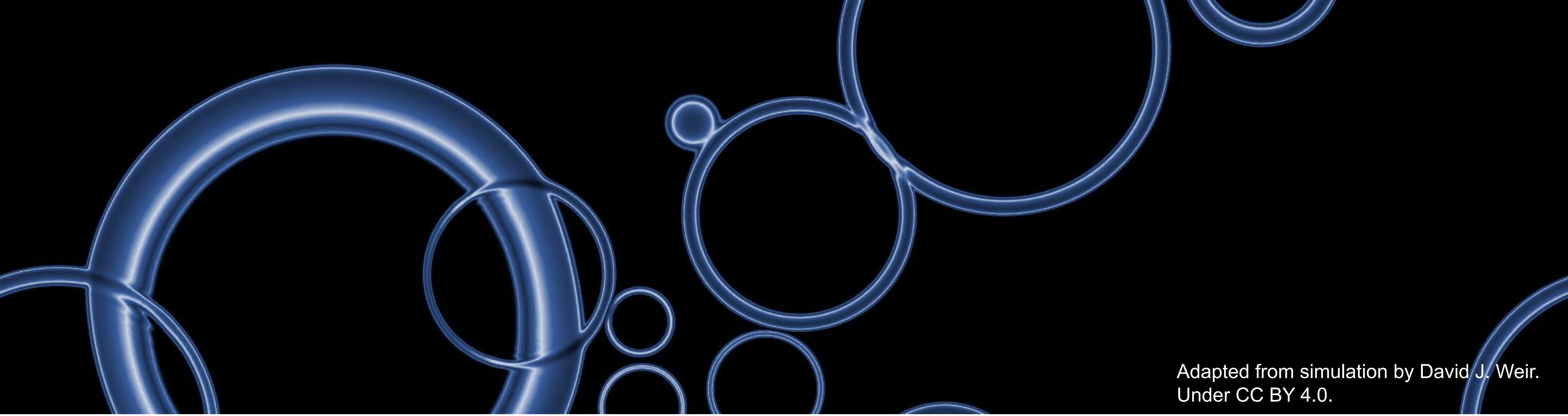


Dimensionally reduced EFTs for cosmological phase transitions

III Joint ICTP-SAIFR/ICTP-Trieste School on Particle Physics

Luis Gil [he/him]

Based (mostly) on [2406.02667], by:

M. Chala, J. C. Criado, LG and J. López Miras

FTAE High Energy Theory
UNIVERSIDAD DE GRANADA
Junta de Andalucía

Thermal field theory

Outline of the Matsubara formalism

Field correlators

- **Generating functional** (J=0) in QFT:

$$\mathcal{Z}[0] = \langle q'\ t | q\ 0 \rangle = \langle q'\ 0 | e^{-i\mathcal{H}t} | q\ 0 \rangle = \mathcal{N} \int \mathcal{D}q \exp(iS)$$

- **Partition function** in quantum statistical mechanics:

$$\mathcal{Z}_{\text{th}} = \text{Tr}\left(e^{-\beta\mathcal{H}}\right) = \sum_q \langle q\ 0 | e^{-\beta\mathcal{H}} | q\ 0 \rangle = \mathcal{N} \int_{q(0)=q(-i\beta)} \mathcal{D}q \exp(-S_E)$$

QFT at finite temperature	=	Euclidean QFT at zero temperature with periodic time

Thermal field theory

Outline of the Matsubara formalism

- In the path integral formalism, at thermal equilibrium, fields decompose in **Matsubara modes** that live in 3D Euclidean space:

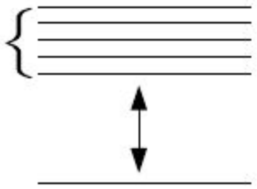
$$\phi(\tau, \mathbf{x}) \equiv T \sum_{n=-\infty}^{\infty} \phi_n(\mathbf{x}) e^{i\omega_n \tau} \qquad \psi(\tau, \mathbf{x}) \equiv T \sum_{n=-\infty}^{\infty} \psi_n(\mathbf{x}) e^{i\omega'_n \tau}$$

$$\omega_n = 2\pi n T \qquad \omega'_n = 2\pi \left(n + \frac{1}{2}\right) T$$

- Each mode acquires a **thermal mass** given by its **Matsubara frequency**.

