Tetraquarks: Successes & Mysteries

Luciano Maiani, CERN

starring:

- BABAR, BELLE, LHCb, BES II/III
- theorists (not many) aficionados to hadron spectrocopy

Twenty years after X(3872) discovery, more than 50 exotic (non $q\bar{q}$ mesons) have been discovered

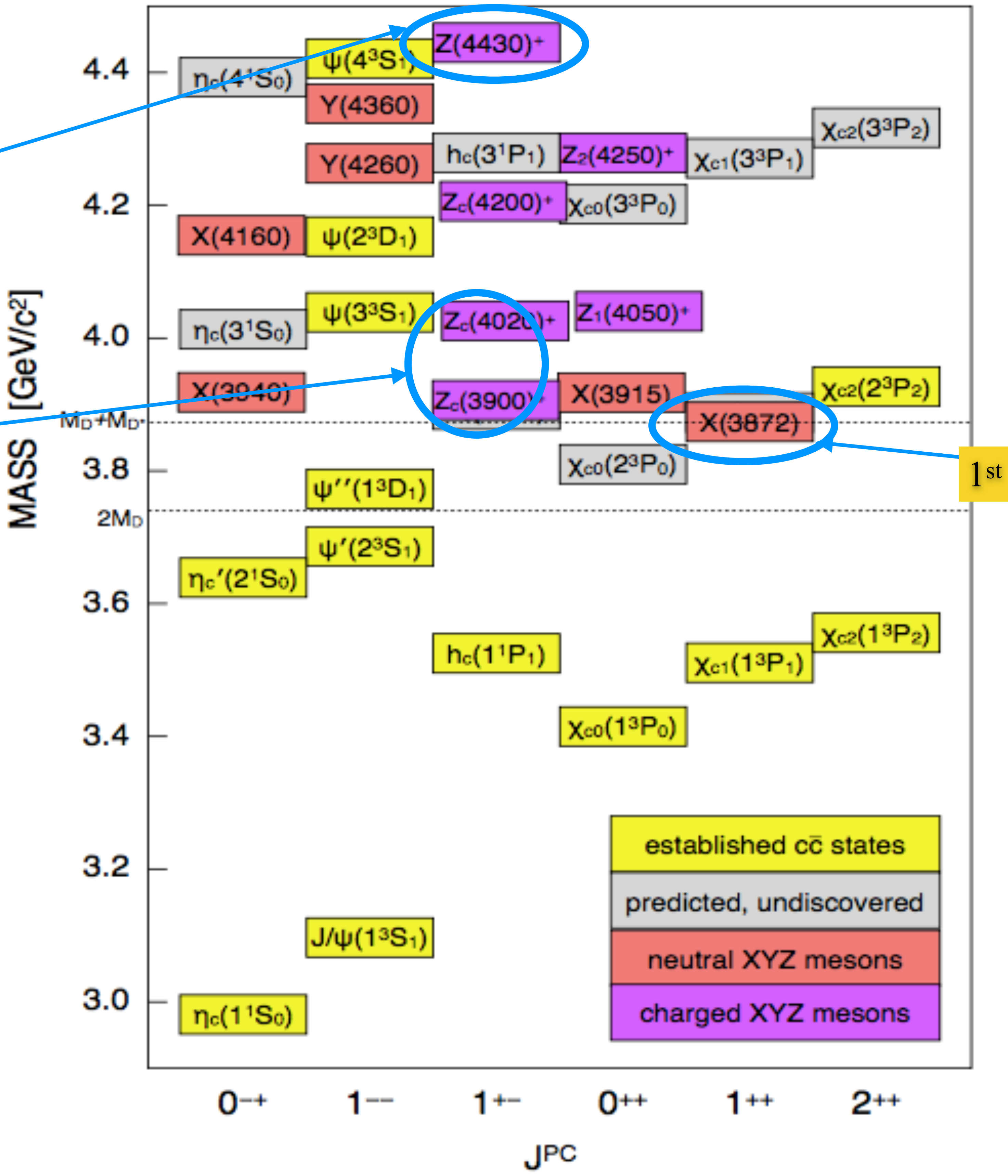
Pentaquark, non qqq baryons, have also been observed

Yet, no consensus on the internal structure of exotic mesons and baryons has been reached

New experiments are called, to get to a satisfactory theory of exotic mesons and pentaquarks

1. Expected and Unexpected Charmonia

figure by:
S. L. Olsen (2015)
arXiv:1511.01589



2nd Unexpected (2007)
a radial excitation?

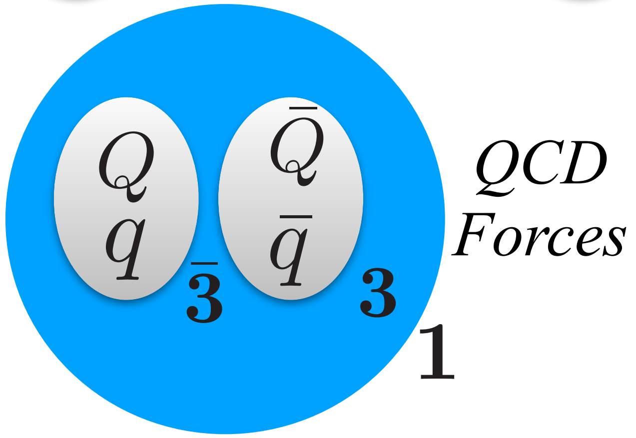
3rd Unexpected (2013):
a multiplet ground
state Tetraquarks?

1st Unexpected (2003)

Hidden Charm Mesons

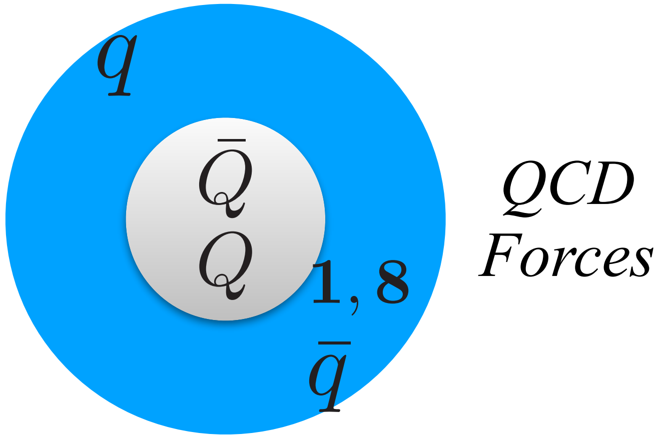
Hadron Molecule

F-K. Guo, C. Hanhart, U-G Meißner, Q. Wang, Q. Zhao, and B-S Zou, arXiv 1705.00141 (2017)



Compact Diquark-Antidiquark

L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. D 71 (2005) 014028; **D 89** (2014) 114010.



HadroCharmonium (1)
Quarkonium Adjoint Meson (8)

S. Dubynskiy, S. and M. B.Voloshin, Phys. Lett. **B 666**,(2008) 344.

E. Braaten, C. Langmack and D. H. Smith, Phys. Rev. **D 90** (2014) 01404

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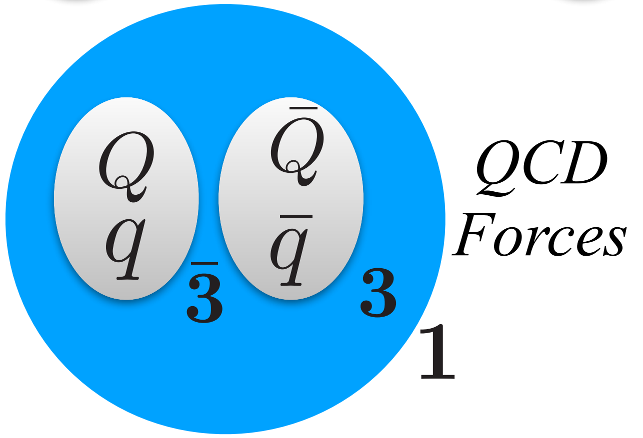

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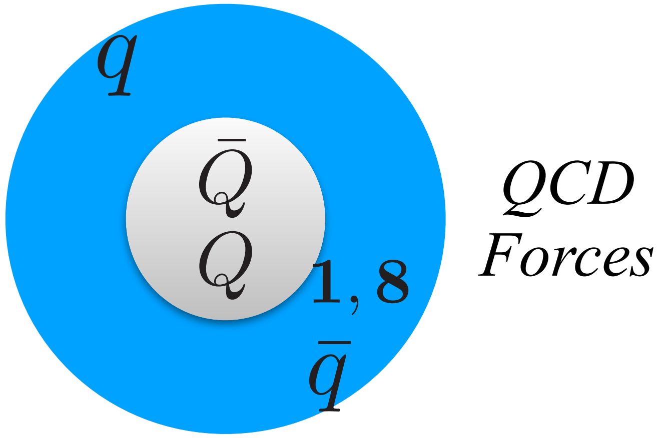

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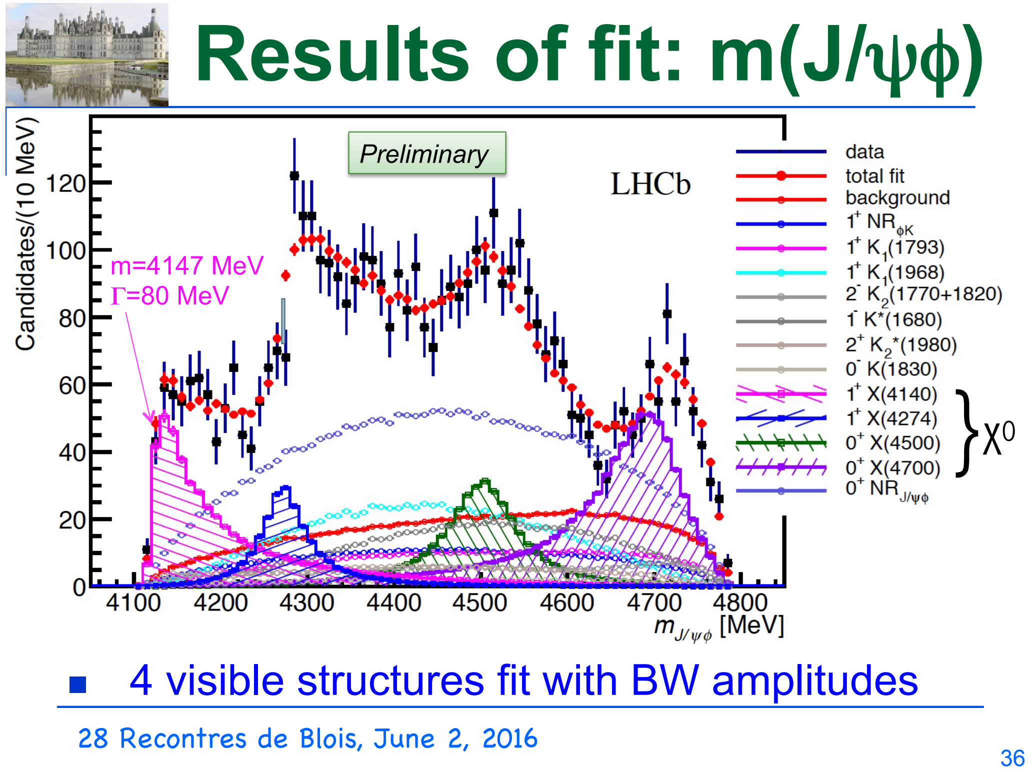
- The compact diquark-antidiquark model makes ***a firm quantitative prediction***: tetraquarks must form ***complete multiplets of flavor SU(3)*** with mass differences determined by the quark mass difference $m_s - m_{u,d}$.
- Comparing tetraquarks with different strangeness ***the rule can be tested and the structure can be deciphered***. A definite shopping list of the missing particles is today available.

Charm-strange exotics

Hidden charm and strangeness and charm-strange tetraquarks have provided crucial steps forward.

LHCb (2016): $\Psi\ \phi$ resonances (***2016***)

$B^+ \to K^+ + X(4140) \to K^+ + \phi\ \Psi$, etc .



