

Functional Matching at Two Loop Order

Adrián Moreno

University of Granada

with J. Fuentes-Martín, A. Palavrić and A. Eller Thomsen



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Effective Field Theory

They represent our best connection between theory and experiment

Wilson
Coefficients

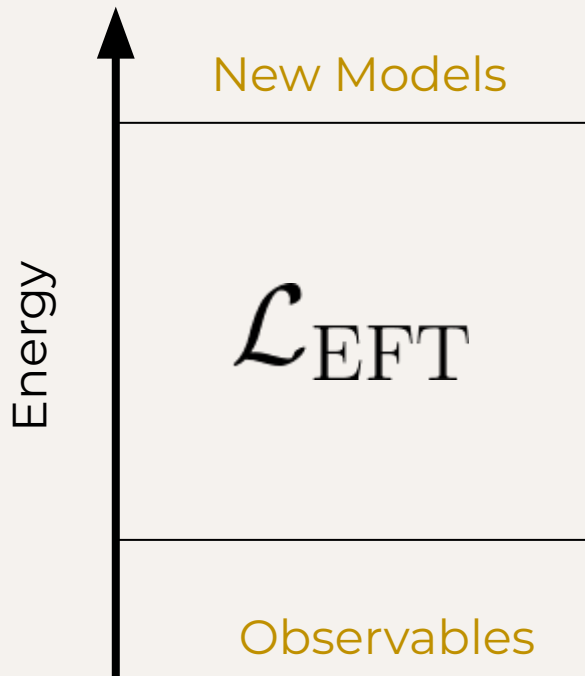
$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{d \leq 4} + \sum_{\ell=0}^{\infty} \sum_{n=5}^{\infty} \frac{C^{(\ell, n)}}{\Lambda^{n-4}} \mathcal{O}_n$$

Bottom up:

Parameterize our lack of knowledge

Top Down: Separate scales for precision measurements

Separation of Scales



BSM requires multiscale physics

- **Matching** of UV theories to low energy observables
- Process of matching automatized up to one loop



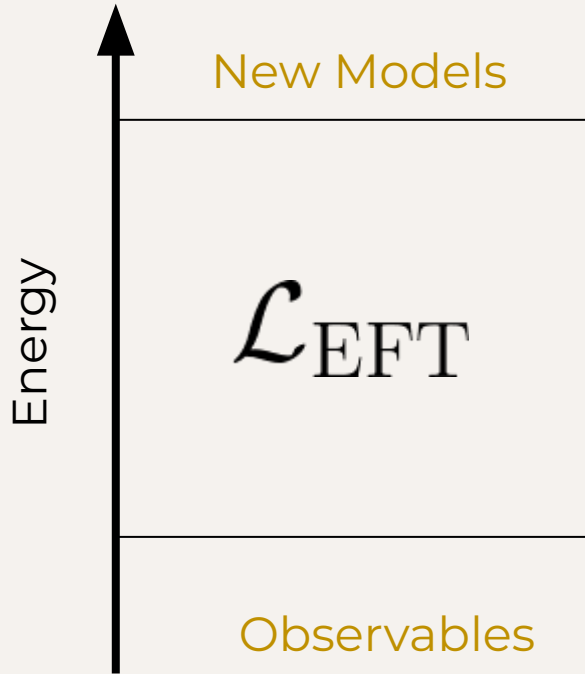
[Carmona et al-2112.10787]



[Fuentes-Martín et al-2212.04510]

- Running of the theory via RG evolution

Why Loops



- There is physics we must not lose at the one loop level
- Scheme independence RG evolution at one loop requires two loops
- Precision test of UV models up to two loops are also interesting

Amplitude Matching

$$\mathcal{L}_{\text{UV}}(z_h, z_l) \xrightarrow{q_i \ll \Lambda} \{\mathcal{A}_{\text{UV}}(q_i)\}$$

Matching:
Determining Wilson
Coefficients

$$\mathcal{L}_{\text{EFT}}(z_l) \longrightarrow \{\mathcal{A}_{\text{EFT}}(q_i)\}$$

