Lesson 2 - "theoretical description of collider events"
getting to the core of collider physics.

Collider events are very complex. (!)

Experimentally: the LHC is producing about 1 billion events/sec ($pp \to X \simeq 30$ mbarn)

$\simeq \underbrace{30 \times 10^{-3}}_{\sigma} \times \underbrace{10^{34}}_{\mathcal{L}} \times \underbrace{10^{-24}}_{\substack{\text{conv}\\ \text{barn} \to \text{cm}^2}}$

$3 \times 10^8 \simeq 10^9$

events from LHC collisions take MB of storage space
+
CPU's have GHz rates
=
not possible to write all the events/sec produced @ LHC
(not to talk about HL-LHC!)

TRIGGERS! (various levels of)

Need to reduce to about 100 events/sec
huge factor!

Ex:
- 1st level → look for hard $e^\pm, \mu^\pm, \gamma$
- higher level → software processing (jet finding algorithms, etc. displaced vertices)

if events do not pass the various trigger levels, they are dropped = GONE!

↳ crucial to have a feeling for what goes into triggering

& also for

what particles look like in a detector.

↳ brief excursus.

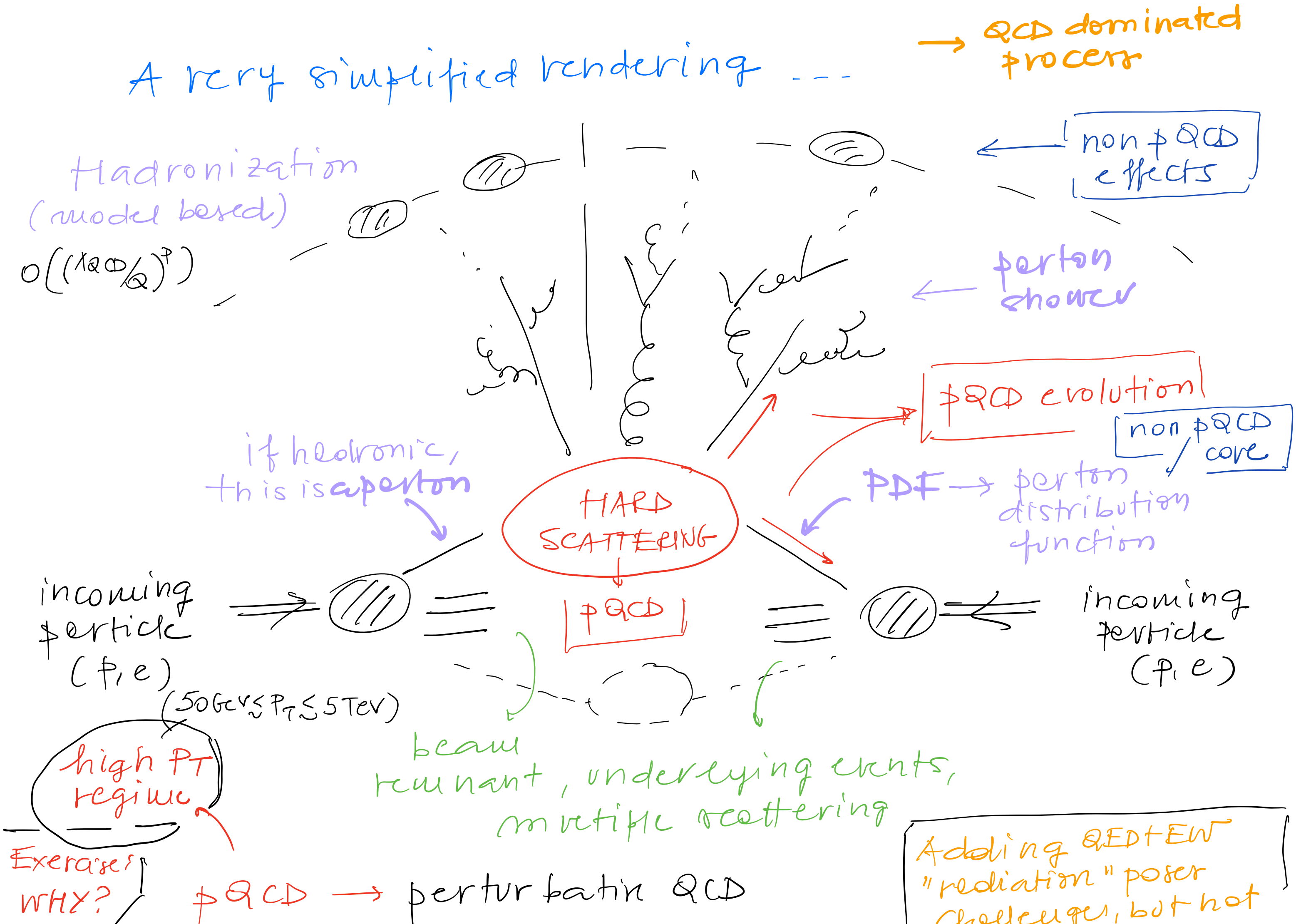
"Can we explain the shape of collider events?"