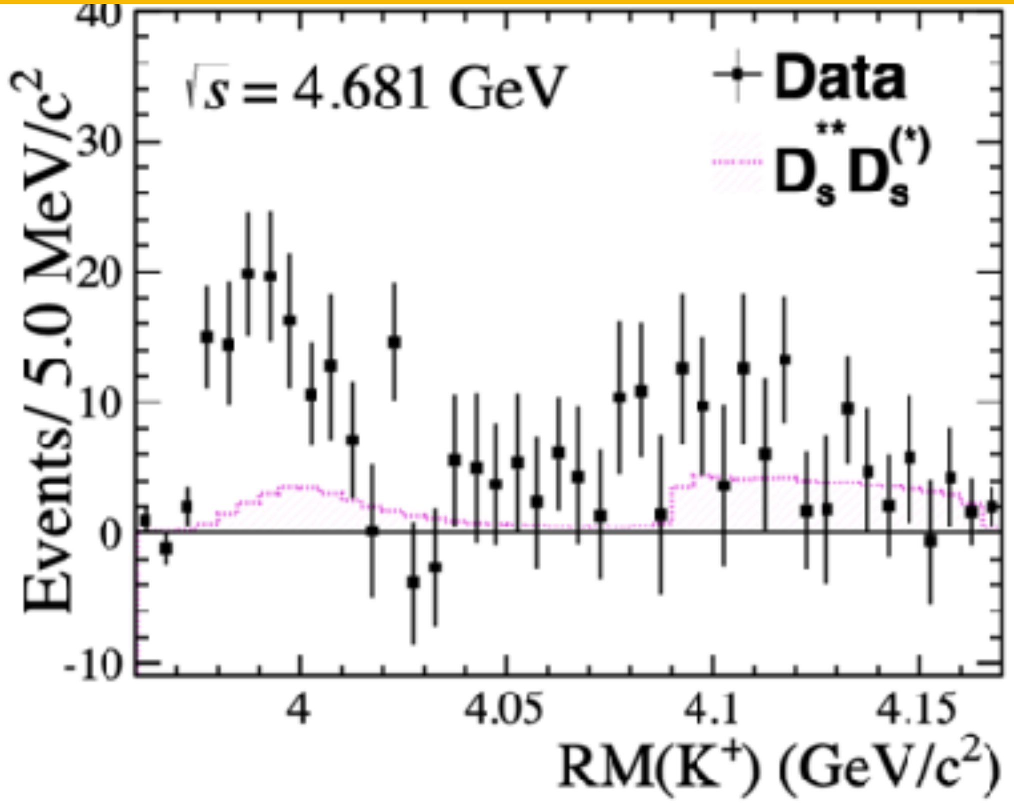
Results of fit: m(J/ψφ)

- 4 visible structures fit with BW amplitudes

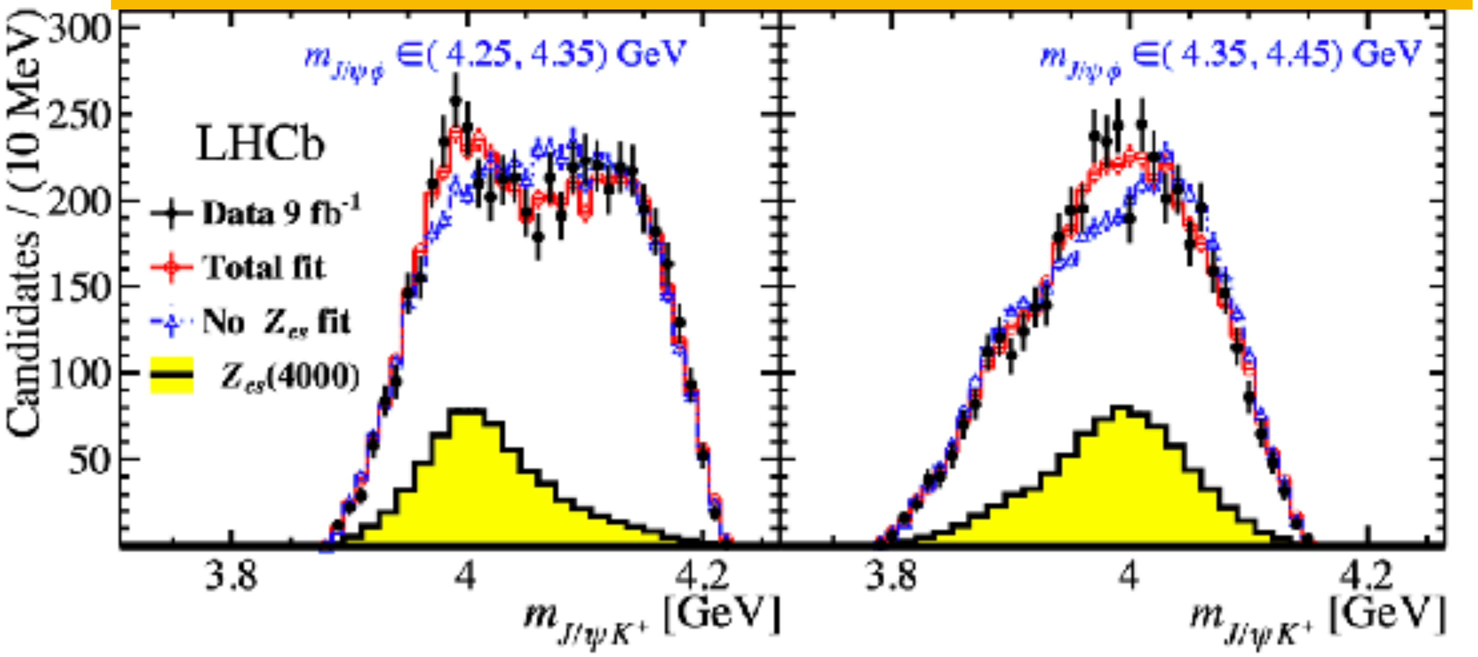
28 Rencontres de Blois, June 2, 2016

36

BES III (***2021***): $e^+e^- \to K^+ + Z_{cs}^-(3985) \to K^+(D_s^{*}D^0 + D_s^- D^{*0})$



LHCb (***2021***): $B \to \Psi + K^+ + \phi \to Z_{cs}(4003) + \phi$



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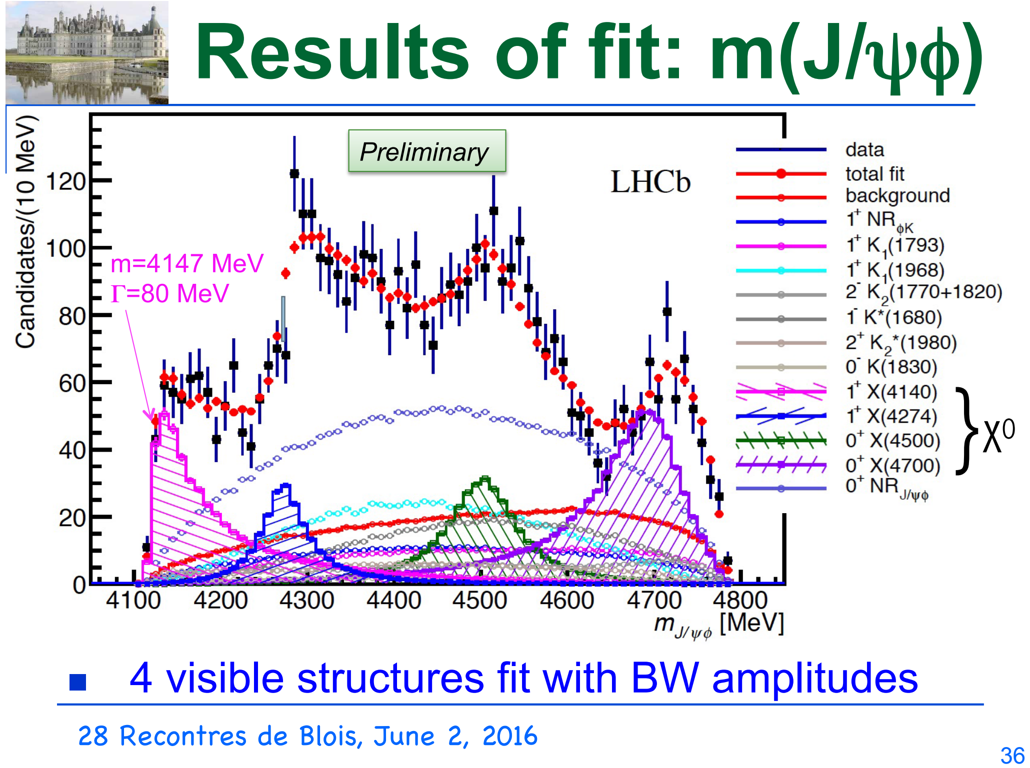

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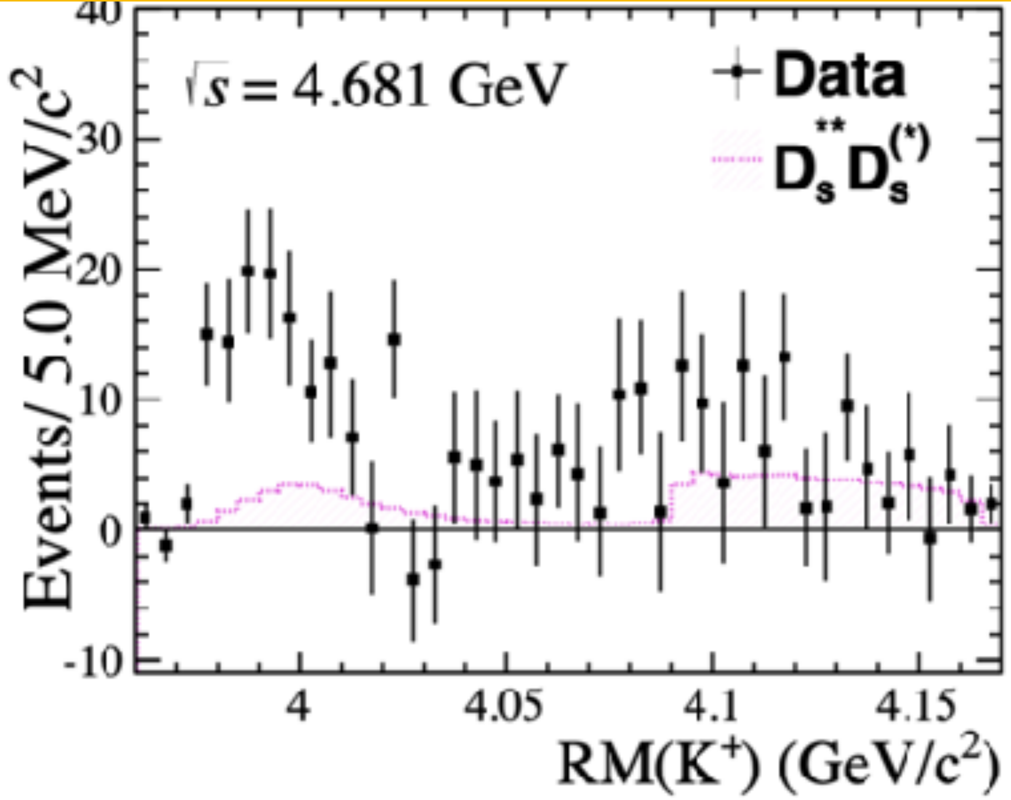

