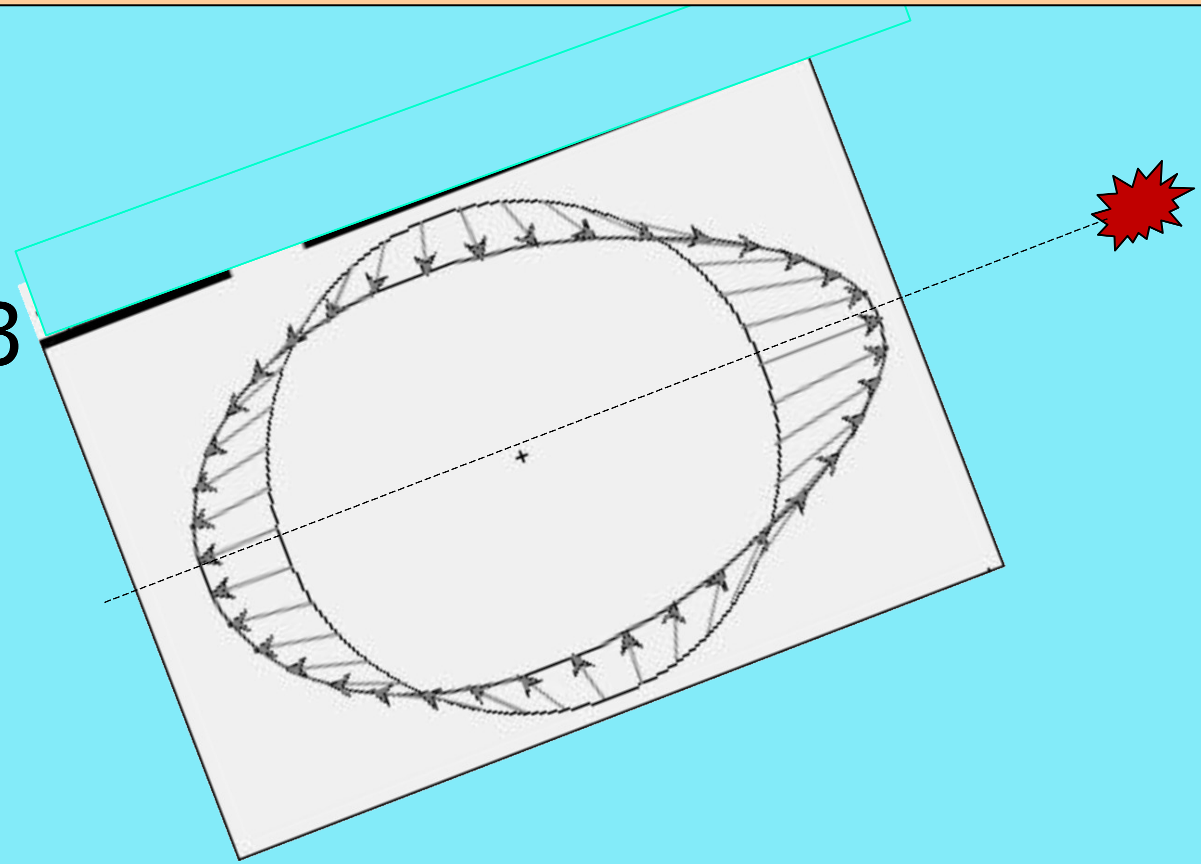
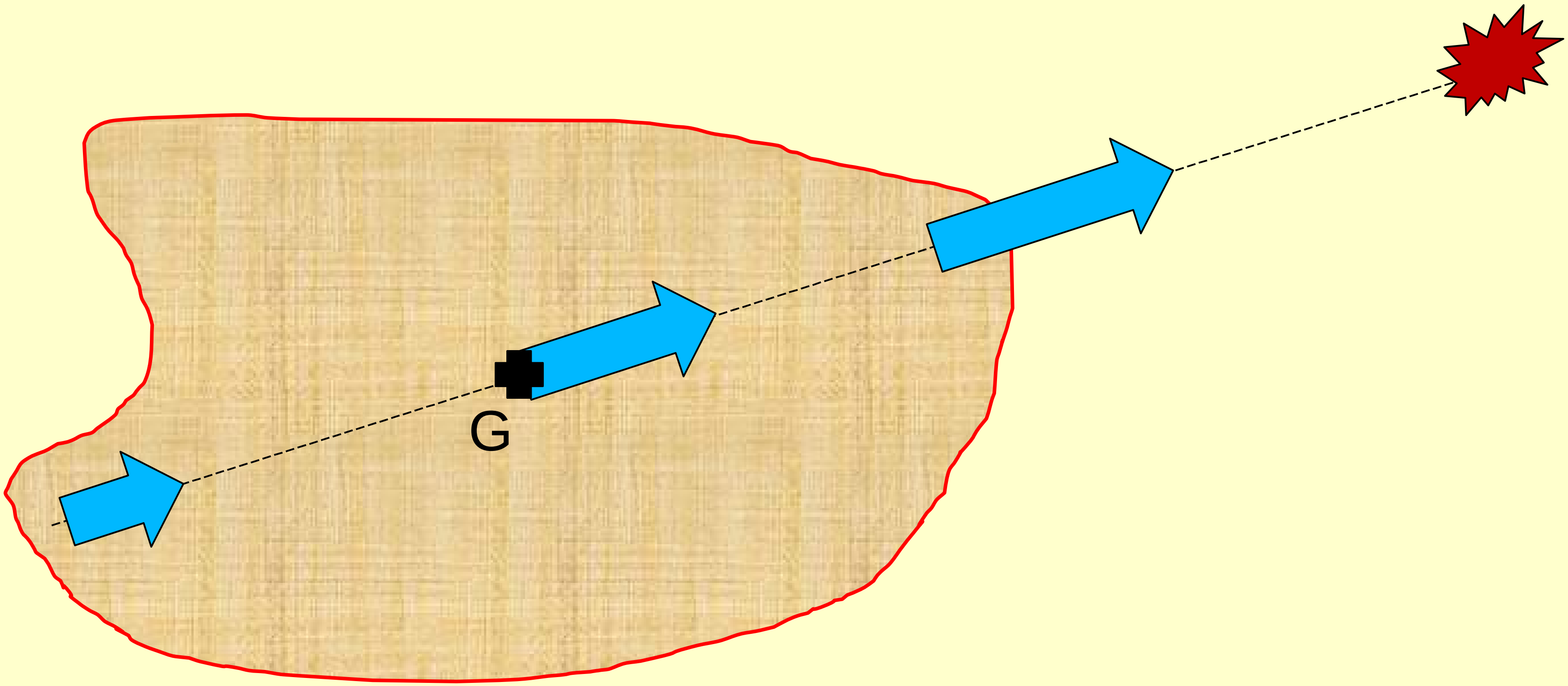
STAR-PLANET TIDAL INTERCTIONS

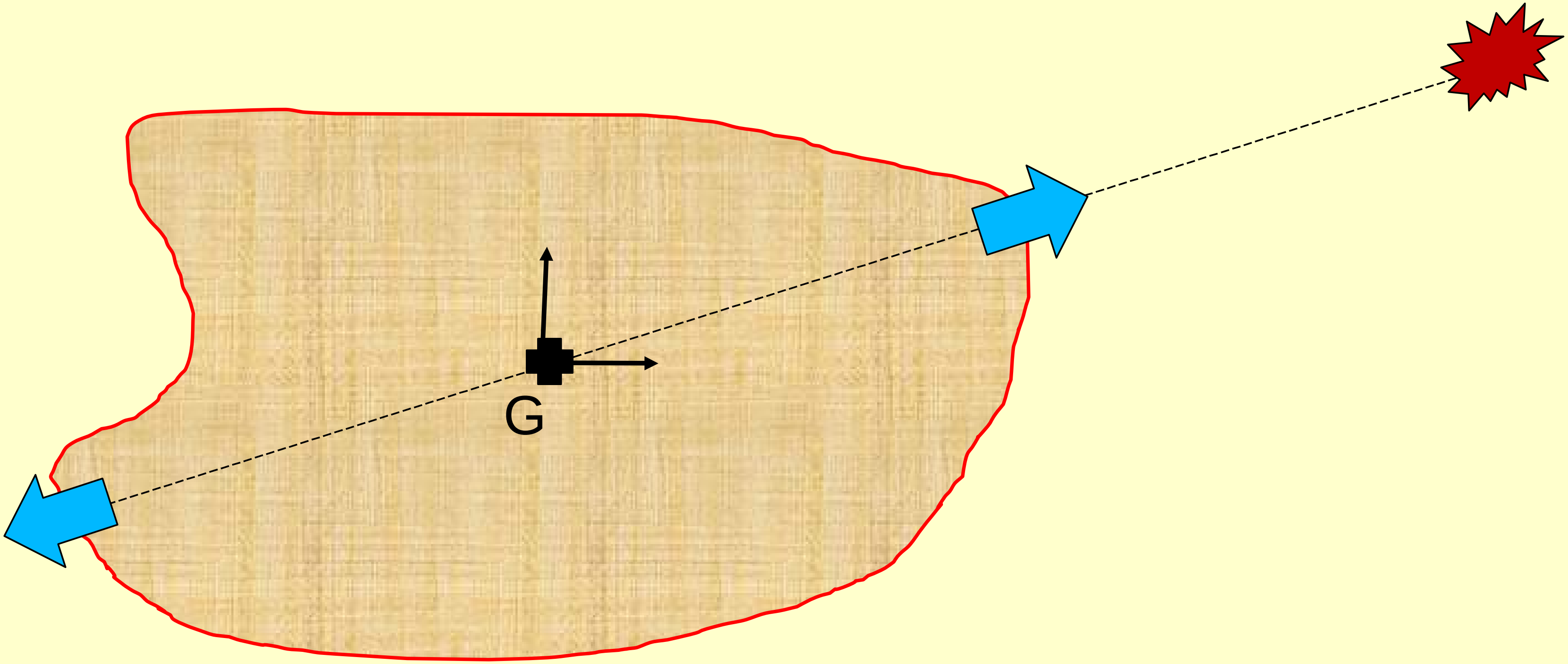
Sylvio Ferraz-Mello
IAG-USP
SAIFR-ICTP Jul.2023



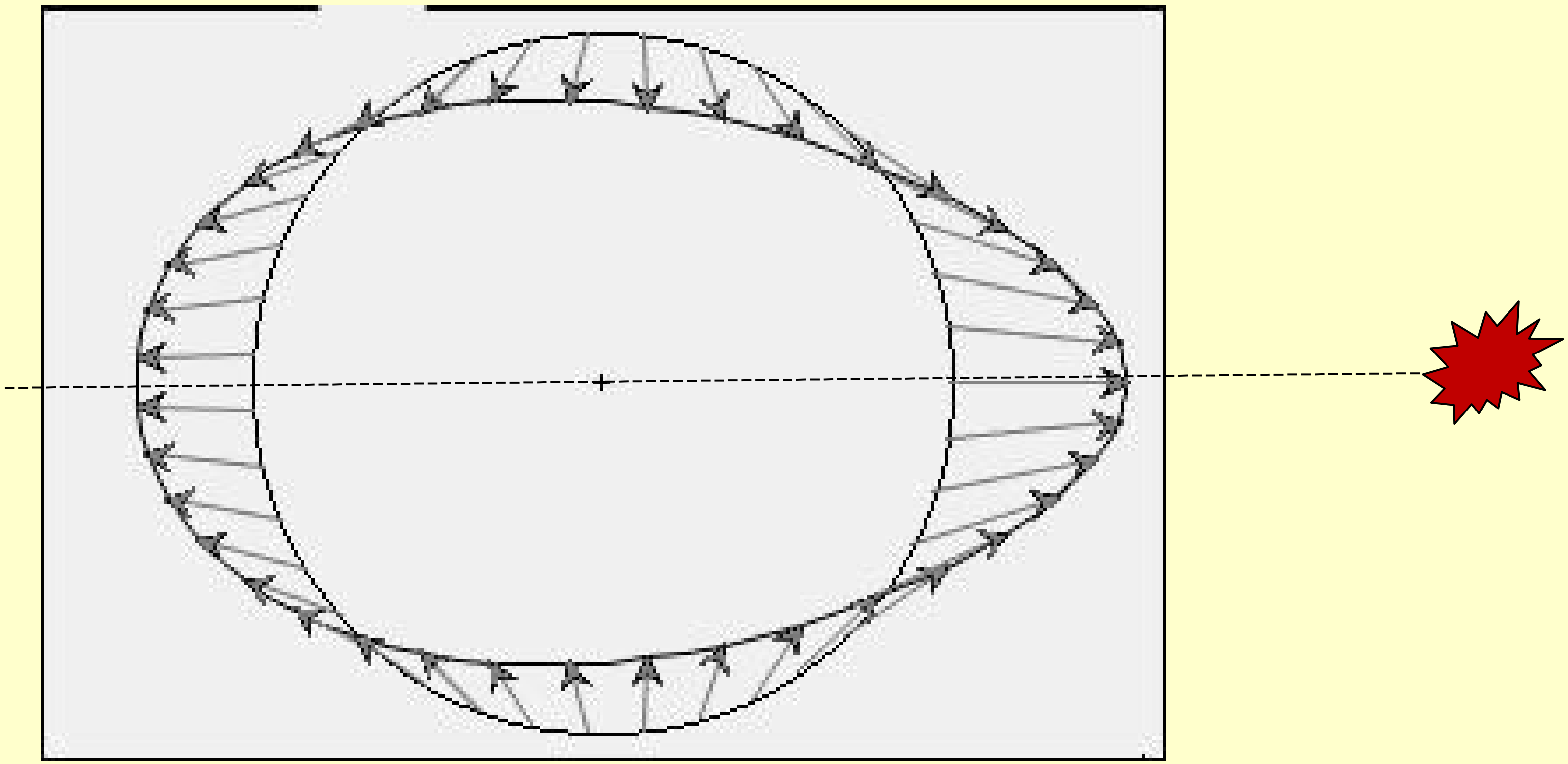
Gravitational forces on extended bodies (TIDAL forces)



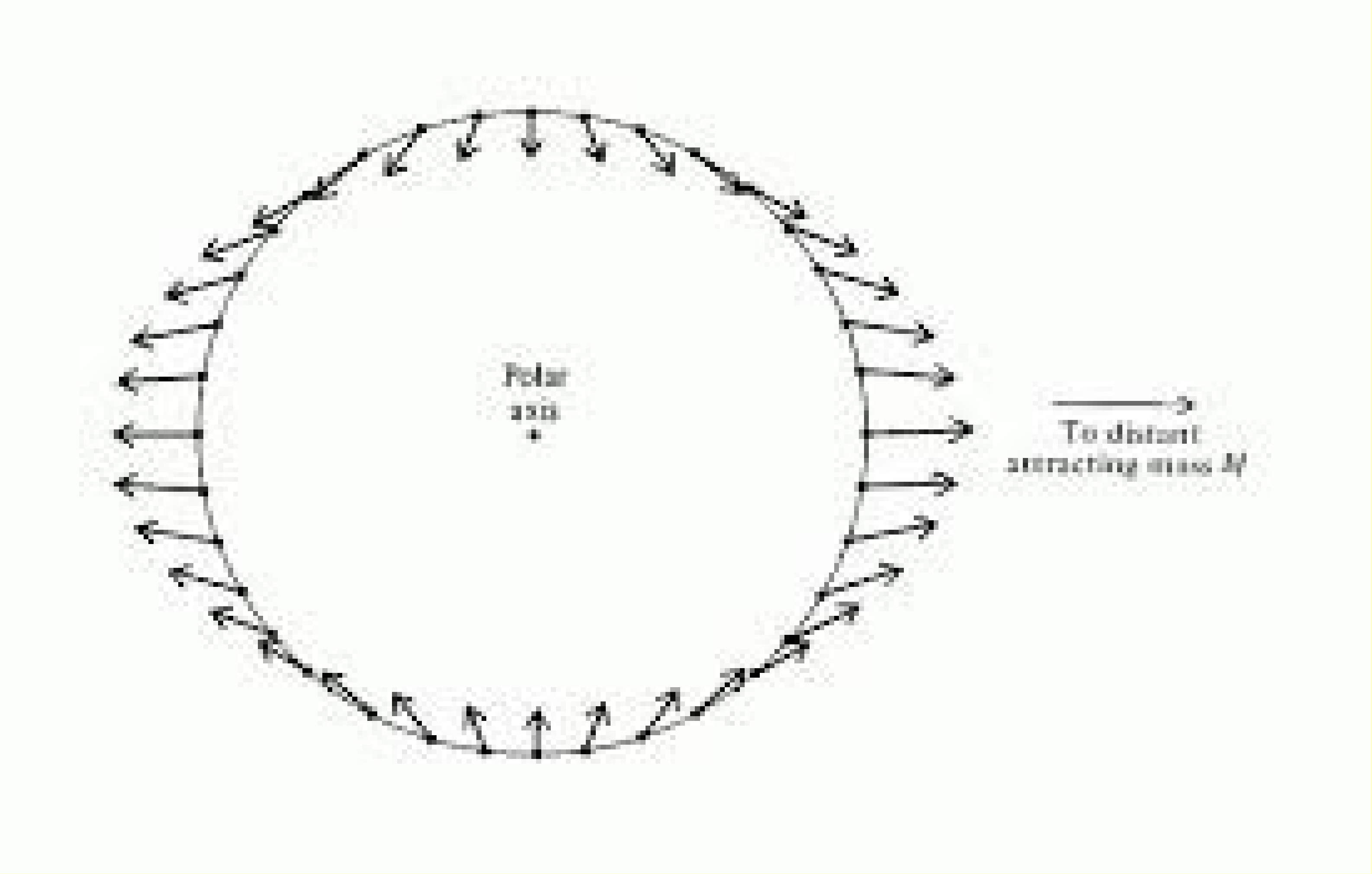
TIDAL FORCES
(differential)