↓ "knowing the rules" ($\equiv \mathcal{L}_{SM}$), we can translate them into systematic algorithms to simulate events.

Events that are eventually recorded are relatively "clean", still very complicated for a theorist to reproduce from first principles.

theoretically : how do we dissect a collider event & what enters its "prediction" or "modelling".

A very simplified rendering ...



Adding QED+EW "radiation" poses challenges, but not conceptually new ones.

pQCD → perturbative QCD
non pQCD → non " QCD

pQCD → this is the part that can be systematically calculated & determines the structure of an event

non pQCD in PDF → fitted from more & more precisely measured / predicted processes
↘ first results from Lattice QCD (first principle calculation)

Master formula ⟷ Factorization

non-pert. effects

desired final-state

$\mu = \mu_R, \mu_F$

$$d\sigma(ab \to F+X) = \sum_{i,j} \int dx_1 dx_2\, f_{a,i}(x_1,\mu)\, f_{b,j}(x_2,\mu)\, d\hat{\sigma}(x_1,x_2,\mu) + \mathcal{O}\left(\left(\frac{\Lambda_{QCD}^2}{Q}\right)^p\right)$$

"inclusive" final state

initial-state particles (partons if hh, leptons if e^+e^-)

"parton distribution functions" (PDF)

(present only for hh collisions)

$h = p, \bar{p}$

$i,j \to q, g$

x_1, x_2 fraction of momentum of parent h.

parton-level cross section

→ why factorization?
think of difference in time scales

"cross-section" < total / differential

depending on degree of integration over final-state phase space.

$$d\hat{\sigma}(p_a, p_b \to \{p_i\}) = \frac{1}{2s}\left(\prod_{i=1}^{N} \frac{d^3p_i}{(2\pi)^3 2E_i}\right)(2\pi)^4 \delta^4\left(p_a+p_b-\sum_i p_i\right)\left|A(p_a,p_b\to\{p_i\})\right|^2$$

$2\to N$ scattering

final-state phase space

universal to $2\to N$ processes

conservation of energy & momentum

A: scattering amplitude

specific of a given process

calculated in SM or any other theory

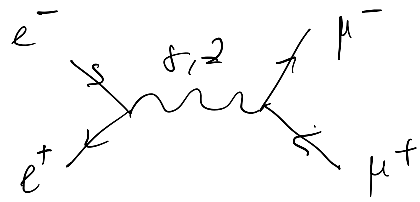
FURTHERMORE

fixed order / with parton shower

To make contact with object you may have seen:

→ consider a simple 2→2 process:

$e^+e^- \to \mu^+\mu^-$



Exercise: show that

$$\frac{d\sigma}{d\cos\vartheta} = \frac{1}{32\pi s}\sqrt{1-\frac{4m_\mu^2}{s}}\;\overline{|A|}^2$$

this is the process dep. part "interesting part"

scattering angle

$s = (p_1+p_2)^2 = (p_3+p_4)^2$

this is phase-space dep., universal

amplitude square, averaged over initial degrees of freedom & summed over the final ones.

and furthermore show that ($m_i \ll \sqrt{s}$)

$$\frac{d\sigma}{d\cos\vartheta} = \frac{\pi\alpha^2}{2s}(1+\cos^2\vartheta)$$ if the muon is a fermion

or $$= \frac{\pi\alpha^2}{4s}\sin^2\vartheta$$ if the muon is a scalar (s-muon?!)

very different distributions!
what do they look like for $\vartheta \in [0,\pi]$?

→ "s"muons cannot be produced in forward/backward direction, while muons can ...

↳ what does the non-observation of scalar muons mean for BSM theories?

Back to dissecting the complexity of collider event! think of them as:

$$\text{hadron} \longrightarrow \text{parton} \longrightarrow \text{hadron}$$

hadron ↓ initial state

$\xrightarrow{\text{PDF}}$ ↓ evolution $\Lambda \to Q$ calculable (in SM)

parton ↓ $\hat{\sigma}_{hard}$ → calculable (in SM)

$\xrightarrow{\text{parton shower}}$ → evolution $Q \to \Lambda$ calculable (in SM)

the evolution

$$\Lambda_{QCD} \longrightarrow Q \longrightarrow \Lambda_{QCD}$$

Can be understood from the basic properties of QCD radiation. QED radiation follows similar pattern. EW radiation is clearly different due to the massivness of the gauge bosons ($\longrightarrow$ will be similar at future Multi-TeV colliders).

Let's start from the hard core ($\hat{\sigma}_{hard}$) and then build the evolution $\longrightarrow$ and $\longleftarrow$, focusing on the properties of QCD radiation!

$\longrightarrow$ in final-state $\to$ { this will give us elements to build the evolution $\longrightarrow$

$\longrightarrow$ in initial-state } { and this one to build the evolution $\longleftarrow$

• Calculating $\hat{\sigma}_{hard}$ –

why does this include QCD "radiation"?

↳ at a given perturbative order, let's say in α_s, but could be in any combination of SM couplings.

⟶ need to consistently include everything that contribute to a given process at a given order of the couplings.

this is why we introduced the notation

$$ab \to F + X$$

F: desired final state

X: anything else that could be there at that order

↳ specify the difference between massless/massive "radiation"

Ex: $e^+e^- \to t\bar{t}$

$$pp \to H + X$$

X are typically jets (= sprays of had. activity)

this is what you are looking for, but events contain everything else that can come along, and it is in our interest to include as much as we can, since "the rest" affects the properties of H.

Hence

we can still require that our events contain H and not demand anything else

inclusive → H+X

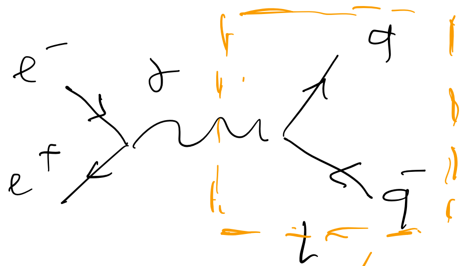
exclusive → H + 1J, H + 2J, etc.

jets defined by "tags" (p_T, η)

- For final-state radiation → can consider e^+e^- initial state

Let's consider a simple case (as we will see the process does not matter!) → e.g. $e^+e^- \xrightarrow[\not{Z}]{\gamma} q\bar{q} \rightarrow \sigma = \sum_n \alpha_s^n(\mu)\cdot\sigma^{(n)}$