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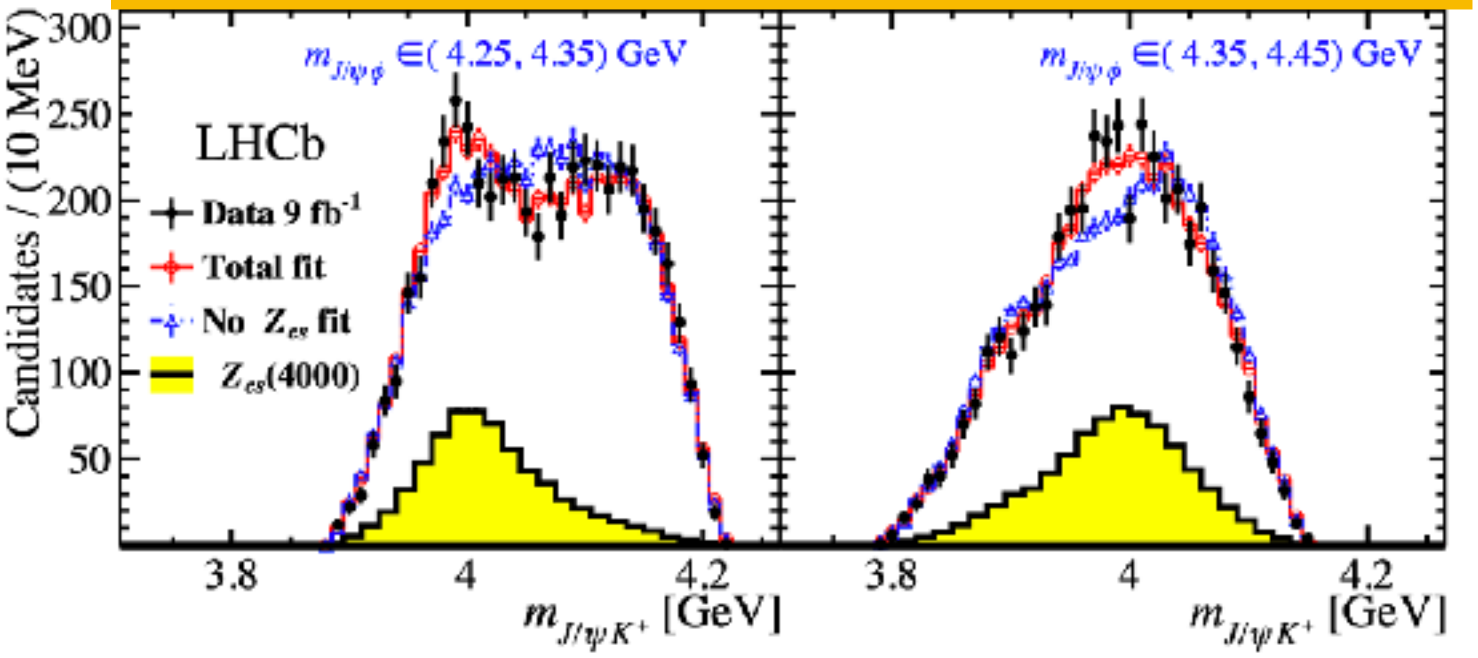
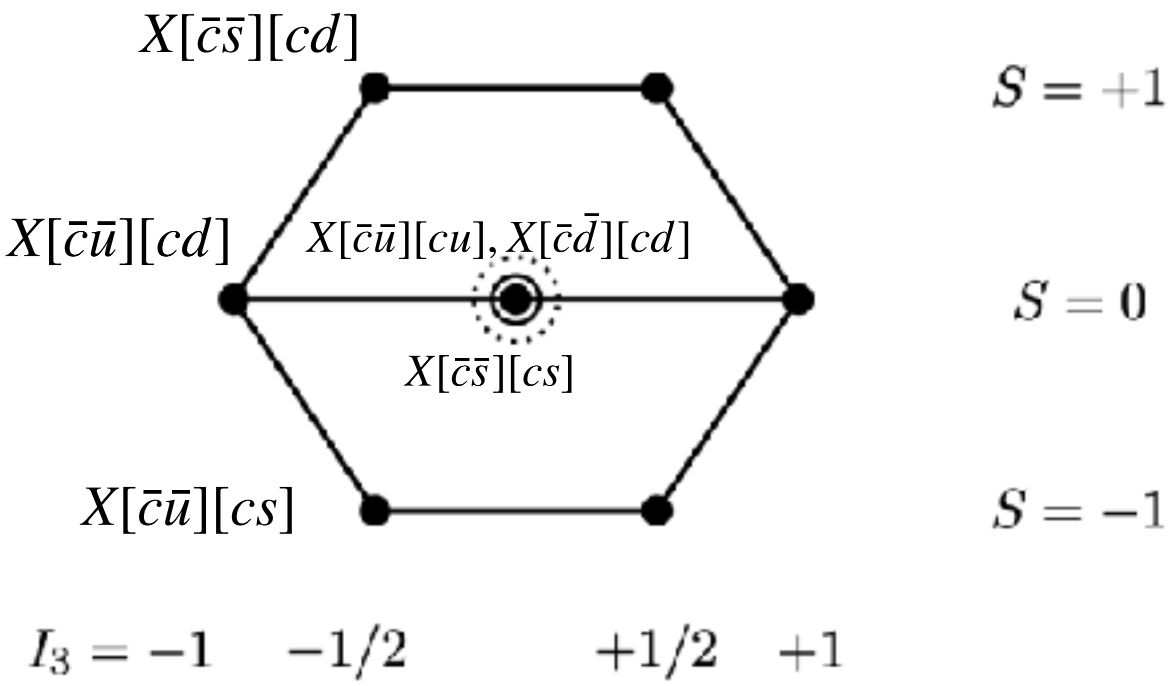


PdG: The incompatible values for the widths reported by AAIJ 2021E and ABLIKIM 2021G could either indicate the existence of two separate states or possibly be explained in a coupled channel model (see ORTEGA 2021

2. Hidden Charm Tetraquarks

Hidden Charm Tetraquarks form ***nonets of flavor SU(3)*** with mass differences determined by the quark mass difference $m_s - m_{u,d}$

with $Z_{cs}(3082)$, $Z_{cs}(4003)$, $Z_{cs}(4220)$ we can almost fill three tetraquark nonets with the expected scale of mass differences



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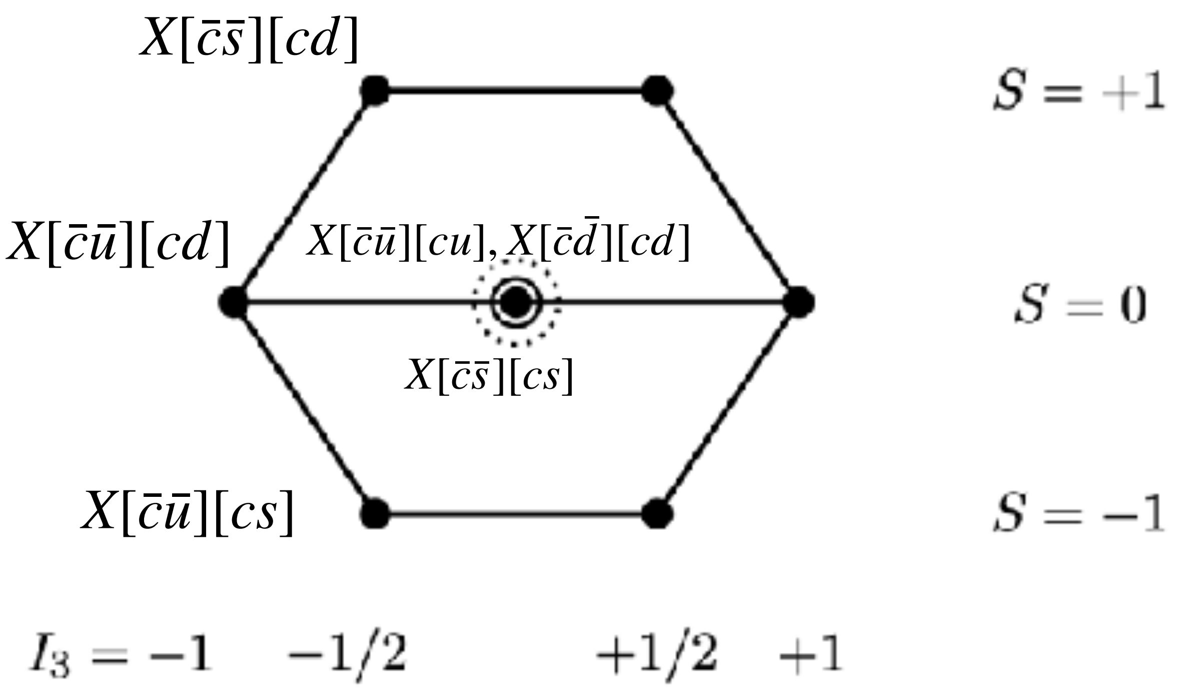
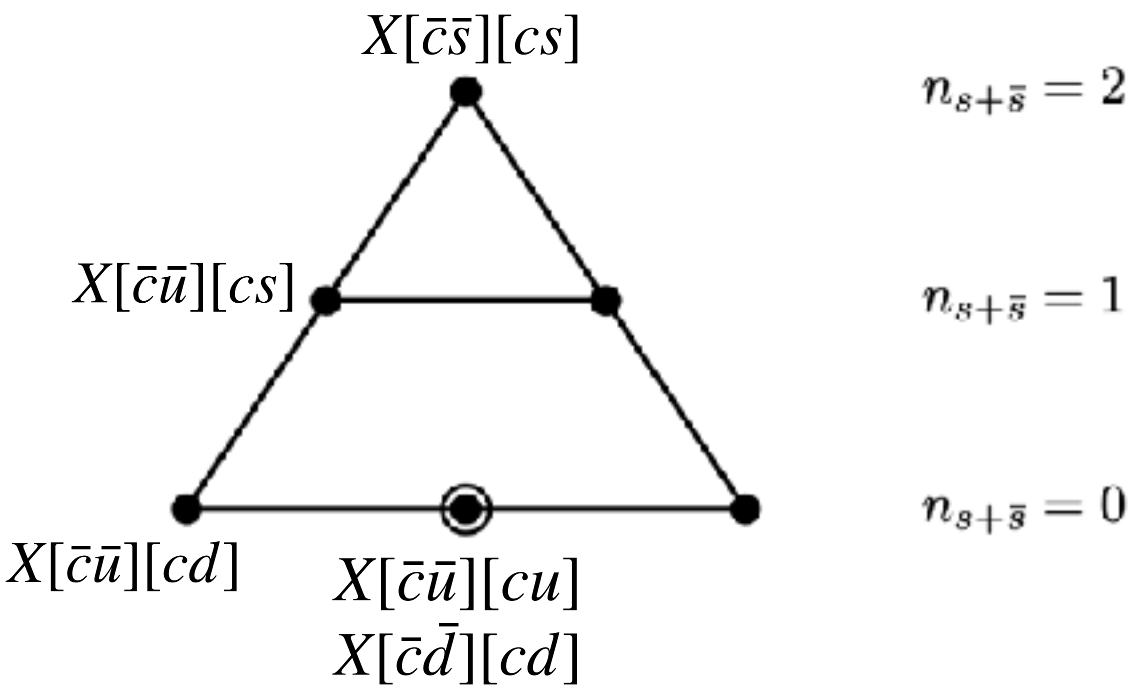




- Octet multiplets can be also represented in function of the total number of s or $\bar{s}$ quarks;
- octet breaking implies ***the equal spacing rule*** of the masses in the ladder.

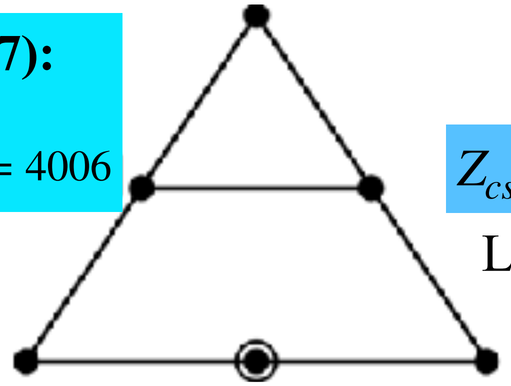
Three nonets

L. Maiani, A. D. Polosa and V. Riquer, Sci. Bull. **66** (2021), 1616, arXiv:2103.08331

$J^{PC} = 1^{++}$

Predicted (2017):

$$\frac{X(4140) + X(3872)}{2} = 4006$$



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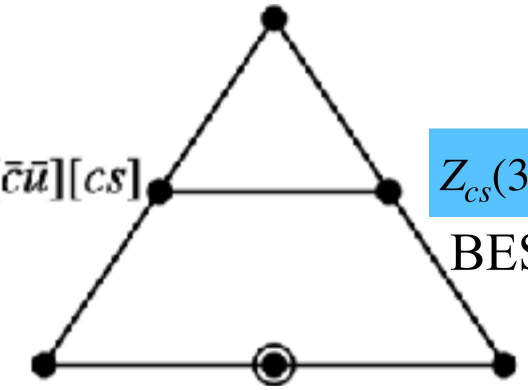

