Introduction to Neutrinos

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Outline of these lectures

1. Neutrino oscillations: 2 neutrinos, 3 neutrinos, CP violation, matter effects
2. How to study neutrinos: production (beam, reactor, solar, ...), detection (ν-e, ν-N, CEνNS, ...)
3. Neutrino oscillation phenomenology: current status, DUNE/HK and MO/CPV, solar/atm,
4. Neutrinos in astrophysics and cosmology: basics of supernova, basics of BBN, cosmogenic neutrinos (IceCube/KM3NET)
5. Overflow lecture - advanced theoretical aspects

Neutrino Oscillations

$\hbar = c = 1$

$c \simeq 3\times 10^{10}\ cm/s = 1$

$s = 3\times 10^{10}\ cm$

$\hbar c = 200\ MeV\,fm = 1$

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle$$

$t = \frac{L}{c} = L$

$$A(\nu_\alpha \to \nu_\beta ; L) = \langle \nu_\beta | e^{-iHL} | \nu_\alpha\rangle$$

$$= \sum_{ij} \langle \nu_j | U_{\beta j}\, e^{-iHL}\, U^*_{\alpha i} | \nu_i \rangle$$

$$= U_{\beta j} U^*_{\alpha i} \langle \nu_j | \exp(-iE_i L) | \nu_i \rangle$$ → Implicit sum

This can't be exact, one should make an argument about the neutrino wave packet here..

$$E^2 = p^2 + m^2 \Rightarrow E = \sqrt{p^2+m^2} = p\left(1+\frac{m^2}{p^2}\right)^{1/2} \simeq p + \frac{m^2}{2p} \sim E + \frac{m^2}{2E}$$

$$A(\nu_\alpha \to \nu_\beta ; L) = U_{\beta j} U^*_{\alpha i} \exp\left[-i\left(E + \frac{m_i^2}{2E}\right) L\right] \underbrace{\langle \nu_j | \nu_i \rangle}_{\delta_{ij}}$$

$$A_{\alpha\beta} = \sum_i e^{-iEL}\, U_{\beta i} U^*_{\alpha i} \exp\left(-i\,\frac{m_i^2}{2E} L\right)$$

$\Delta m^2_{ij} \equiv m_i^2 - m_j^2$

$$P_{\alpha\beta} = |A_{\alpha\beta}|^2 = \sum_{ij} U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j} \exp\left[i\, \frac{\overbrace{(m_i^2 - m_j^2)}^{} L}{2E}\right]$$

① Derive this formula

$$\boxed{\begin{aligned} P_{\alpha\beta} = \delta_{\alpha\beta} &- 4 \sum_{i<j} Re\left(U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j}\right) \sin^2\left(\frac{\Delta m^2_{ij} L}{4E}\right) \\ &+ 2\, Im\left(U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j}\right) \sin\left(\frac{\Delta m^2_{ij} L}{2E}\right) \end{aligned}}$$

Nice, but too general. Let's be concrete. Take two neutrinos

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \underbrace{\begin{array}{c|cc} & 1 & 2 \\ e & \cos\theta & \sin\theta \\ \mu & -\sin\theta & \cos\theta \end{array}}_{U_{\alpha i}} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$\alpha = \mu$

$\beta = e$

$$\Rightarrow P_{\mu e} = -4\,\mathrm{Re}\left(U_{e1} U^*_{\mu 1} U^*_{e2} U_{\mu 2}\right) \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$P_{\mu e} = \underbrace{-4\, c_\theta (-s_\theta) s_\theta c_\theta}_{\sin^2 2\theta} \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$\boxed{P_{\mu e} = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)}$$

This formula is surprisingly useful

Plugging in numbers:

$$\frac{\Delta m^2 L}{4E} = \frac{(\Delta m^2/eV^2)(L/km)}{(E/GeV)} \frac{eV^2 km}{4 GeV} \frac{1}{\underbrace{\hbar c}_{0.197\,GeV\,fm}}$$

$$\Rightarrow \boxed{\frac{\Delta m^2 L}{4E} = 1.27\,\frac{(\Delta m^2/\mathrm{eV}^2)(L/\mathrm{km})}{(E/\mathrm{GeV})}}$$

More on measurements later!

Two neutrinos is actually a very special case: there is no CP violation.

⊛ CPT is conserved and sum of probs. = 1

$$\begin{tikzcd}
\nu_\mu \to \nu_e \arrow[rr, leftrightarrow, "CP"] \arrow[d, leftrightarrow, "T"] \arrow[drr, leftrightarrow, "CPT"] & & \bar\nu_\mu \to \bar\nu_e \arrow[d, leftrightarrow, "T"] \arrow[dll, leftrightarrow] \\
\nu_e \to \nu_\mu \arrow[rr, leftrightarrow, "CP"'] & & \bar\nu_e \to \bar\nu_\mu
\end{tikzcd}
\qquad
\begin{tikzcd}
\nu_\mu \to \nu_\mu \arrow[rr, leftrightarrow, "CP"] \arrow[d, leftrightarrow, "T"] & & \bar\nu_\mu \to \bar\nu_\mu \arrow[dll, leftrightarrow, "CPT"] \\
\nu_\mu \to \nu_\mu & &
\end{tikzcd}$$

(similar for $\nu_e \to \nu_e$)

$$P_{\mu\mu} + P_{\mu e} = 1$$

$$\bar{P}_{\mu\mu} + \bar{P}_{\mu e} = 1 \overset{CPT}{\Rightarrow} P_{\mu\mu} + \bar{P}_{\mu e} = 1 \Rightarrow 1 - P_{\mu e} + \bar{P}_{\mu e} + 1 = 0$$

$$\Rightarrow P_{\mu e} = \bar{P}_{\mu e}$$

No CP violation!

