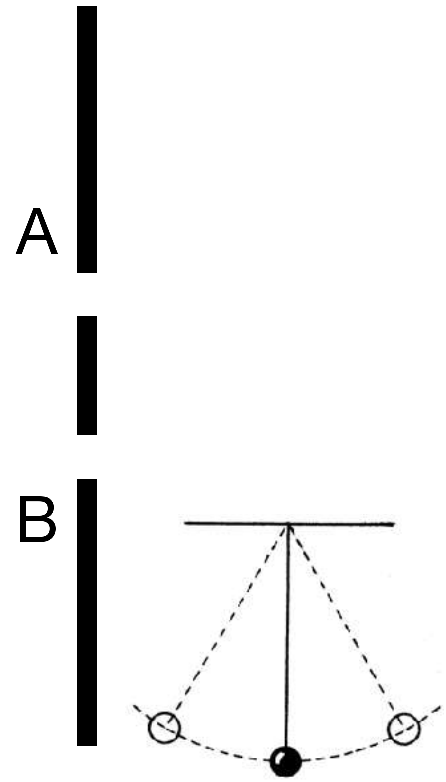
Precision mechanical tests of gravity's quantumness?

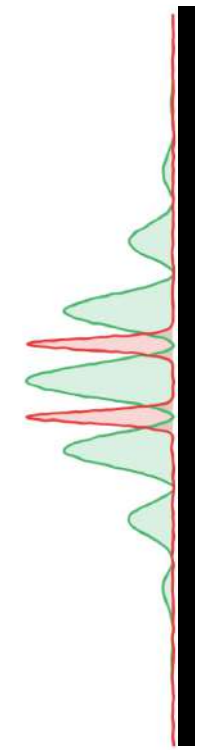
Vivishek Sudhir
Massachusetts Institute of Technology
precision.mit.edu

LIGO

MITMECHE

Is gravity quantum?





$$|A\rangle + |B\rangle$$

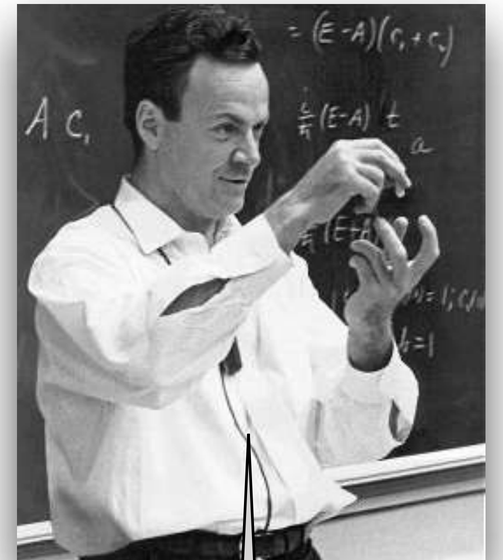
$$P \propto \| |A\rangle + |B\rangle \|^2$$
$$= \langle A|A\rangle + \langle B|B\rangle + 2\mathrm{Re}\,\langle A|B\rangle$$

$$|A, \theta\rangle + |B, \theta'\rangle$$

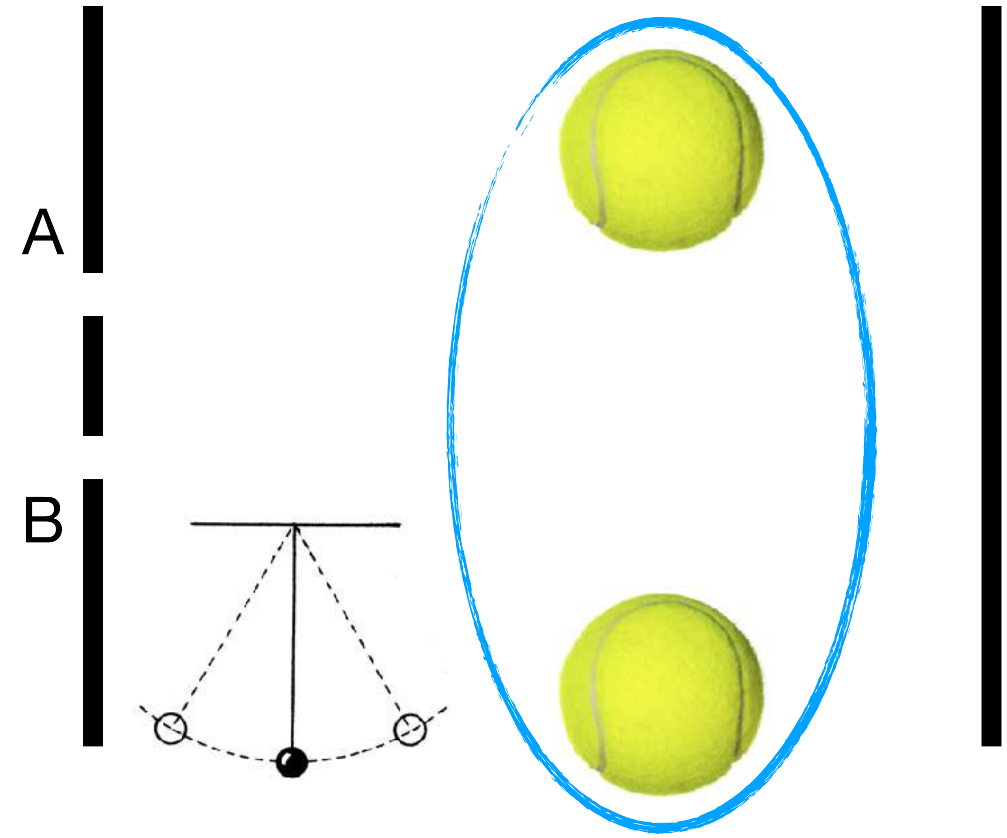
$$P \propto \| |A, \theta\rangle + |B, \theta'\rangle \|^2$$
$$= \langle A|A\rangle\langle\theta|\theta\rangle + \langle B|B\rangle\langle\theta'|\theta'\rangle$$
$$+ 2\mathrm{Re}\,\langle A|B\rangle\langle\theta|\theta'\rangle$$

If quantum mechanics holds for large masses, then gravity shouldn't convey which-path information

Is gravity quantum?