Feynman Diagrams

- Well-established
- Ansatz: Redundancies, redefinitions...
- Explicit break of Gauge Symmetry in intermediate steps
- Normally uses off-shell computations

Functional Matching

Background Field Method

$$\phi = \bar{\phi} + \hat{\phi}$$

$\bar{\phi}$

Classical Configuration: Tree Level

$\hat{\phi}$

Quantum Fluctuation: Loops

Quantum Effective Action: We are going to “**Integrate Out**” the heavy fields

$$e^{i\Gamma_{UV}[\hat{\phi}]} = \int [D\phi] \exp \left(\int d^d x \mathcal{L}_{UV}(\bar{\phi} + \hat{\phi}) \right)$$

Functional Matching

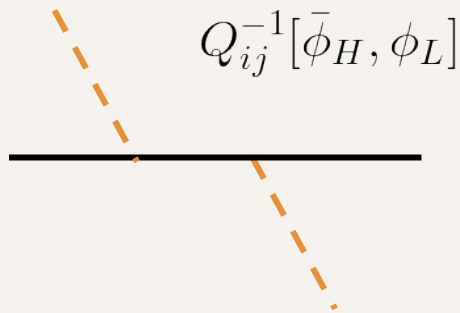
Expanding Lagrangian

Fluctuation Operator $Q_{ij}(\phi(x), \partial_x)$

$$\mathcal{L}_{\text{UV}}(\bar{\phi} + \hat{\phi}) = \mathcal{L}_{\text{UV}}(\bar{\phi}) + \frac{1}{2} \phi_i \phi_j \frac{\delta^2 \mathcal{L}_{\text{UV}}}{\delta \phi_i \delta \phi_j} \Big|_{\phi = \bar{\phi}}$$

- **Tree Level:**

$$\mathcal{L}_{\text{EFT}}(\phi_L) = \mathcal{L}_{\text{UV}}(\phi_L, \bar{\phi}_H[\phi_L])$$



- **One-loop:**

$$\exp(i\Gamma_{\text{UV}}^{(1)}[\hat{\phi}]) = \int [D\phi] \exp \frac{1}{2} \left(\int d^d x \phi_i Q_{ij} \phi_j \right)$$



Gaussian Integration

$$\Gamma_{\text{UV}}^{(1)} = \frac{i}{2} \text{STr} \ln Q$$

- **Two Loops:** More Topologies involved

$$\exp(i\Gamma_{UV}^{(2)}[\hat{\phi}]) = \int [D\phi] \exp\left(\int d^d x \frac{1}{2} \phi_i Q_{ij} \phi_j\right) \times$$

$$\left[-\frac{1}{2} \phi_i \phi_j B_{ij} - \frac{i}{24} \phi_i \phi_j \phi_k \phi_l D_{ijkl} - \frac{i}{2} \phi_i \phi_j \phi_k \phi_l \phi_m \phi_n C_{ijk} C_{lmn} + \dots\right]$$



$$\Gamma_{UV}^{(2)}[\bar{\phi}] = \frac{i}{2} Q_{ij}^{-1} B_{ij} - \frac{1}{8} Q_{ij}^{-1} D_{ijkl} Q_{kl}^{-1} + \frac{1}{12} C_{ijk} Q_{il}^{-1} Q_{jm}^{-1} Q_{kn}^{-1} C_{lmn}$$

$$\frac{i}{2} \text{(1)} + \frac{1}{12} \text{---} - \frac{1}{8} \text{---}$$

Method of Regions

[Beneke, Smirnov,-9711391]

[Jantzen-1111.2589]

Multiscale ($m \ll M$) integrals can be separated in regions

$$\int \frac{d^d k}{(k^2 - m^2)(k^2 - M^2)} \begin{cases} \int \frac{d^d k}{k^2(k^2 - M^2)} \left[1 - \frac{m^2}{k^2} + \dots \right] & \text{soft (} k \sim m \text{)} \\ \int \frac{d^d k}{-M^2(k^2 - m^2)} \left[1 - \frac{k^2}{M^2} + \dots \right] & \text{hard (} k \sim M \text{)} \end{cases}$$