For 3 neutrinos, there is no way to show that due to

$$\left.\begin{aligned} P_{\mu\mu} + P_{\mu e} + P_{\mu\tau} = 1 \\ \bar{P}_{\mu\mu} + \bar{P}_{\mu e} + \bar{P}_{\mu\tau} = 1 \end{aligned}\right\}$$ Too much freedom!

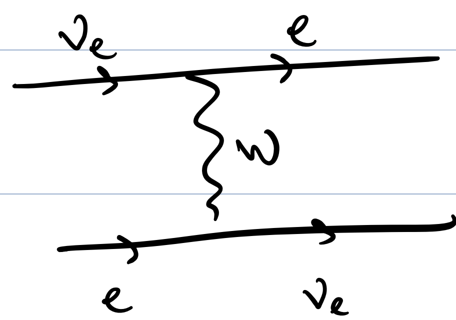
Expressions for 3 neutrinos are a bit lengthy, but *everyone* (and I mean it!) uses this parametrization

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -c_{23} & s_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} e^{i\eta_1} & 0 & 0 \\ 0 & e^{i\eta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Majorana phases (do not affect oscillation)

Matter Effects

When neutrinos travel through matter, there is an overall "weak charge" which generates a potential for neutrinos. Here is the right way to calculate matter effects:



1) Write down the neutrino Lagrangian
2) Integrate out the W and Z bosons to get terms like

$$\mathcal{L} \supset \frac{g^2}{2} \bar{\nu}_e \gamma_\mu P_L e \, \bar{e} \gamma^\mu P_L \nu_e + \dots$$

3) Fierz what needs to be Fierz-ed $2\sqrt{2} G_F (\bar{\nu}_{eL} \gamma_\mu \nu_{eL})(\bar{e}_L \gamma^\mu e_L)$
4) Integrate (trace) over the background

$$\int d^3p \underbrace{f(p)}_{\text{momentum dist.}} \langle \underbrace{\Omega}_{\text{background}} | \bar{e}_L \gamma^\mu e_L | \Omega \rangle = \int d^3p \, f(p) \langle \Omega | e_L^\dagger \gamma^0 \gamma^\mu e_L | \Omega \rangle$$

assuming isotropic medium: $\gamma^0\gamma^i$ terms cancel out

$$= \int d^3p\, f(p) \langle \Omega | \underbrace{e_L^\dagger e_L}_{\text{number operator}} | \Omega \rangle = n_e/2$$

n_e: number density

$$\Rightarrow 2\sqrt{2}G_F (\bar{\nu}_{eL}\gamma_\mu \nu_{eL})(\bar{e}_L \gamma^\mu e_L) \longrightarrow \sqrt{2}G_F n_e (\bar{\nu}_{eL}\gamma^0 \nu_{eL})$$

5) Derive equations of motion, it will be clear that this term behaves as energy such that

$$H = H_0 + V = H_0 + \begin{pmatrix} \sqrt{2}G_F n_e & & \\ & \ddots & \\ & & \ddots \end{pmatrix}$$

I know, it is a long, complicated calculation, BUT it pays off to do it once. By the end of it, you will feel great.

② Derive the matter potential without cutting corners (this may be challenging). Here is some small (fat) help

$$\mathcal{L}_{\text{kin}}^{\text{fer}} = \sum_{\psi}^{\text{fermions}} \overline{\psi} i \not{D} \psi \quad \Rightarrow \quad \mathcal{L}_{\text{kin}}^{\text{fer}} \supset \frac{g}{\sqrt{2}}\left(J_W^\mu W_\mu^+ + J_W^{\mu\dagger} W_\mu^-\right) + \frac{g}{\cos\vartheta_W} J_Z^\mu Z_\mu$$

with
$$\begin{cases} J_W^\mu \equiv \sum_{\text{gen.}} \bar{u}\gamma^\mu P_L d + \bar{\nu}\gamma^\mu P_L \ell, \\ J_Z^\mu \equiv \sum_f \bar{f}\gamma^\mu \left(I_3^f P_L - \sin^2\vartheta_W Q_f\right) f \end{cases}$$

$1 - \frac{M_W^2}{M_Z^2} \simeq 0.23$ (weak mixing angle)

electric charge

$\pm\frac{1}{2}$ or 0

Equations of motion $\left(\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Phi)} - \frac{\partial \mathcal{L}}{\partial \Phi} = 0\right)$, set kinetic terms of W/Z to zero

$$\frac{\partial \mathcal{L}}{\partial W_\mu^+} = \frac{g}{\sqrt{2}} J_W^\mu + M_W^2 W^{-\mu} = 0, \qquad \frac{\partial \mathcal{L}}{\partial Z_\mu} = \frac{g}{\cos\vartheta_W} J_Z^\mu + M_Z^2 Z^\mu = 0$$

Let's take a deep breath. How big is V? Focus on the W diagram (that is, V_{CC} or charged current V)

$$V = \sqrt{2}\, G_F\, n_e \sim \sqrt{2} \times 1.1 \times 10^{-5}\, GeV^{-2} \times \underbrace{6 \times 10^{23}}_{\text{Avogadro}}\, cm^{-3}$$

$$V \sim \overbrace{\sqrt{2} \times 7}^{\sim 10} \times 10^{18}\, GeV^{-2}\, cm^{-3}\, (\hbar c)^3$$

$$\sim 10^{19}\, GeV^{-2}\, cm^{-3}\, (0.2\, GeV\, 10^{-13}\, cm)^3$$

$$\sim 8 \times 10^{19} \times 10^{-42}\, GeV \sim 8 \times 10^{-23}\, 10^{9}\, eV$$

$$\sim 8 \times 10^{-14}\, eV$$

This is super tiny. Neutrino masses should be (one of them) larger than $\sqrt{|\Delta m^2_{31}|} \simeq 0.08\, eV$. But the potential is relevant to neutrino oscillations when $V \sim \Delta m^2/E$. So, oscillations are sensitive to very small potentials!