$$M \sim \pi T^2$$

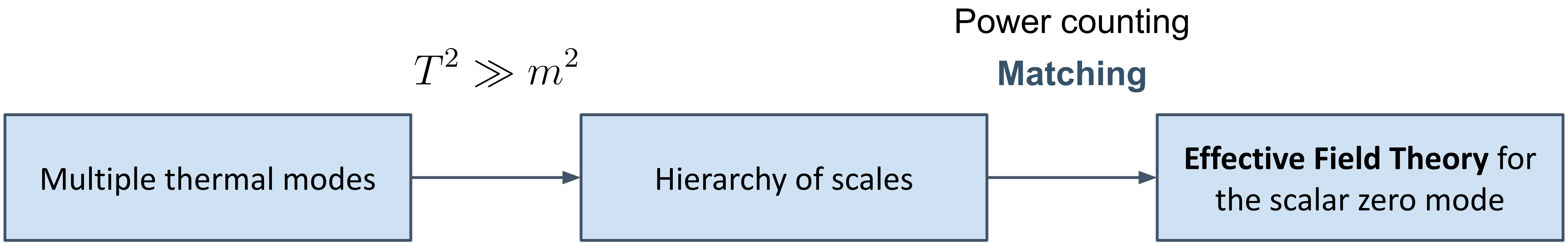
This introduces a **hierarchy of scales**.



Thermal field theory

The 3D EFT approach

See talk by Fuensanta



Thermal field theory

The 3D EFT approach

See talk by Fuensanta

$T^2 \gg m^2$

Power counting

Matching

Multiple thermal modes → Hierarchy of scales → **Effective Field Theory** for the scalar zero mode

We have a simplified theory to study cosmological phase transitions in the high temperature limit

Bubbles and gravitational waves

Thermally induced field phase transitions

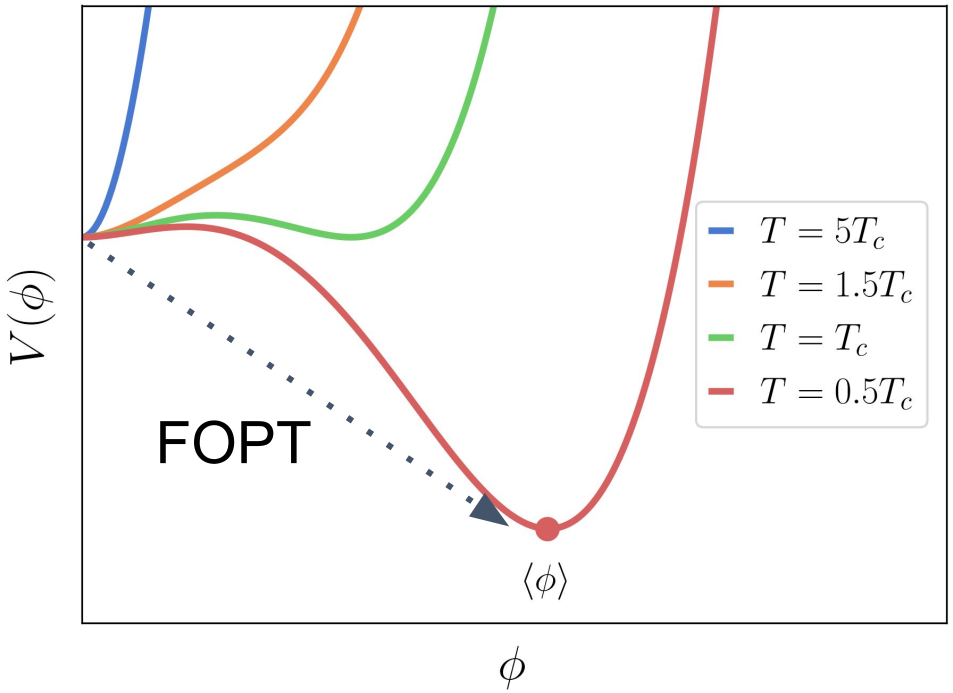


Fig. 1: Temperature evolution of scalar potential

Transition rate: $\Gamma = A(T)e^{-S_E[\varphi_b](T)}$

Depends on static, non-homogeneous solutions to the (Euclidean) EoMs:

$$\left.\frac{\delta S_E}{\delta \varphi}\right|_{\varphi=\varphi_b} = 0$$

These are the so-called **bounce solutions**.