In our case, we want to integrate out **hard momentum modes**

$$\Gamma_{UV} = \Gamma_{UV} \Big|_{\text{hard}} + \Gamma_{UV} \Big|_{\text{soft}}$$

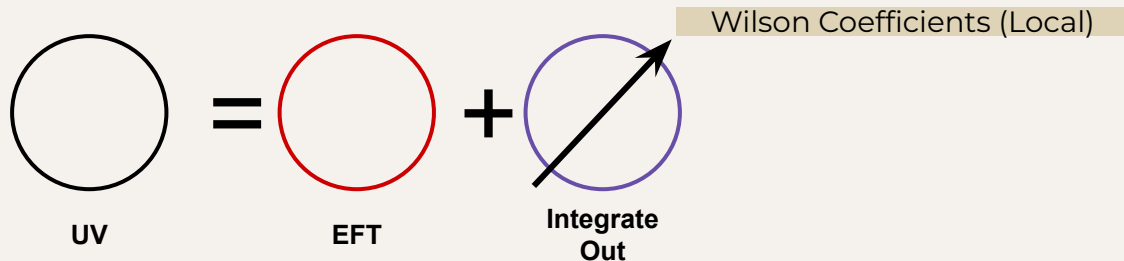
Matching Condition

[Fuentes-Martin, Palavrić, Eller Thomsen-2311.13630]

The soft loops explicitly cancel with the EFT matrix elements

$$S_{\text{EFT}} = \Gamma_{\text{UV}}[\bar{\phi}_H[\hat{\phi}_L], \hat{\phi}_L] \Big|_{\text{hard}} \frac{\delta \Gamma_{\text{UV}}|_{\text{hard}}[\bar{\phi}_H, \phi_L]}{\delta \phi_H} = 0$$
$$\Gamma_{\text{UV}}|_{\text{soft}} = \Gamma_{\text{EFT}}$$

We just have to consider hard loops (with heavy and light fields)



Evaluation of Traces

[Fuentes-Martín, Palavrić, Eller Thomsen, AM]

Differential operators under a Gauge Symmetry

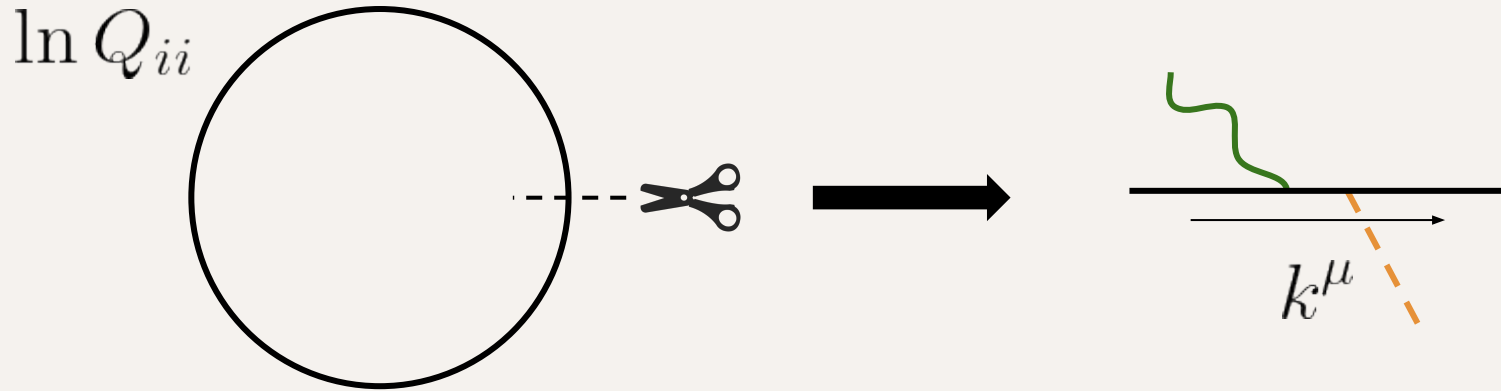
$$Q_{ij}^{ab}(x, y) = \frac{\delta^2 S}{\delta \phi_i^a(x) \phi_j^b(y)} = Q_{ij}^{ac}(x, P_x) \delta_c^b(x, y) \quad \delta_c^b(x, y) = \delta(x - y) U_c^b(x, y)$$