Back to 2-neutrinos, here is why matter effects are important.

$$H = \frac{1}{2E} U^\dagger \begin{pmatrix} 0 & \\ & \Delta m^2 \end{pmatrix} U + \begin{pmatrix} V_{cc} & \\ & 0 \end{pmatrix}$$ $\rightarrow$ Two flavor, flavor basis

$A = U^\dagger A_{diag} U$

$$P_{\alpha\beta} = \left| \langle \nu_\beta | e^{-iHt} | \nu_\alpha \rangle \right|^2$$

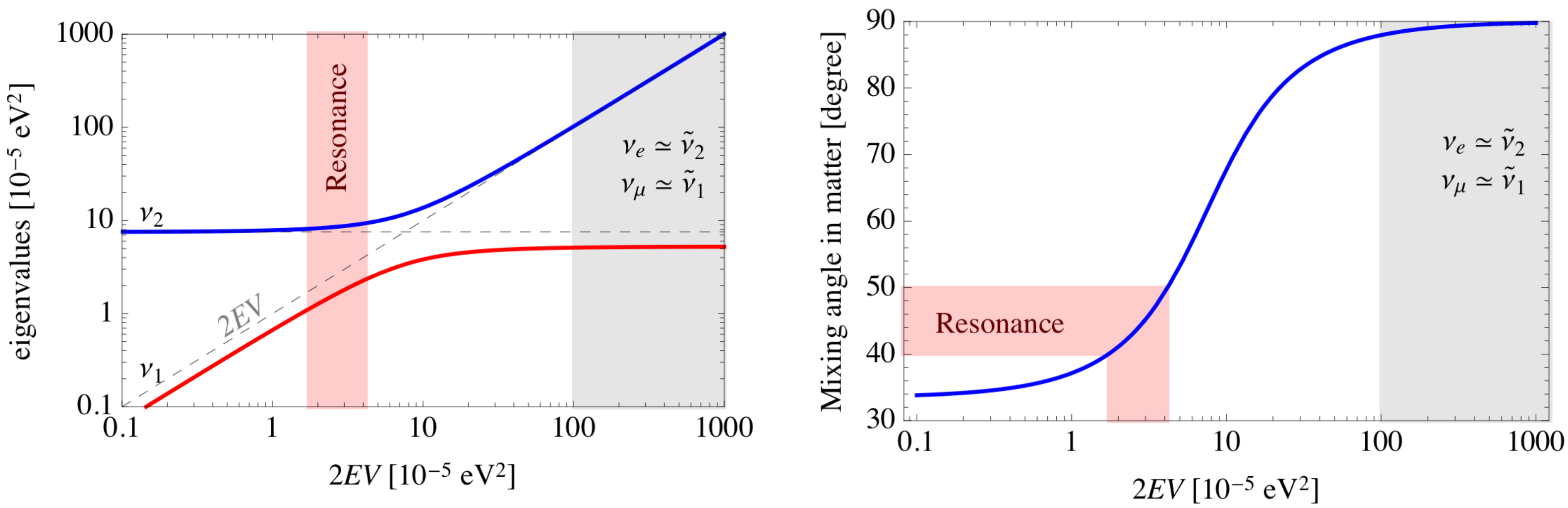
$$e^A = U^\dagger U e^A U^\dagger U = U^\dagger e^{UAU^\dagger} U$$
$$= U^\dagger \exp(A_{diag}) U$$

Diagonalization $\underset{\text{diagonal}}{\tilde{H}} = \tilde{U} H \tilde{U}^\dagger$ leads to

$$\Delta \tilde{m}^2 = \sqrt{A^2 - 2A\Delta m^2 \cos(2\vartheta) + (\Delta m^2)^2},$$
$$\tan(2\tilde{\vartheta}) = \frac{\Delta m^2 \sin(2\vartheta)}{\Delta m^2 \cos(2\vartheta) - A},$$

$\rightarrow A \equiv 2EV_{cc}$

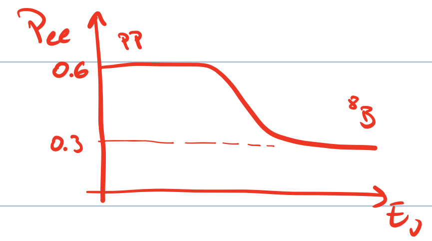
$$\Rightarrow \boxed{P(\nu_e \rightarrow \nu_\mu) = \sin^2 2\tilde{\theta} \, \sin^2\left(\frac{\Delta \tilde{m}^2 L}{4E}\right)}$$



High E solar ν are produced as $\nu_e \simeq \tilde{\nu}_2$, evolve adiabatically, leave the sun as ν_2

$P_{ee} \simeq |\langle \nu_e | \nu_2 \rangle|^2 = |U_{e2}|^2 = \sin^2\theta_{12} \simeq 0.3$