[Coleman - PhysRevD.15.2929]

Bubbles and gravitational waves

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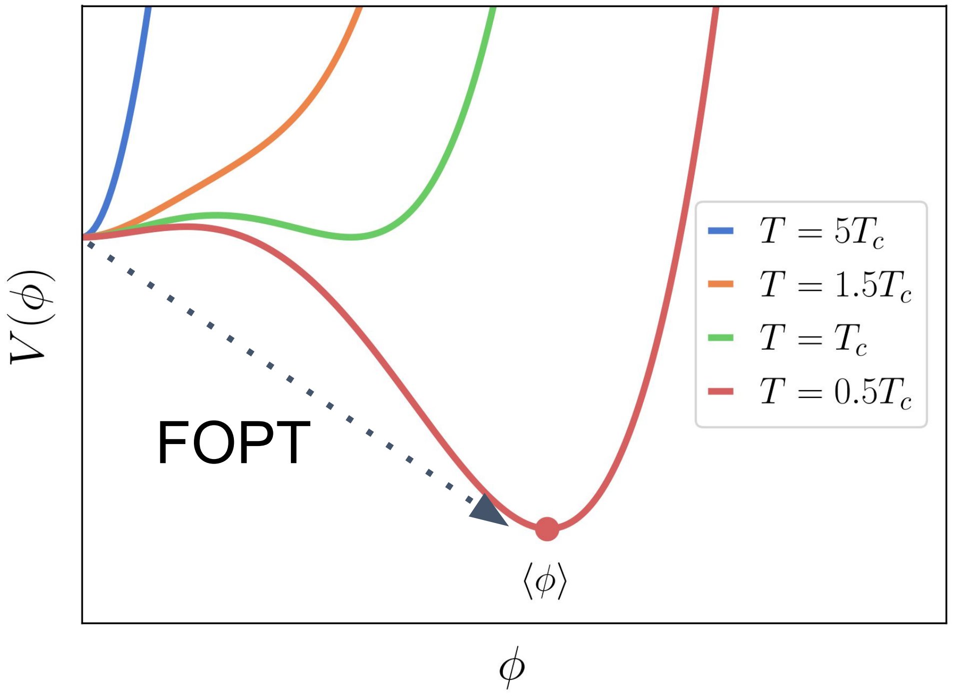

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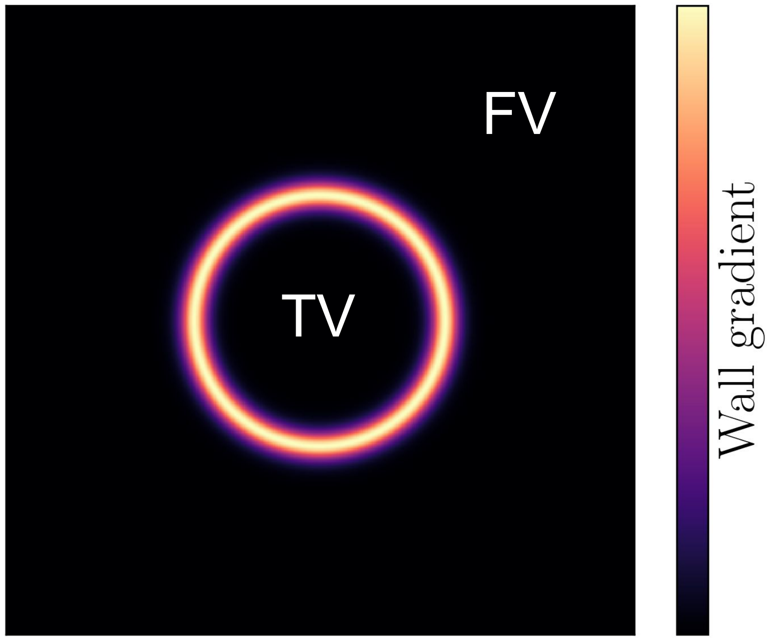

Fig. 2: O(3) symmetric bounce solution

Bubbles and gravitational waves

[Caprini *et al.* - 1512.06239]

From PTs to GWs

As the transition rate grows, in a **first-order PT (FOPT)**:

1) Bubbles of true vacuum nucleate and expand in a hot plasma
2) Bubble fronts collide
3) Sound waves
4) Turbulence
5) Single bubble source **[Blum, Mirbabayi - 2403.2016]**

Bubbles and gravitational waves

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Items 2)–4):

$$h\Omega_{\rm GW} \simeq h\Omega_\phi + h\Omega_{\rm sw} + h\Omega_{\rm turb}$$

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Probe

Answers for electroweak baryogenesis

Bubbles and gravitational waves

From PTs to GWs

How do we connect a QFT model to these GW spectra?