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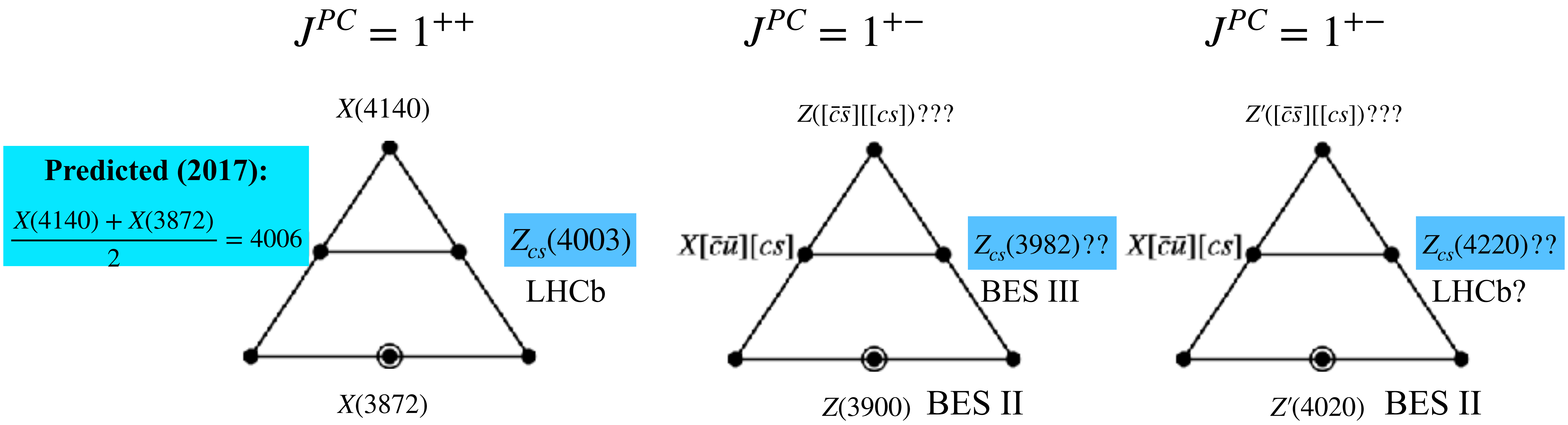

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$X(4140)$

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$\frac{X(4140) + X(3872)}{2} = 4006$

$Z_{cs}(4003)$

LHCb

$X(3872)$

$J^{PC} = 1^{+-}$

$Z([\bar{c}\bar{s}][[cs])???$

$X[\bar{c}\bar{u}][cs]$

$Z_{cs}(3982)??$

BES III

$Z(3900)$ BES II

$J^{PC} = 1^{+-}$

$Z'([\bar{c}\bar{s}][[cs])???$

$X[\bar{c}\bar{u}][cs]$

$Z_{cs}(4220)??$

LHCb?

$Z'(4020)$ BES II

- There is a ***third nonet associated with*** $Z_c(4020)$, $J^{PC} = 1^{+-}$: a third Z_{cs} is required, with Mass=4150 - 4170
- LHCb sees a $Z_{cs}(4220)$**,** $J^P = 1^+$***: is it too heavy ?***
- A bold proposal: ***two nonets mixing***

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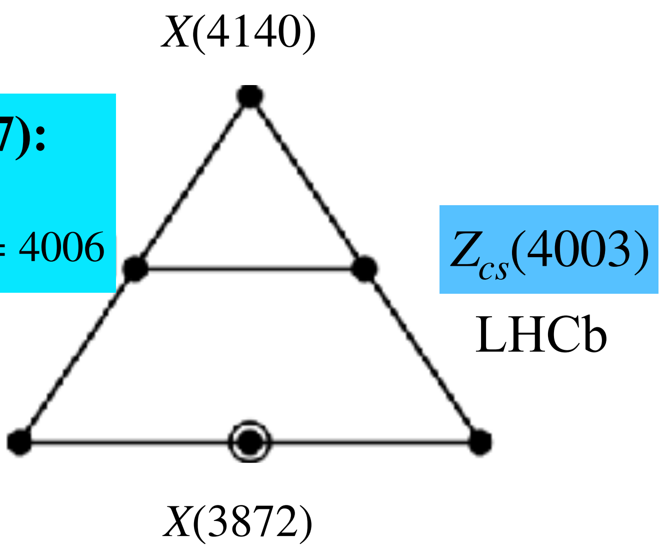


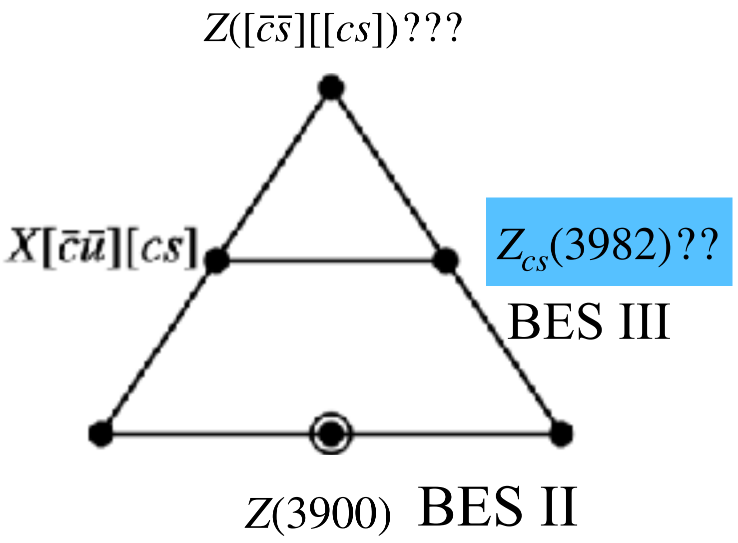
$X(4140)$

$Z_{cs}(4003)$

LHCb

$X(3872)$

$J^{PC} = 1^{+-}$



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$Z_{cs}(3982)??$

BES III

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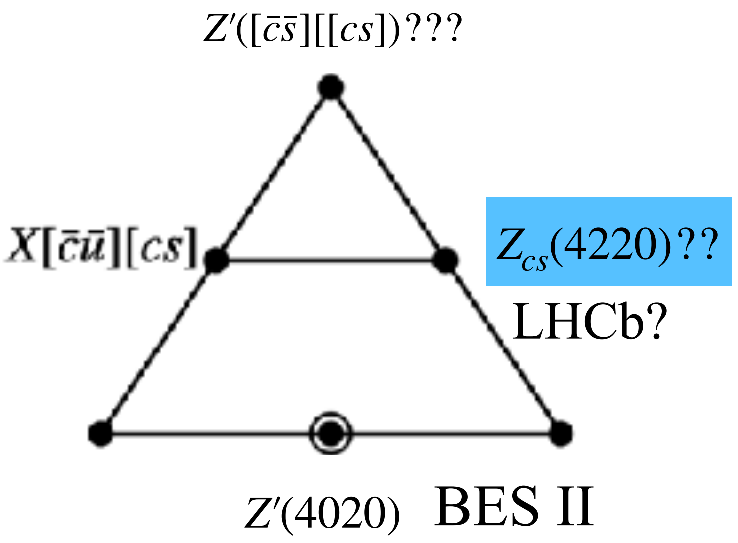

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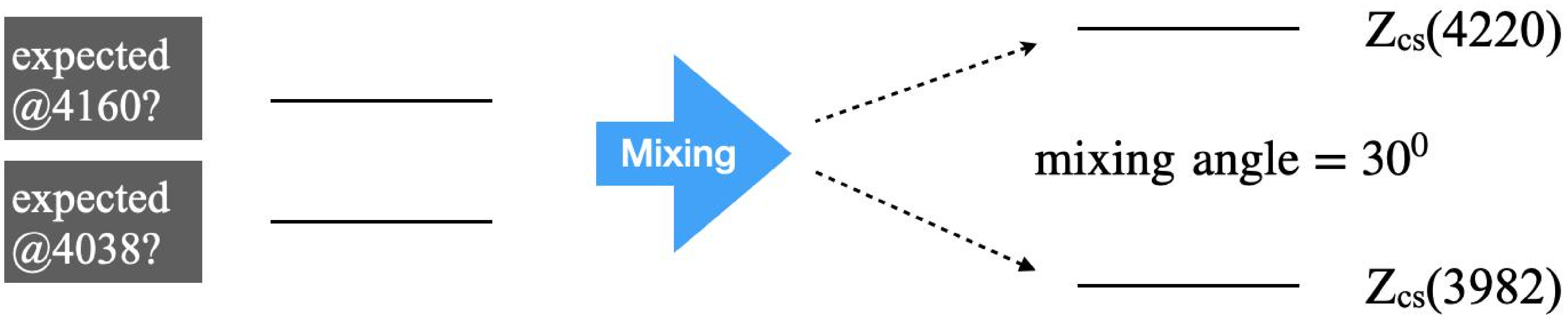


expected @4160?

expected @4038?

Mixing

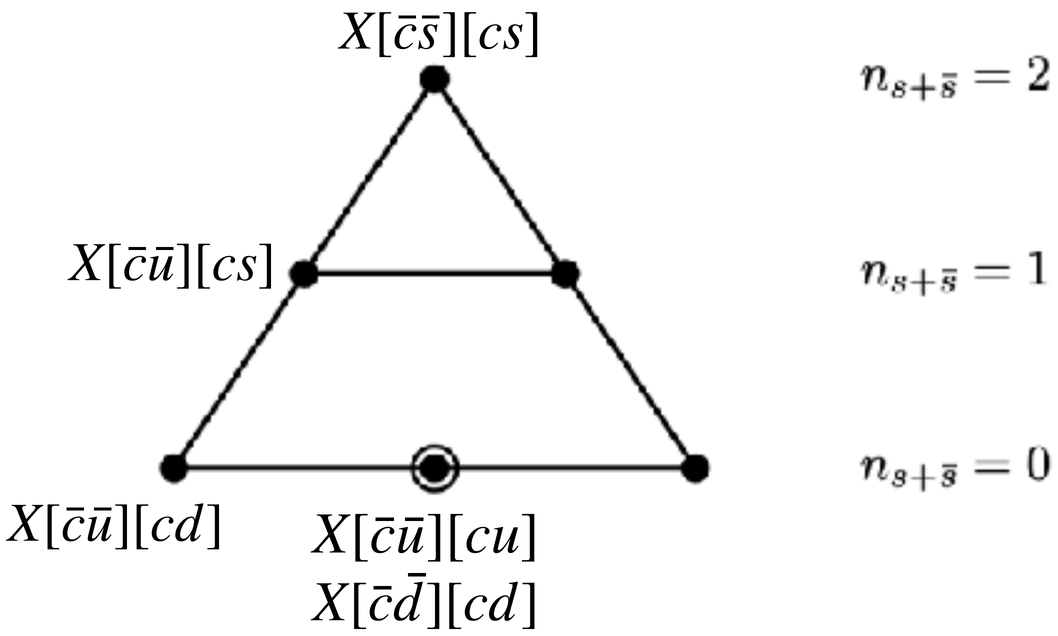
$Z_{cs}(4220)$

mixing angle $= 30^0$

$Z_{cs}(3982)$

Missing particles to complete the hidden charm nonets, $J^P = 1^+$

The shopping list towards completion of the ***hidden charm nonets***:



- two $X_{[\bar{c}\bar{s}][cs]}$, expected at:

$M \sim 4170$ for $Z_c(3900)$ and $M \sim 4290$ for $Z_c(4020)$

with decays: $\eta\psi$, $\eta_c\ \phi$, $D_s^*\bar{D}_s$ (threshold: 4080 MeV)

- the I=1 partner of X(3872), with decay:

$$X^+ \to J/\psi\ \rho^\pm \to J/\psi\ \pi^+\pi^0 \text{ with the bounds:}$$

$$0.057 < R_{2\pi}^{(0+,00)} = \frac{\Gamma(B^0 \to K^+X^- \to K^+\psi\ \pi^0\pi^-)}{\Gamma(B^0 \to K^0X(3872) \to K^0\psi\ \pi^+\pi^-)} < 0.50$$

L. Maiani, A. D. Polosa and V. Riquer, Phys. Rev. D **102** (2020) 034017

- the I=0 partners of Z$_c$(3900) and Z$_c$(4020), possibly decaying into:

$$J/\psi + f_0(500) \text{ (aka } \sigma(500))$$

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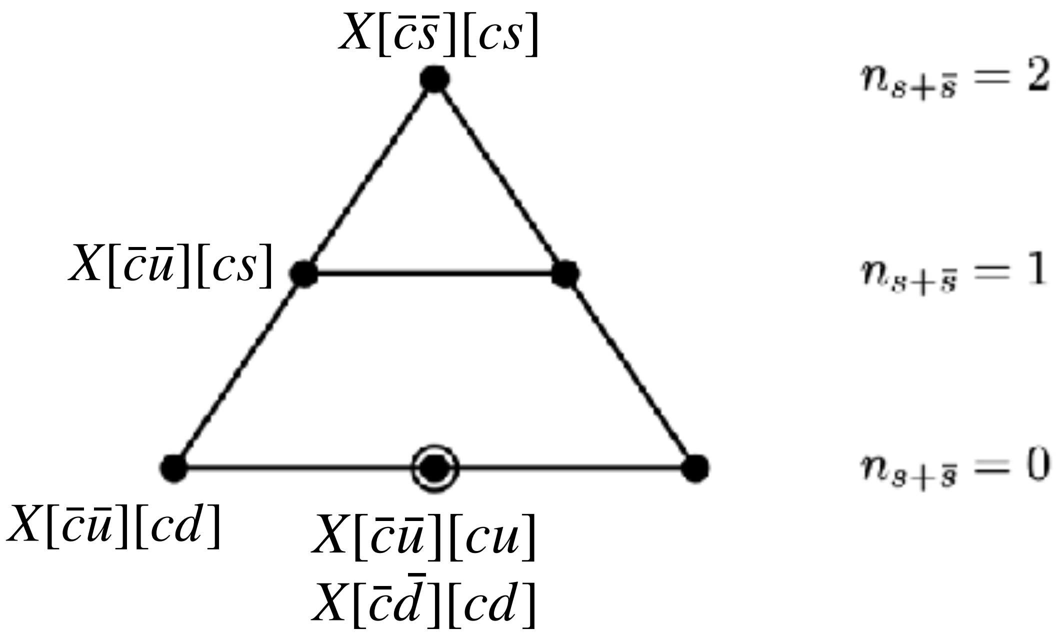