“The necessity of gravitational quantization”
Chapel Hill conference (1957)



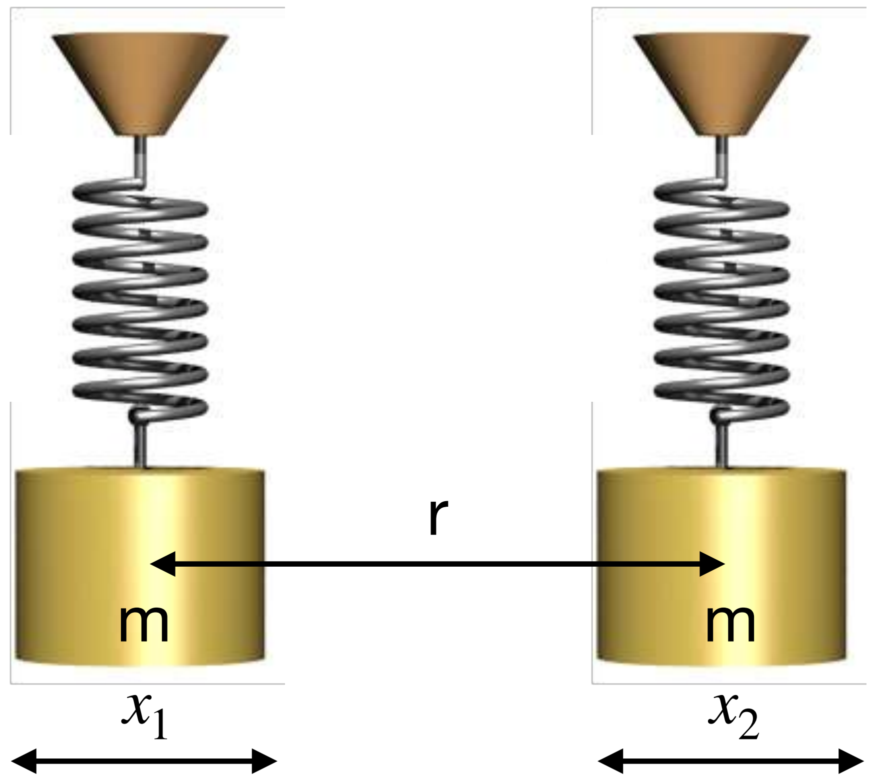
$$|\psi\rangle = |A, \theta\rangle + |B, \theta'\rangle$$

Relies on gravitational entanglement between source and test mass

Erase which-path information: $(|\theta_+\rangle\langle\theta_+|)|\psi\rangle = |\theta_+\rangle(|A\rangle + |B\rangle)$

recover interference

Quantum fluctuation of the gravitational field obscures which-path information



$$H \approx \left(\sum_i \frac{p_i^2}{2m} + \frac{m\omega^2}{2} x_i^2 \right) - \frac{Gm^2}{r^3} x_1 x_2$$

Tree-level calculations motivate the quantized Newtonian interaction:

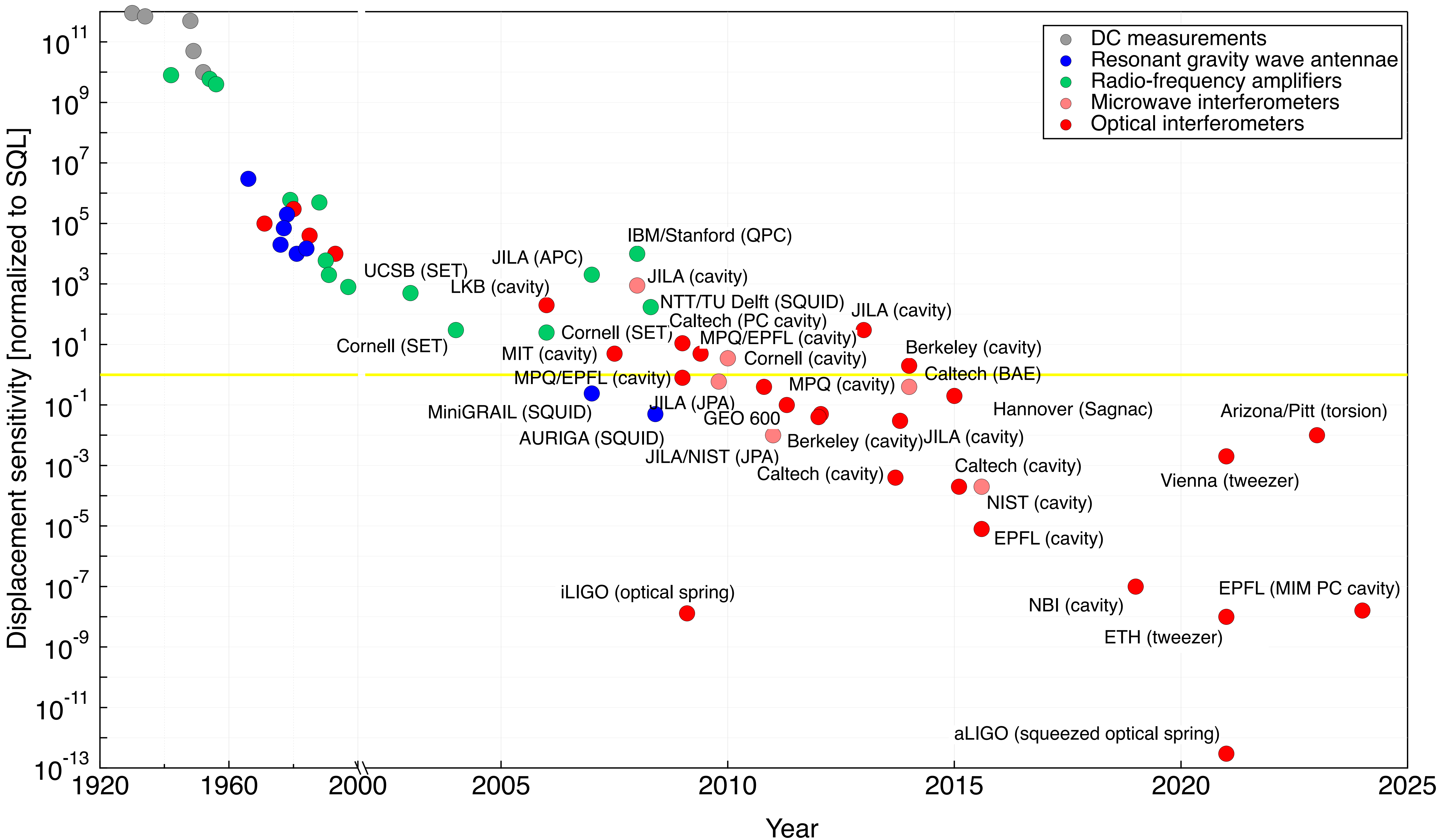
Burgess, Living Rev. Relativity 7, 5 (2004)
Carney, Phys. Rev. D 105, 024029 (2022)
Bose et al., Phys. Rev. D 105, 106028 (2022)

Quantum corrections to Newton?

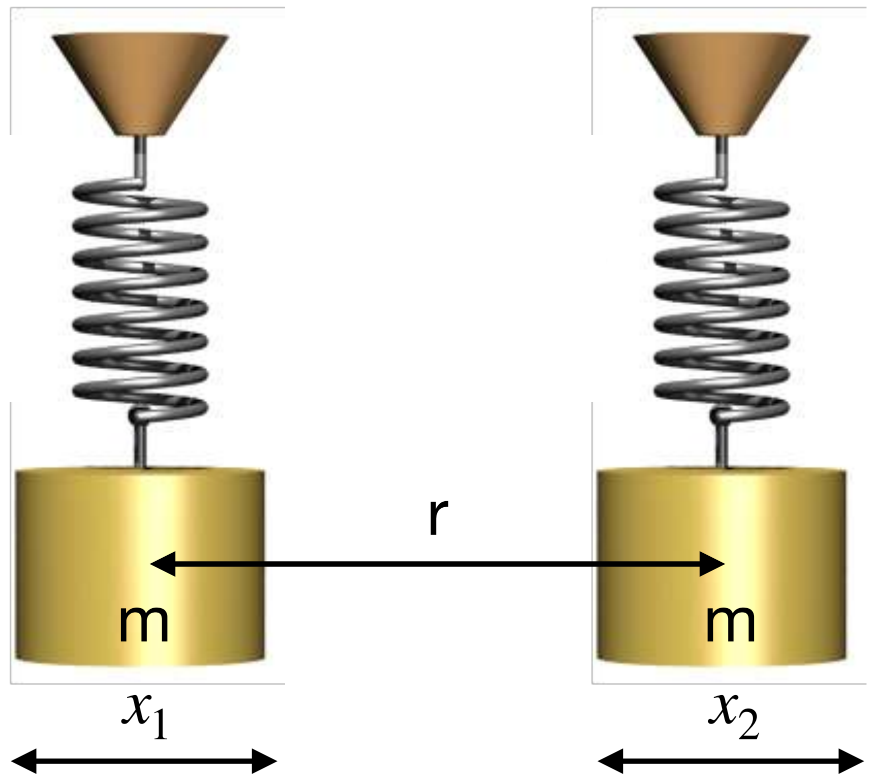
$$V(r) = -\frac{Gm_1m_2}{r}\left[1 - \frac{G(m_1+m_2)}{rc^2} - \frac{127}{30\pi^2}\frac{\hbar G}{r^2c^3}\right]$$

$\sim 10^{-60}$

Donoghue, Phys. Rev. D 50, 3874 (1994)



"Only" 24 orders of magnitude in 100 years



$$H \approx \left(\sum_i \hbar\omega_i \hat{a}_i^\dagger \hat{a}_i \right) - \hbar \cdot \frac{Gm^2}{r^3}\frac{x_{zp}^2}{\hbar}(\hat{a}_1 + \hat{a}_1^\dagger)(\hat{a}_2 + \hat{a}_2^\dagger)$$