Bubbles and gravitational waves

From PTs to GWs

How do we connect a QFT model to these GW spectra?

Nucleation temperature

$$S_3[\varphi_c] \sim 100 - 4\log\frac{T_*}{100\,\mathrm{GeV}}$$

Strength parameter

$$\alpha \approx -0.03\frac{V_3(\varphi_T)}{T_*^3}$$

Inverse duration of PT

$$\frac{\beta}{H_*} = T_*\frac{\mathrm{d}S_3[\varphi_c]}{\mathrm{d}T}\bigg|_{T_*}$$

Terminal bubble wall velocity

A bit more difficult…

[Laurent, Cline - 2204.13120]

Bubbles and gravitational waves

What is the state of the art?

This list is NOT exhaustive!

- Several studies of FOPTs in BSM extensions and in the SMEFT:
  - N^4LO Higgs potential at finite T **[Ekstedt *et al.* - 2405.18349]**
  - EWPT in dim-6 SMEFT **[Camargo-Molina *et al.* - 2103.14022]**
  - EWPT in dim-6 XSM **[Oikonomou *et al.* - 2403.01591]**
  - EWPT in ΣSM **[Niemi *et al.* - 1802.10500]**

- Tools for PT-related computations:
  - *CosmoTransitions* **[Wainwright - 1109.4189]**
  - *FindBounce* **[Guada *et al.* - 2002.00881]**
  - *DRalgo* **[Ekstedt *et al.* - 2205.08815]**
  - *BubbleDet* **[Ekstedt *et al.* - 2308.15652]**

Effective operators with derivatives **not included**

- Theoretical uncertainties persist from the particle physics side:
  - LISA **[Caprini *et al.* - 1910.13125]**

3D EFT approach

Building our high-T EFT

UV theory in 4D Minkowski:

$$\mathscr{L}_4 = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \kappa\phi^3 - \lambda\phi^4 + \overline{\Psi} i \partial\!\!\!/ \Psi - g\phi\overline{\Psi}\Psi$$

[Gould, Xie - 2310.02308]

At temperature T, in Euclidean space:

$$S_0 = (-i)\frac{1}{T}\int \mathrm{d}^3\mathbf{x} \sum_{n=-\infty}^{\infty} \left[\frac{1}{2}(\partial_i \varphi_n)^2 + \frac{1}{2}\left(m^2 + (2\pi n T)^2\right)\varphi_n^2 + \overline{\psi}_n \partial\!\!\!/ \psi_n + \left(2\pi\left(n+\frac{1}{2}\right)T\right)^2 \overline{\psi}_n \psi_n \right]$$

Multiple thermal modes → ($T^2 \gg m^2$) → Hierarchy of scales → (Power counting, **Matching**) → **Effective Field Theory** for the scalar zero mode

3D EFT approach

Building our high-T EFT

[Kajantie *et al.* - 9508379]
[Criado - 1901.03501]

We find an off-shell basis of operators up to (4D) dim-8 with the help of *BasisGen* and match at 1-loop (fermion only loops)[1]:

$$\mathscr{L}_3 = \frac{1}{2}K_3(\partial\varphi)^2 + \frac{1}{2}m_3^2\varphi^2 + \kappa_3\varphi^3 + \lambda_3\varphi^4 \\ + \alpha_{61}\varphi^6 + \beta_{61}\partial^2\varphi\partial^2\varphi + \beta_{62}\varphi^3\partial^2\varphi \\ + \alpha_{81}\varphi^8 + \alpha_{82}\varphi^2\partial_\mu\partial_\nu\varphi\partial^\mu\partial^\nu\varphi + \beta_{81}\varphi\partial^6\varphi + \beta_{82}\varphi^3\partial^4\varphi + \beta_{83}\varphi^2\partial^2\varphi\partial^2\varphi + \beta_{84}\varphi^5\partial^2\varphi$$

[1] Scalar loop contributions to effective operators are subleading (see Appendix A in [2406.02667]).