↦ for the moment



focus on this part

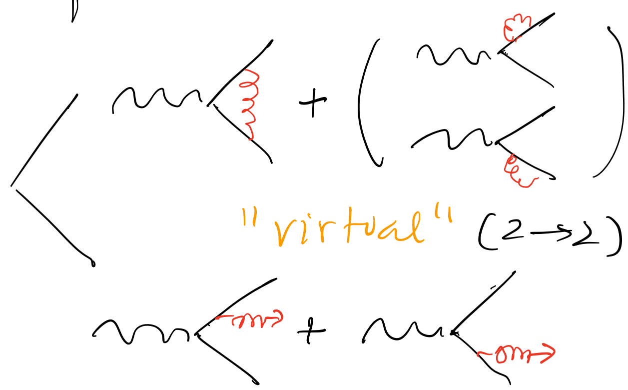
α_s^0

tree level

$+\ O(\alpha_s)$

α_s

1st order



"virtual" $(2\to2)$

"real" $(2\to3)$

both contribute to the same order at the σ level

$$\sigma(e^+e^-\to q\bar{q}) \simeq \underbrace{\int dPS_{2\to2}\,\left|M_{2\to2}\right|^2}_{\sigma_{tree}\ (\text{Born})\ +\ \sigma_{virtual}} + \underbrace{\int dPS_{2\to3}\,\left|M_{2\to3}\right|^2}_{\sigma_{real}}$$

$O(\alpha_s)$ "tree + virtual" "real"

$M_{q\bar{q}}$

$$\left|M^{(0)}_{2\to2} + M^{(1)}_{2\to2}\right|^2 = \left|M^{(0)}_{2\to2}\right|^2 + \left(M^{(0)*}M^{(1)} + M^{(1)*}M^{(0)}\right) + O(\alpha_s^2)$$

$M_{q\bar{q}g}$

$\left|M_{2\to3}\right|^2 \longrightarrow O(\alpha_s)$

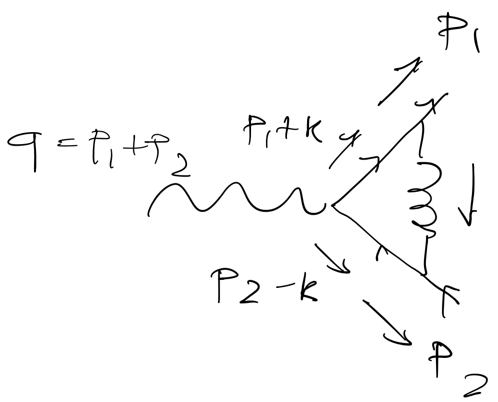
Need to consider both if inclusive on final state

• $\sigma_{virtual}$ $\to$ $O(\alpha_s)$ corrections to $\gamma^* \to q\bar{q}$

UV divergences $\longrightarrow$ canceled by renorm. procedure

but, there are IR divergences

Ex.



$d = 4 - 2\epsilon_{IR}$ (dimensional regularization)

$$\longrightarrow \int \frac{d^4k}{(2\pi)^4} \frac{N(k, p_1, p_2)}{k^2 (k+p_1)^2 (k-p_2)^2}$$

$k \equiv (\omega_k, \vec{k})$

$$k^2 (k\cdot p_1)(k\cdot p_2)$$

$$\simeq \omega_k^4 (1-\cos\theta_{kp_1})(1-\cos\theta_{kp_2})$$

physical & mathematical meaning for having to consider $2\to n$ and $2 \to n+1$ together

$\omega_k \longrightarrow 0$ "soft" IR divergence

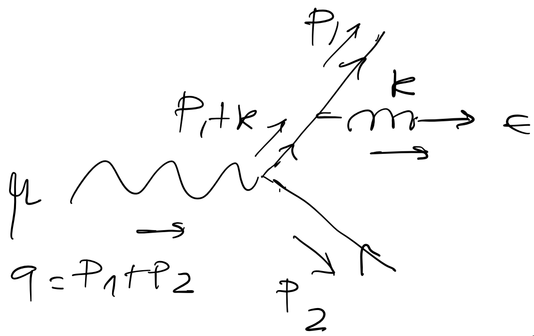
θ_{kp_1}, θ_{kp_2} $\longrightarrow 0$ "collinear" IR divergence

$$\sigma_{virtual} = \sigma^{IR}_{virtual} + \sigma^{finite}_{virtual}$$

$$\propto \alpha_s \left(\frac{C_2}{\epsilon_{IR}^2} + \frac{C_1}{\epsilon_{IR}} \right) \sigma_{q\bar{q}}$$

