{\mu\nu}B^{\mu\nu}(H^\dagger\sigma^I H)$ | $\mathcal{O}_H$ | $(H^\dagger H)^3$ |
| | | **$H^2XD^2$** | | | |
| | | $\mathcal{O}_{WDH}$ | $D_\nu W^{I\mu\nu}(H^\dagger i\overleftrightarrow{D}^I_\mu H)$ | | |
| | | $\mathcal{O}_{BDH}$ | $\partial_\nu B^{\mu\nu}(H^\dagger i\overleftrightarrow{D}_\mu H)$ | | |

# Future work

**BOSONIC SECTOR**

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| $\mathcal{O}_{3W}$ | $\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$ | $\mathcal{O}_{HW}$ | $W_{\mu\nu}^I W^{I\mu\nu}(H^\dagger H)$ | $\mathcal{O}_{H\Box}$ | $(H^\dagger H)\Box(H^\dagger H)$ |
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| | | $\mathcal{O}_{BDH}$ | $\partial_\nu B^{\mu\nu}(H^\dagger i\overleftrightarrow{D}_\mu H)$ | | |

$$m_0^2 \to m_0^2$$

$$\lambda \to \lambda - 2m_0^2 r'_{HD}$$

$$c^{(1)}_{H^4D^4} \to 2r^2_{BDH}$$

$$c^{(2)}_{H^4D^4} \to -2r^2_{BDH}$$

$$c^{(3)}_{H^4D^4} \to 0$$

$$c_{H\Box} \to c_{H\Box} + \frac{1}{2}g' r_{BDH} + \frac{1}{2}r'_{HD}$$

$$c_{HD} \to c_{HD} + 2g' r_{BDH}$$

$$g' \to g'$$

$$D_\mu = \partial_\mu - ig' B_\mu$$

$$c_{HB} \to c_{HB}$$

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$$D_\mu = \partial_\mu - ig' B_\mu$$
$$g' \to g'$$
$$c_{HB} \to c_{HB}$$

**Fermions** ✓

**Evanescent operators**

# Notice that …

**Reduction of the Green's basis**

- Field redefinitions and EOMs
  - Non-systematic
  - Highly non-lineal
- On-Shell matching approach
  - Systematic
  - Algebraic system of linear equations

# Notice that ...

**Reduction of the Green's basis**

- Field redefinitions and EOMs
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  - Highly non-trivial
- On-Shell matching approach
  - Systematic
  - Algebraic system of linear equations

The reduction of **ANY** theory to **ANY** physical basis will be completely **AUTOMATIC**

Universidad de Granada

FTAE High Energy Theory

# THANKS FOR YOUR ATTENTION !

# MUITO OBRIGADA!

# Generation of random momenta

$$SL(2,\mathbb{C}) \cong SU(2)_L \times SU(2)_R \quad \left\{ \begin{array}{c} \lambda \in SU(2)_L \\ \tilde{\lambda} \in SU(2)_R \end{array} \right\} \quad \begin{array}{l} \lambda^\alpha = \varepsilon^{\alpha\beta}\lambda_\beta \\ \tilde{\lambda}_{\dot{\alpha}} = \varepsilon_{\dot{\alpha}\dot{\beta}}\tilde{\lambda}^{\dot{\beta}} \end{array}$$

**Massless momenta :** $P_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$ ➡ $P = p_\mu \sigma^\mu = \begin{pmatrix} p_0 + p_3 & p_1 - ip_2 \\ p_1 + ip_2 & p_0 - p_3 \end{pmatrix}$

**Massive momenta :** $P^\mu := q^\mu + \frac{m^2}{2q \cdot k} k^\mu$

$$\begin{aligned} q^2, k^2 &= 0 \\ q_{\alpha\dot{\alpha}} &= \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}} \\ k_{\alpha\dot{\alpha}} &= \mu_\alpha \tilde{\mu}_{\dot{\alpha}} \end{aligned}$$

# Evanescent operators

$\mathcal{R} = \alpha\,\mathcal{O}$ $\xrightarrow{d=4-2\epsilon}$ $\mathcal{R} = \alpha\mathcal{O} + \mathcal{E}$

$\mathcal{O}(\epsilon)$

Additional finite local contributions in loop amplitudes

$$IR^{(0)} + IR^{(1)} = UV^{(0)} + UV^{(1)}$$

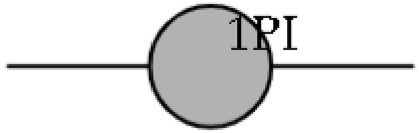
$$IR^{(0)} + IR^{(1)}_{soft} = UV^{(0)} + UV^{(1)}_{hard} + UV^{(1)}_{soft}$$

We take the hard region

$$\int \mathcal{R} - \mathcal{O} = \frac{1}{\epsilon}(b_{\mathcal{R}}\epsilon - b_{\mathcal{O}}\epsilon) = b$$

$$\int \mathcal{O} = \frac{1}{\epsilon}(a + b_{\mathcal{O}}\epsilon)$$

$$\int \mathcal{R} = \frac{1}{\epsilon}(a + b_{\mathcal{R}}\epsilon)$$




$$= \frac{i}{p^2 - m^2 - \Pi(p^2)} = \frac{iZ}{p^2 - m^2_{phys}} + \dots\,,$$

$$p^2 - m^2 - \Pi(p^2)\Big|_{p^2 - m^2_{phys}} = 0$$

$$\Pi(p^2) = \Pi(m^2_{phys}) + \Pi'(m^2_{phys})(p^2 - m^2_{phys}) + \dots$$

$$\frac{i}{p^2 - m^2 - \Pi(p^2)} = \frac{i}{p^2 - m^2 - \left(\Pi(m^2_{phys}) + \Pi'(m^2_{phys})(p^2 - m^2_{phys}) + \dots\right)}$$

$$= \frac{i}{\left(p^2 - m^2_{phys}\right)\left(1 - \Pi'(m^2_{phys}) + \dots\right)} \sim \frac{i\left(1 - \Pi'(m^2_{phys})\right)^{-1}}{\left(p^2 - m^2_{phys}\right)}$$