Deformation of a spherical body due to tidal forces

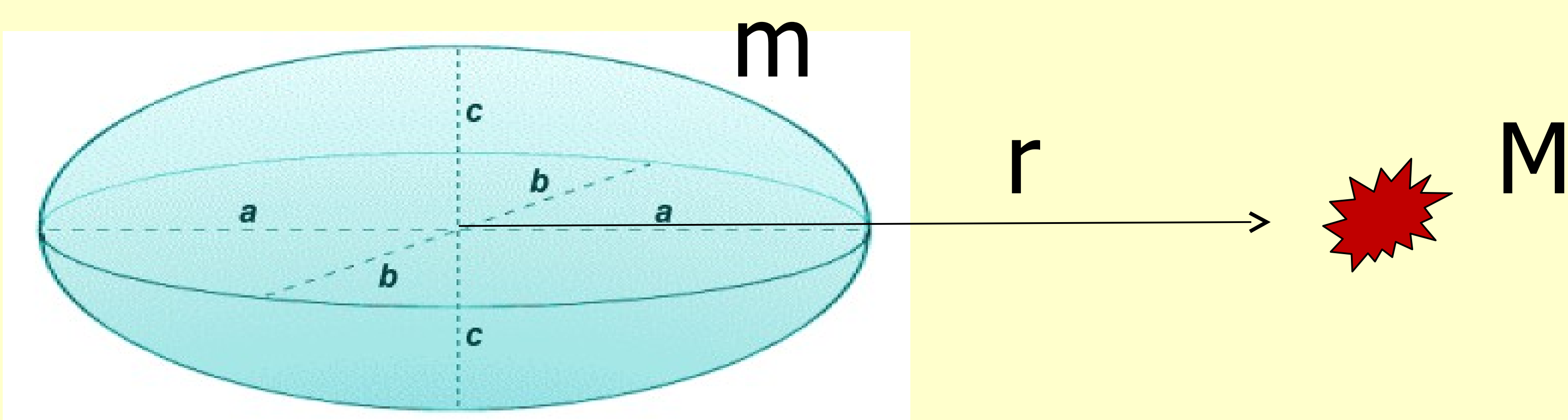


If the distance to the external body is much arger than the dimmensions of the body, the deformation tends be symmetrical.
Classsical figure: The Roche Ellipsoid.



If the rotation is not takan into account ===>

Jeans spheroid



Equatorial prolateness

$$\epsilon_\rho = \frac{15}{4}\mathcal{H}_n\left(\frac{M}{m}\right)\left(\frac{R_e}{r}\right)^3$$

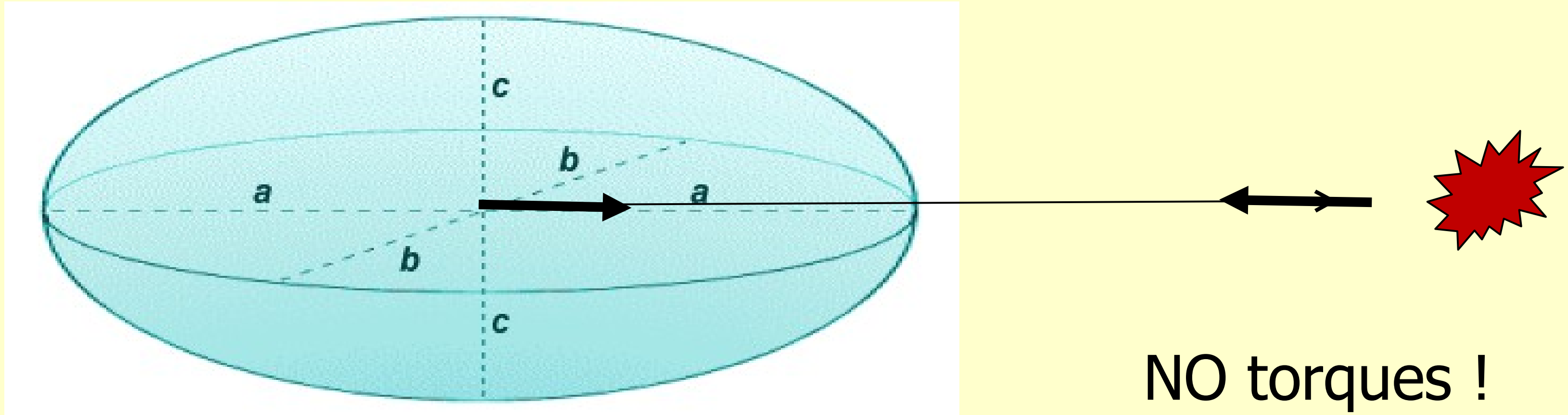
$$R_e = \sqrt{ab}$$

$$\epsilon_\rho = \frac{a}{b} - 1.$$

H_n = Clairaut number

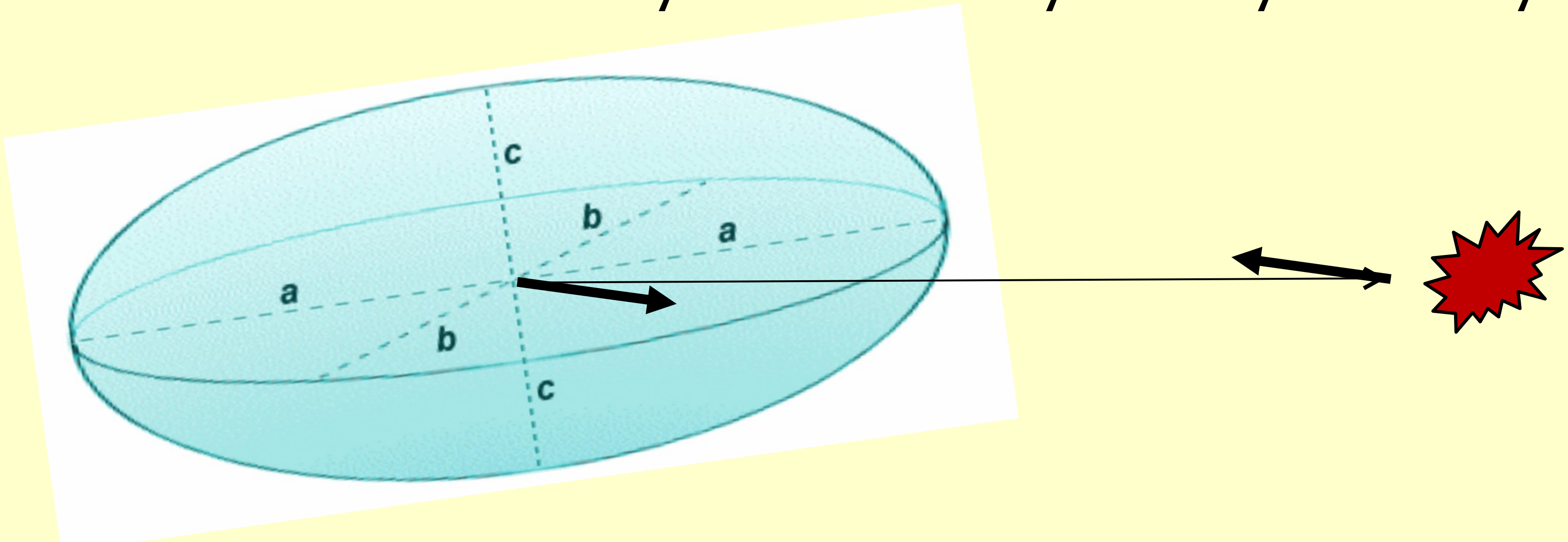
Depends on the mass concentration

Static tide: Symmetrical



NO torques !

The body is not a perfect fluid; It has viscosity.
The relative rotation of the body breaks the symmetry of the system.



Main consequences

The loss of mechanical energy - (transformed into heat due to the tidal stretching of the bodies)

Exchange of angular momentum between the relative orbital motion of the bodies and their rotations.

Examples in the solar system:

- The EARTH-MOON system

The MOON is synchronized: $P_{orb} = P_{rot}$

On the EARTH the duration of the day grows by 2.395 ms/cy

The orbit changes:

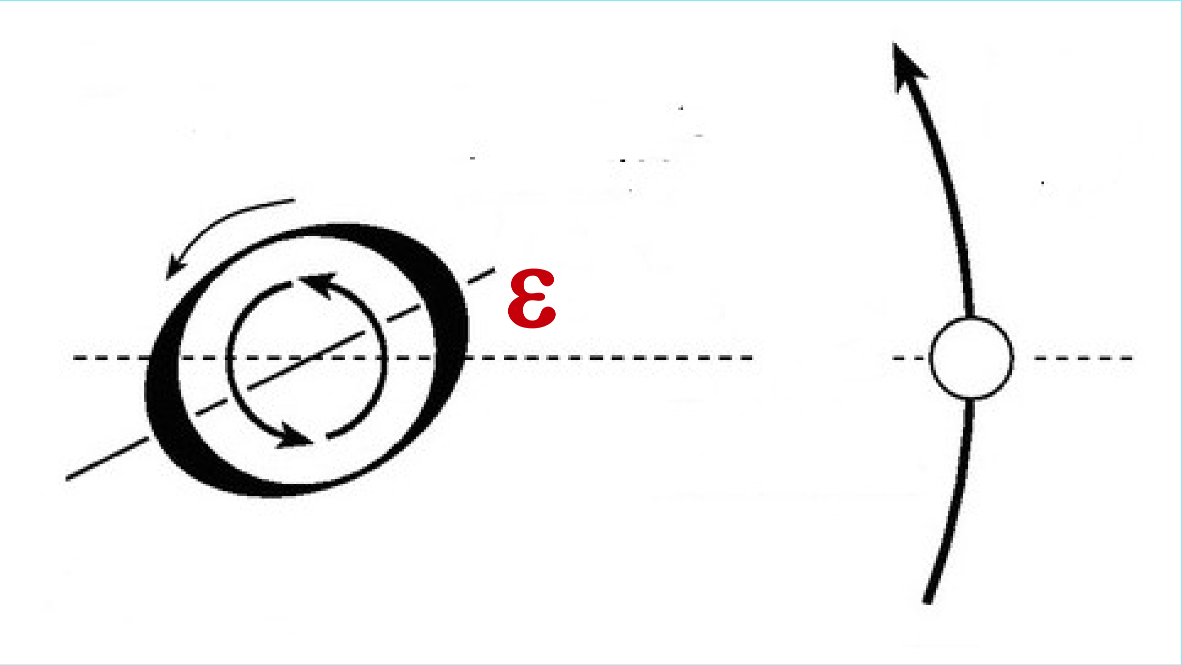
$$da/dt = 38.30 \pm 0.08 \text{ mm/year}$$

$$de/dt = (1.50 \pm 0.10) \times 10^{-11}\text{/year}$$

- The internal oceans in the satellites of Jupiter and Saturn

The first theory was established by G.E.Darwin (1879-1880)

Darwin's **ad hoc tidal lag** $\varepsilon = \nu\tau$



$\nu = 2\Omega - 2n$ (semi-diurnal frequency)

$\tau =$ time lag (constant)

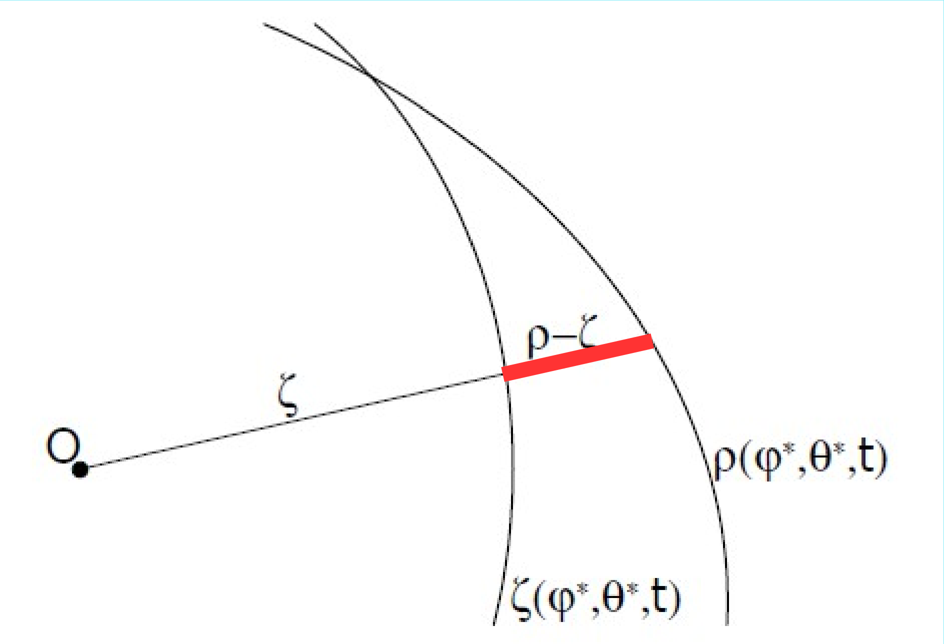
CREEP TIDE Theory

Aim: To get rid of the ad-hoc lags

To obtain the tidal evolution equations from physical laws

S.Ferraz-Mello,
DDA 2012 (astro-ph 1204.3957) &
Cel.Mech.Dyn.Astron. **116**, 109 (2013)

CREEP TIDE Theory



ζ …. Actual surface of the body
ρ …. Instantaneous equilibrium ellipsoid (**VIRTUAL**)

$$\dot{\zeta} = \gamma(\rho - \zeta).$$

Newtonian creep

$$\dot{\zeta} = \gamma(\rho - \zeta).$$

Newtonian creep = Approximated solution of the Navier-Stokes equation

$$\gamma = \frac{wR}{2\eta} = \frac{3gm}{8\pi R^2\eta}$$

Relaxation factor

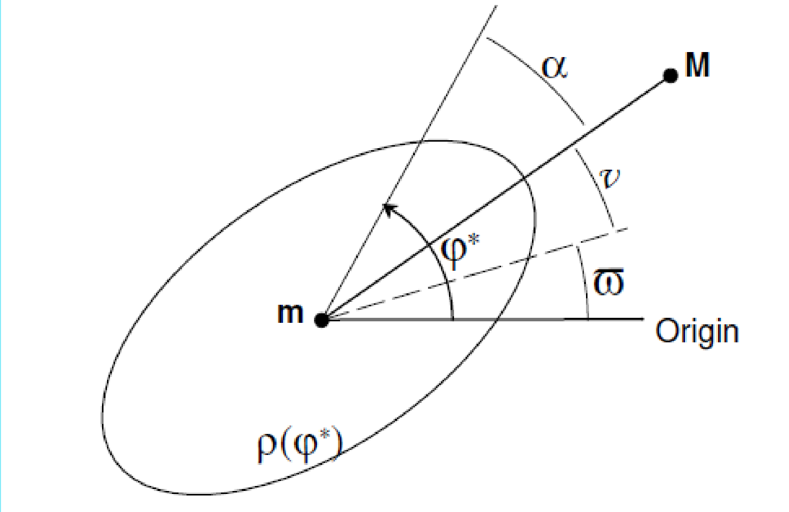
(η = viscosity)

Ref: Happel and Brener,
Low Reynolds number Hydrodynamics,
Kluwer, 1973
+ Darwin, 1879

In the more simple case (homogeneous body and coplanarity of orbit and equator)

$$\dot{\zeta} + \gamma\zeta = \gamma R' + \frac{15\gamma R \sin^2\theta^*}{8} \left(\frac{M}{m}\right)\left(\frac{R}{a}\right)^3\left(\frac{a}{r}\right)^3 \cos(2\varphi^* - 2\varpi - 2v).$$

$$\varpi = \varpi_0 - \Omega t.$$



Ordinary Differential Equation

Recipe

1. Integrate the creep differential equation

$$\dot{\zeta} + \gamma\zeta = \gamma\rho(t).$$

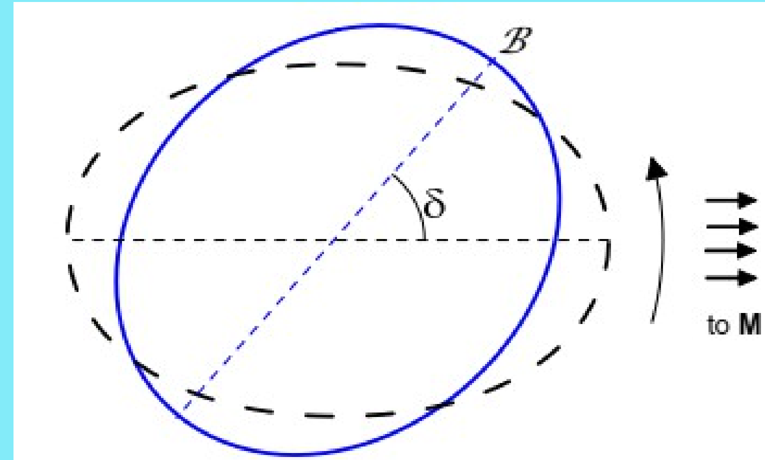
$$\zeta = e^{-\gamma t}\int_t \gamma\rho(t)e^{\gamma t}dt.$$

N.B.