Low E solar ν evolve in vacuum $\Rightarrow P_{ee} = \sum_i |U_{ei}|^4 \sim 0.6$



How to study neutrinos

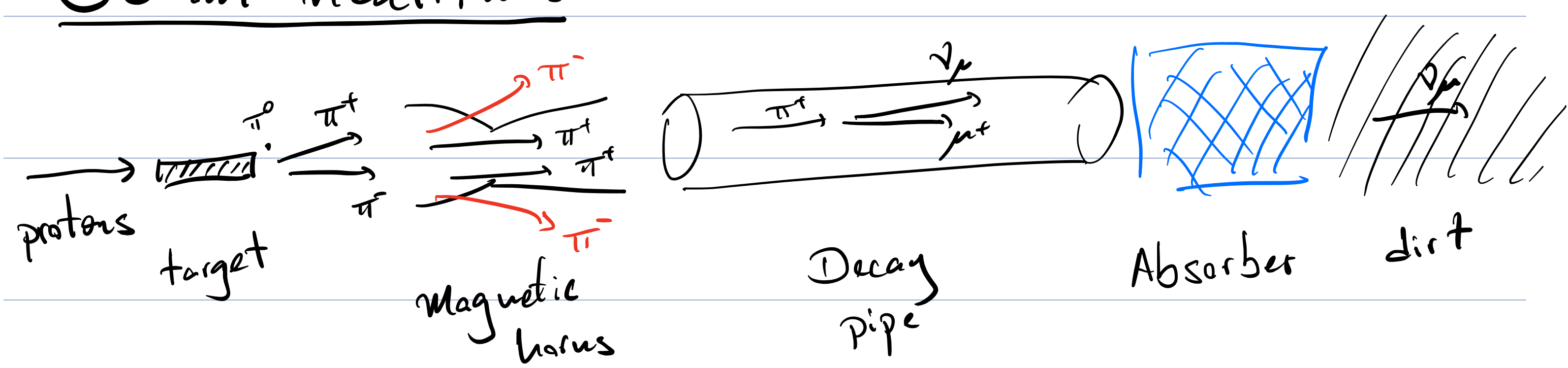
The crux of neutrinos: weak interactions only

→ Need intense sources

→ Need massive detectors

Let's go through some examples on how to produce and detect neutrinos.

Beam neutrinos

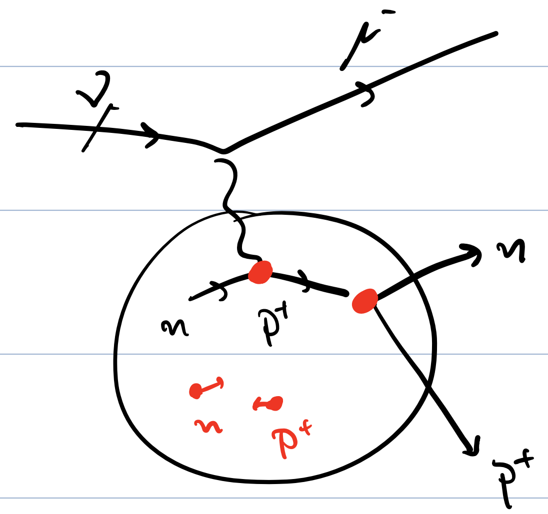


- Production via strong interactions: large flux, but also large uncertainties. Mostly ν_μ due to $\pi^\pm$ decays.

$\pi^+ \to \mu^+ \nu_\mu$ $\qquad$ $K^+ \to \pi^0 e^+ \nu_e$ $\qquad$ pdg $\to$ live pdg

$\pi^- \to \mu^- \bar{\nu}_\mu$ $\qquad$ $K^- \to \pi^0 e^- \bar{\nu}_e$

- Detection: neutrino-nucleus scattering - relatively large cross section, but large uncertainties. Neutrino-nucleus interaction in the GeV scale is very hard to model



Challenges:

① Initial state of nucleus

② Nucleon form factor

③ Propogation throughout nucleus

Final state interactions

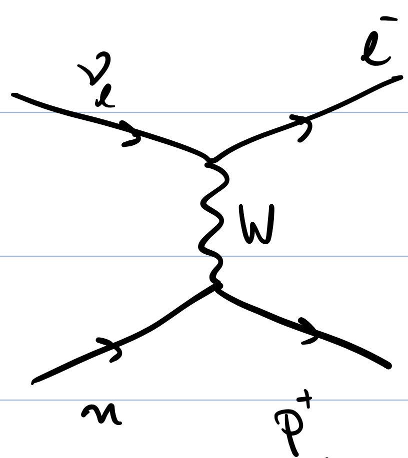
Intranuclear cascade

④ No clear separation of scales

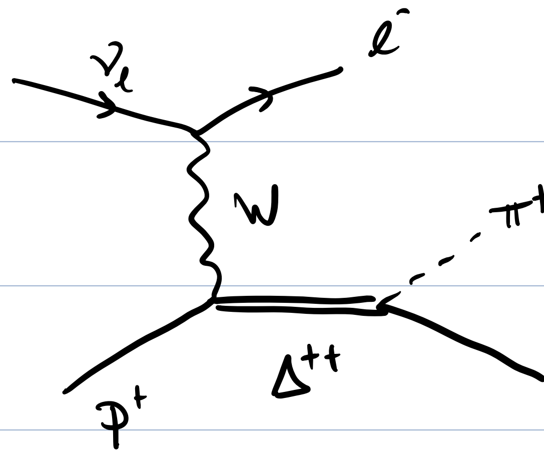
⑤ Nonperturbative physics

Why GeV scale? If you want to see muons, you need $E_\nu > 100$ MeV, above 300 MeV is better. But $\Delta m^2 L/E$ requires $E_\nu \lesssim 10$ GeV, or else we need a detector on orbit (not very easy...)