$\sigma_{q\bar{q}} \longrightarrow$ tree level

- $\sigma_{real} \longrightarrow \gamma^* \to q\bar{q} + g$ at $O(\alpha_s)$

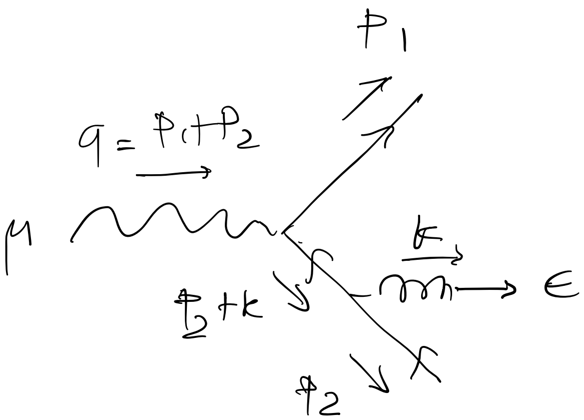


$$\bar{u}(p_1)\, i g_s \not{\epsilon}\, T^A \frac{i}{(\not{p}_1 + \not{k})}\, i e_q \gamma^\mu v(p_2)$$

$$\Big\downarrow \text{ soft limit } (\omega_k \to 0)$$

$$\ldots\ \bar{u}(p_1) \not{\epsilon} \frac{(\not{p}_1 + \not{k})}{2 p_1 \cdot k} \gamma^\mu v(p_2)$$

$$\ldots\ \frac{\cancel{2} p_1 \cdot \epsilon}{\cancel{2} p_1 \cdot k}\, \underbrace{\bar{u}(p_1) \gamma^\mu v(p_2)}_{\text{tree-level structure}}$$



$$-\bar{u}(p_1)\, i e_q \gamma^\mu \frac{i}{(\not{p}_2 + \not{k})}\, i g_s T^A \not{\epsilon}\, v(p_2)$$

$$\Big\downarrow \text{ soft limit}$$

$$\ldots\ -\frac{\cancel{2} p_2 \cdot \epsilon}{\cancel{2} p_2 \cdot k}\, \underbrace{\bar{u}(p_1) \gamma^\mu v(p_2)}_{\text{tree-level structure}}$$

such that :

$$M_{q\bar{q}+g} \underset{\text{soft limit}}{\simeq} \bar{u}(p_1)\, i e_q \gamma^\mu T^A v(p_2) \cdot \left(\frac{p_1 \cdot \epsilon}{p_1 \cdot k} - \frac{p_2 \cdot \epsilon}{p_2 \cdot k} \right)$$

$Tr(T^A T^A) = N \cdot \frac{N^2-1}{2N} = N C_F$

$\hookrightarrow |M_{q\bar{q}}|^2$

$$\sum_{\substack{\text{color} \\ \text{pol'ztn.}}} |M_{q\bar{q}+g}|^2 = |M_{q\bar{q}}|^2 \cdot C_F \cdot g_s^2 \cdot \underbrace{\frac{2 p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)}}$$

gluon emission has factored out !