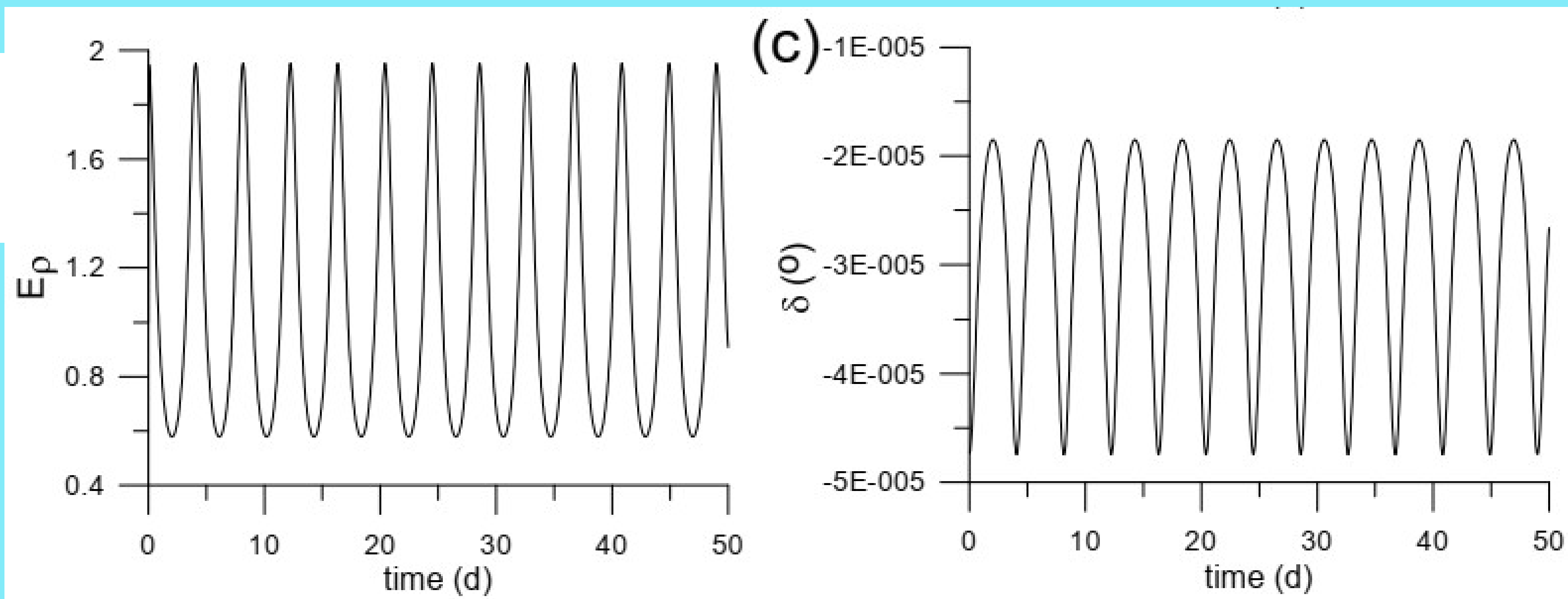
$\rho = \rho\ (\phi,\theta;\ t).$

Example

Slow rotating Sun-like star (P=30d) hosting a hot Jupiter in a 4-day eccentric orbit (e=0.2)



$$E_\rho = \frac{\mathcal{E}_\rho}{\bar{\epsilon}_\rho}.$$



a=0.05 AU
γ=50/s
(homogeneous)

N.B. δ<0

Different of the planet-satellite case

2. Compute the force and torques due to the tidal deformation of the bodies, using the solution $\zeta = \zeta\ (\phi,\theta;\ \mathbf{t})$.

3. Integrate the variational equations of the orbital elements (e.g. Gauss equations) and of the rotation of the body (angular momentum variation).

Example. From: $C\dot{\Omega} = M_2$

$$\dot{\Omega} = -\frac{3GM\overline{\epsilon}_\rho}{2a^3} \sum_{k\in\mathbb{Z}} E_{2,k} \cos \overline{\sigma}_k \sum_{j+k\in\mathbb{Z}} E_{2,k+j} \sin(j\ell + \overline{\sigma}_k),$$

Where $\sin 2\overline{\sigma}_k = \frac{2\gamma(\nu + (2-k)n)}{\gamma^2 + (\nu + (2-k)n)^2}$ and

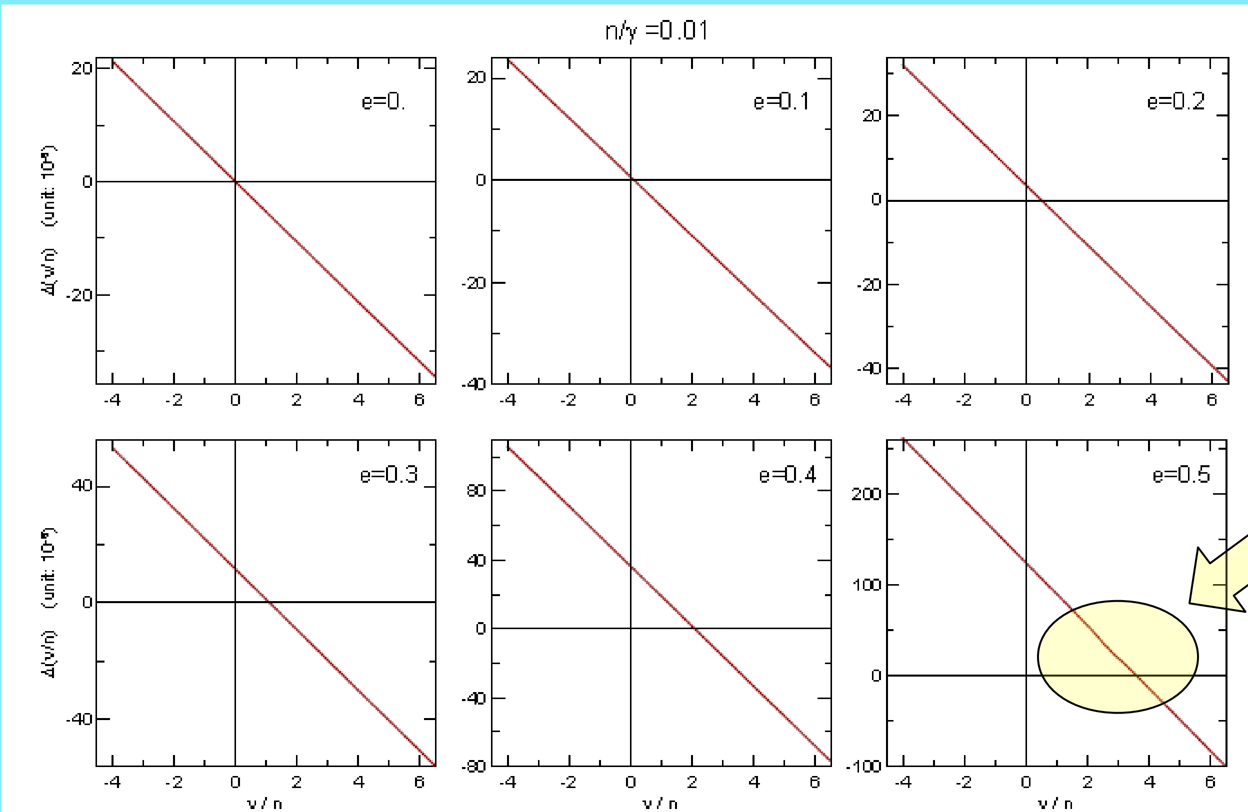
ν is the semi-diurnal frequency = $2\Omega - 2\dot{\lambda}$

Similar equations for da/dt, de/dt etc.

see Ferraz-Mello, S., Proc. Intl. Astron. Union. Vol. 364, p. 20 (2022) (updated set)

Graphs: $d\nu/dt$ vs ν (in units of n)

Case $\gamma >> n$ (ex: **stars**, hot Jupiters)



The intersections with the axis $\Omega'=0$ are attractors

N.B.

These attractors are **supersynchronous**

$$\Omega = n + 6ne^2$$

Ref: SFM, DDA 2014 (astro-ph 1204.3957) & CMDA (2015); Correia et al. A&A 2013.

Examples of near synchronous companions:

CoRoT 15b BD (m=63.3 Jup) around a F7V star

Orbital period: 3.06 d

Star rotation: 2.9 – 3.1 d

KELT 1b BD (=27.4 Jup) around a F5 star

Orbital period: 1.217 d

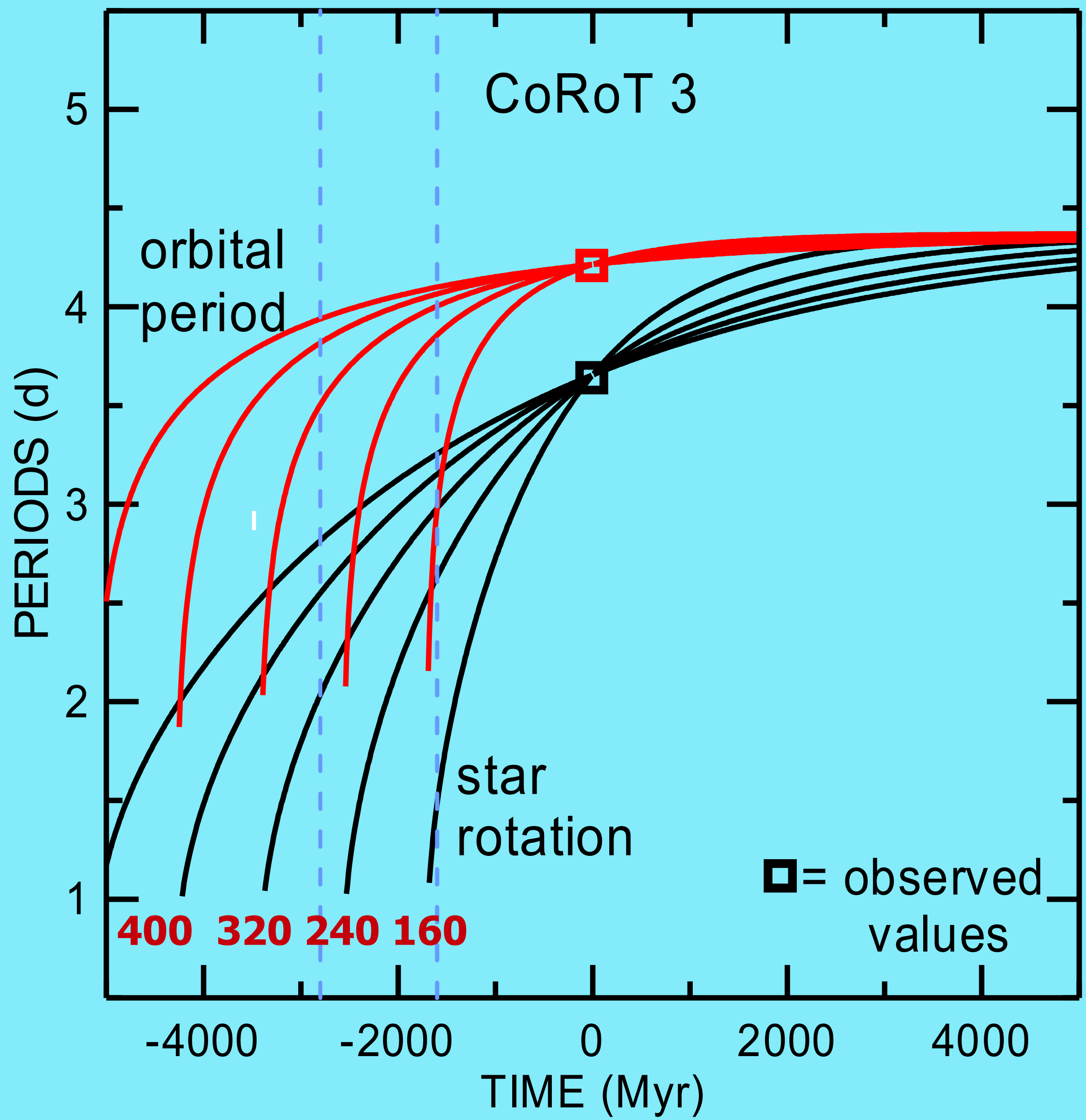
Star rotation: 1.348 ± 0.4 sin I

Example:

CoRoT-3
F3V
M~1.37 Sun
age~[1.3-2.8] Gyr

CoRoT-3 b
M~22 Jup (BD)
P~4.25 days
e~0.012

$\gamma/k_2 \sim$ [**150-300**] s^{-1}

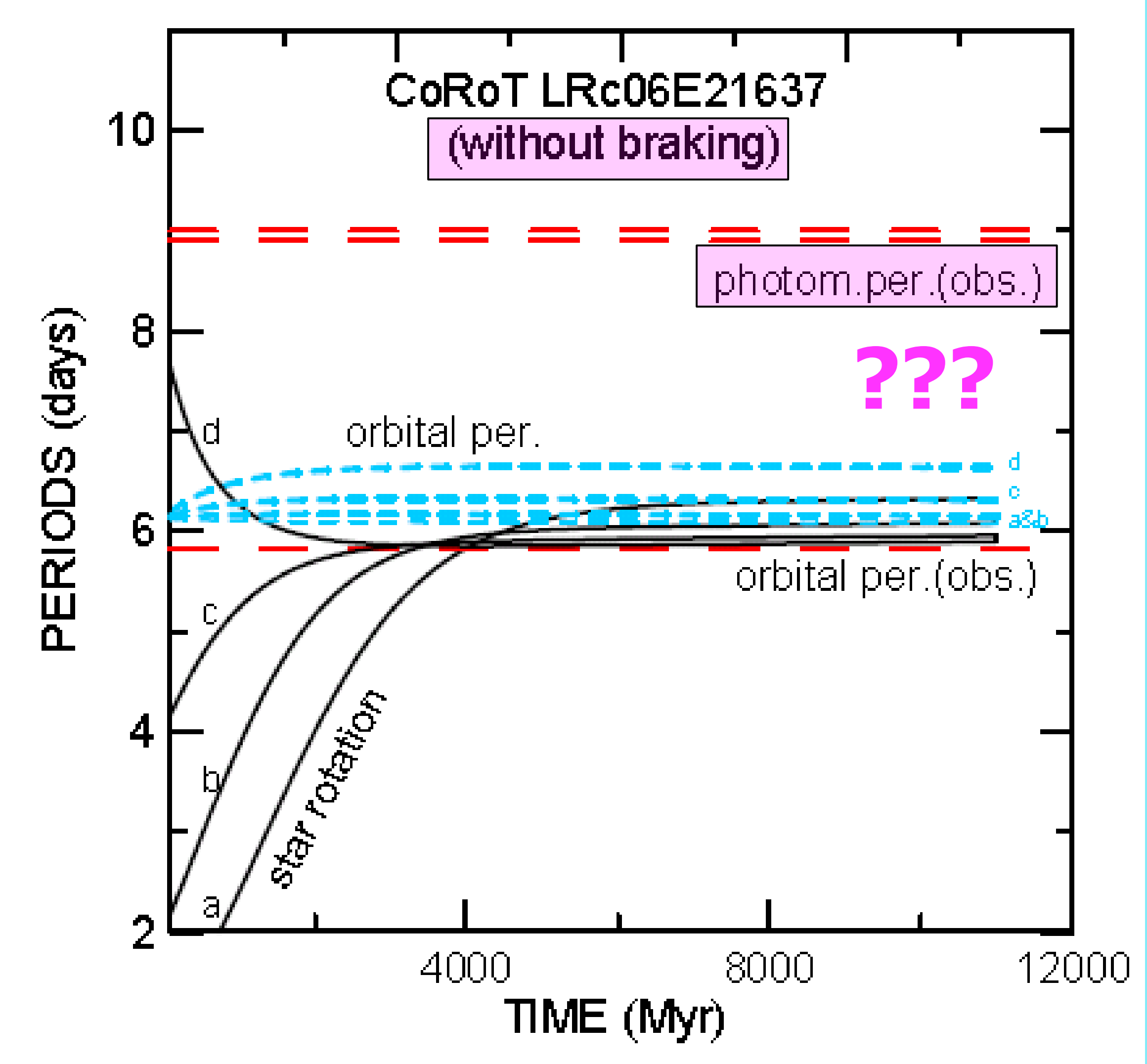


From SFM, in CoRoT legacy book, 2016.