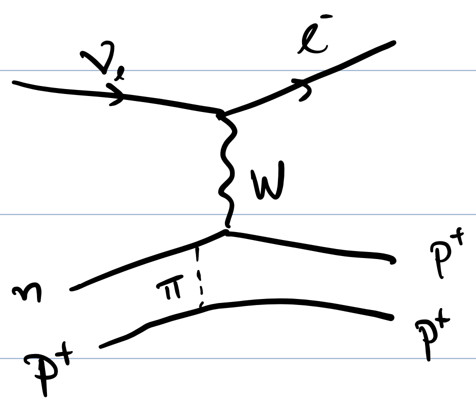
- Interaction channels



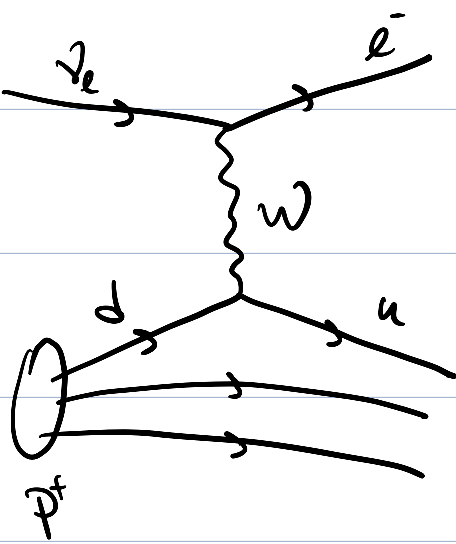
quasi-elastic



resonant



Meson exchange current



Deep inelastic scattering

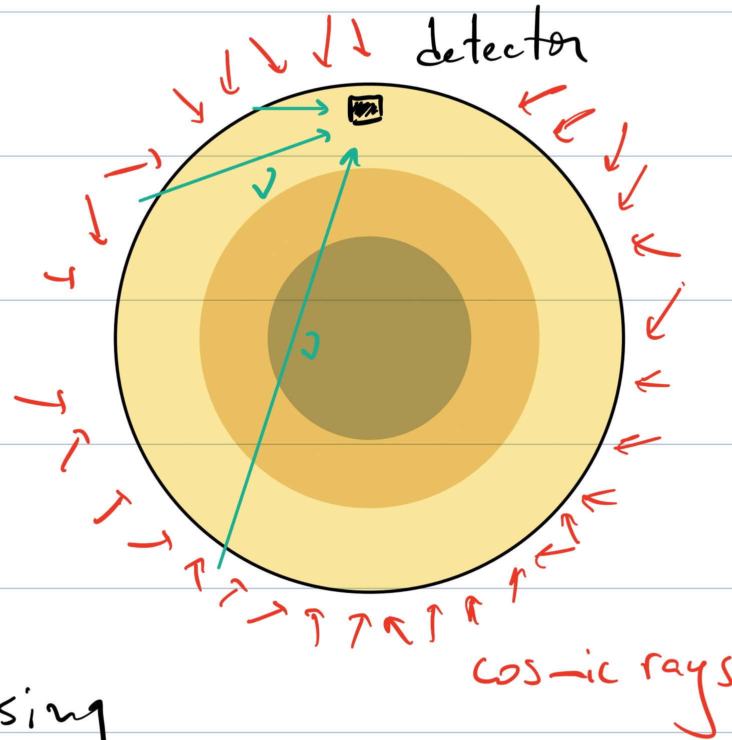
- ⊛ Hard to disentangle RES from DIS
- ⊛ MEC vs QE + FSI should interfere...
- ⊛ Difficult to describe initial distribution of nucleons

Atmospheric Neutrinos

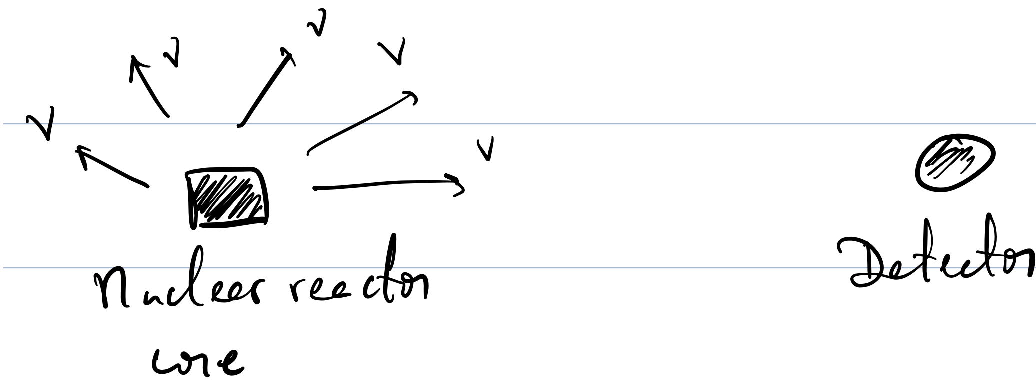
Some similarities with beam neutrinos: the beam are cosmic rays, the target is Earth's atmosphere.

Additional complications:

1) Neutrino production is fairly uncertain
2) Neutrino direction needs to be reconstructed
3) Matter effects from crossing mantle and core can be highly relevant (good for mass ordering
4) Neutrino flavor is dominated by ν_e and ν_μ



Reactor Neutrinos



- Production:

$n \to p^+ e^- \bar{\nu}_e$: Beta decays are the key production, though not so simple

$(A, Z) \to (A, Z+1)\, e^- \bar{\nu}_e$ or more complicated to to chain reaction.

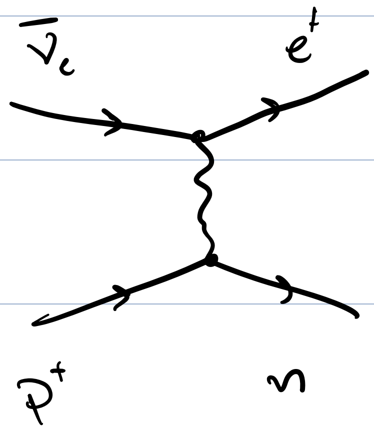
Predicting reactor neutrino fluxes is not easy.

Neutrino energies are given by β-branches, typically around MeV.