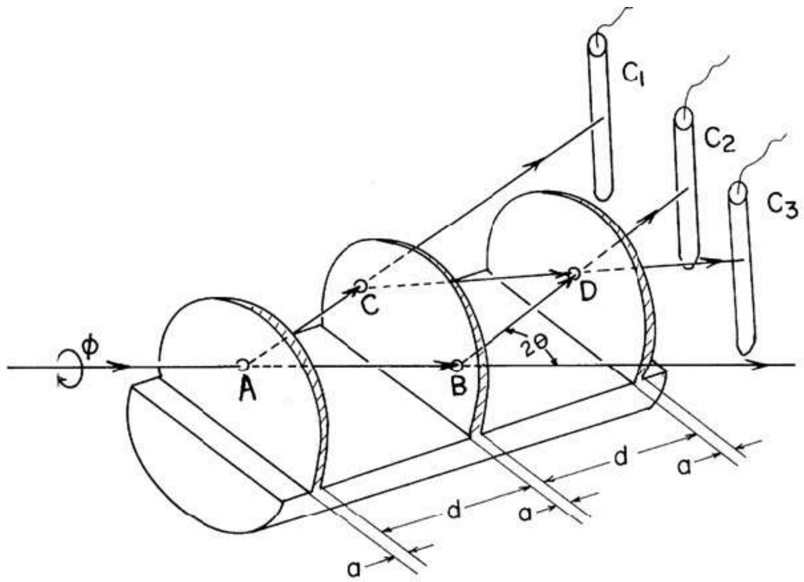
gravitational coupling rate

$$\Omega_G = \frac{Gm}{2\omega r^3}$$

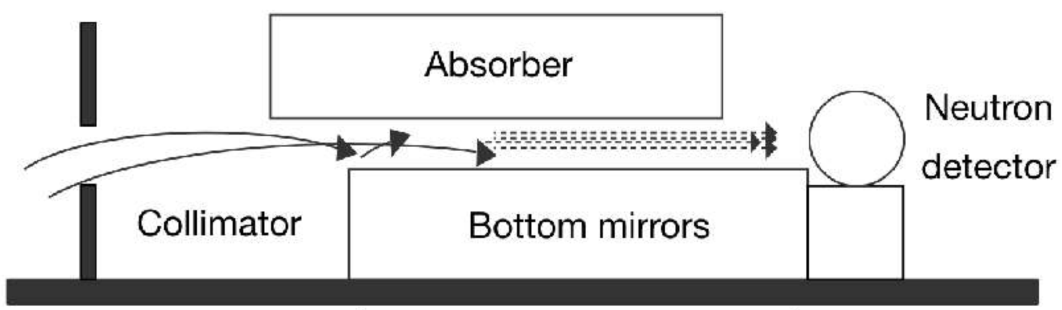
- Prepare pair of large masses in pure quantum states of motion
- Isolate them so that interaction is dominated by mutual gravity

What does it take to observe gravitational entanglement?

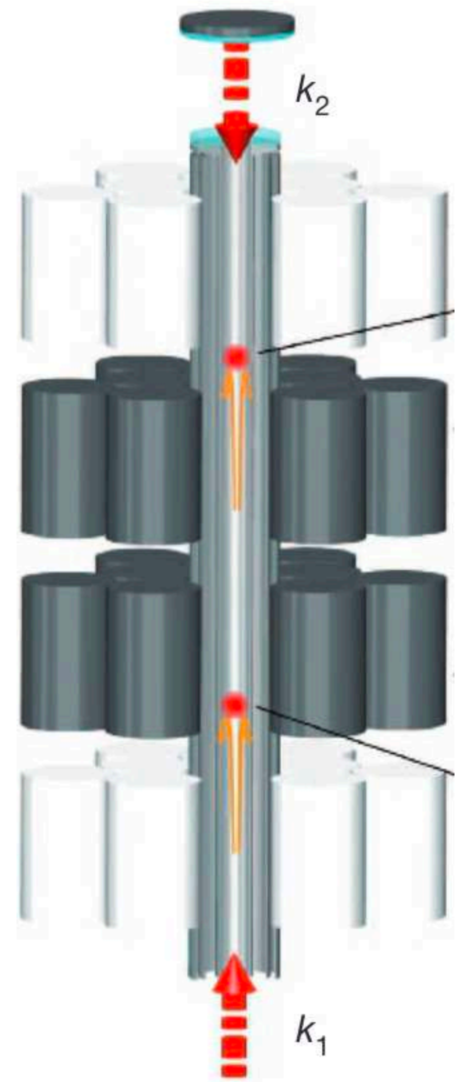
- Prepare pair of large masses in pure quantum states of motion
- Isolate them so that interaction is dominated by mutual gravity



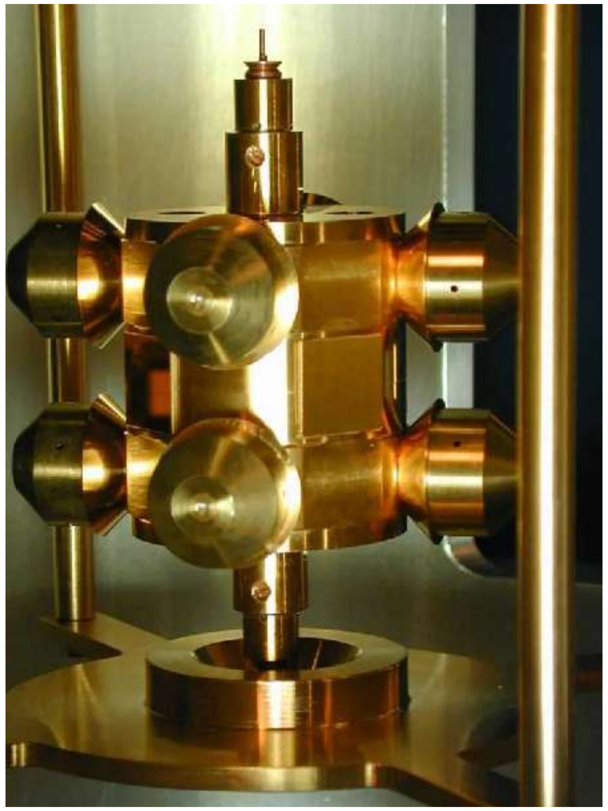
Collela et al., Phys Rev Lett 34, 1472 (1975)