CoRoT 33b

m = 62±5 Jup (BD)
a = 0.0624 au

star:
Sp G3V
Prot=8.946±0.05 d
age > 4.6 Gyr
(Csizmadia et al. A&A 2015)



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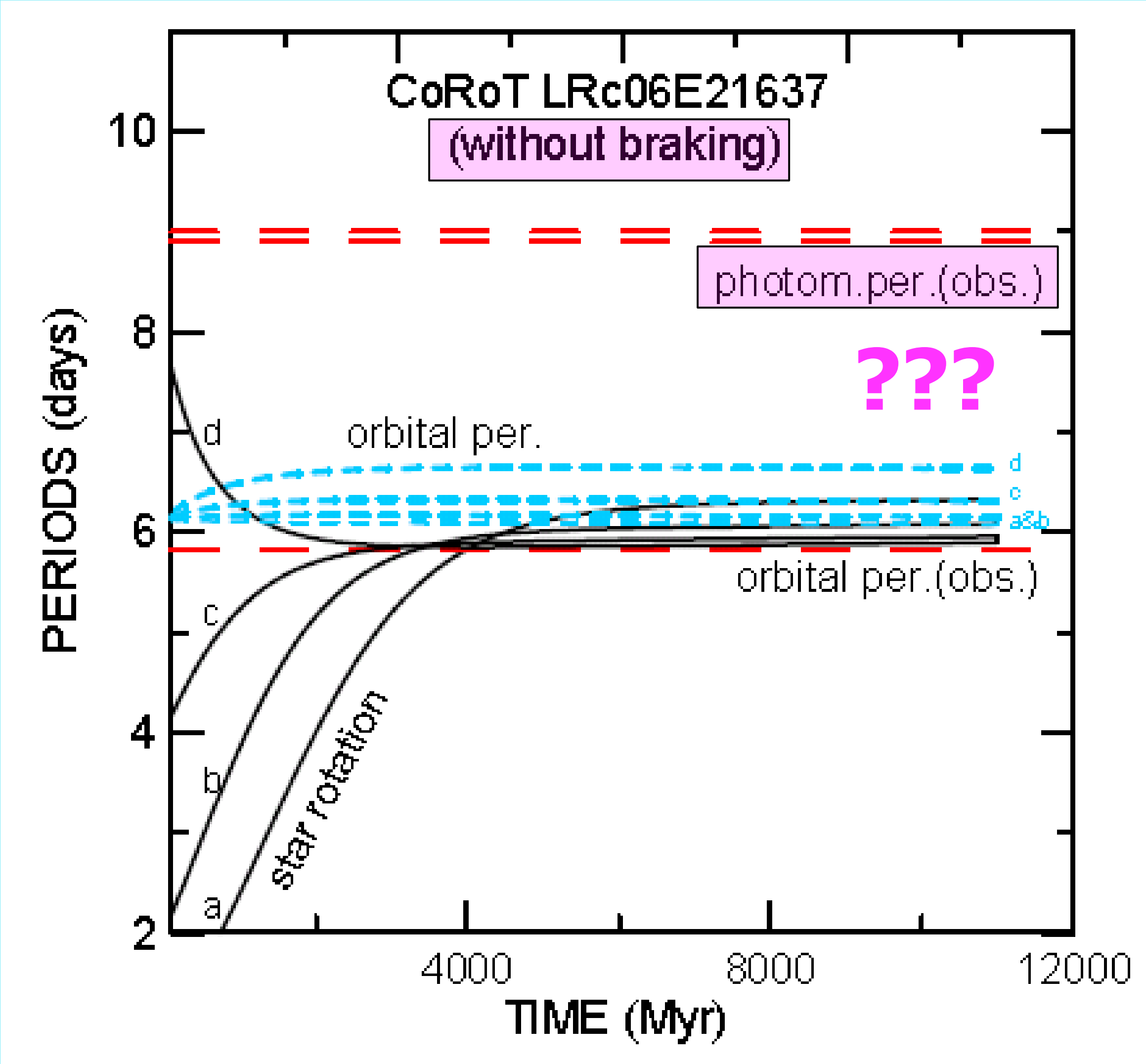
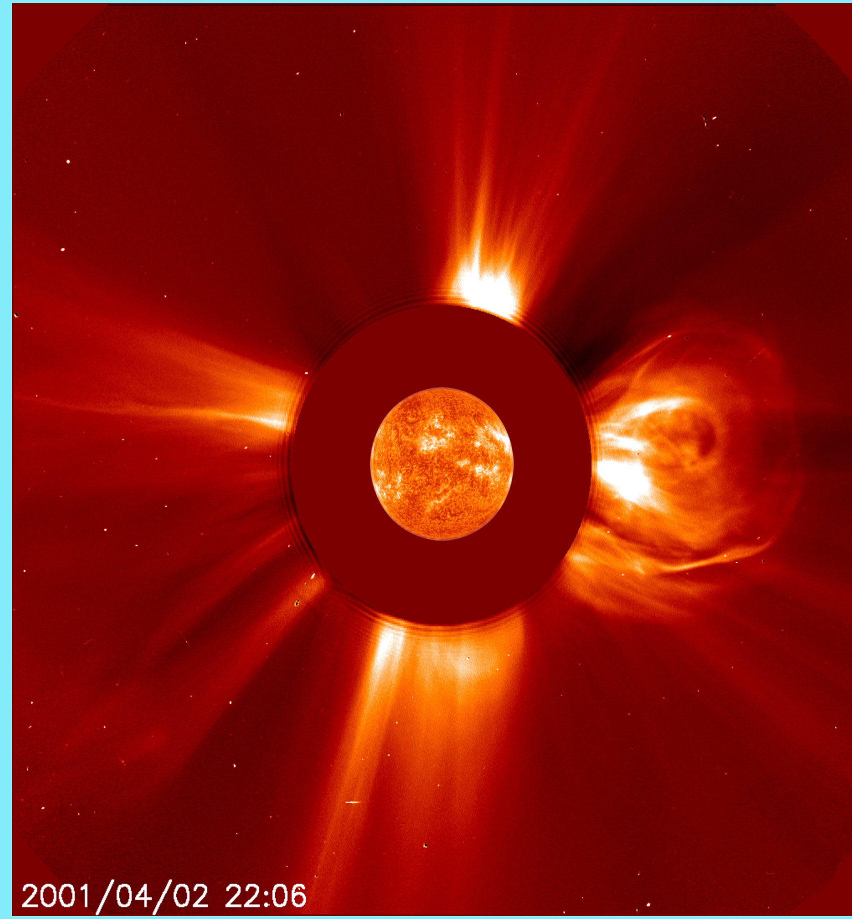


In evolution studies, the rotation of the host stars must take into account that

$$\nu = 2\Omega - 2n$$

is affected by the **Angular Momentum Leakage** due to stellar winds



$$\dot{\Omega}_s = \begin{cases} -B_W \Omega_s^3 & \text{when} \quad \Omega_s \leq \omega_{\rm sat} \\ -B_W \omega_{\rm sat}^2 \Omega_s & \text{when} \quad \Omega_s > \omega_{\rm sat} \end{cases}$$

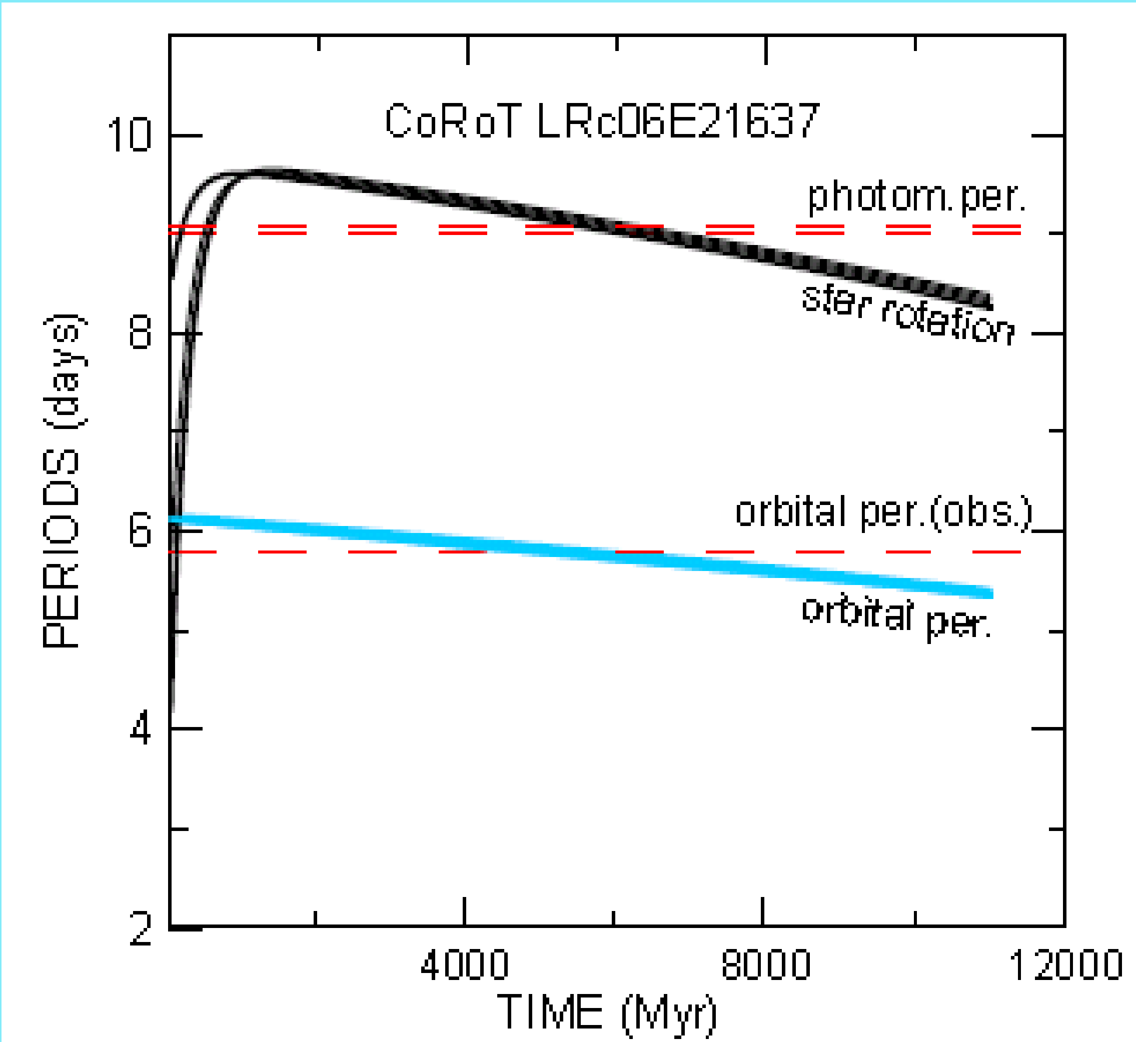
Ref: for stars with $0.5 < M < 1.1\ M_\odot$

(Bouvier et al. 1997)

For M stars see Engle & Guinan 2023

$$B_W = 2.7 \times 10^{47} \frac{1}{C_s} \sqrt{\left(\frac{R_s}{R_\odot} \frac{M_\odot}{M_s}\right)} \qquad \text{(cgs units).}$$

CoRoT 33: A paradigm



$m_{comp}=62$ jup

$m_{star}=0.86$sun

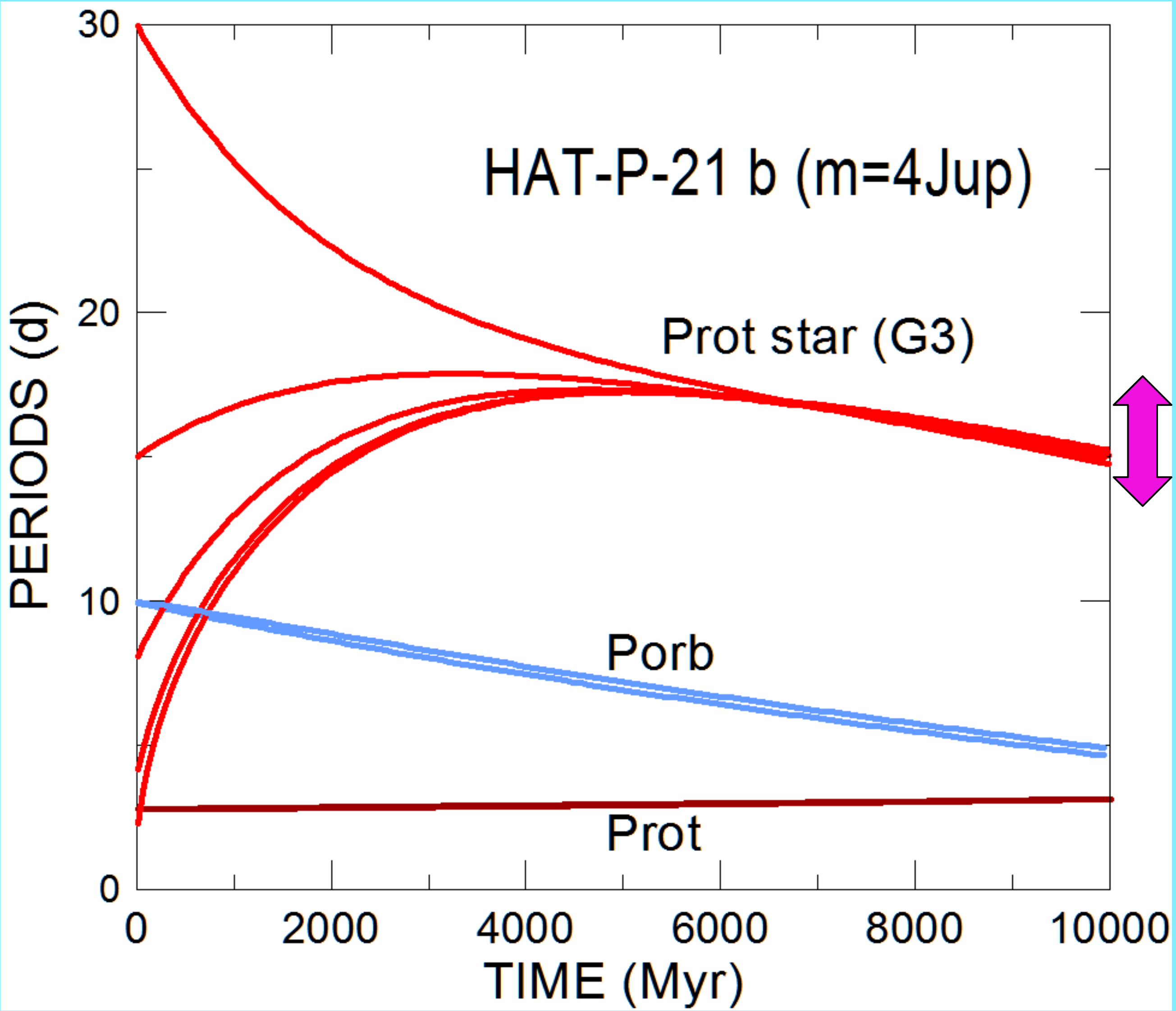
$\gamma/k_2 = 210\ s^{-1}$

age > 4.6 Gyr

After ~1 Gyr the tidal interaction is stronger than the magnetic braking and the stellar rotation accelerates

SFM et al. Ap.J. **807** (2015)