- Detection:

Oscillation experiments need to detect flavor, so inverse beta decay it is

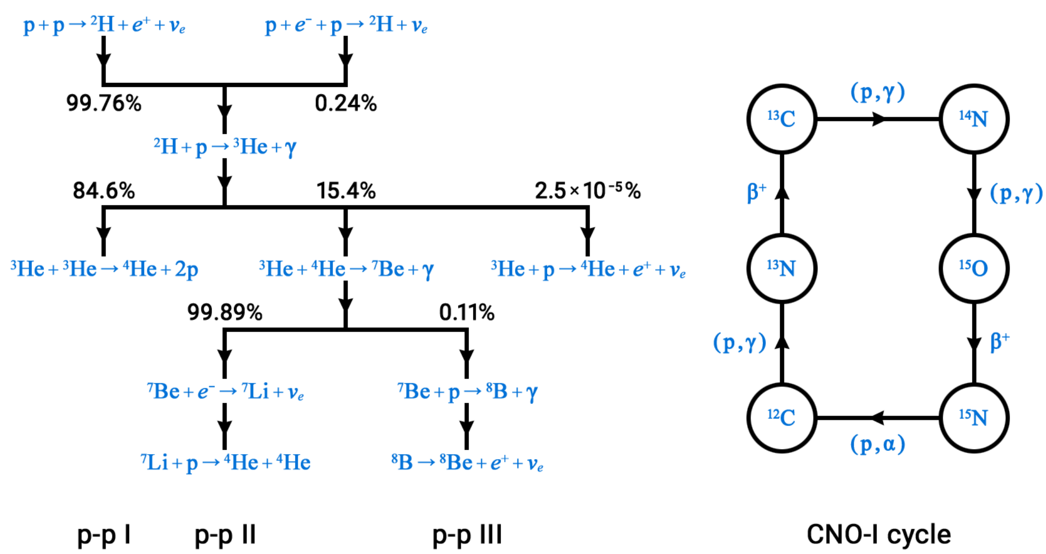
$$\bar{\nu}_e + p^+ \to e^+ + n$$



⊛ Calculate the IBD cross section. See 2206.13449 for details

Solar Neutrinos

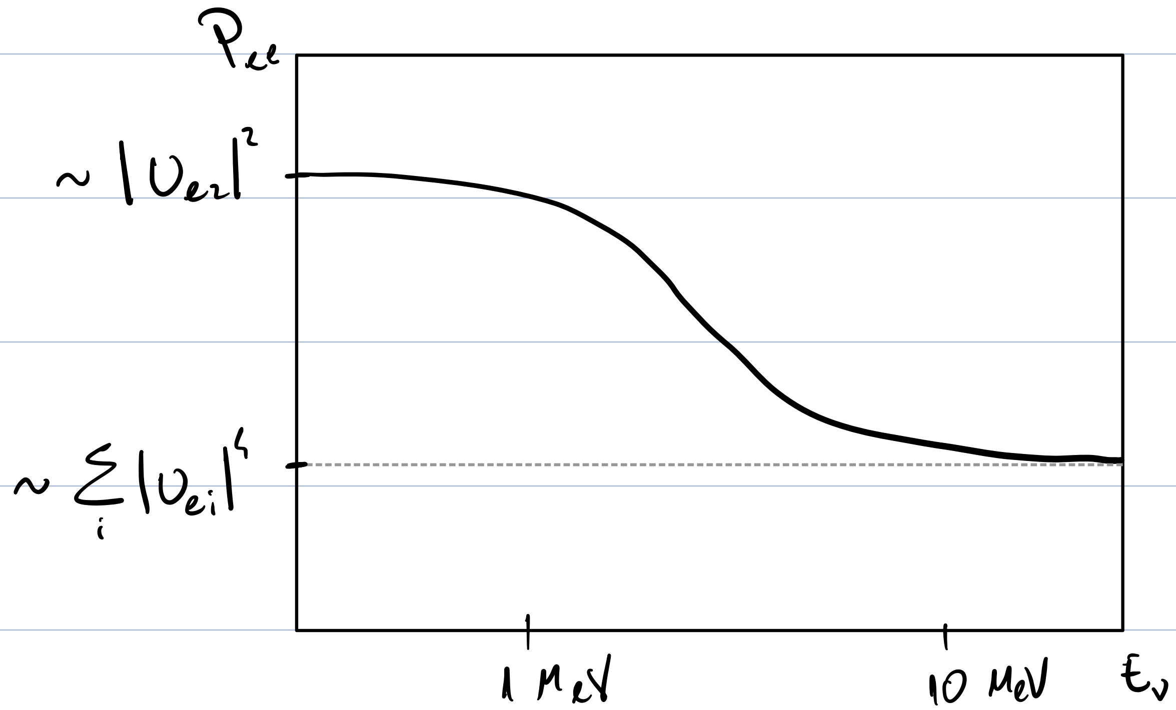
Neutrinos are produced in reactions in the Sun



Matter effects are *very* important: adiabatic transition

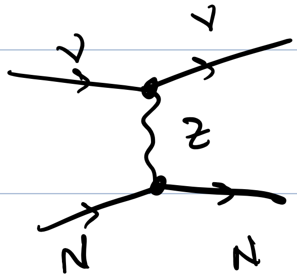
Low energy solar neutrinos are produced as ν_2 in matter and remain as ν_2 as they exit the Sun.

High energy neutrinos oscillate as in vacuum.