Nesvizhevsky et al., Nature 415, 297 (2002)



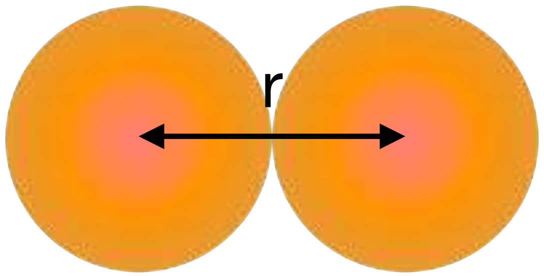
Rosi et al., Nature 510, 518 (2014)



Adelberger et al., Ann. Rev. Nucl. Phys. 53, 77 (2003)

Precision measurements of static gravity

- Prepare pair of large masses in pure quantum states of motion
- Isolate them so that interaction is dominated by mutual gravity



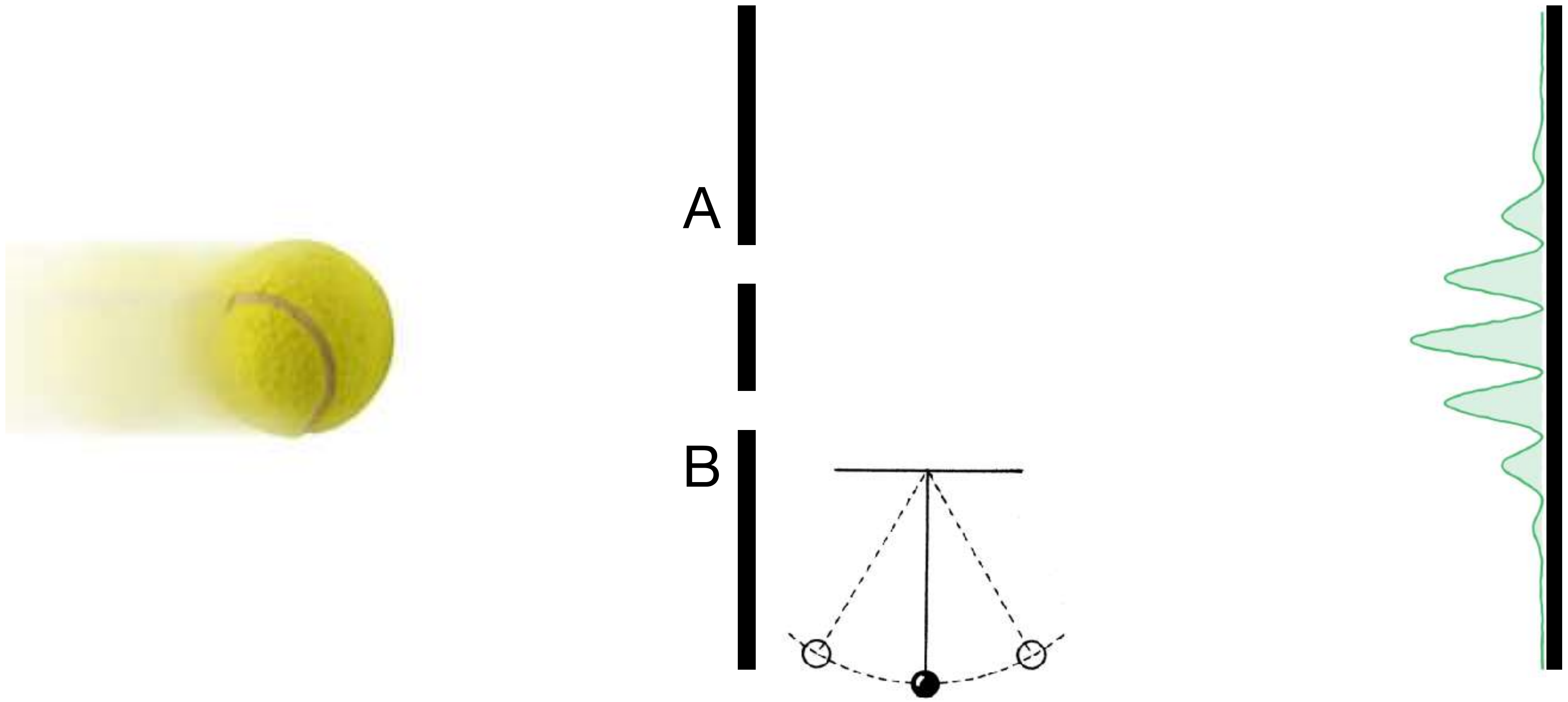
$$\Omega_G = \frac{Gm}{2\omega r^3} \lesssim \frac{G\rho}{\omega}$$

Need to beat thermal decoherence

$$\frac{\Omega_G}{\Gamma_{th}} \sim \frac{G\rho/\omega}{\gamma(n_{th}+1)} \begin{cases} 10^{-26} & \text{(cold atom clouds)} \\ 10^{-15} & \text{(solid-state mechanics)} \end{cases}$$