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$$+\alpha_{61}\varphi^6 + \beta_{61}\partial^2\varphi\partial^2\varphi + \beta_{62}\varphi^3\partial^2\varphi$$

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dim-6

Measure eff. ops. relevance

dim-8

Control EFT validity

[1] Scalar loop contributions to effective operators are subleading (see Appendix A in [2406.02667]).

3D EFT approach

Computing relevant PT magnitudes (perturbatively)

[Chala, Criado, LG, López Miras - 2406.02667]

Coleman (1977) proved the existence of bounces for generic "kinetic + potential" actions. But we have higher derivative terms…

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We expand the action and bounce **perturbatively**,

dim-6 dim-8

$$\varphi_c = \varphi_c^{(0)} + \epsilon\varphi_c^{(1)} + \epsilon^2\varphi_c^{(2)} + \cdots\,, \quad S_3 = S_3^{(0)} + \epsilon S_3^{(1)} + \epsilon^2 S_3^{(2)} + \cdots$$

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obtaining:

$$S_3[\varphi_c] = S_3^{(0)}[\varphi_c^{(0)}] + \varepsilon S_3^{(1)}[\varphi_c^0] + \varepsilon^2 \left\{ S_3^{(2)}[\varphi_c^{(0)}] + 2\pi \int_0^\infty dr\, r^2 \varphi_c^{(1)} \frac{\delta S_3^{(1)}}{\delta\varphi}\bigg|_{\varphi_c^{(0)}} \right\} + \mathcal{O}(\varepsilon^3)$$

Solve w/ *CosmoTransitions*

[Wainwright - 1109.4189]

Solve another diff. eq.

3D EFT approach

Computing relevant PT magnitudes

Eff. ops. depend solely on the Yukawa (g)

↓

Fix all other UV parameters in scan m^2, κ, λ

[Chala *et al.* - 1905.00911]

&

Study all PT magnitudes as functions of the Yukawa $T_*, \alpha, \beta/H_*$

Bounce solution must be computed perturbatively!

Results

PT magnitudes and effective interactions

[Chala, Criado, LG, López Miras - 2406.02667]

$(m^2, \kappa, \lambda)_A = (20\,000\ \text{GeV}^2, -40\ \text{GeV}, 0.01)$ $(m^2, \kappa, \lambda)_B = (31\,643.5\ \text{GeV}^2, -71.1\ \text{GeV}, 0.045)$