An old star: HAT-P-21



Observed value

$\gamma/k_2 = 100\ s^{-1}$

$e = 0.228$

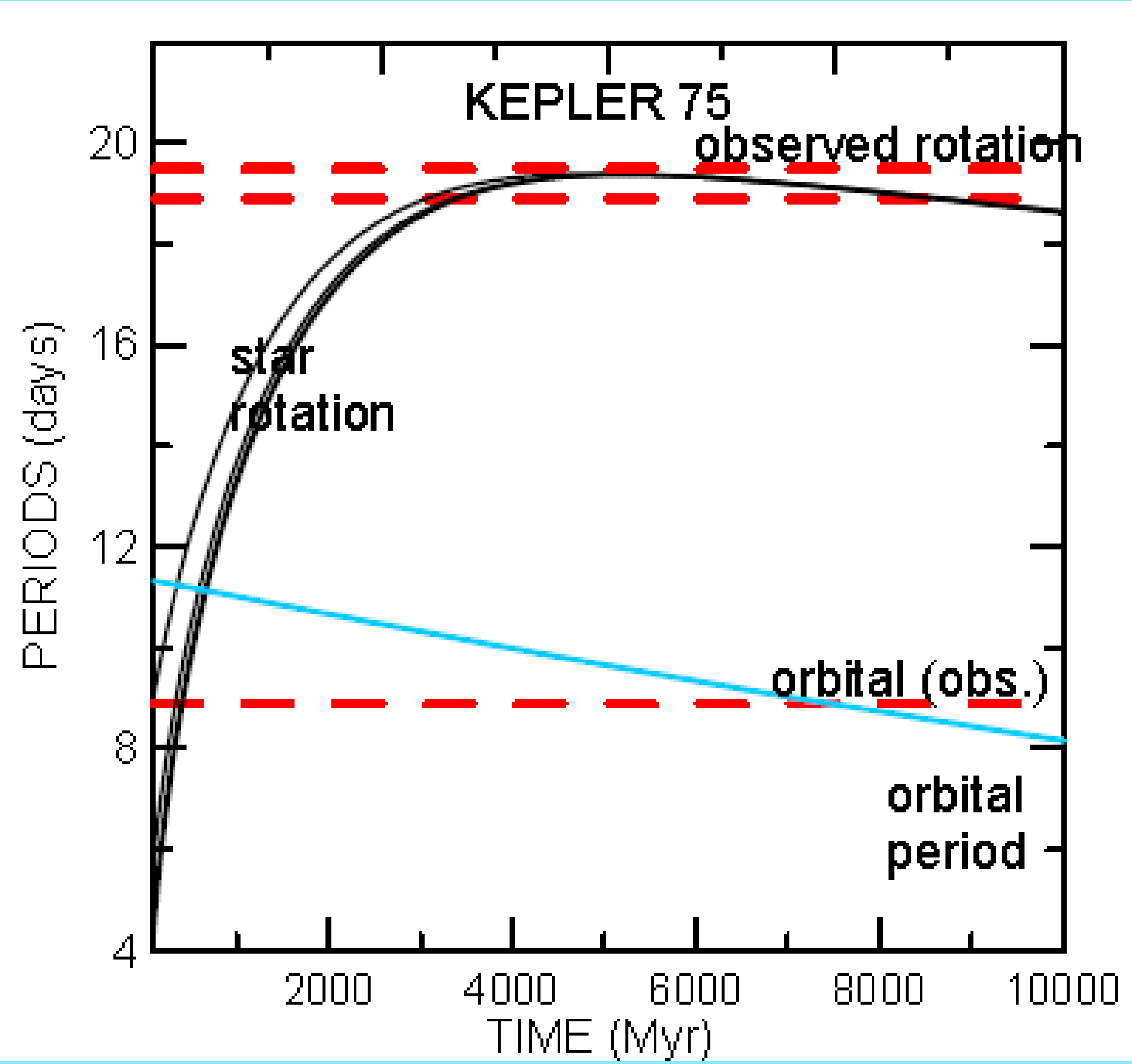
Age~10 Gyr

KEPLER 75b

m = 9.9± 0.5 Jup
a = 0.080 au

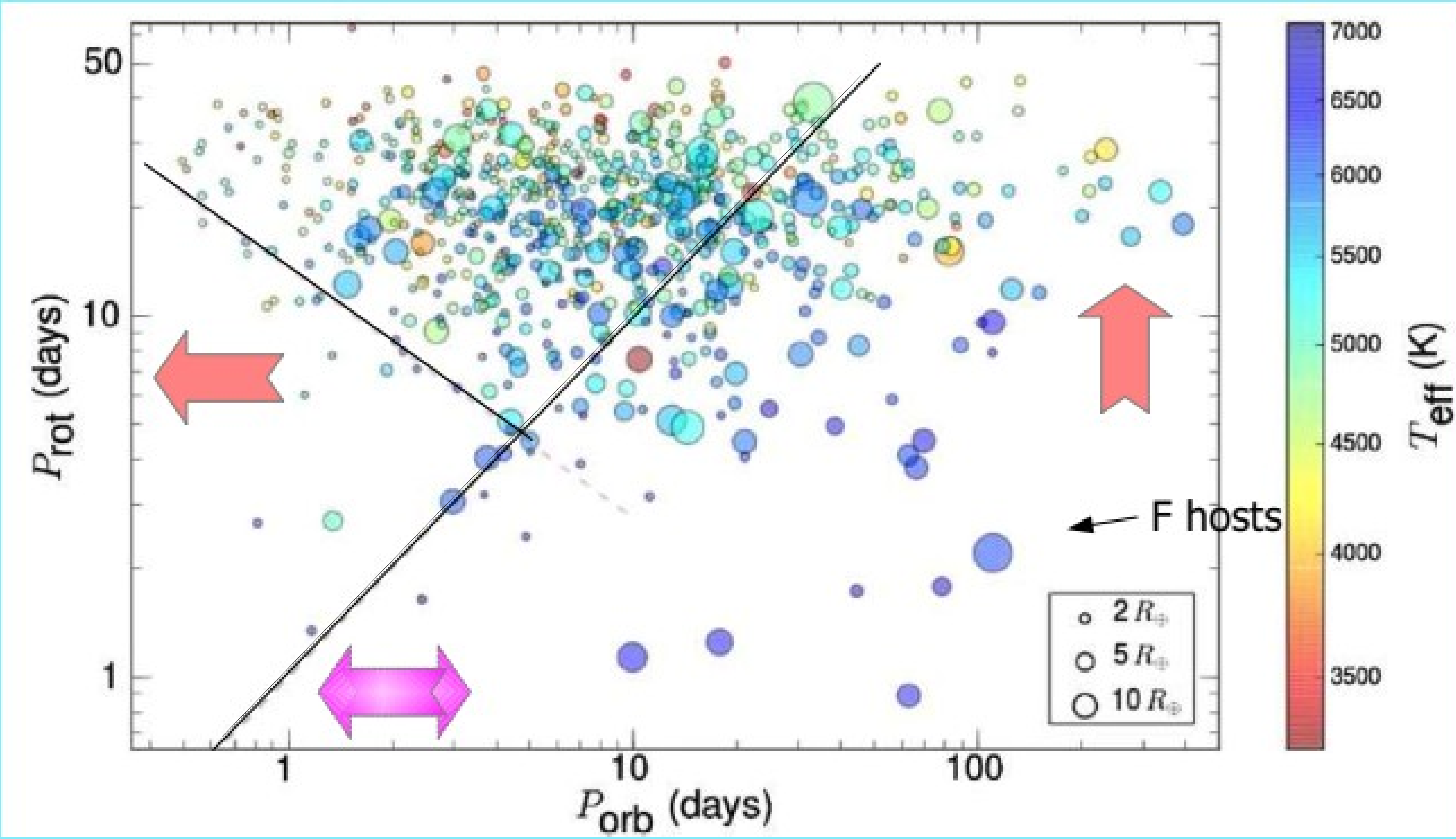
star:
Sp. G8V
Prot=19.2±0.3 d
age = 6 ± 3 Gyr

(Hébrard+ 2013)



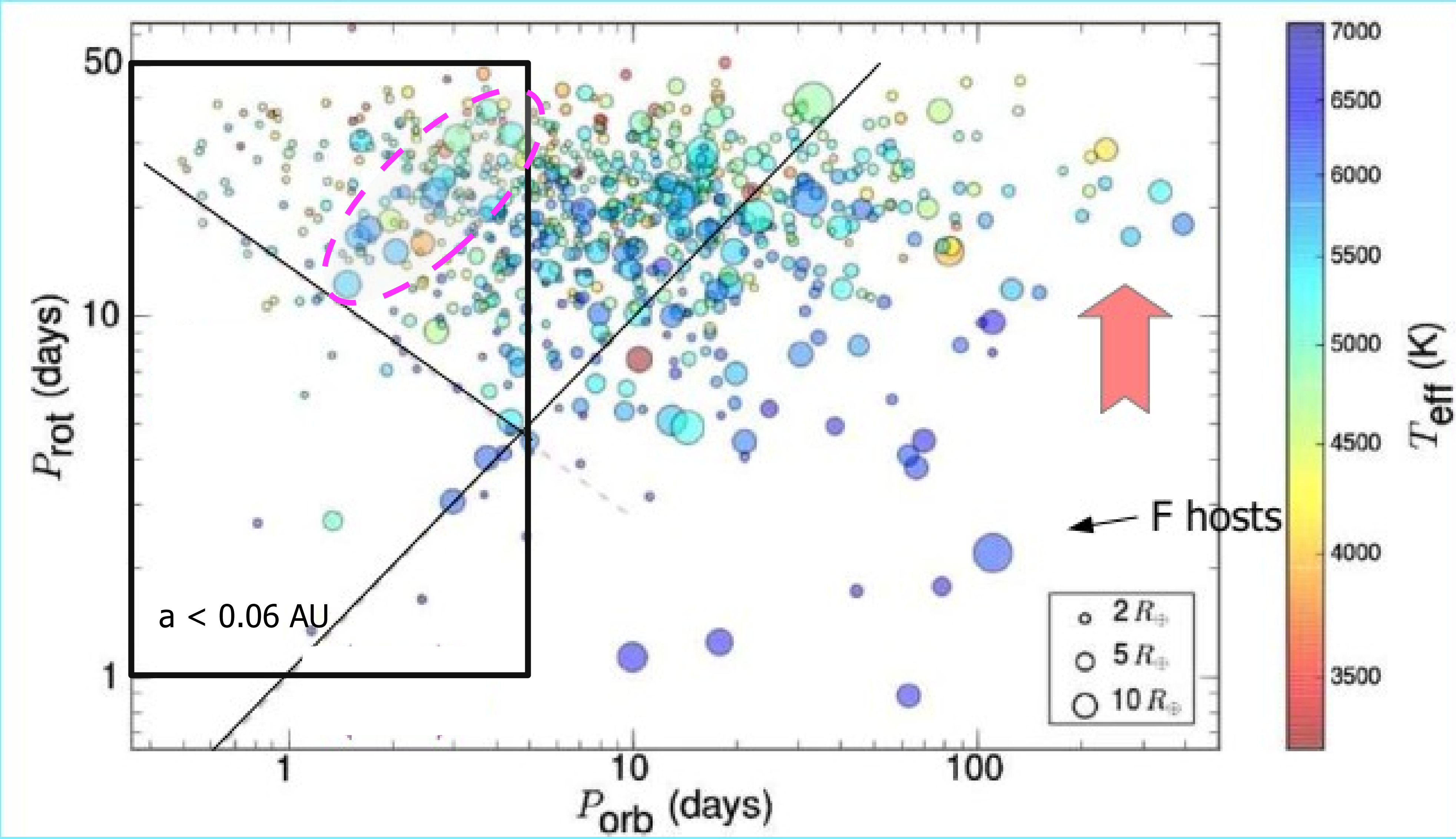
SFM et al. Ap.J. **807** (2015)

Kepler KOI population (2013)



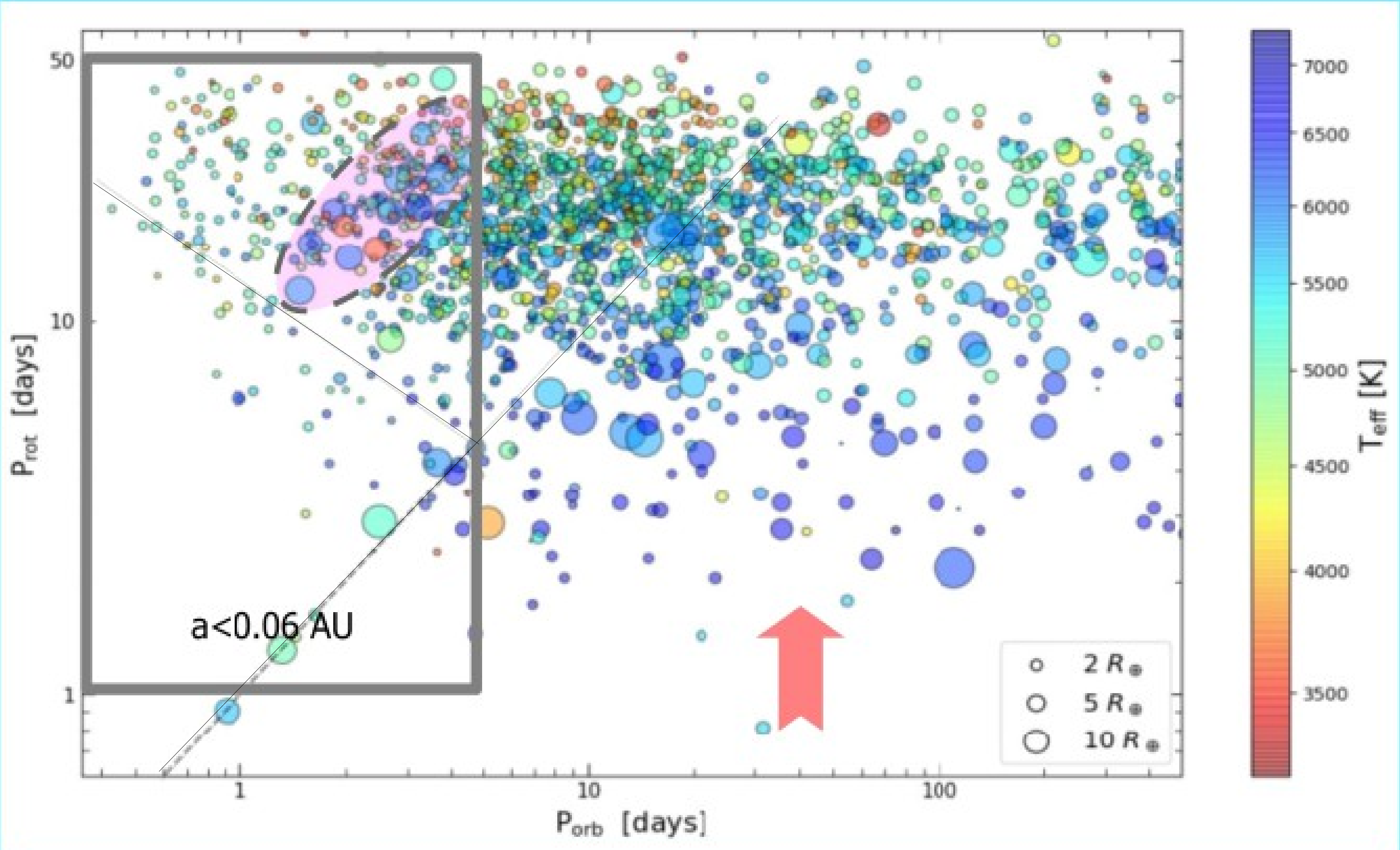
Ref: McQuillan et al. ApJL 2013

Accumulation of close-in hot Jupiters



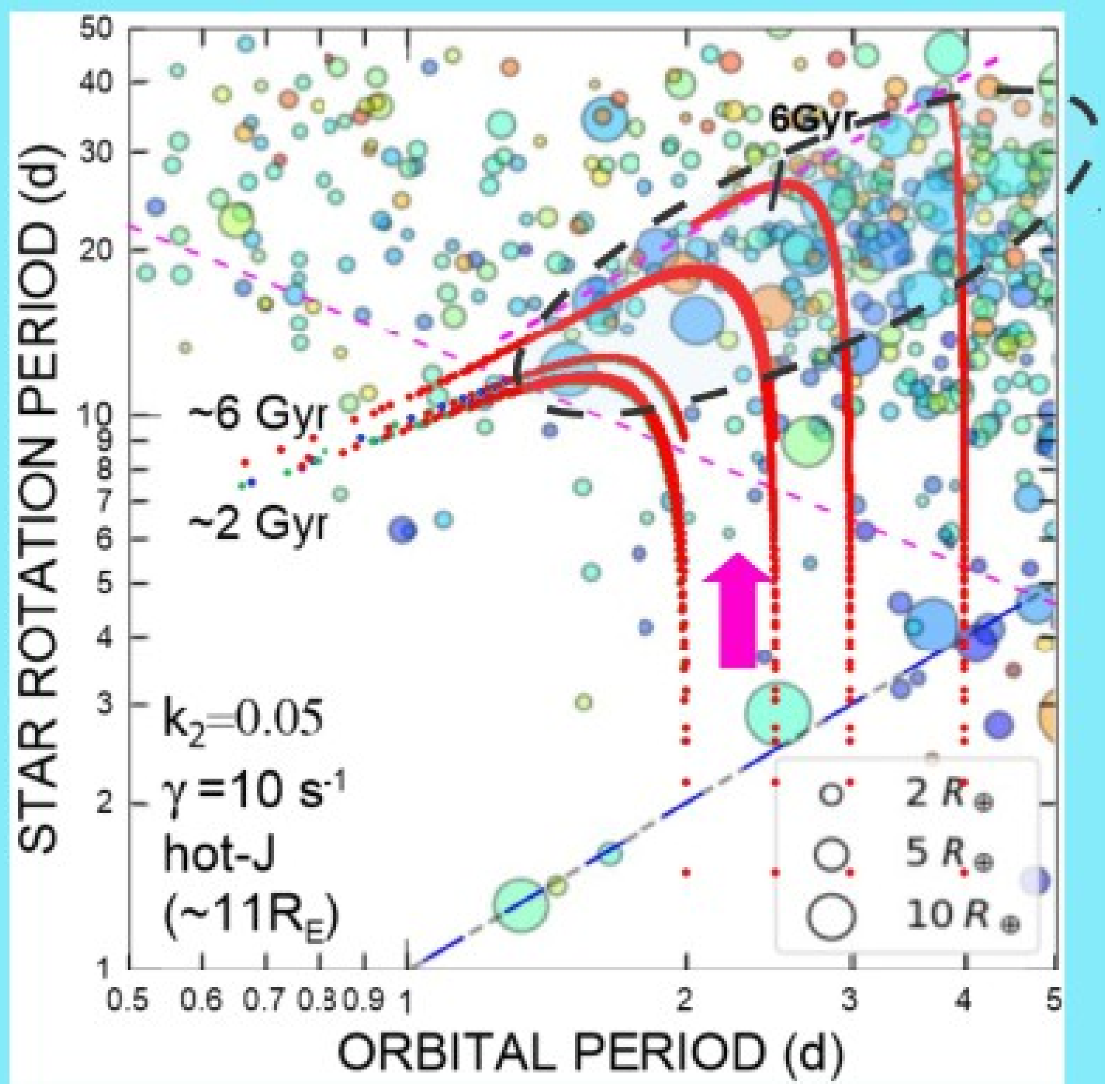
N.B. Fastest rotators are mostly false positives

Update by C.Beaugé 2022 (data from Santos et al. 2021)



N.B. Rjup ~ 11 $R_\oplus$

Evolutionary Tracks of hot Jupiters + host stars in P-P diagram with Tidal Evolution and Wind Braking (Ferraz-Mello & Beaugé, MNRAS, 2023 June.)