Coherent Neutrino-Nucleus Scattering (CEνNS)

say it like "sevens"

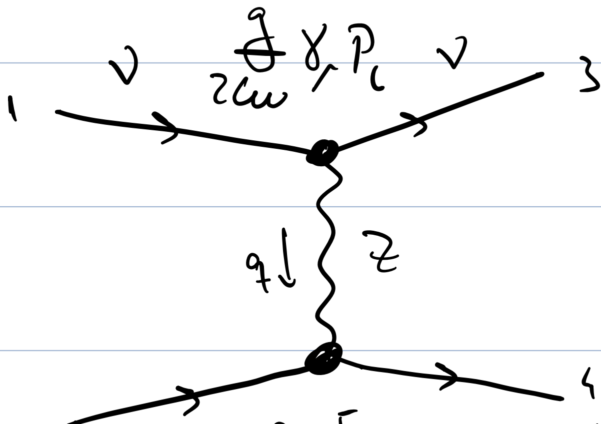


Effective Lagrangian

$$\mathcal{L} \supset \frac{g}{2c_W}\bar{\nu}\gamma_\mu P_L \nu + \frac{g}{c_W}\bar{n}\gamma_\mu\left(-\frac{1}{4}\right)n$$

$$+ \frac{g}{c_W}\bar{p}\gamma_\mu\left(+\frac{1}{4} - s_W^2\right)p + \dots$$

At low energies, neutrinos scatter coherently with protons and neutron in the nucleus. The $\gamma_\mu\gamma_5$ operator couples to spin, so we drop it (subleading). The nucleus "weak charge" is the sum of protons plus neutrons.



$$\frac{g}{c_W}\left[N_p\left(\frac{1}{4} - s_W^2\right) + N_n\left(-\frac{1}{4}\right)\right]\gamma_\mu F(q^2)$$

Form factor that describes the degree of coherence

$$\mathcal{A} = \left(\bar{u}_3 \frac{g}{2c_w}\gamma_\mu P_L u_1\right) \underbrace{\frac{g^{\mu\nu} - q^\mu q^\nu/M_Z^2}{q^2 - M_Z^2}}_{|q^2| \ll M_Z^2 \;\Rightarrow\; g^{\mu\nu}/M_Z^2} \left(\bar{u}_4 \frac{g f}{c_w}[\dots]\gamma_\nu u_2\right)$$

$$M_Z = M_W/c_w \quad \Rightarrow \quad g^{\mu\nu} c_w^2/M_W^2$$

$$G_F = \frac{\sqrt{2}}{8}\frac{g^2}{M_W^2}$$

$$\mathcal{A} = \underbrace{\frac{g^2}{4c_w^2}\,\frac{c_w^2}{M_W^2}}_{\to\ \sqrt{2}\,G_F} F\left[N_p\left(1/2 - s_w^2\right) + N_n\left(-1/2\right)\right] \times g_{\mu\nu} \cdot$$

$$\times \left(\bar{u}_3 \gamma_\mu P_L u_1\right) g^{\mu\nu} \left(\bar{u}_4 \gamma_\nu u_2\right)$$

$$\frac{1}{2}|\mathcal{A}|^2 = G_F^2\,[\dots]^2 F\, \mathrm{Tr}\left\{\not{p}_3 \gamma_\mu P_L \not{p}_1 \gamma_\alpha P_L\right\}$$

$$\times\ \mathrm{Tr}\left\{(\not{p}_4 + m_N)\gamma_\nu(\not{p}_2 + m_N)\gamma_\beta\right\} g^{\mu\nu} g^{\alpha\beta}$$

$$\Rightarrow \boxed{\frac{d\sigma}{dT} \simeq \frac{G_F^2}{4\pi} m_N F^2(q^2)\left[Z(4s_w^2 - 1) + N\right]^2 \left(1 - \frac{m_N E_r}{2E_\nu^2}\right)}$$

Current Status of Oscillation Physics

θ_{13}: Reactor neutrinos ~ km baseline Daya Bay RENO

θ_{12}: Solar neutrinos (see plot above) Super-K SNO Borexino

θ_{23}: Atmospheric and beam Super-K IceCube/DeepCore NOvA + T2K ($\nu_\mu \to \nu_\mu$)

δ_{CP}: Poorly known, accelerator NOvA T2K ($\nu_\mu \to \nu_e$)

Δm^2_{21}: KamLAND (reactor @ 200km) + Solar

Δm^2_{atm}: Reactor (km) + accelerator + atmospherics

↳ Mass ordering still open question

Future

DUNE + HK → atmospheric parameters

JUNO → Solar parameters

Observatories → atmospheric ν

Astro Neutrinos

Too vast subject to be covered in one lecture. Let's focus on one specific topic: ultra high energy ($\gg 1\,\mathrm{TeV}$) neutrinos. Here are the main points

1) Neutrinos offer a great probe of extreme environment. Charged particles are deflected by magnetic fields, photons may scatter on CMB.

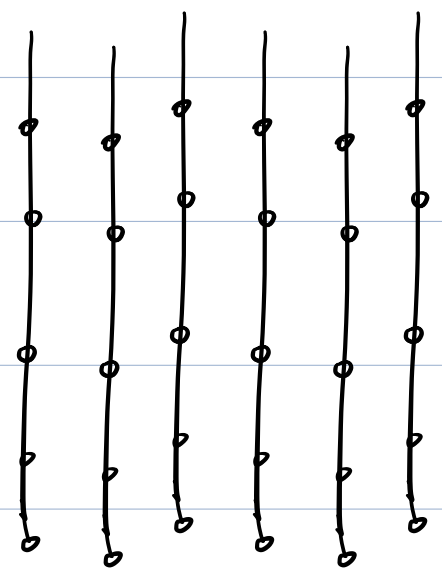
2) We do not understand the mechanism of acceleration of ultra high energy cosmic rays. Neutrino flavor could help us understanding mother particle acceleration (e.g. $\pi^+ \to \mu^+ \nu_\mu \to e^+ \bar{\nu}_\mu \nu_e \nu_\mu$)

3) Cosmic ray physics above the GZK cutoff - CRs above ~ 10^{19} eV should interact with CMB sourcing a neutrino flux

4) Extreme physics can be window to BSM

Let's focus on the phenomenology.