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- two $X_{[\bar{c}\bar{s}][cs]}$, expected at:

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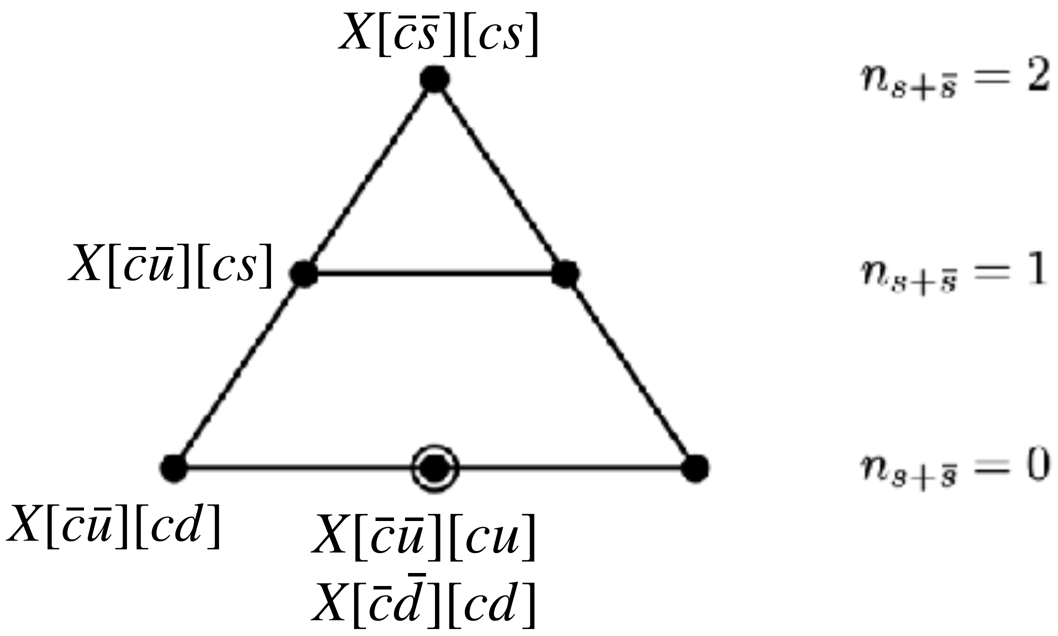

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- the I=0 partners of Zc(3900) and Zc(4020), possibly decaying into:

$$J/\psi + f_0(500) \text{ (aka } \sigma(500))$$

Only few particles are missing, in well defined mass regions and with identified decay modes.

3. Single charm tetraquarks, with three SU(3)-flavour light mesons: the case of $J^P = 0^+$

L.Maiani, A. Polosa, V.Riquer, ArXiv:2405.08545

- In a recent lattice QCD calculation the $SU(3)_{flavor}$ configurations of possible bound states in the $\bar{D}K, J^P = 0^+$, channel are studied;
- the allowed $SU(3)_{flavor}$ channels are those appearing as irreducible components of the tensor product

$$\bar{D}K = \mathbf{3} \otimes \mathbf{8} = \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15}$$

- Yeo *et al.* find attraction in $\mathbf{3}$ and $\bar{\mathbf{6}}$ but not in $\mathbf{15}$.

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- Consider now four-quark mesons in the simplest diquark-antidiquark model restricting to all spin zero case:

$$[\bar{c}\bar{q}]^{\mathbf{3}}_{S_{c3}}[q_1q_2]^{\bar{\mathbf{3}}}_{S_{12}}$$

$$S_{c3} = S_{12} = 0; \;\; J^P = S_{c3} + S_{12} = 0^+$$

- The product $[q_1q_2]^{\bar{\mathbf{3}}}_{S_{12}=0}$ is antisymmetric in spin (to get total spin 0) and color (to obtain a $\bar{\mathbf{3}}_c$).

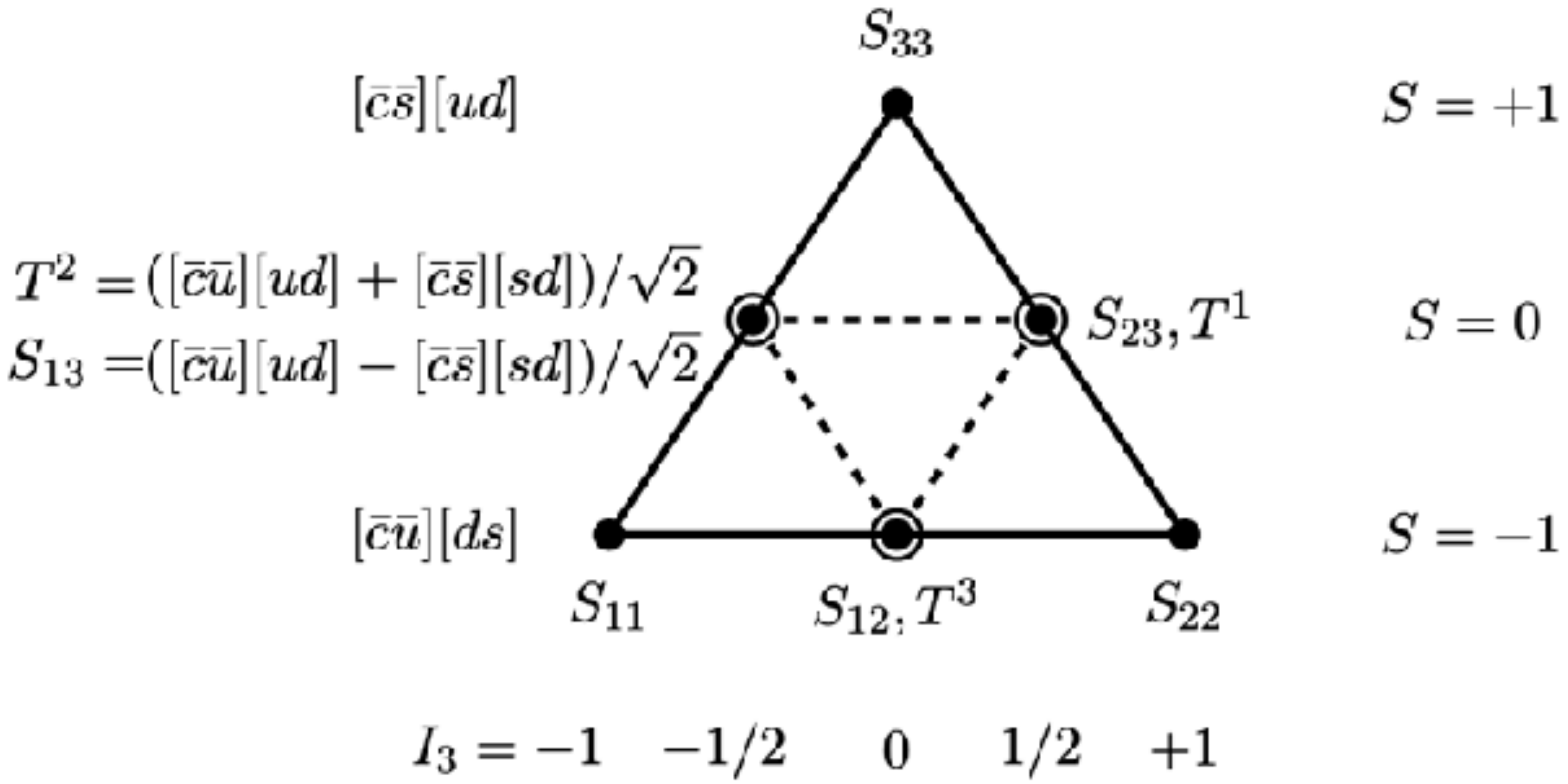
- Fermi statistics: *quarks in the light diquark must be antisymmetric in flavour, i.e must be in a* $\bar{\mathbf{3}}$ *of* $SU(3)_{flavor}$.
- combining with the light antiquark $\bar{q} \propto \bar{\mathbf{3}}$, the tetraquark must be in a $SU(3)_{flavor}$ multiplet:

$$\bar{\mathbf{3}} \otimes \bar{\mathbf{3}} = \mathbf{3} \oplus \bar{\mathbf{6}} \text{ , no } \mathbf{15}$$

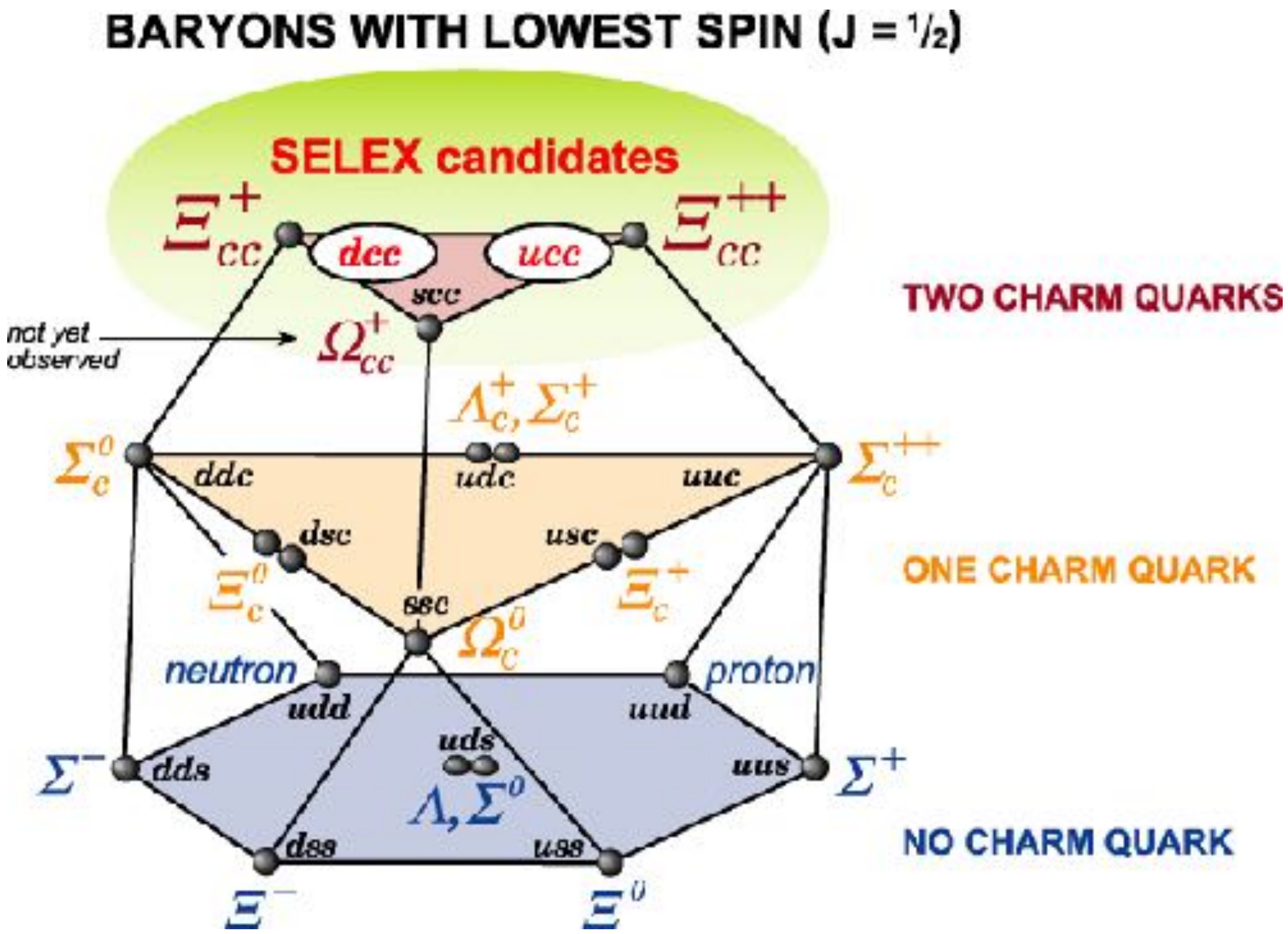
in agreement with the lattice indication.