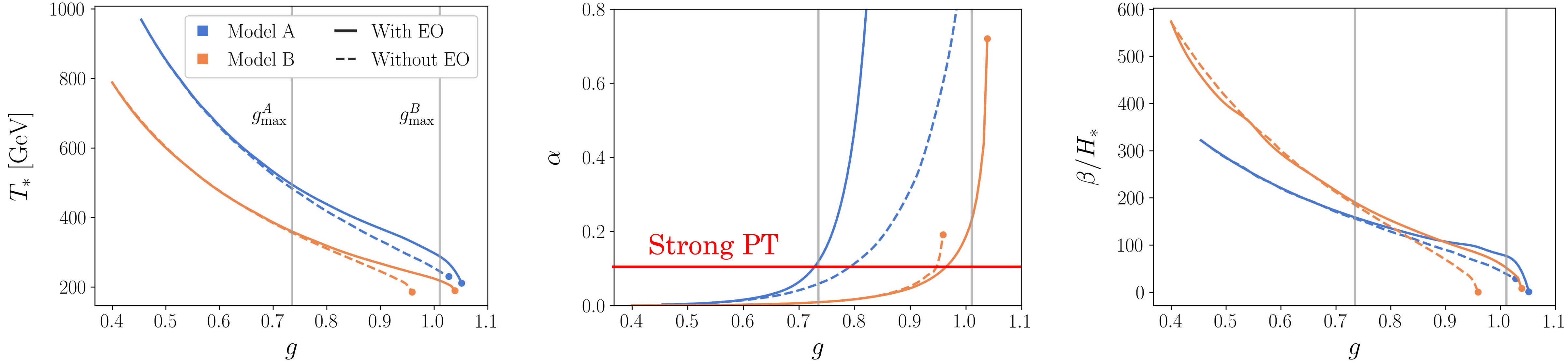

Fig. 5: PT magnitudes in two models, with and w/o effective operators

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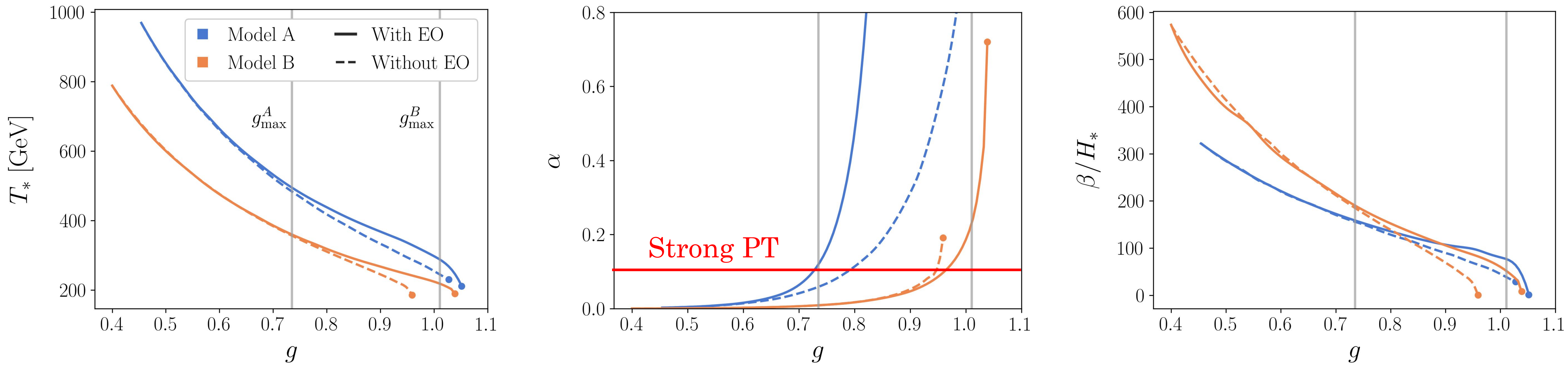

Fig. 5: PT magnitudes in two models, with and w/o effective operators

Two observations:

- Eff. ops. **allow for PTs in a wider range** of values of the Yukawa
- Including eff. ops. yields **very different estimations** at large Yukawas

Results

GW power spectra and effective interactions

[Chala, Criado, LG, López Miras - 2406.02667]

$$(m^2, \kappa, \lambda)_A = (20\,000\ \mathrm{GeV}^2, -40\ \mathrm{GeV}, 0.01) \qquad (m^2, \kappa, \lambda)_B = (31\,643.5\ \mathrm{GeV}^2, -71.1\ \mathrm{GeV}, 0.045)$$