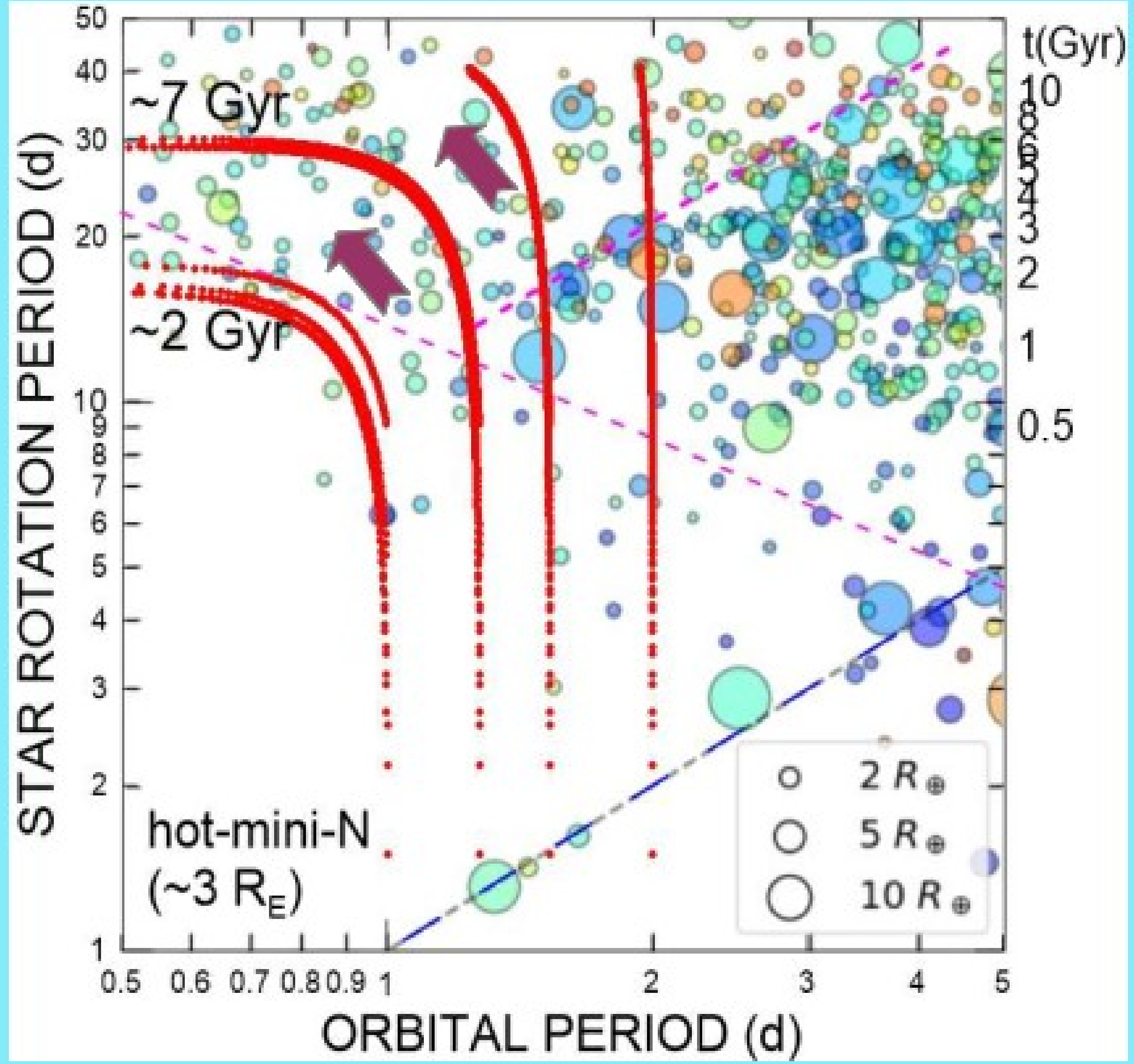


Accumulation of Hot Jupiters (*moraine-like*)

Hot Jupiters around Sun-like star

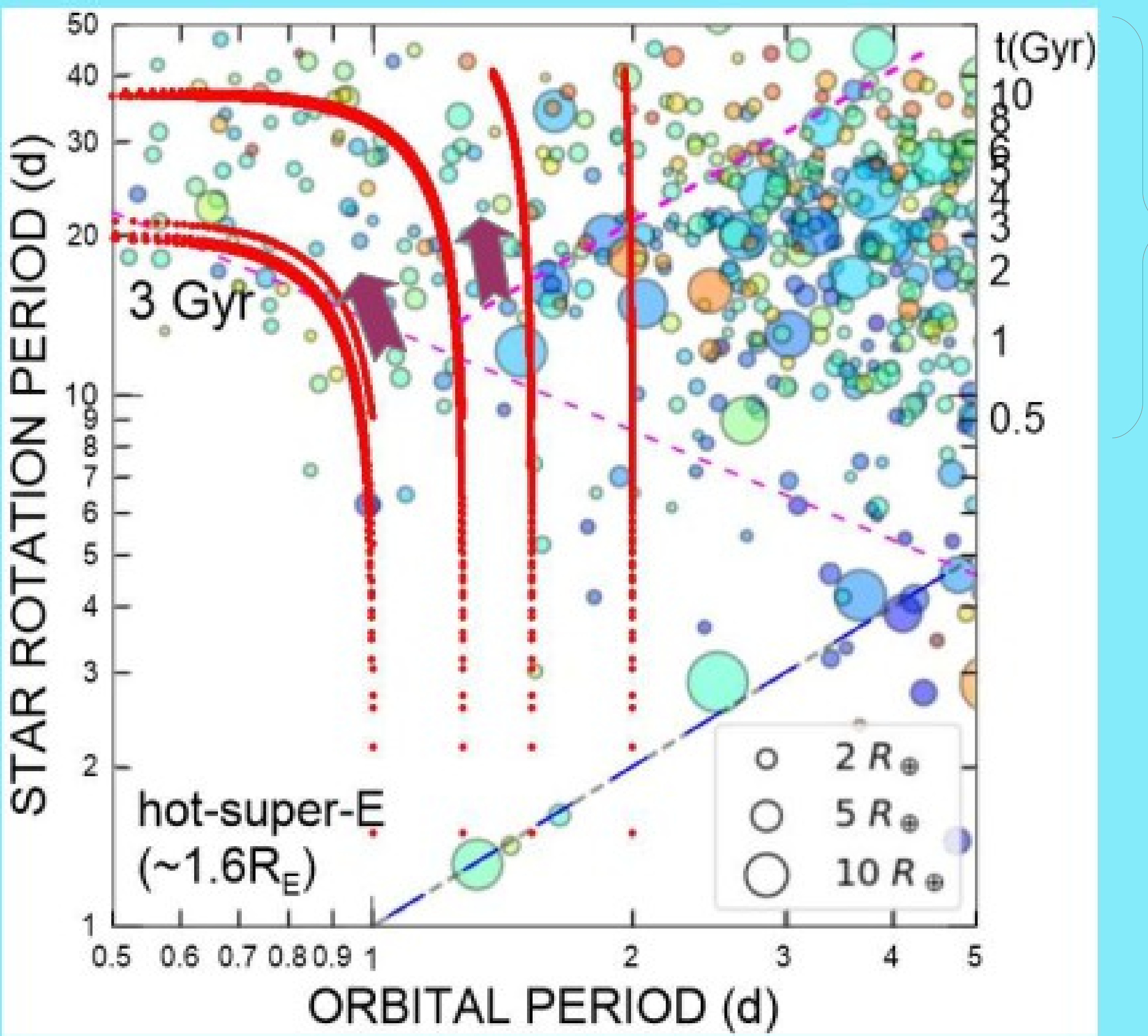
γ=10 /s

Evolutionary tracks of close-in mini-Neptunes



Timescale.
$P_{orb} \sim kt^{1/2}$
Skumanich law

Evolutionary tracks of close-in super Earths

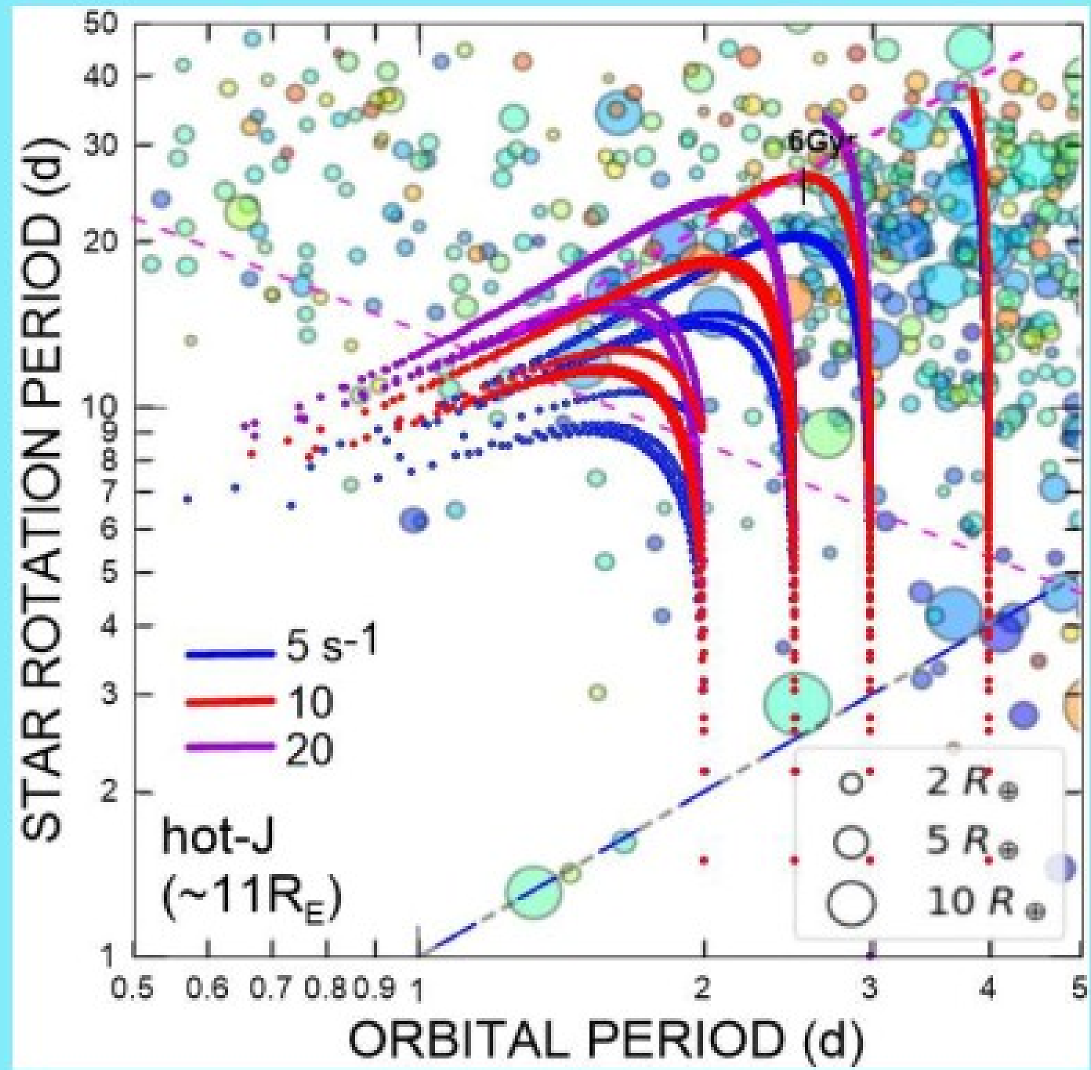


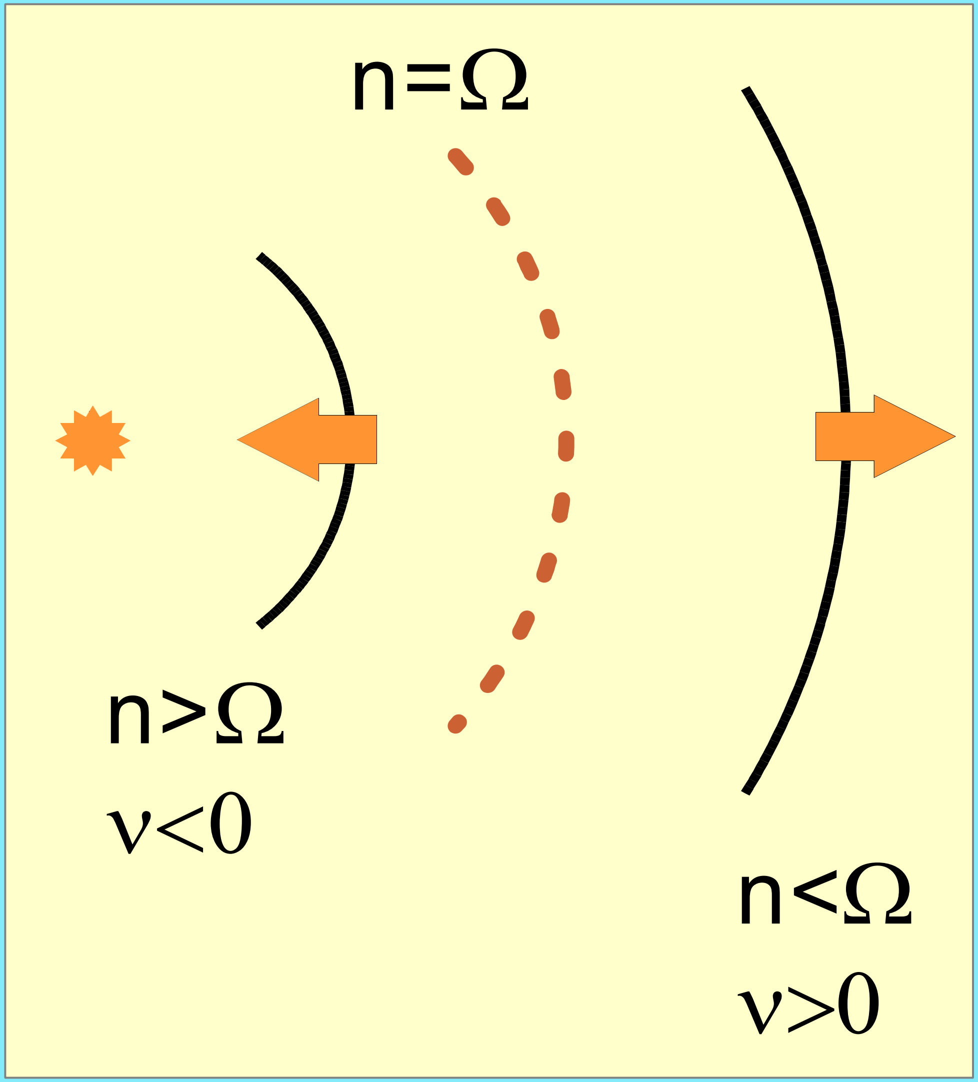
Timescale
$P_{orb} \sim kt^{1/2}$

End of Lecture

Thank you

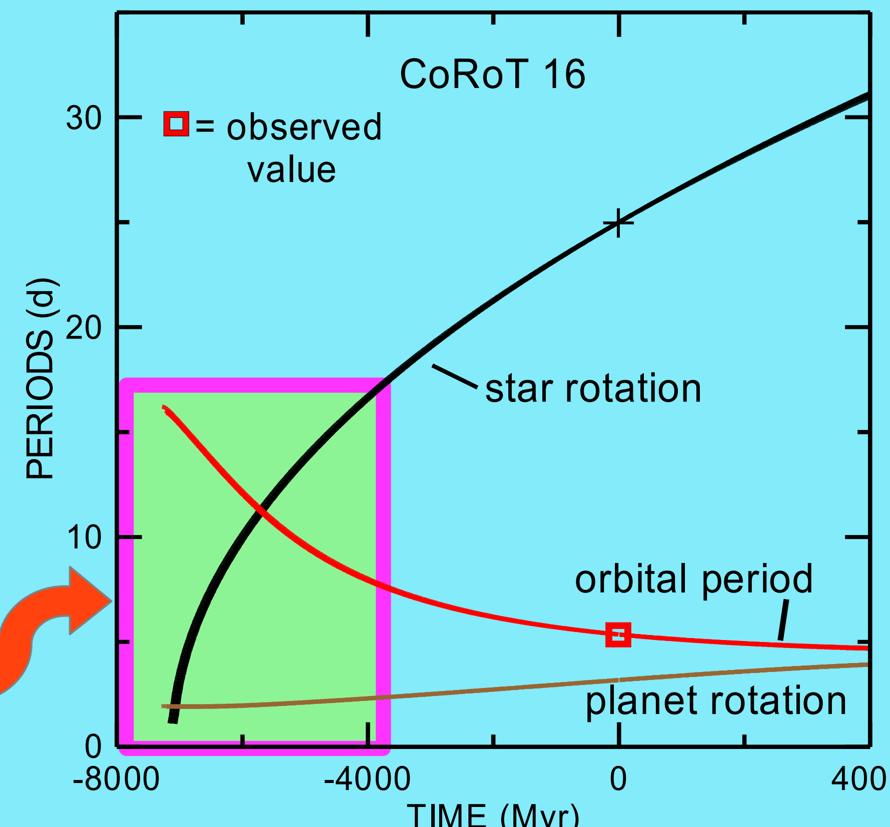
hot Jupiters - Relaxation factors γ = 5 – 10 - 20 s^{-1} (Love number k_2=0.05)