(cold atom clouds): low density, low Q, $n_{th} \ll 1$

(solid-state mechanics): high density, high Q, $n_{th} \gg 1$

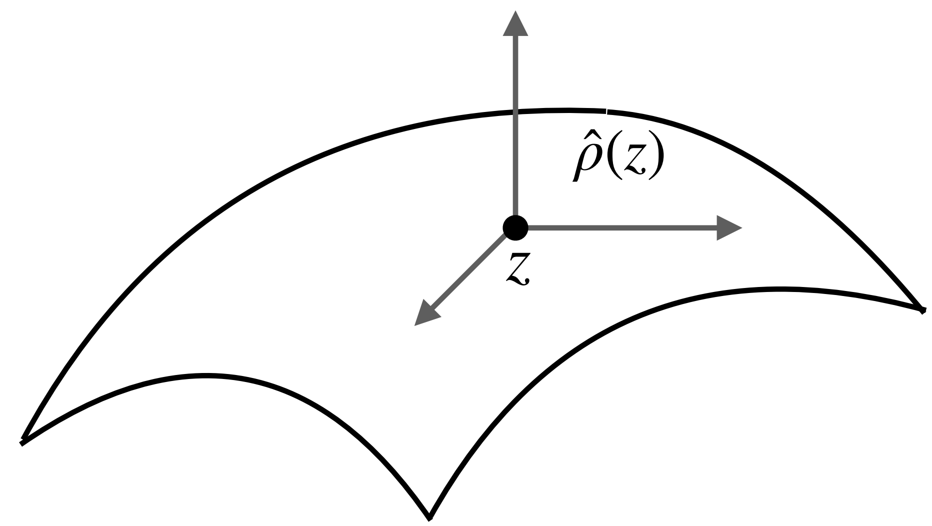


Which-path information is obscured if gravity is classical but stochastic

Poulin/Preskill, KITP (2017)
Oppenheim, PRX 13, 041040 (2023)
Kryhin/Sudhir, arXiv:2309.09105

What if gravity is classical?

Need a **consistent** theory of quantum matter sourcing classical gravity



1. A joint state of quantum matter and classical gravity

$$\mathrm{Tr}\,\hat{\rho}(z) = p_C(z) \qquad \int \hat{\rho}(z)\,\mathrm{d}z = \hat{\rho}_Q$$

2. A consistent equation of motion for the joint state

$$\partial_t \hat{\rho}(z) = -i[\hat{H}, \hat{\rho}] + Q_{\alpha\beta}\left(\hat{L}_\alpha \hat{\rho} \hat{L}_\beta^\dagger - \tfrac{1}{2}[\hat{L}_\beta^\dagger \hat{L}, \hat{\rho}]_+\right)$$

$$+\partial_{z_i}(C_i \hat{\rho}) + \partial_{z_i z_j}(C_{ij}\hat{\rho})$$

$$+\partial_{z_i}(M_{\alpha i}\hat{\rho}\hat{L}_\alpha^\dagger + h.c.)$$

3. Also consistent with general relativity

In the Newtonian limit, quantum source masses obey

Coherent Newtonian coupling $\sim \hbar\Omega_G$

$$\partial_t \hat{\rho}_Q = -i\left[\hat{H}_0 - \frac{G}{2}\int \frac{\hat{T}^{00}(x)\hat{T}^{00}(x')}{|x-x'|}\mathrm{d}x\,\mathrm{d}x', \hat{\rho}_Q\right]$$

$$-\epsilon\frac{G}{2}\int \frac{\mathrm{d}x\,\mathrm{d}x'}{|x-x'|}[\hat{T}^{00}(x), [\hat{T}^{00}(x'), \hat{\rho}_Q]]$$ + (non-gravitational terms)

Experimentally distinguish the hypotheses that $\epsilon = 0$ or $\epsilon > 0$

Kryhin/Sudhir, arXiv:2309.09105
See also: Poulin/Preskill, KITP (2017), Oppenheim, PRX 13, 041040 (2023)

Lindbladian is "natural"

Only dimensional constant at lowest order in d^{-1}

$$\mathcal{L}[\rho] = \frac{Gm^2}{\hbar d}[O_1(x_1 - x_2), [O_2(x_1 - x_2), \rho]]$$

For small fluctuations of mass positions

$$O_i(x_1 - x_2) \approx \epsilon_i(x_1 - x_2)$$

$$\mathcal{L}[\rho] = \frac{Gm^2}{\hbar d}\left(\sum_i \epsilon_i^2 [x_i, [x_i, \rho]] + \epsilon_1\epsilon_2[x_1, [x_2, \rho]]\right)$$