$\to$ indendent PS integration

$$\sigma_{real} \xrightarrow[\text{limit}]{\text{soft}} \int dPS_3 \, \overline{|M_{q\bar{q}+g}|^2}_{\text{soft limit}}$$

$$= \underbrace{\int dPS_2 \, |M_{q\bar{q}}|^2}_{\substack{\text{Hard} \\ \gamma^* \to q\bar{q} \\ \text{(tree level)}}} \; \underbrace{\frac{1}{q^2} \frac{d^3k}{(2\pi)^3 2\omega_k} C_F g_s^2 \frac{2p_1 \cdot p_2}{(p_1\cdot k)(p_2 \cdot k)}}_{dS_{eik} \to \text{soft gluon emission}}$$

q^2

(also contain collinear piece)

$$dS_{eik} = \omega_k \, d\omega_k \, d\cos\theta \, \frac{d\phi}{2\pi} \frac{2\alpha_s C_F}{\pi} \underbrace{\frac{2p_1 \cdot p_2}{(p_1\cdot k)(p_2\cdot k)}}_{\frac{1}{\omega_k^2 (1-\cos^2\theta)}}$$

$$\simeq \frac{d\theta}{\theta}$$

$$= \frac{2\alpha_s C_F}{\pi} \frac{d\omega_k}{\omega_k} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi}$$

$$\begin{cases} \omega_k \to 0 & \text{IR soft divergence} \\ \theta \to 0, \pi & \text{IR collinear divergence} \end{cases}$$

$$\sigma_{real} = \sigma_{real}^{IR} + \sigma_{real}^{finite}$$

$$\alpha_s \left(-\frac{c_2}{\epsilon_{IR}^2} - \frac{a}{\epsilon_{IR}} \right) \sigma_{q\bar{q}} = -\sigma_{virtual}^{IR} \; ?$$

Such that:

$$\sigma(e^+e^- \to q\bar{q}) = \sigma_{q\bar{q}} \left\{ 1 + \frac{2\alpha_s C_F}{\pi} \int \frac{d\omega_k}{\omega_k} \int \frac{d\theta}{\sin\theta} \left[R\left(\frac{\omega_k}{Q}, \theta\right) - V\left(\frac{\omega_k}{Q}, \theta\right) \right] \right\}$$

$+$ finite (hard)

ex.
$Q \to$ center of mass energy of the process

✓
$$\lim_{\omega_k \to 0} (R - V) = 0$$
$$\lim_{\theta \to 0, \pi} (R - V) = 0$$

↳ Several things to notice:

Note 1: PS is flat, but matrix elements are enhanced in soft and collinear regions

↳ finite parts contain residual large soft and collinear logs

think of resumming it

↳ radiation preferentially emitted collinear to hard partons.

↳ this explains the "jetty" pattern of QCD radiation

+ cloud of soft activity

Note 2: IR divergences cancel in inclusive observables.

↳ careful when considering exclusive observables

↳ need to check IR safety

ESW: "For an observable to be calculable at fixed-order in pQCD the observable should be "IR-safe", i.e. it must be invariant under

$\vec{p}_i \to \vec{p}_j + \vec{p}_k$ $\quad \begin{cases} \vec{p}_j \parallel \vec{p}_k \\ \text{or } \vec{p}_j, \vec{p}_k \text{ small} \end{cases}$

Examples of non IR-safe observables are :