Locality of the action: Functional Traces are dressed loop integrals

$$\begin{aligned} \text{Str} \ln Q|_{\text{hard}} &= \int_{x,y} \delta_b^a(x, y) \ln Q_{ii}^{ab}(x, y) \\ &= \int_{x,k} \ln Q_{ii}^{ab}(x, P_x + k) U_b^a(x, y)|_{x=y} \end{aligned}$$

OPE around $k \sim \Lambda$
=
Explicit Gauge
Invariance

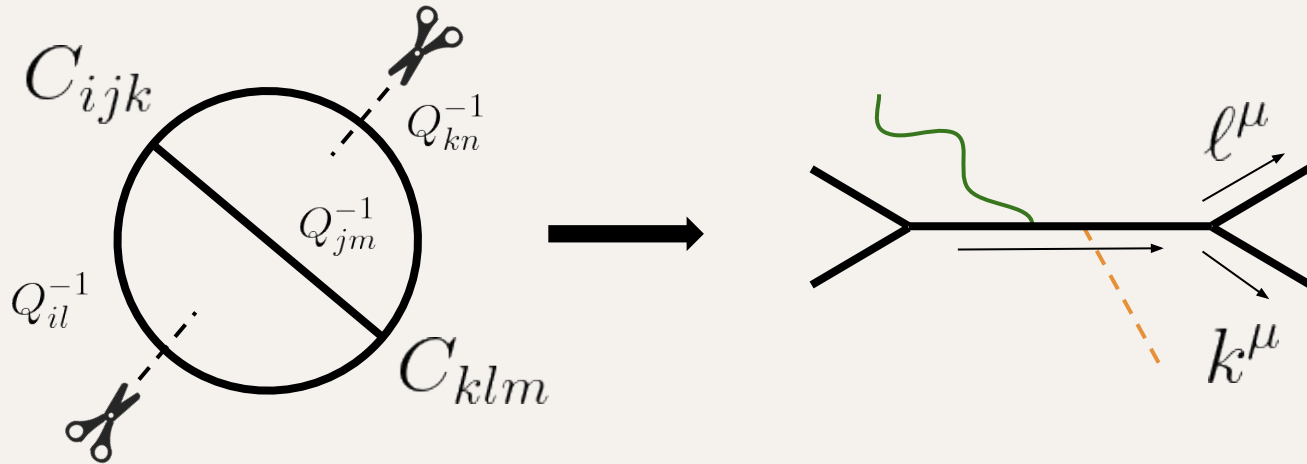
Diagrammatically,



Operators traced in different points of spacetime remain local by a **momentum shift** operation

Two Loops Traces

$$G_{\text{ss}}|_{\text{hard}} = \sum_{n,m,n',m'} (-1)^{n+m} \int_x \int_{k,\ell} C_{abc}^{(n,m)} \mathcal{Q}_{aa'}^{-1}(y, P_y - k - \ell) C_{a'b'c'}^{(n',m')}(y) \\ \times [(P_x + k)^m \mathcal{Q}_{be}^{-1}(x, P_x + k) (P_x + k)^{m'} U_b^e(x, y)] \\ \times [(P_x + \ell)^n \mathcal{Q}_{cf}^{-1}(x, P_x + \ell) (P_x + \ell)^{n'} U_c^f(x, y)]|_{x=y}$$



Outlook

- EFT is given by the formalism (not by an ansatz)
- Basis reduction is still needed
- Better for Automation: Matchete
- Future study of applications of interest to the community



Thank You!
Muito Obrigado!