Quantum numbers and states of $[\bar{c}\bar{q}]^{\mathbf{3}}_{S_{c3}=0}[q_1q_2]^{\bar{\mathbf{3}}}_{S_{12}=0}$ (cont'd)

The $\mathbf{3} \oplus \bar{\mathbf{6}}$ representation in the $I_3 - S$ plane:



Single Charm Baryons & Fermi Statistics



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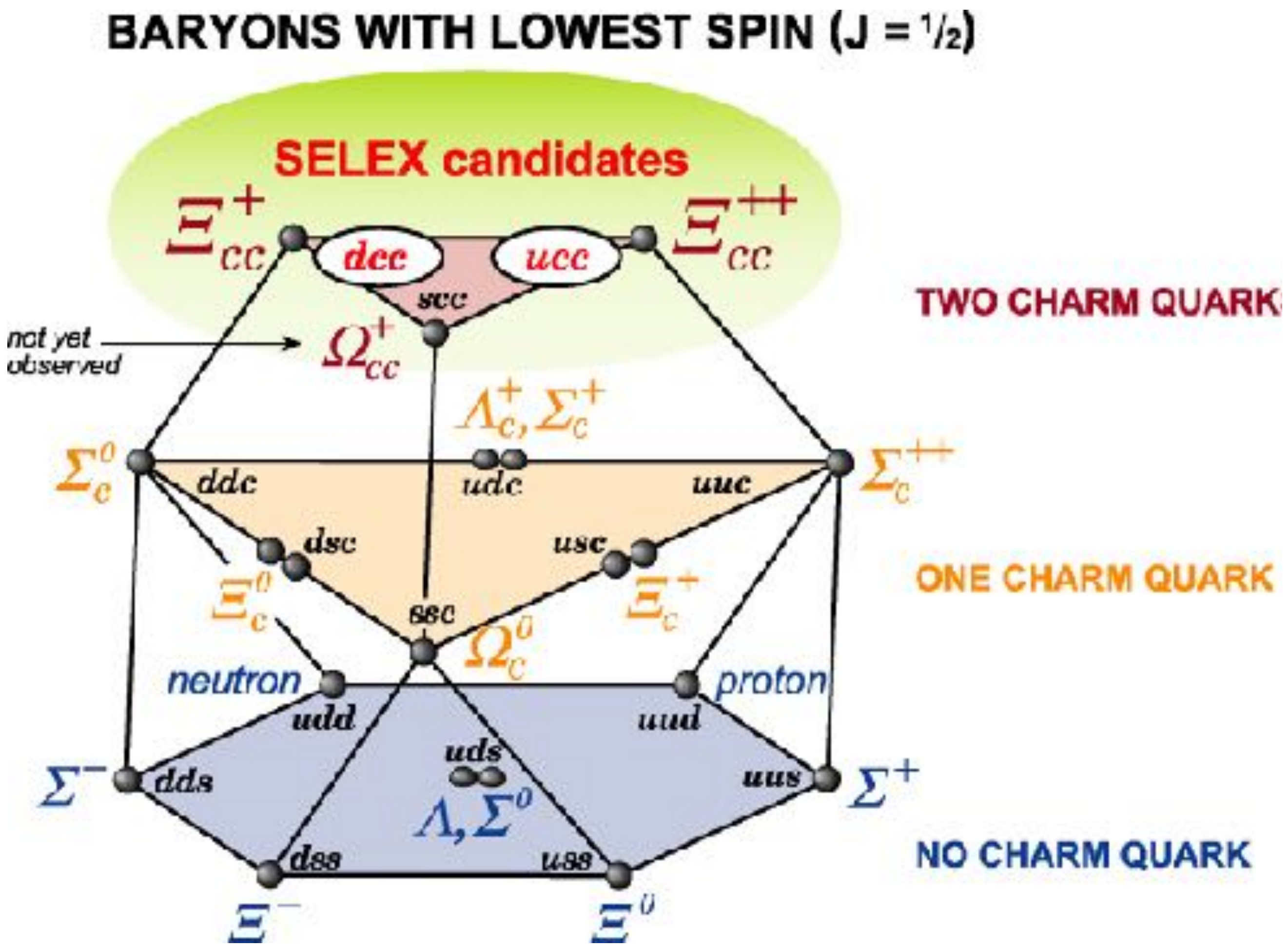


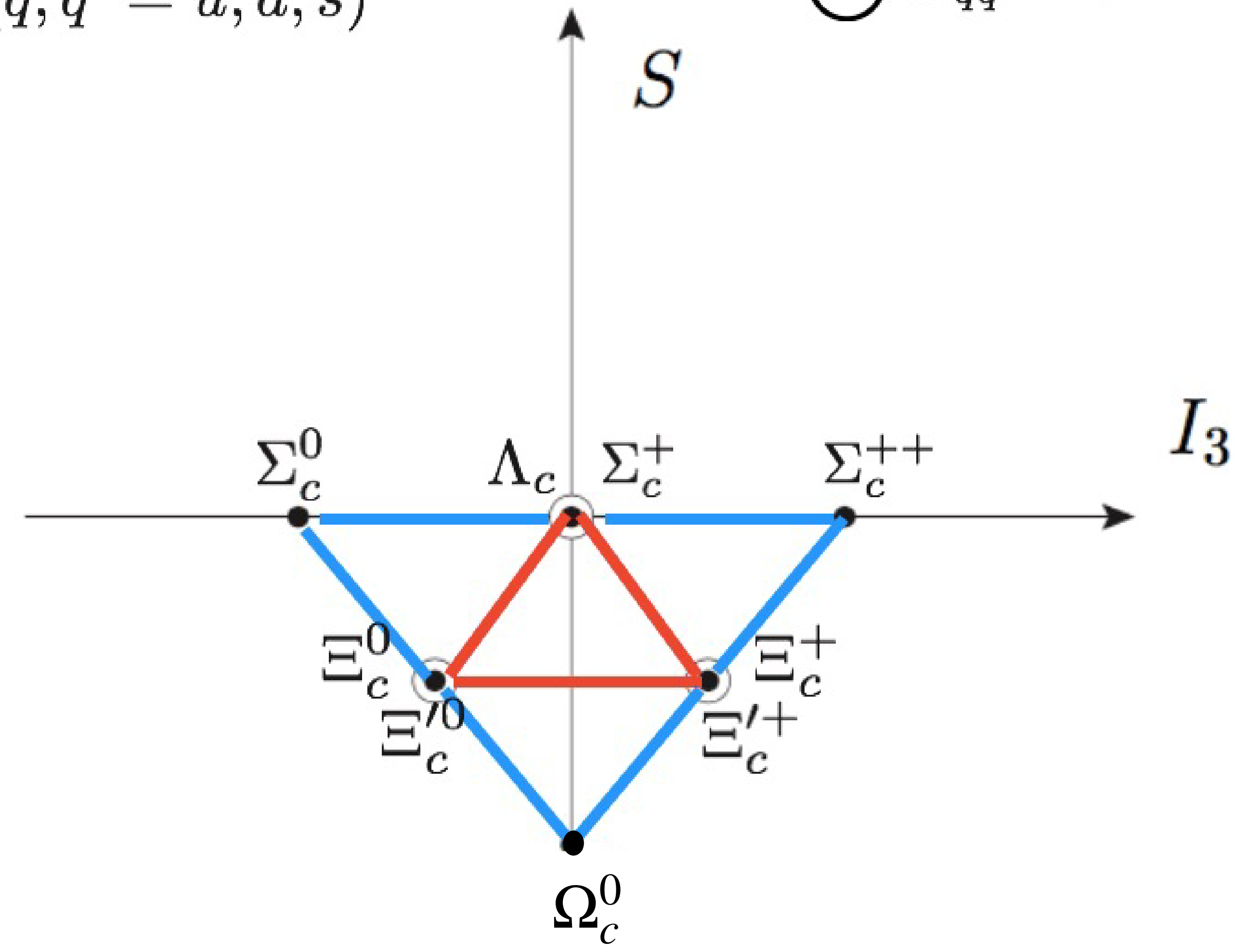
$q,\ q'$: color $= \mathbf{3} \otimes \mathbf{3} \to \bar{\mathbf{3}}$

$\mathbf{J} = 1/2 :\ \mathbf{6} \oplus \bar{\mathbf{3}}$

$(q, q' = u, d, s)$

● $S_{qq'} = 1$

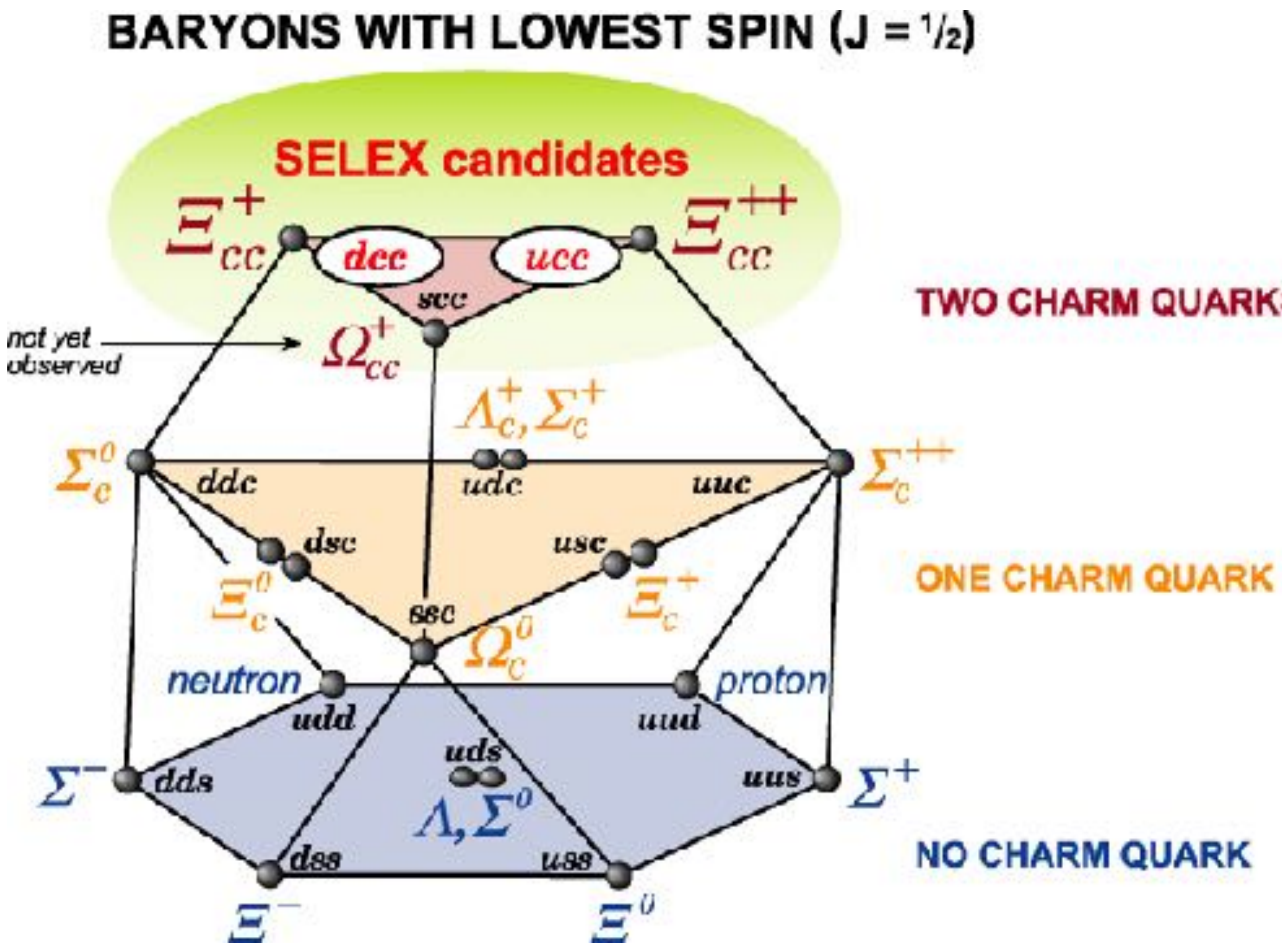
○ $S_{qq'} = 0$



$$I = 1 \ \leftrightarrow \ spin = 1,\ (\Sigma_c^{0,+,++})$$

$$I = 0 \ \leftrightarrow \ spin = 0 \ \ (\Lambda_c^+)$$

Single Charm Baryons & Fermi Statistics



The masses of spin 1 diquarks in the **6** increase linearly with Strangeness, i.e. with the number of strange quarks:

$M(\Omega_c) - M(\Sigma_c) \simeq 270$ MeV

$q,\ q'$: color $= \mathbf{3} \otimes \mathbf{3} \to \bar{\mathbf{3}}$

$\mathbf{J} = 1/2: \ \mathbf{6} \oplus \bar{\mathbf{3}}$

$(q, q' = u, d, s)$

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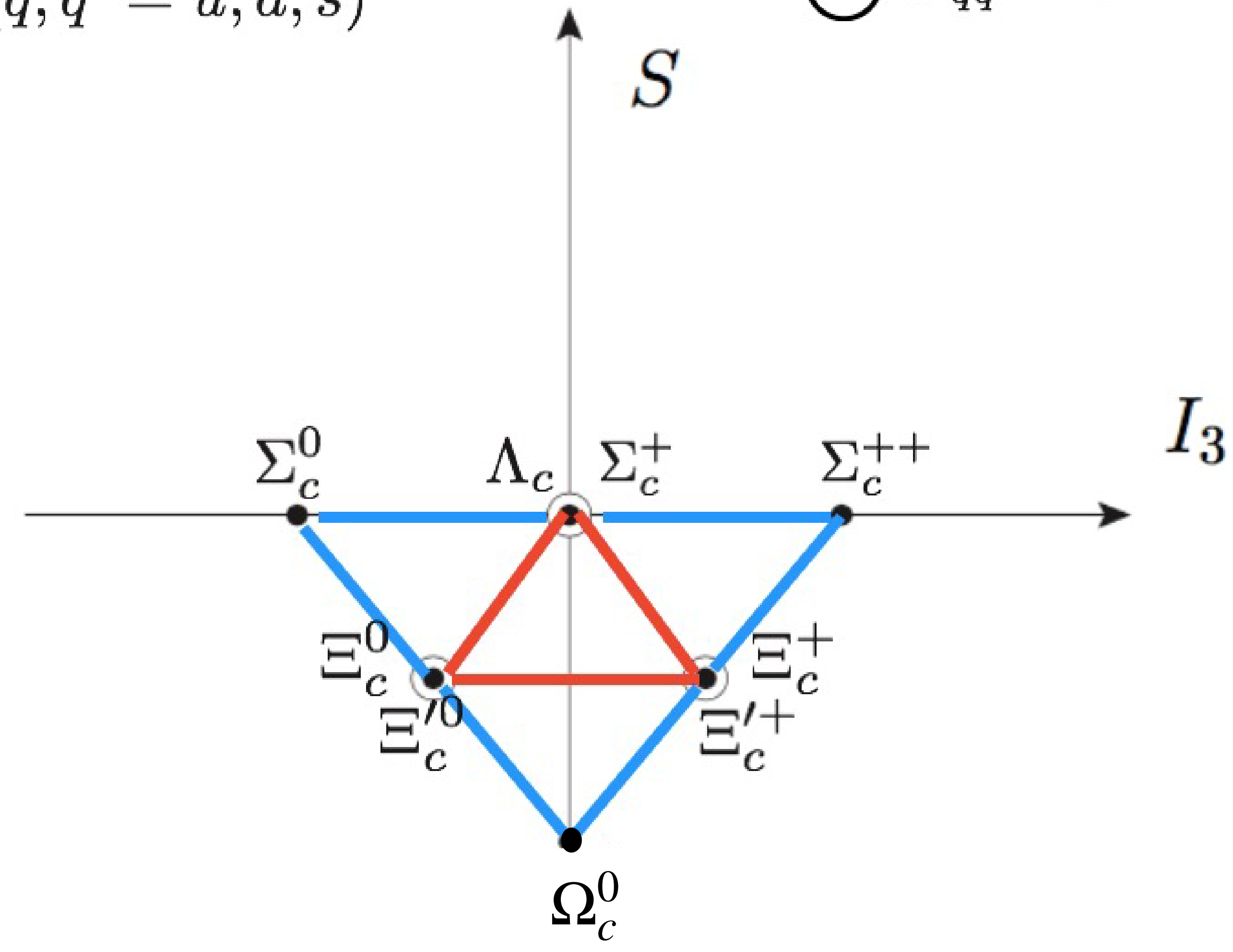

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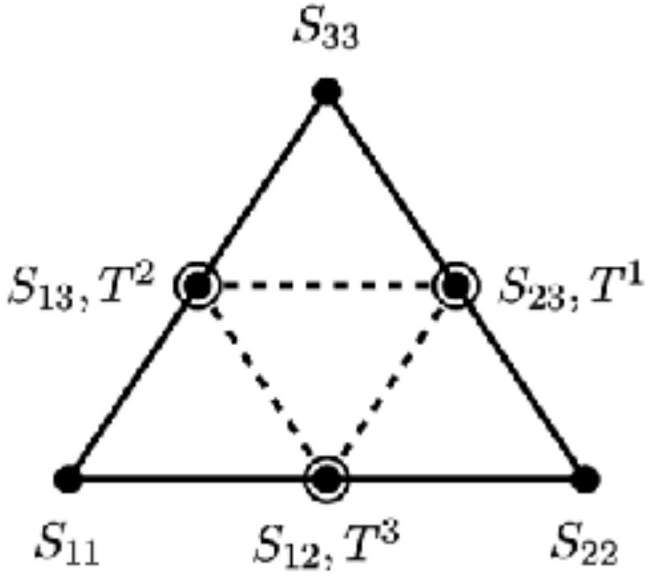
A remarkable regularity



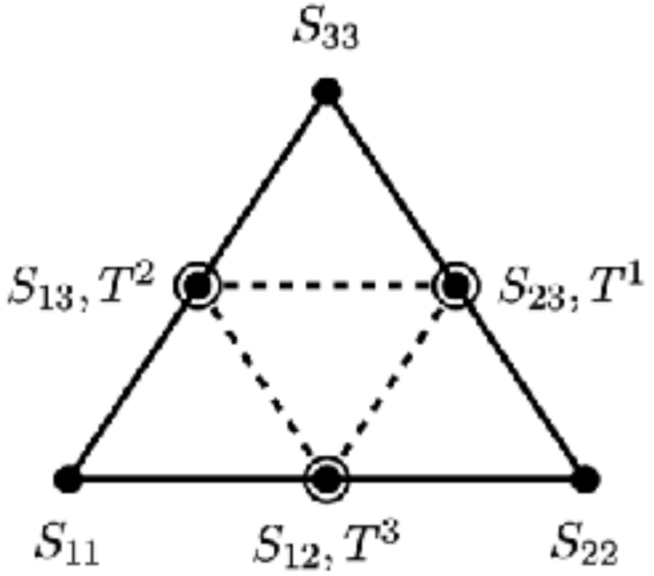
Like the masses of single charm baryons, the masses of single charm tetraquarks are equally spaced in Strangeness, with a slope given by a parameter α.

However, ***unlike charmed baryons, the lower indices in S_{11} correspond to the quark-diquark antisymmetric configuration $\bar{u} \otimes [ds]_A$, while the lower indices in S_{33} correspond to $\bar{s} \otimes [ud]_A$,***

which have obviously the same content in quark masses, two light and one heavy.

- Exact equality $M(S_{33}) = M(S_{11})$ corresponds to $\alpha \sim 0$: same masses at the upper vertex and lower corners of the triangle in the figure.
- In this case, symmetry breaking is restricted to the mass difference between the two $S = 0$, $I = 1/2$ multiplets induced by the $\mathbf{3} - \bar{\mathbf{6}}$ mixing and of order $\delta \sim 2(m_s - m_q)$, with all other masses degenerate at M.
- A small, non vanishing value of α arises from differences in the hyperfine interactions, which are between different pairs in the two cases.

A remarkable regularity



Like the masses of single charm baryons, the masses of single charm tetraquarks are equally spaced in Strangeness, with a slope given by a parameter α.
However, ***unlike charmed baryons, the lower indices in*** S_{11} ***correspond to the quark-diquark antisymmetric configuration*** $\bar{u} \otimes [ds]_A$***, while the lower indices in*** S_{33} ***correspond to*** $\bar{s} \otimes [ud]_A$***,***
which have obviously the same content in quark masses, two light and one heavy.

- Exact equality $M(S_{33}) = M(S_{11})$ corresponds to $\alpha \sim 0$: same masses at the upper vertex and lower corners of the triangle in the figure.
- In this case, symmetry breaking is restricted to the mass difference between the two $S = 0$, $I = 1/2$ multiplets induced by the $\mathbf{3} - \bar{\mathbf{6}}$ mixing and of order $\delta \sim 2(m_s - m_q)$, with all other masses degenerate at M.
- A small, non vanishing value of α arises from differences in the hyperfine interactions, which are between different pairs in the two cases.

In charmed baryons, two light quarks in spin one are in a **6** *representation. In this case, indices 1 or 3 correspond univocally to u or s quarks. Group theory disentangles efficiently the ambiguity in these two* **6** *representations* ***making use of the parameter allowed by the Wigner-Eckart theorem***.

4. The $\mathbf{3} \oplus \bar{\mathbf{6}}$ (n=2) radially excited multiplet

- $X_0(2900)$ and $D_{s0}^0(2900)$, $D_{s0}^{++}(2900)$ observed by LHCb

$X_0(2900)$ R. Aaij et al. [LHCb], Phys. Rev. D 102 (2020), 112003

$D_{s0}^{0,++}(2900)$ R. Aaij et al. [LHCb], Phys. Rev. Lett. 131 (2023) 041902;

are too heavy to be included in the basic $\mathbf{3} \oplus \bar{\mathbf{6}}$ multiplet of $D_{s0}^*(2317)$.

- The mass difference: $M(2900) - M(2317) = 583$ MeV is similar to the mass gaps

$$M(J/\Psi) - M(J/\Psi') = 590 \text{ MeV}, \ M(X(3872) - M(Z(4430)) = 558 \text{ MeV}$$

- we interpret the LHCb resonances as the ***first radial excitations*** ($n = 2$) of the basic $D_{s0}^*(2317)$ multiplet.

The n=2 multiplet:

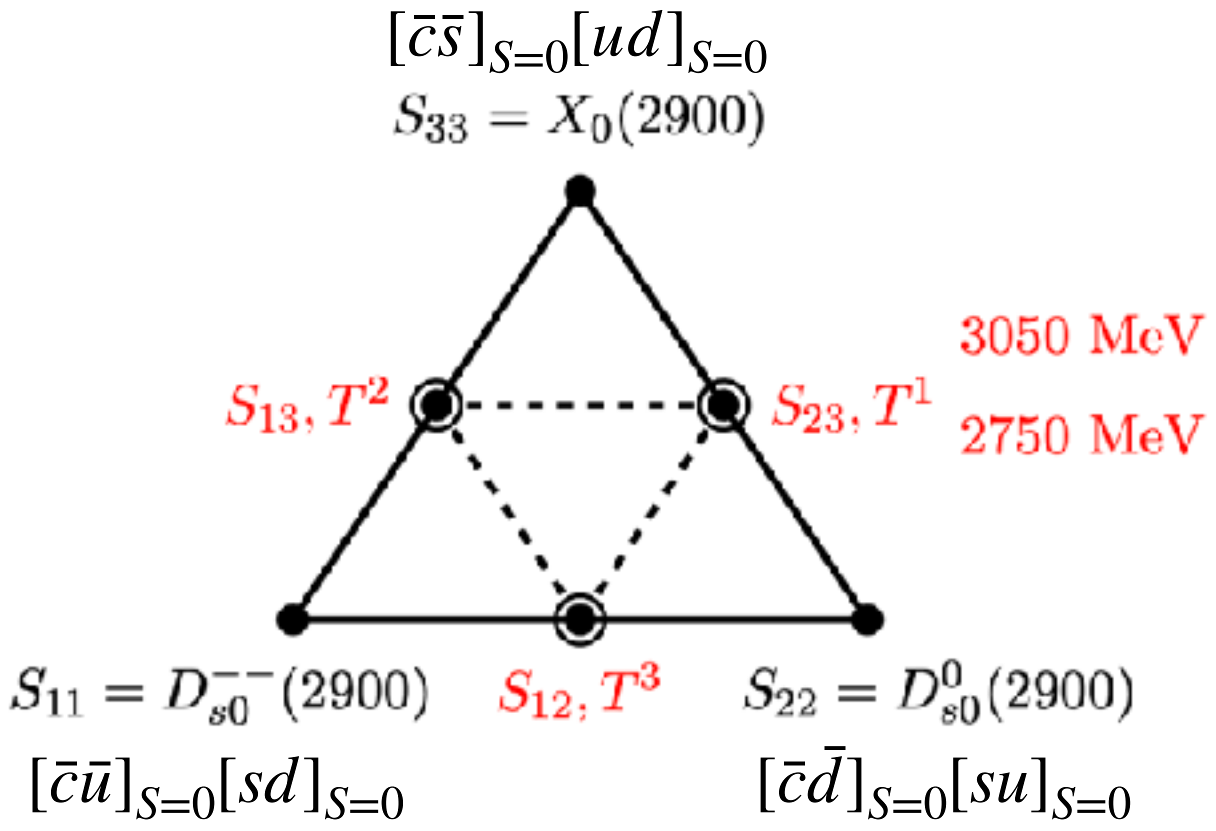
(i) in black the resonances observed by LHCb;

(ii) in red the missing S=0 states and their estimated masses (indicative only) .