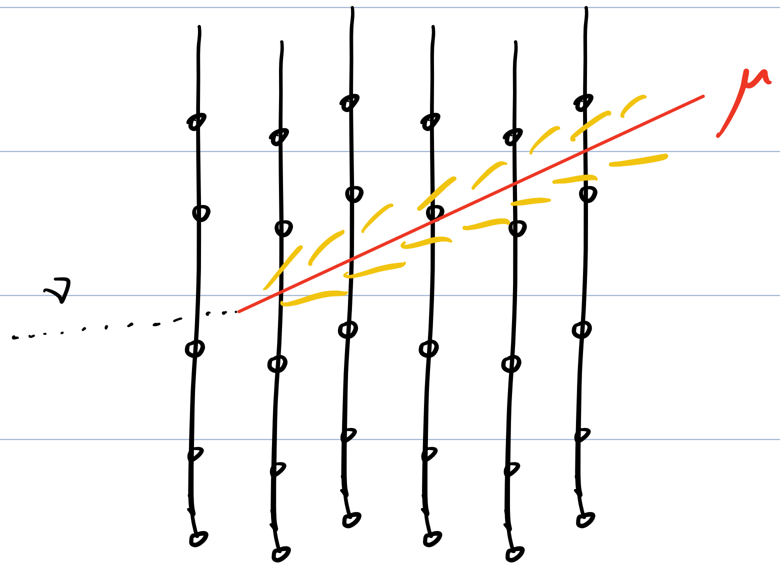
Neutrino Observatories (IceCube, KM3NeT, P-ONE, ...)



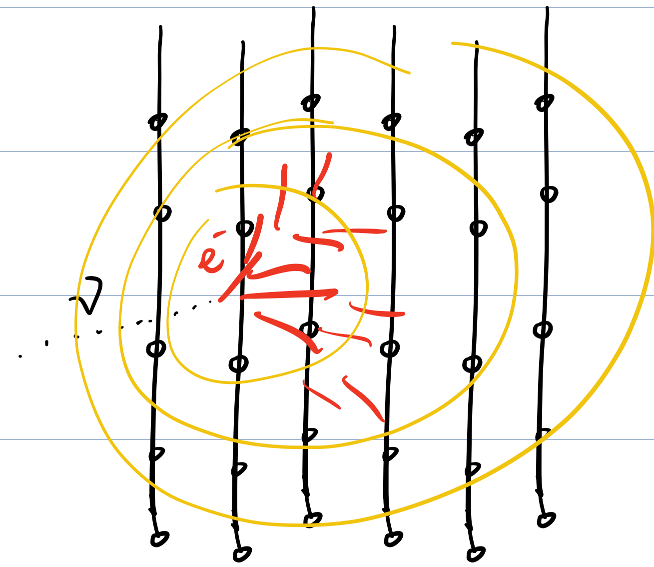
Typical setup is a number of strings below ice or water with PMTs attached to detect Cherenkov light.

Experimental observables

Tracks



Showers

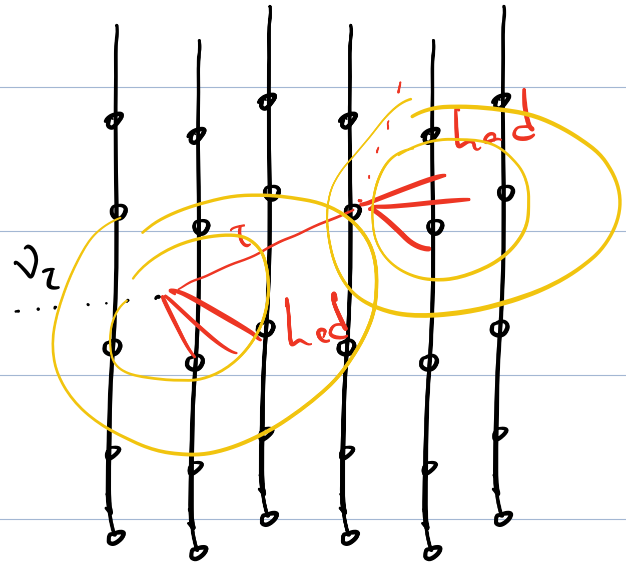


Track vs. shower allow for some PID.

Tracks: muons only; great pointing

Showers: electrons, photons, hadrons; some pointing

Some particles could be long lived enough to lead to two distinct signals: double bangs, typical for ν_τ with $E_{\nu_\tau} \gtrsim 100$ TeV.



These allow for some flavor ID. But how do we predict the flavor of UHE ν at detection?

Imagine you produce some flavor composition at the source, f_α. For example π decays could lead to $f_\alpha = 1/3 : 2/3 : 0$ $(\nu_e : \nu_\mu : \nu_\tau)$. As they propagate oscillations average out $(\nu_1 : \nu_2 : \nu_3)$

$$f_i^{prop} = \sum_\alpha |U_{\alpha i}|^2 f_\alpha .$$

But at detection, we detect flavor states, that is we project $\langle \nu_\beta | \nu_i \rangle$, so the flavor composition at detection is

$$\boxed{f_\beta^{det} = \sum_{\alpha, i} |U_{\beta i}|^2 |U_{\alpha i}|^2 f_\alpha .}$$

Notice that, because the mixing angles are large, $|U_{\beta i}|$ are all approximately of the same size, except for $|U_{e3}|$, and therefore at detection we tend to have a democratic admixture of flavors.

⊛ Make a flavor triangle plot for different source compositions ($1/3 : 2/3 : 0$, $1:0:0$, $0:1:0$, $x : 1-x : 0$)