→ gluon multiplicity

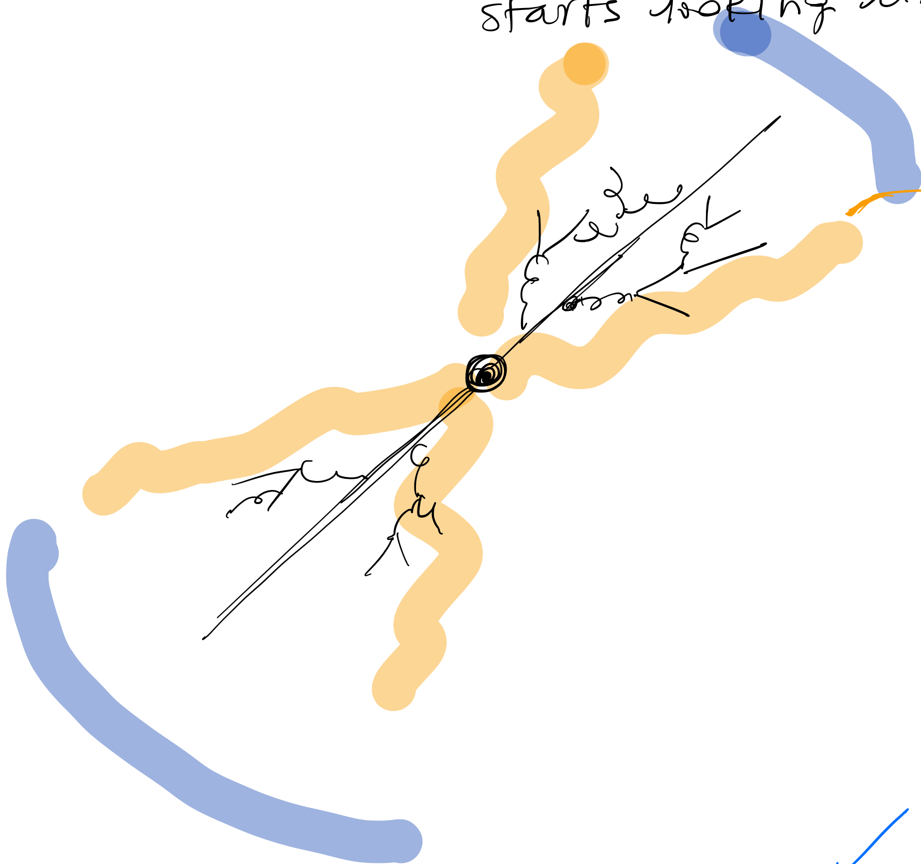
→ energy of the hardest particle in an event

both modified by collinear splittin

While the "total energy flow" into a given cone is an IR-safe observable since soft emission do not modify the energy flow & collinear emission do not modify its direction.

<u>Note 3</u> $e^+e^- \to q\bar{q} \ldots \to$ hadrons

starts looking like this :



hadronization

collimated regions of hadronic activity

confined to a narrow region around the original hard process (hard partons)

pictorially already suggesting the idea of "jets"

Parton shower Event Generators translate this property of pQCD into a systematic algorithm

→ Caveat: Parton Shower Ev. Generts. only based on soft / collinear dynamics

↳ they cannot reproduce hard emission correctly

*

they take off from the hard process

(→ processes that can be generated through multiple channels may be missing some)

Good / Necessary to push $\hat{\sigma}_{\substack{\text{fixed}\\\text{order}}}$ to include first order of radiation

i.e. to push the interface with PS to a later stage, in so doing generating more of the correct "shape" of the hard process.

↳ "the more hard partons the better"

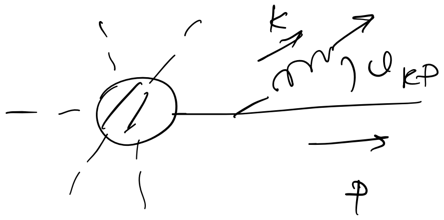
→ Existing methods { MC@NLO, POWHEG } ↔ interface NLO QCD (and QED) calculation with PYTHIA, HERWIG, SHERPA

both methods avoid double counting in matching NLO calculation to PS.

↳ first order of radiation already included.

Parton-Shower in a nutshell

"Radiation kernels"

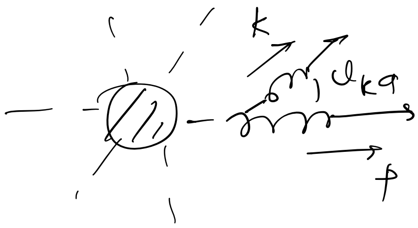


$\sim \dfrac{2\alpha_s C_F}{\pi}\,\dfrac{d\omega_k}{\omega_k}\,\dfrac{d\theta_{kp}}{\theta_{kp}}$