(iii) expected decay modes:

$$[\bar c\bar s][sd]_{(n=2)}(3050) \to \bar D^- \eta,\ \bar D_s^- K^0,\ \bar D^{*-}\phi, \ldots$$

$$[\bar c\bar u][ud]_{(n=2)}(2750),\ [\bar c\bar d][ud]_{(n=2)} \to \bar D\pi, \ldots$$



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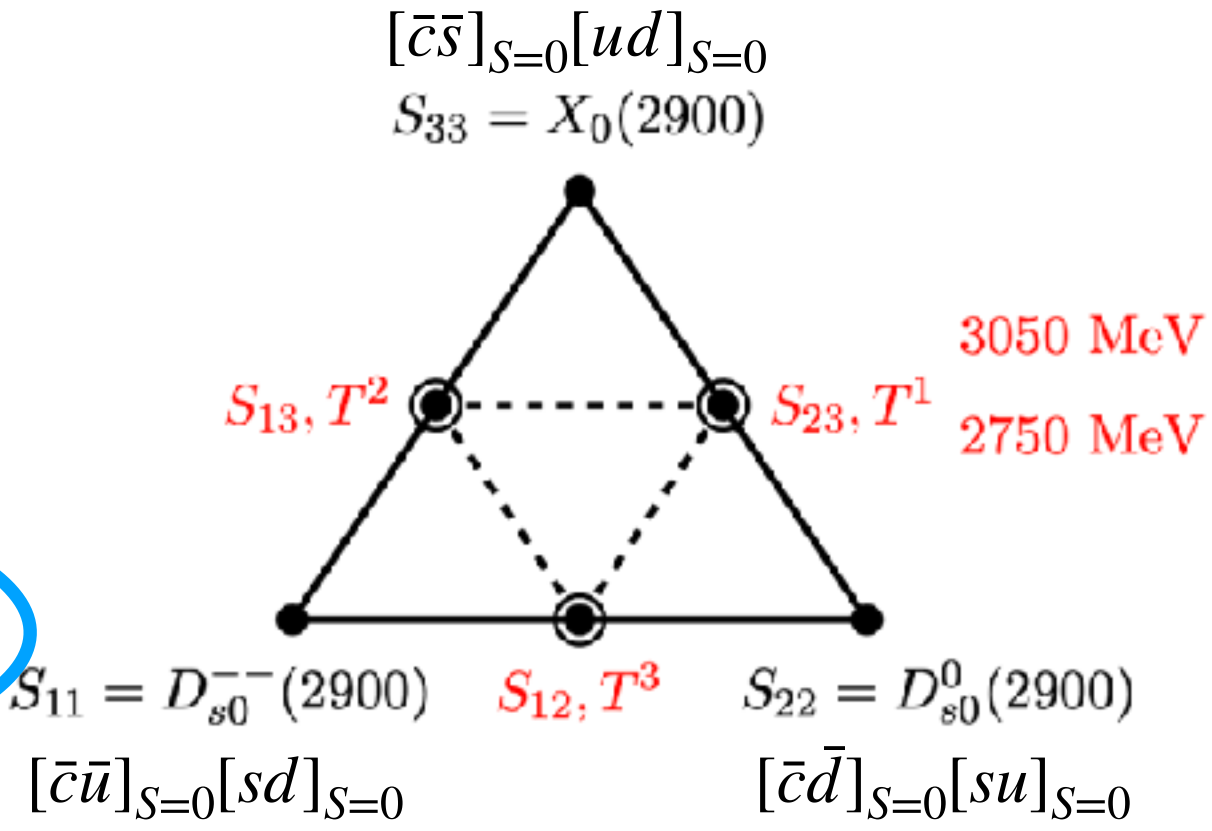

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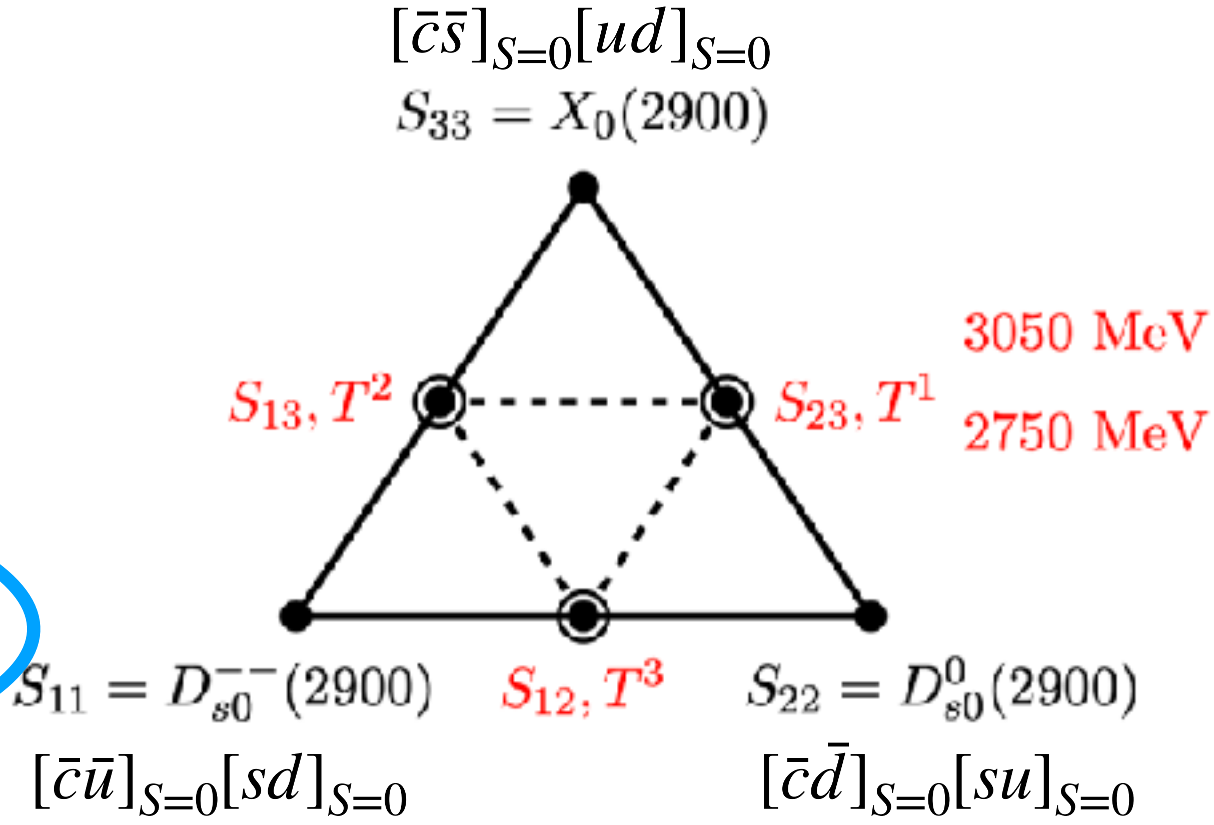

- ***Mass degeneracy between*** $\mathbf{X_0(2900)}$ ***(S=+1) and*** $\mathbf{D^{--,0}_{s0}(2900)}$ ***(S=-1) is the footprint of the tetraquark compositions:***

$$[\bar{c}\bar{s}]_0[ud]_0 \text{ and } [\bar{c}\bar{u}]_0[sd]_0$$

6. Heavy particle spin conservation and Fermi Statistics of light quark pairs: QCD tetraquarks vs hadron molecules

- For molecular tetraquarks, in the limit of very massive charm quark, the light quark total spin is a separatly conserved quantity (this is the *light quark spin symmetry in the static quark approximation* introduced by Isgur and Wise)
- For hidden charm molecules $(\bar{c}q)(\bar{q}'c)$, flavour symmetry, e.g. Isospin, is also an independent (commuting) conserved quantity. The possible combinations of light and heavy spin generate six states with definite Isospin, total angular momentum and charge conjugation:

Z. H. Zhang *et al.*, arXiv:2404.11215 [hep-ph]

$$J_I^{PC} = 0_I^{++},\ 1_I^{+-},\ 1_I^{\prime +-},\ 1_I^{++},\ 0_I^{\prime ++},\ 2_I^{++}.$$

- These are the same J_I^{PC} states predicted for diquark-antidiquark tetraquarks of the form $[cq]^{\bar{\mathbf{3}}}[\bar{c}\bar{q}']^{\mathbf{3}}$. Noticeably, they include the I=1 partner of $X(3872)$, i.e. X^+.

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- Concerning Fermi Statistics, the situation for the molecular structure $(\bar{c}q_1)(\bar{s}q_2)$ is different with respect to diquark-antidiquark situation.
- q_1 and q_2, sit in different color singlets and the color of the pair $q_1 \otimes q_2$ is not determined (in fact it is a superposition of $\bar{\mathbf{3}}_c$ and $\mathbf{6}_c$). There is no definite restriction to their behaviour under flavor exchange and no forbidden $\mathbf{15}$.

Final questions (to LHCb and BESIII)

- Are $Z_{cs}(3986)$ and $Z_{cs}(4003)$ two different states? is there a third $Z_{cs}(4220)$?
- Can X^+ near $X(3872)$ be found in B decays?
- can we find the missing partners of the $\bar{\mathbf{6}} \oplus \mathbf{3}$, (n=2) multiplet:

$$[\bar{c}\bar{s}][sd]_{(n=2)}(3050) \to \bar{D}^-\eta,\ \bar{D}_s^-K^0,\ \bar{D}^{*-}\phi, \dots$$

$$[\bar{c}\bar{u}][ud]_{(n=2)}(2750),\ [\bar{c}\bar{d}][ud]_{(n=2)} \to \bar{D}\pi, \dots$$

- LHCb has used efficiently the channel $B \to (J/\Psi)\phi K + \dots$to study $X_{ss}(4140)$ etc.., and Z_{cs} etc. of SU(3) nonet tetraquarks…
- Can the study of $B \to \bar{D}_s D\phi$ channel be similarly used to study single charm $[\bar{c}\bar{s}]_{S=1}[ss]_{S=0}$, $J^P = 1^+$ tetraquarks of the interesting $\mathbf{15} \oplus \mathbf{3}$, $J^P = 1^+$ multiplet ?
- Reconsider K-like states which decay into $K\phi$ (e.g. $K_1(2650)$), therefore unlikely to be $(s\bar{q})$ excited Kaons: could they be zero-charm $[\bar{u}\bar{s}][ss]$ tetraquarks?

A suggestion for a joint LA Facility

- Charm-tau Factories in Japan and China have greatly contributed to the discovery of exotic hadrons, together with LHC-CERN
- A Super-Charm-tau Factory, high luminosity, c.o.m. energy in the 2-7 GeV range

could be decisive to advance the field

- A facility like that is discussed in China.

Would it be realistic to consider the Super-Charm-tau Factory

to start a common LA experimental activity in Particle Physics

as it has been the CERN-Proton Syncrotron in Europe, 70 years ago?