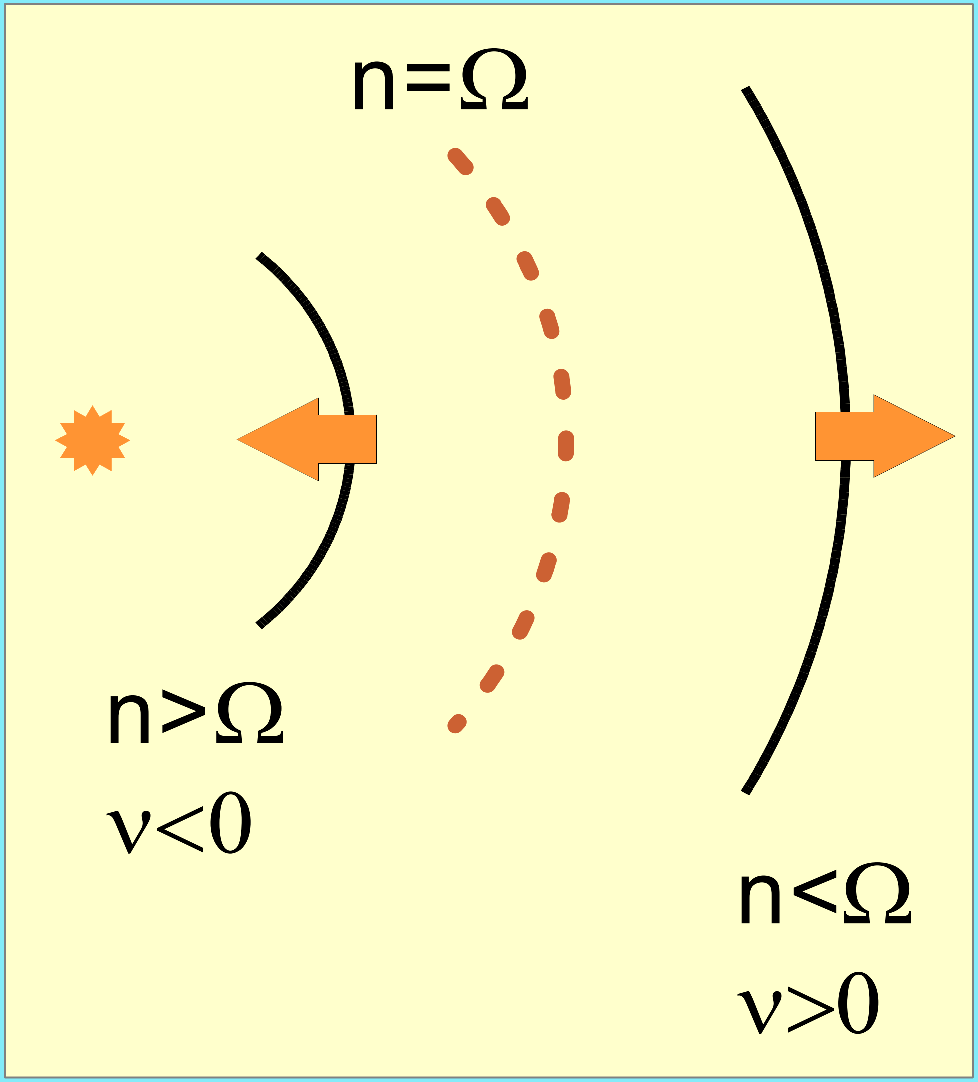


Digression:

This is true only for small eccentricities



watch here



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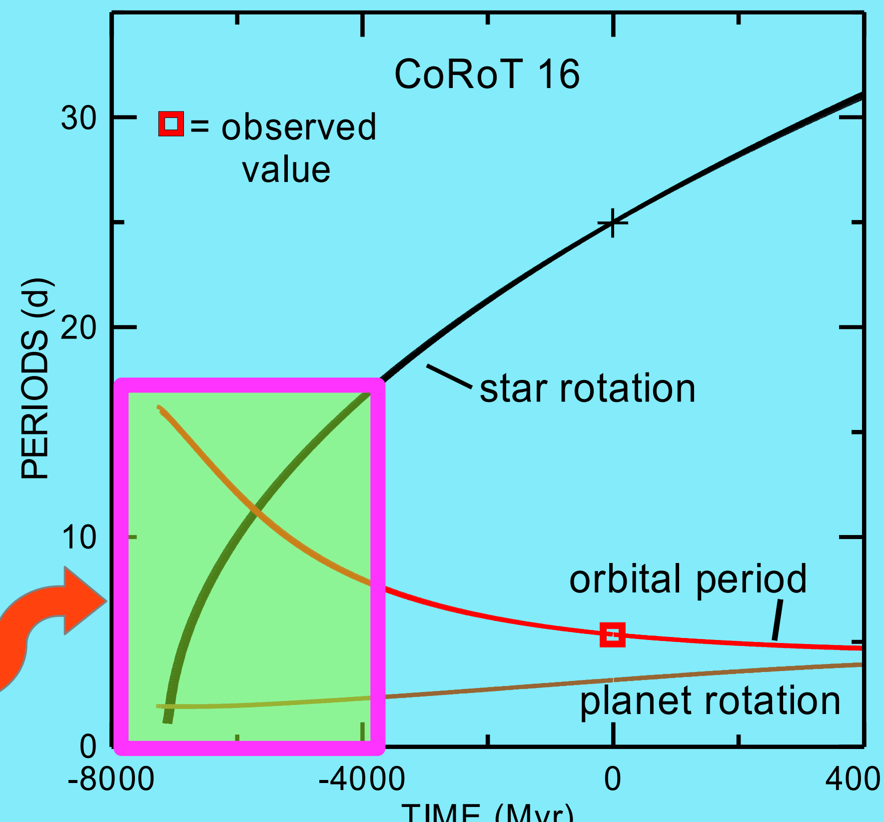


watch here

$$[\langle \dot{a} \rangle]_i = \frac{3k_{2i} n m_j R_i^5}{m_i a^4} \left((1-5e^2)\frac{\gamma_i \nu_i}{\gamma_i^2+\nu_i^2} - \frac{3e^2}{4}\frac{\gamma_i n}{\gamma_i^2+n^2} + \frac{e^2}{8}\frac{\gamma_i(\nu_i+n)}{\gamma_i^2+(\nu_i+n)^2} + \frac{147e^2}{8}\frac{\gamma_i(\nu_i-n)}{\gamma_i^2+(\nu_i-n)^2} \right) + \mathcal{O}(e^4).$$

i=star (tide in the star)
j=planet

$\nu = 2\Omega - 2n$

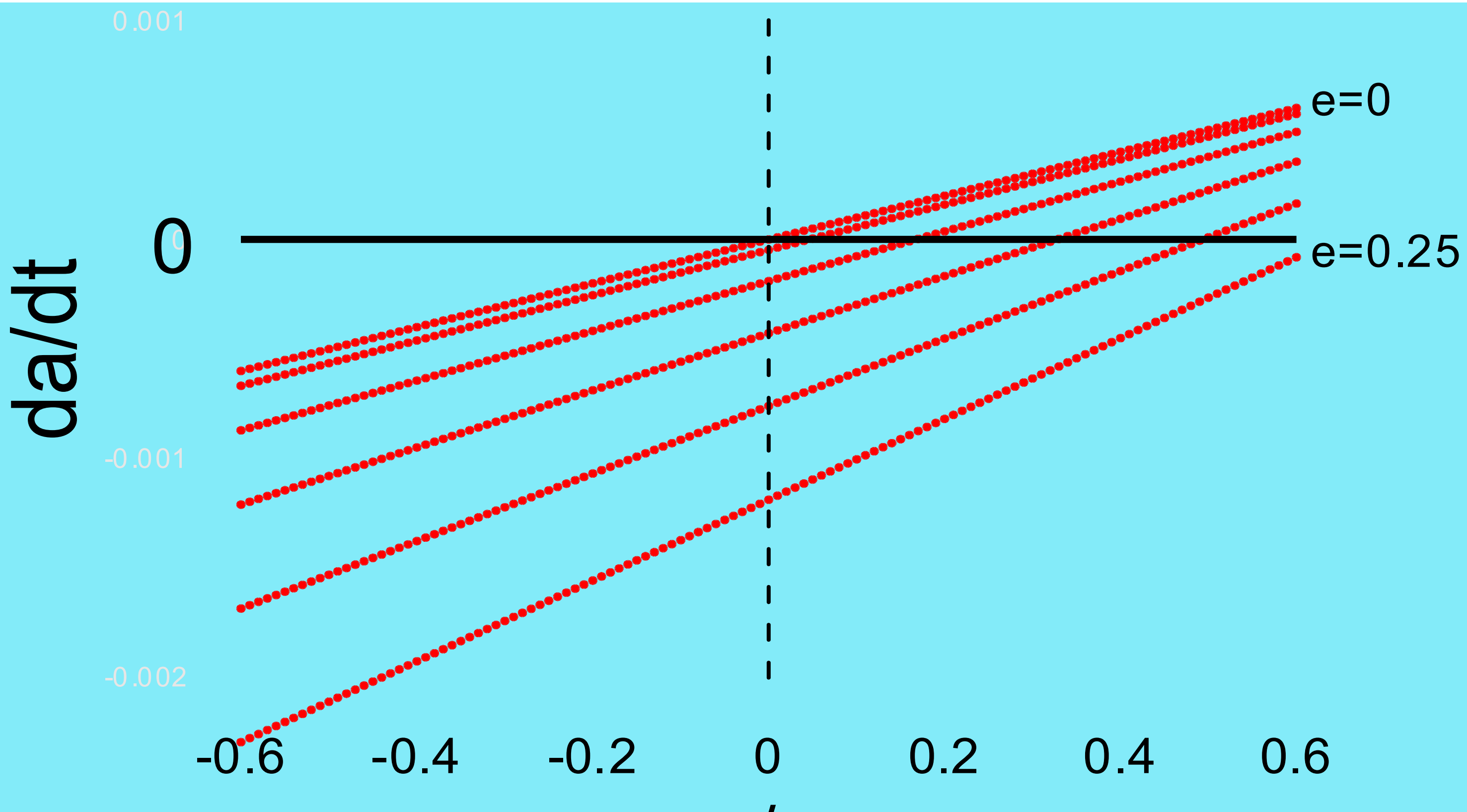
Relaxation Factors (STARS)

- from Rotations + Ages: **$20\text{-}100\ s^{-1}$**
  (SFM, 2015, 2018)

- from the orbital decay of WASP-12 b
  29 ms/year ==> **$\sim 18\ s^{-1}$** (Yee et al 2019)

- from the analysis of survival of short-period planets **$5\text{-}45\ s^{-1}$** (Hansen, 2010)

$$[\langle \dot{a} \rangle]_i = \frac{3k_{2i} n m_j R_i^5}{m_i a^4} \left((1-5e^2)\frac{\gamma_i \nu_i}{\gamma_i^2+\nu_i^2} - \frac{3e^2}{4}\frac{\gamma_i n}{\gamma_i^2+n^2} + \frac{e^2}{8}\frac{\gamma_i(\nu_i+n)}{\gamma_i^2+(\nu_i+n)^2} + \frac{147e^2}{8}\frac{\gamma_i(\nu_i-n)}{\gamma_i^2+(\nu_i-n)^2} \right) + \mathcal{O}(e^4).$$



$$[\langle \dot{a} \rangle]_i = \frac{3k_{2i} n m_j R_i^5}{m_i a^4} \left((1-5e^2)\frac{\gamma_i \nu_i}{\gamma_i^2+\nu_i^2} - \frac{3e^2}{4}\frac{\gamma_i n}{\gamma_i^2+n^2} + \frac{e^2}{8}\frac{\gamma_i(\nu_i+n)}{\gamma_i^2+(\nu_i+n)^2} + \frac{147e^2}{8}\frac{\gamma_i(\nu_i-n)}{\gamma_i^2+(\nu_i-n)^2} \right) + \mathcal{O}(e^4).$$

tide in the planet

$$[\langle \dot{a} \rangle]_i \simeq -\frac{21 k_{2i} n e^2 C_i m_j R_i^5}{m_i a^4} \frac{\gamma_i n}{\gamma_i^2+n^2}$$

i=planet; j=star

$$[\langle\dot{a}\rangle]_i = \frac{3k_{2i}nm_jR_i^5}{m_ia^4}\left((1-5e^2)\frac{\gamma_i\nu_i}{\gamma_i^2+\nu_i^2} - \frac{3e^2}{4}\frac{\gamma_i n}{\gamma_i^2+n^2} + \frac{e^2}{8}\frac{\gamma_i(\nu_i+n)}{\gamma_i^2+(\nu_i+n)^2} + \frac{147e^2}{8}\frac{\gamma_i(\nu_i-n)}{\gamma_i^2+(\nu_i-n)^2}\right) + \mathcal{O}(e^4).$$

tide in the planet

$$[\langle\dot{a}\rangle]_i \simeq -\frac{21k_{2i}ne^2C_im_jR_i^5}{m_ia^4} \quad \frac{\gamma_i n}{\gamma_i^2+n^2}$$

i=planet; j=star

Example:

CoRoT 16

G5V (=Sun)

M~1.1 Sun

age~7.6κ2.8Gyr

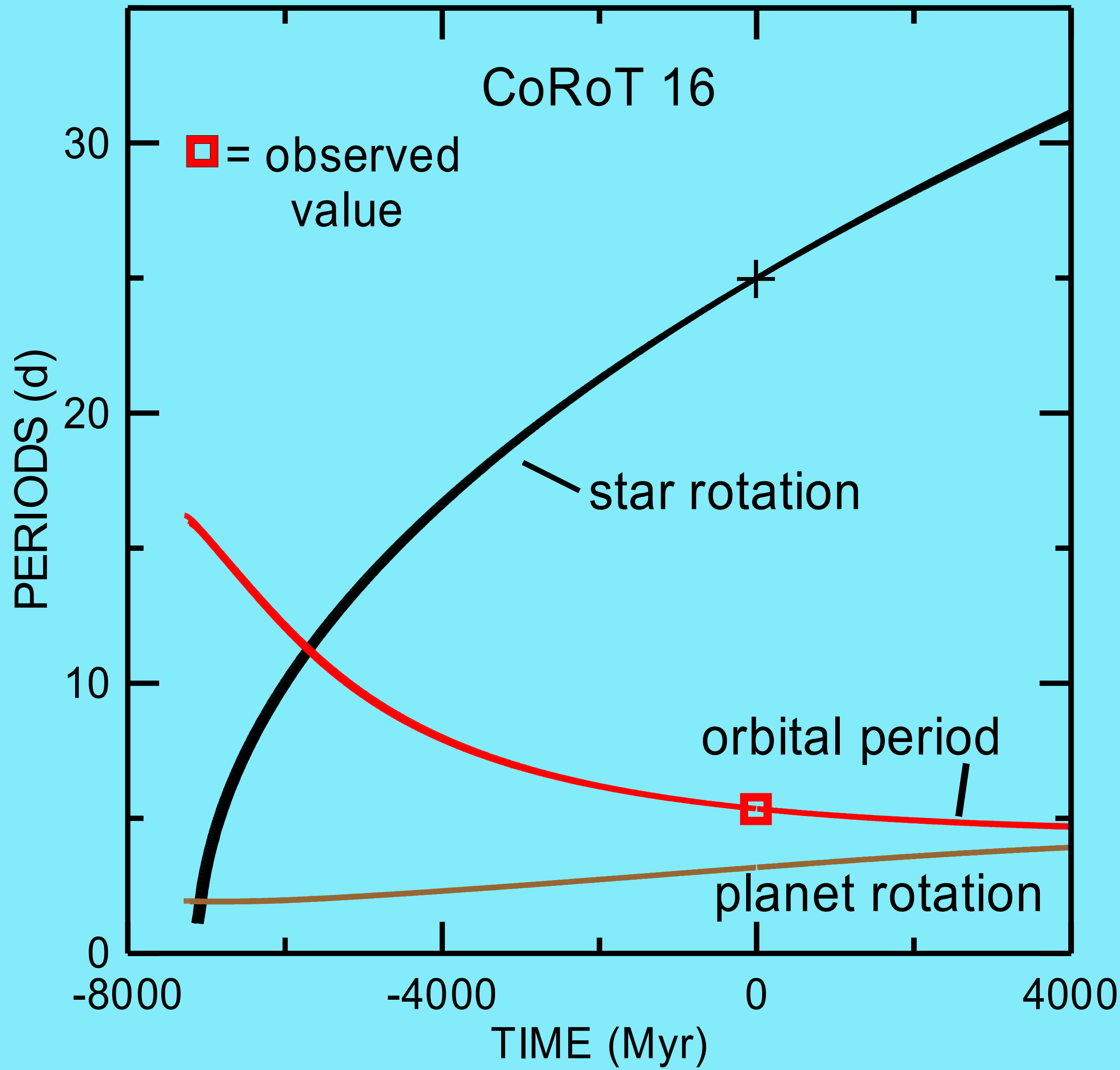
CoRoT 16b

M~0.5 Jup

P~5.3 days

e~0.33

γ~[30,80] s^{-1}



Example:

CoRoT-3

F3V

M~1.37 Sun

age~[1.3-2.8] Gyr

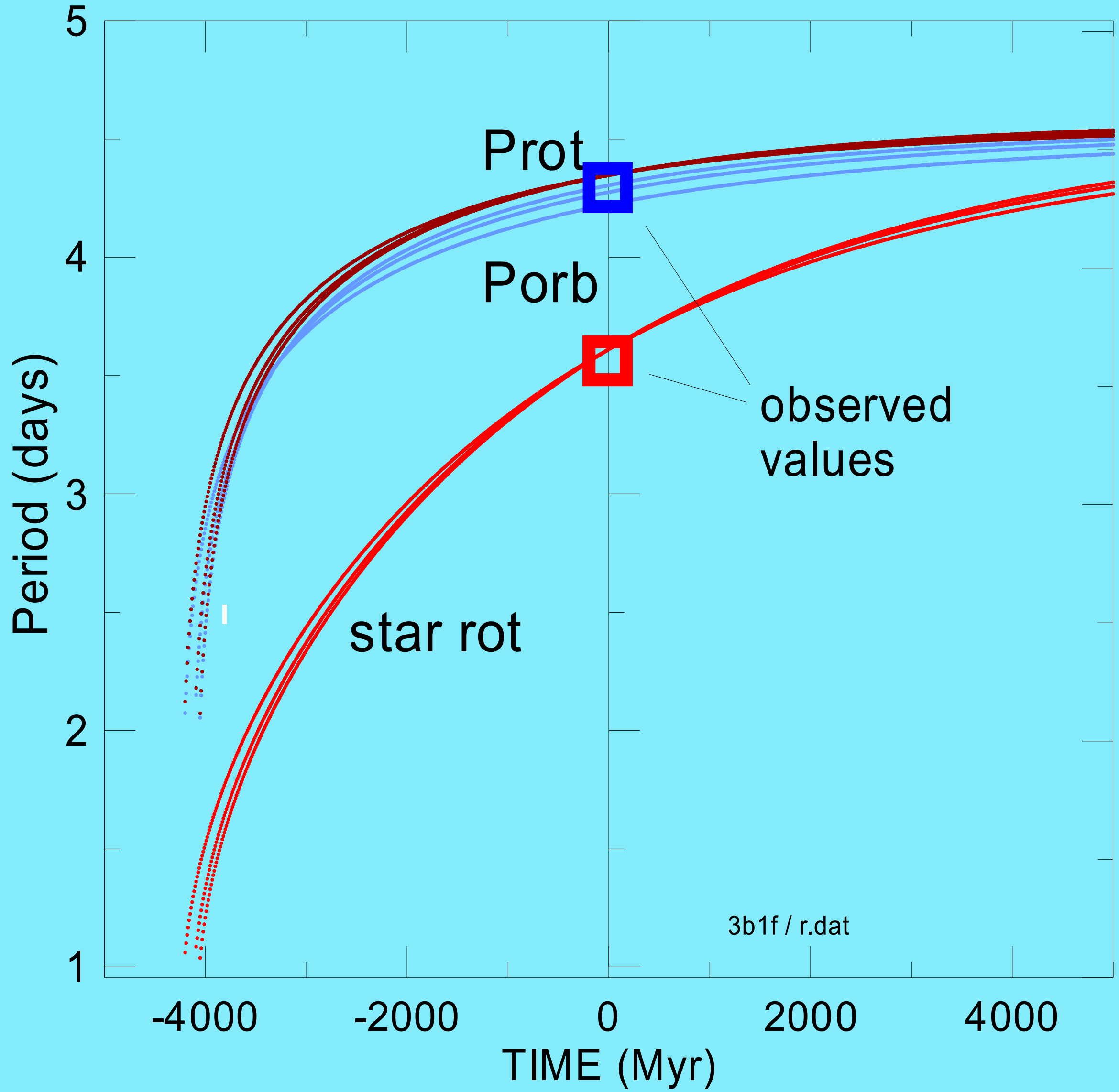
CoRoT-3 b

M~22 Jup (BD)

P~4.25 days

e~0.012

$\gamma \sim 100$ s^{-1}



Relaxation Factors (hot Jupiters)

- from the analysis of survival of WASP-17 b, CoRoT-5 b and Kepler-6 b **10-60 s^{-1}** (Hansen, 2010)

- from the putative eccentricity evolution of CoRoT-15 b **200 s^{-1}** (SFM, 2013)

Relaxation Factors (earths & super-earths)

Only solar system results

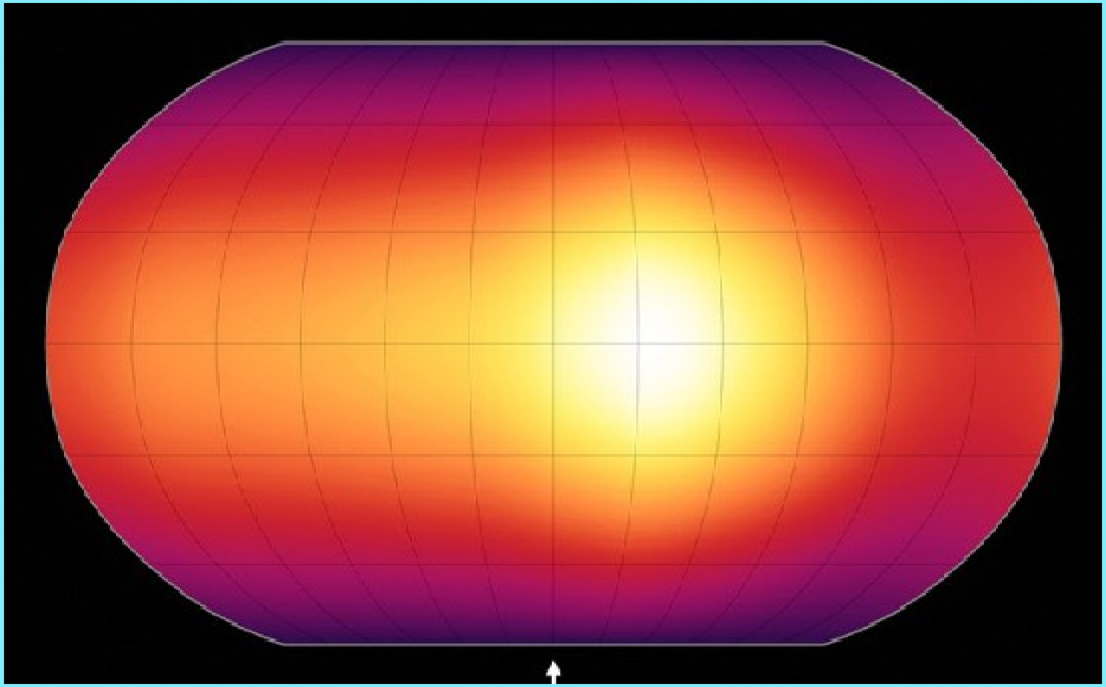
Mercury: **$4\text{-}30 \times 10^{-9}\ s^{-1}$** (SFM, 2015)

solid Earth **$1\text{-}4 \times 10^{-7}\ s^{-1}$** (typical)

whole Earth (from the acceleration of the Moon)

$\sim 1.2 \times 10^{-5}\ s^{-1}$ (Mignard, 1979)

It is not wise just extend Solar System values to close-in exoplanets. The stellar radiation power creates extreme physical environments.



HD 189733
a~3 million km
P~2.2 d

very hot spot

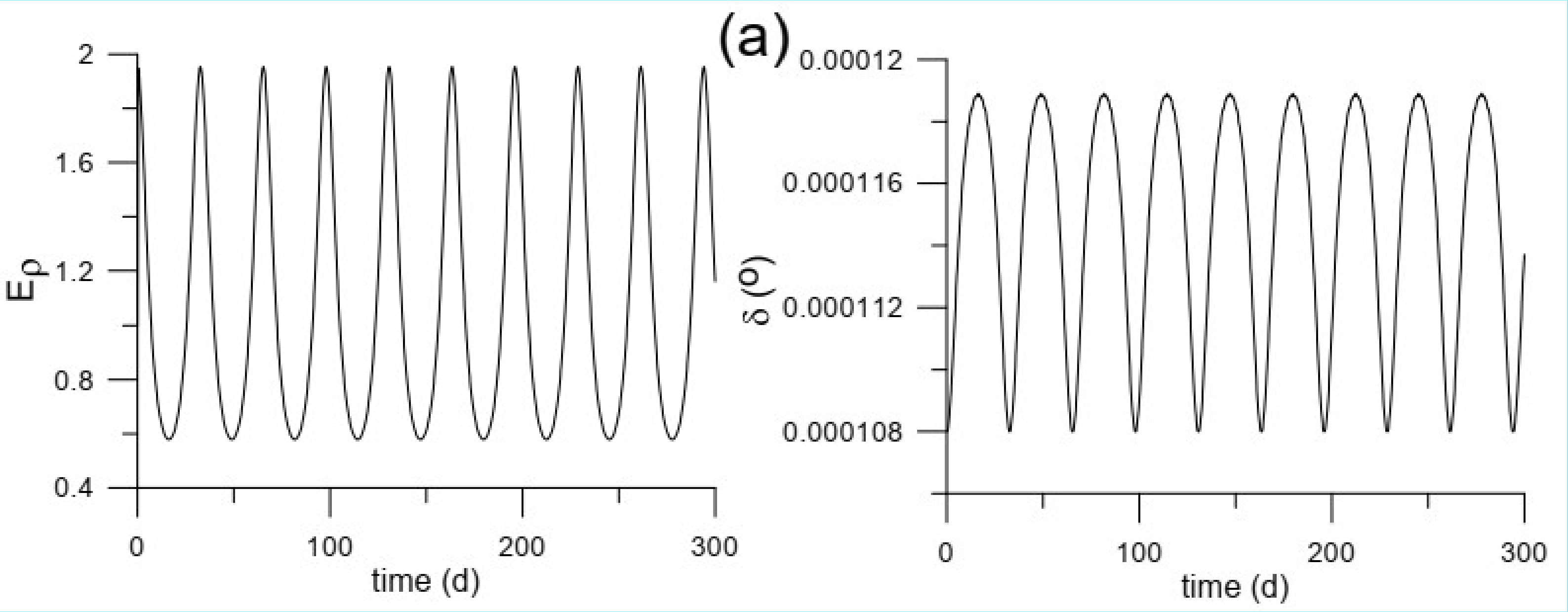
Example: CoRoT-7 b
first super Earth ever discovered
from estimated physical parameters
(Léger et al. 2011)

$\gamma < 6 \times 10^{-7}\ s^{-1}$

End of Lecture

Preprint:
www.astro.iag.usp.br/~sylvio/Review-21.pdf

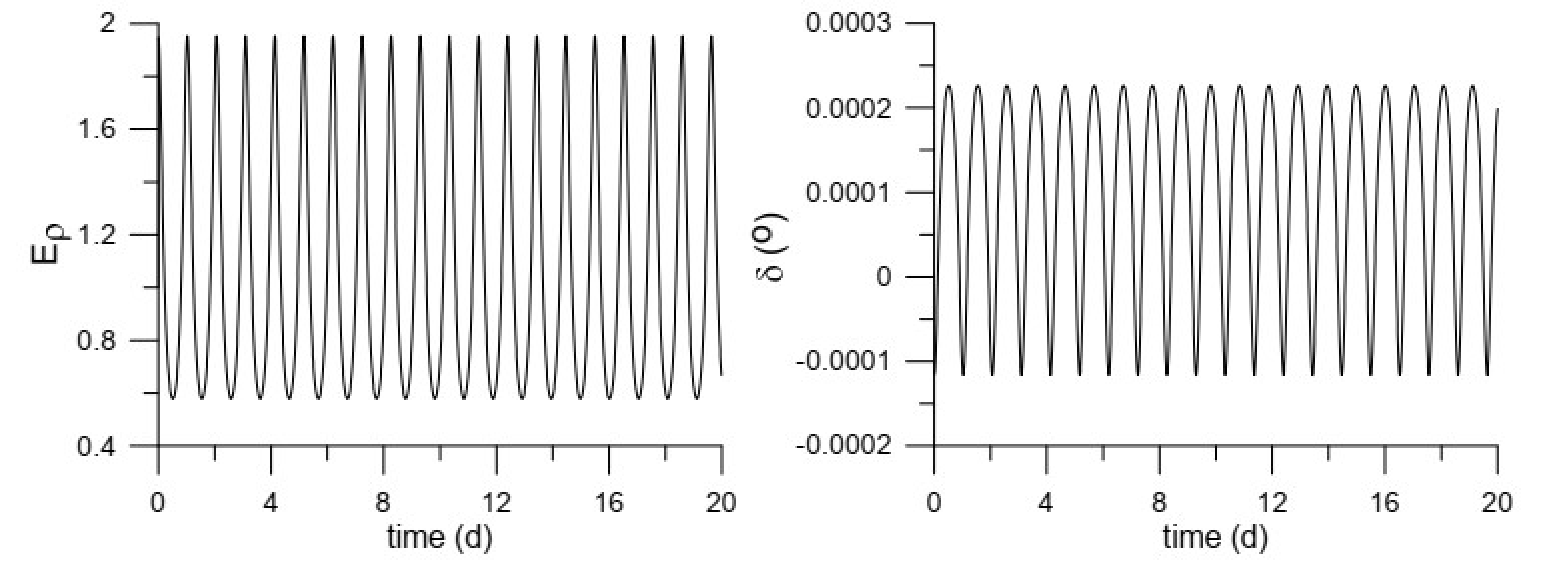
2. Fast rotating Neptune-like planet in a 33-day eccentric orbit around a Sun-like star. (e=0.2)



γ=10/s a=0.2 AU

P_{rot}=3.27d

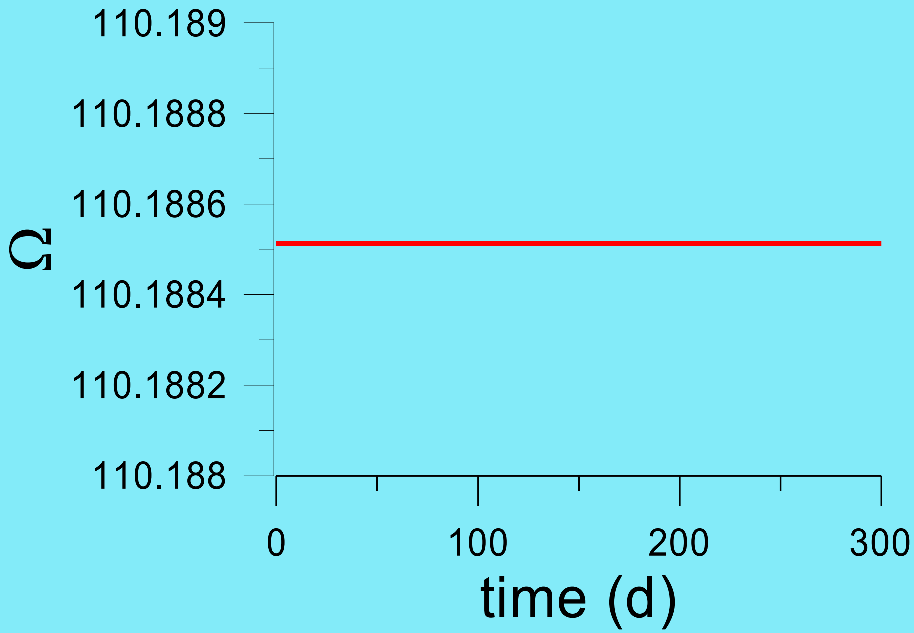
3. Neptune-like planet in super-synchronous stationary rotation ($<d\Omega/dt>=0$) around a Sun-like star, in an eccentric orbit (e=0.2)

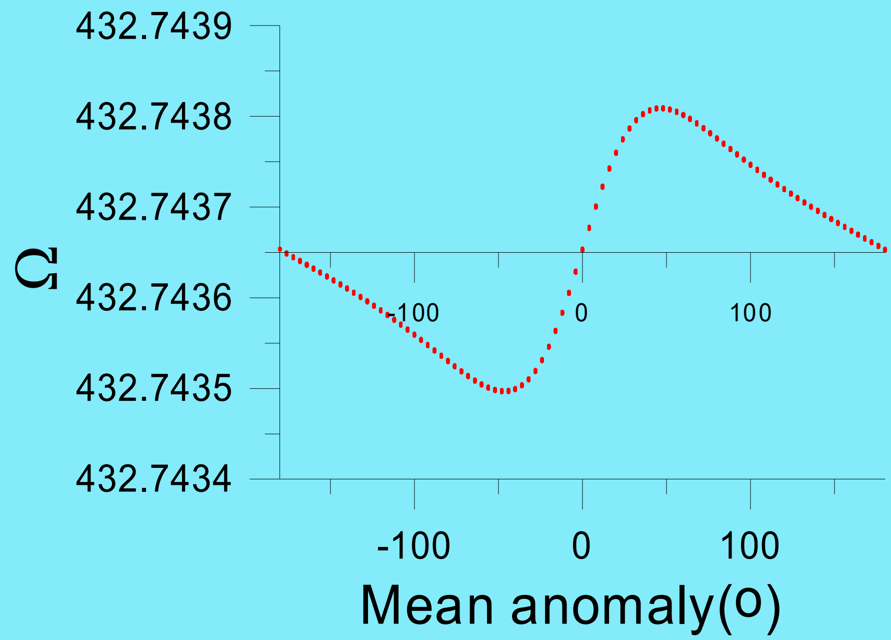


P_{rot}=1 day

γ=10/s

Neptune's rotation in the two examples (°/day)





33-day orbit (nonstationary rotation)

Relative variation less than 10^{-8} in one year

Supersynchronous stationary rotation

Relative variation less than 10^{-6} in one period.