$$g \lesssim g_{\max}$$

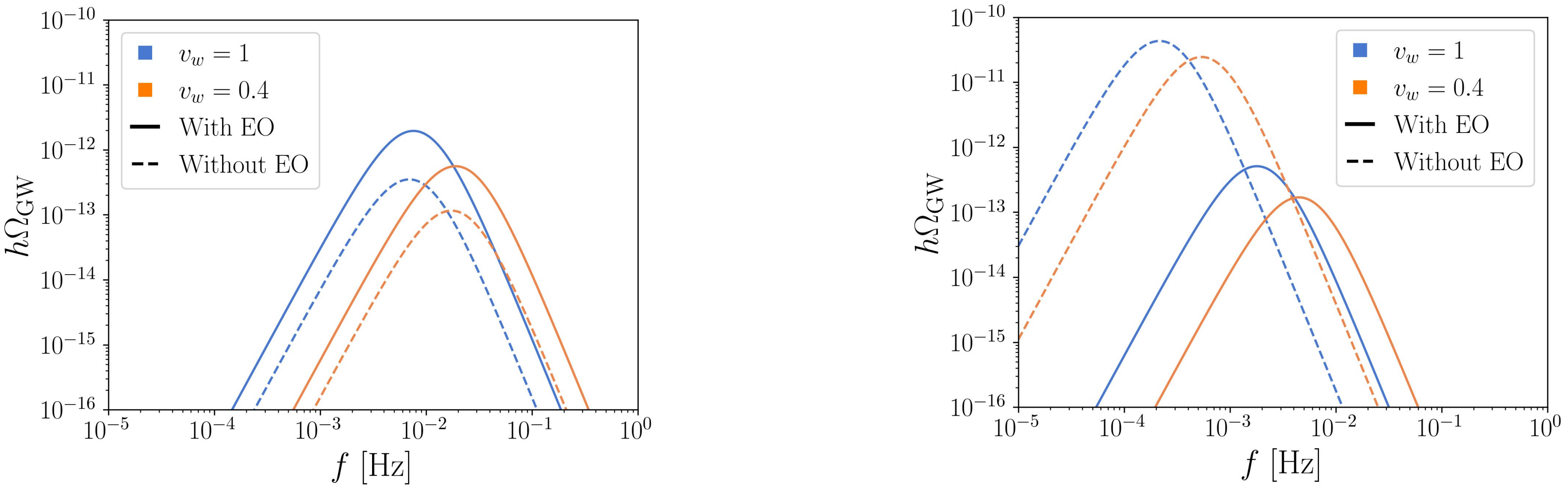

Fig. 6: GW power spectra in two models, with and w/o effective operators

(*) Generated with **PTPlot**

[Caprini *et al.* - 1910.13125]

Results

GW power spectra and effective interactions

[Chala, Criado, LG, López Miras - 2406.02667]

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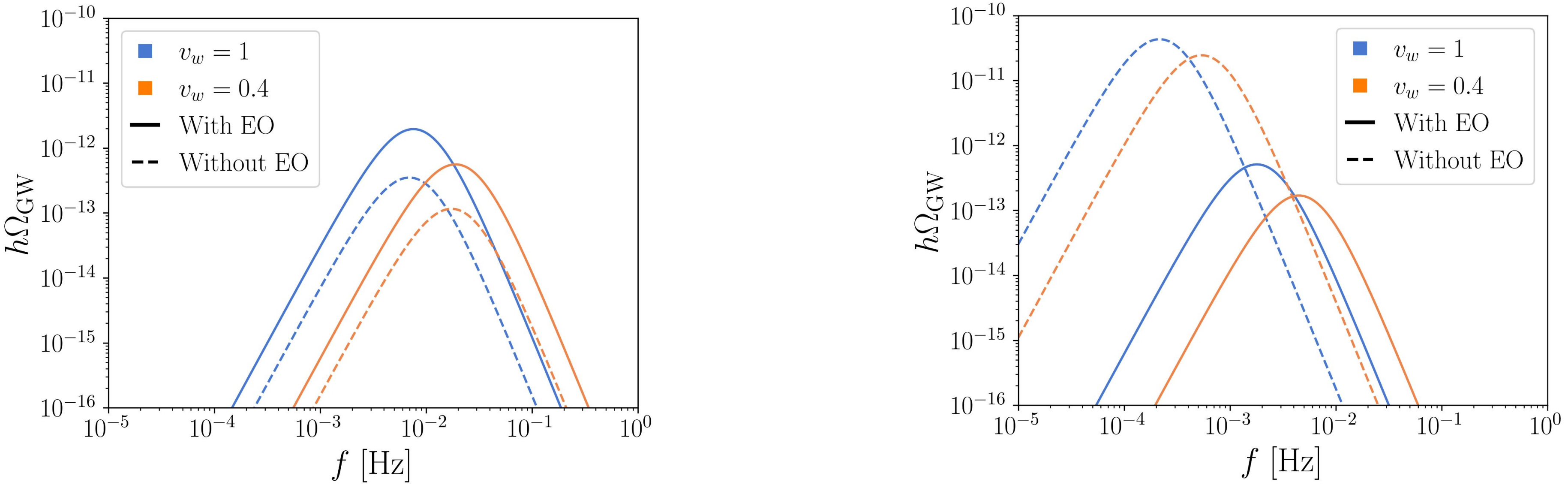

Fig. 6: GW power spectra in two models, with and w/o effective operators

Including eff. ops. can change the peak amplitude and frequency by **one order of magnitude**

Take home messages

We have learned

- Strong FOPTs occur in regions of parameter space **close the limit of validity** of the 3D EFT
- Higher dimensional effective operators **are thus relevant in the estimation of PT-related magnitudes** (order of magnitude differences in GW spectra)
- **Bounce solutions** can be obtained perturbatively in the presence of higher derivative terms

Future work

- Application to well-motivated BSM models
- Going beyond the semiclassical approximation to cosmological phase transitions

Thank you for your attention!

¡Gracias por vuestra atención!