$C_F = \dfrac{N^2-1}{2N}$

$C_A = N$

such that: $d\sigma_{IR} = \left(\quad \right)\cdot\sigma_0$



$\sim \dfrac{2\alpha_s C_A}{\pi}\,\dfrac{d\omega_k}{\omega_k}\,\dfrac{d\theta_{kp}}{\theta_{kp}}$

collinear kinematics can be included more precisely

→ "splitting functions"

Same behavior replicated at all orders

↓

Parton Shower Event Generators → implement this dynamics (dictated by exact QCD!) with a sequential algorithm based on an "ordered" emission

→ in angle

→ in energy ($k_T = \omega_k \theta$)

Need to calculate

→ emission probability at each splitting

<u>Ex.</u> : $k_{T,1} > k_{T,2} > \dots$

each k_T integration is cut off by the nested one, till the final cutoff

($Q \simeq \Lambda_{QCD}$) ↔ map to a given hadronization model.

$P(\text{emission} > k_T) = 1 - P(\text{non-emission} > k_T)$

↳ this is the soft/collinear part we here calculated

RESUM at all orders

$\Delta(k_T, Q)$

↳ $\boxed{\text{Sudakov form factor}}$

ex $e^+e^- \to q\bar{q}$

$$\simeq \exp\left\{ -\frac{2\alpha_s C_F}{\pi} \int^{Q} \frac{d\omega_k}{\omega_k} \int^{\pi/2} \frac{d\theta}{\theta} \Theta(\omega_k \theta - k_T) \right\}$$

Transv. moment. of the emitted parton for $\theta \ll 1$

Forming jets.

→ We saw that energy flow is an IR-safe observable

↓

when?

↓

in a given "cone"

Need a "jet" definition

↓

jet algorithms

Based on a "distance" definition

$\forall$ "i" parton or "seed"

d_{ij} = distance w.r.t. other partons

d_{iB} = " " beam (initial partons)

↳ many options

- naive: $d_{ij} = \Delta R_{ij} = \left((y_i - y_j)^2 + (\phi_i - \phi_j)^2 \right)^{1/2}$
- k_T, anti-k_T: $d_{ij} = \min\left(p_{T,i}^{2n}, p_{T,j}^{2n}\right) \dfrac{\Delta R_{ij}^2}{R_j^2}$ $(n = -1, 0, 1)$

Side remark:

Notice: under a longitudinal boost:

$$E \longrightarrow E\cosh\beta + p_z \sinh\beta$$

$$\longrightarrow \vec{p}_T \quad \begin{cases} p_x \longrightarrow p_x \\ p_y \longrightarrow p_y \end{cases}$$

$$p_z \longrightarrow p_z \cosh\beta + E\sinh\beta$$

(p_x, p_y)
transverse momentum
$\downarrow$
boost invariant

$\longrightarrow \phi \equiv \tan^{-1} \dfrac{p_x}{p_y}$ also boost-invariant

$\longrightarrow y = \dfrac{1}{2} \ln \dfrac{E + p_z}{E - p_z}$ "rapidity" is not boost-invariant

$$\frac{E + p_z}{E - p_z} \longrightarrow \frac{E + p_z}{E - p_z} (\cosh\beta + \sinh\beta)^2$$

$$y \longrightarrow y + \ln(\cosh\beta + \sinh\beta)$$

$\hookrightarrow$ Δy is boost invariant

$\hookrightarrow$ Hence: $\Delta R = \left[\Delta y^2 + \Delta\phi^2\right]^{1/2}$ is boost-invariant

For massless particles

$$y = \frac{1}{2}\ln\frac{E+p_z}{E-p_z} = \frac{1}{2}\ln\frac{1+\cos\vartheta}{1-\cos\vartheta} = \frac{1}{2}\ln\frac{\cancel{2}\cos^2\vartheta/2}{\cancel{2}\sin^2\vartheta/2} =$$

$$= \frac{1}{\cancel{2}}\cdot\cancel{2}\ln\cot g\frac{\vartheta}{2} \equiv \eta$$

"pseudorapidity"

$\sim$ "geometric" quantity ($\longleftrightarrow$ angle)

Collider geometry