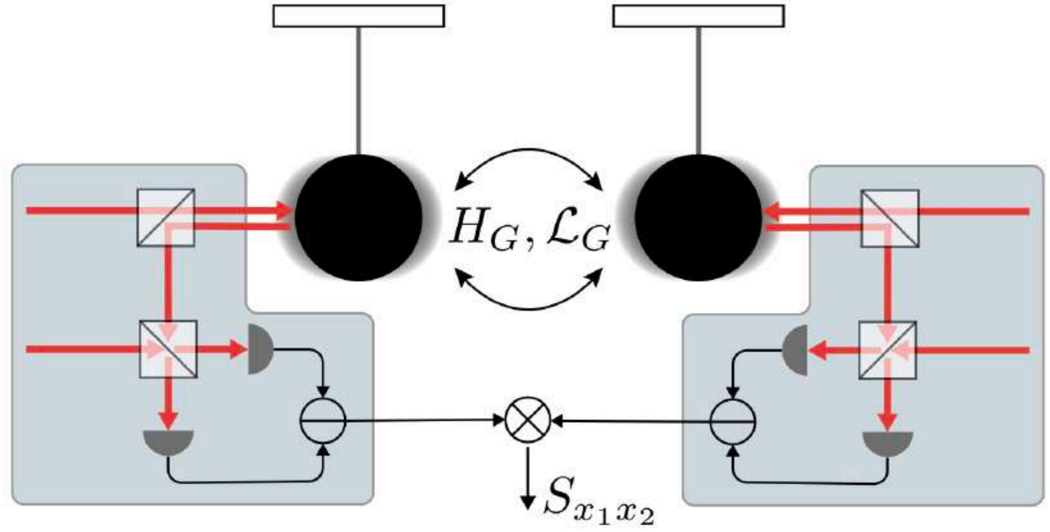
Decoherence of individual masses

3. Also consistent with general relativity

In the Newtonian limit, quantum source masses obey

$$\partial_t \hat{\rho}_Q = -i\left[\hat{H}_0 - \frac{G}{2}\int \frac{\hat{T}^{00}(x)\hat{T}^{00}(x')}{|x-x'|}\mathrm{d}x\,\mathrm{d}x', \hat{\rho}_Q\right]$$
$$-\epsilon\frac{G}{2}\int \frac{\mathrm{d}x\,\mathrm{d}x'}{|x-x'|}[\hat{T}^{00}(x), [\hat{T}^{00}(x'), \hat{\rho}_Q]]$$

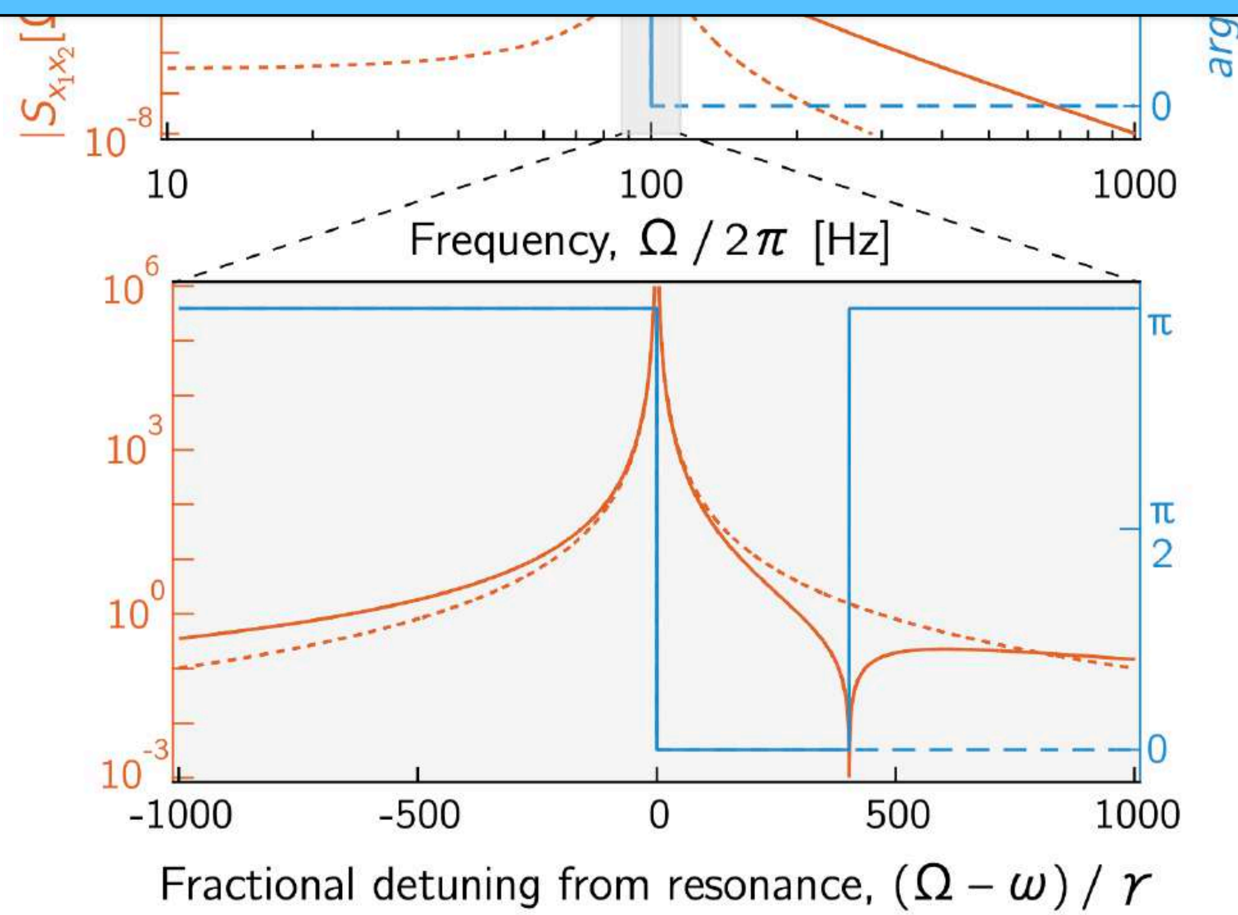
Experimentally distinguish the hypotheses that $\epsilon = 0$ or $\epsilon > 0$



Experimental setup similar to gravitational entanglement:

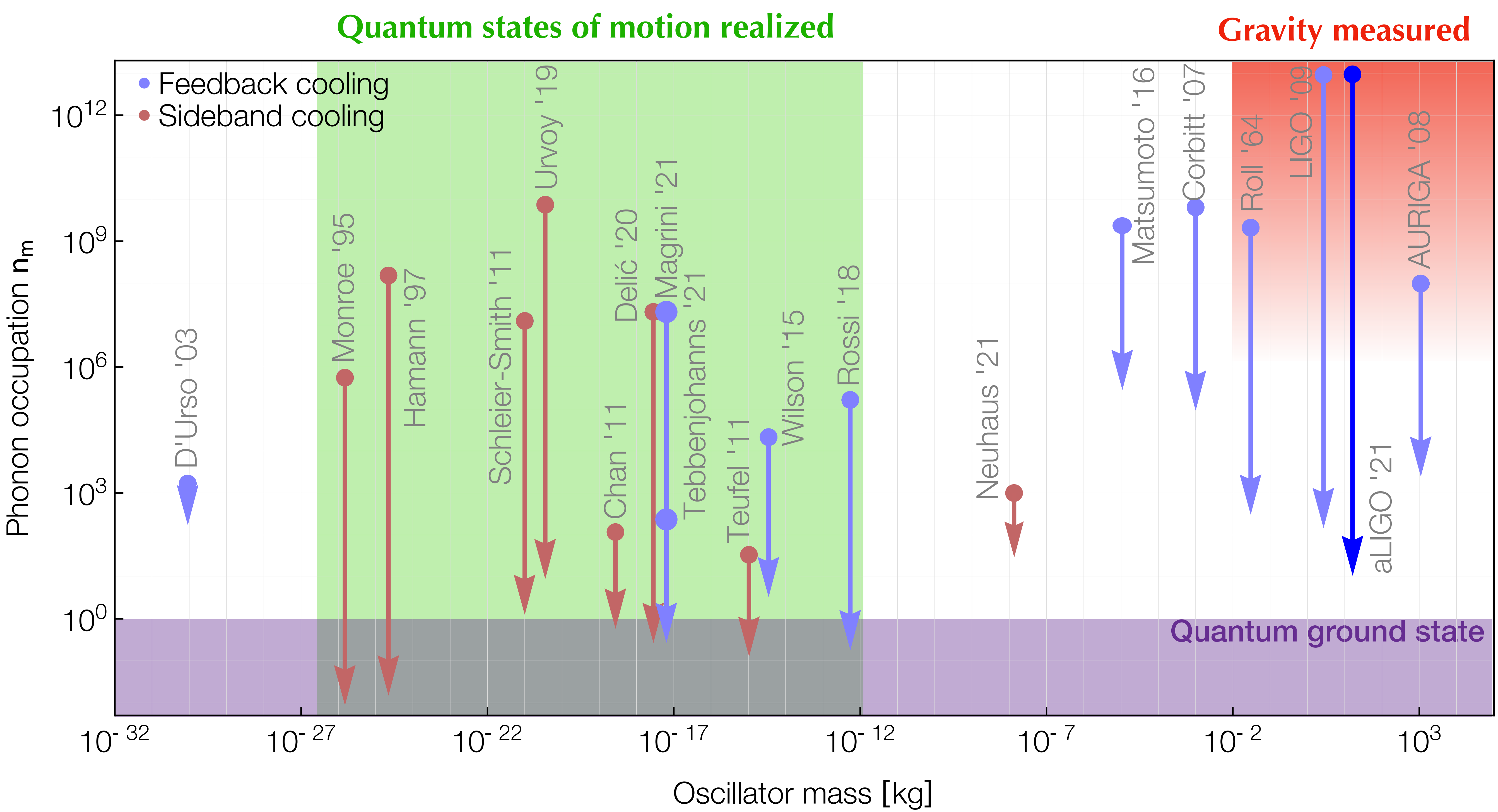
- Pair of large masses in **nearly**-pure quantum states
- Interaction dominated by mutual gravity

Don't need $\Omega_G \approx \Gamma_{\text{th}}$

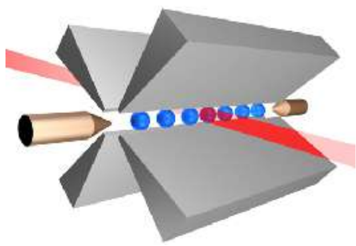


Kryhin/Sudhir, arXiv:2309.09105

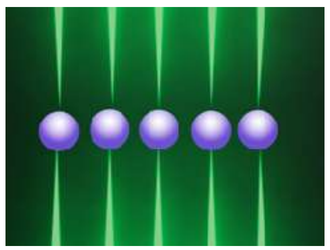
- Pair of large masses in nearly-pure quantum states
- Interaction dominated by mutual gravity



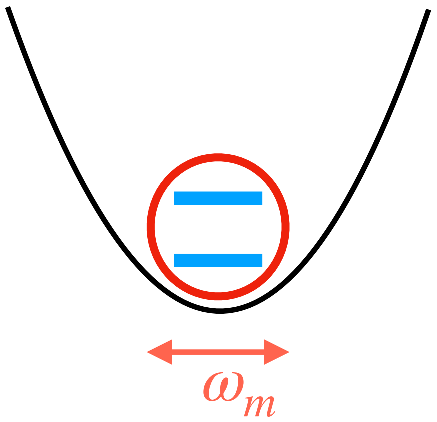
Inefficiency of laser cooling for state preparation

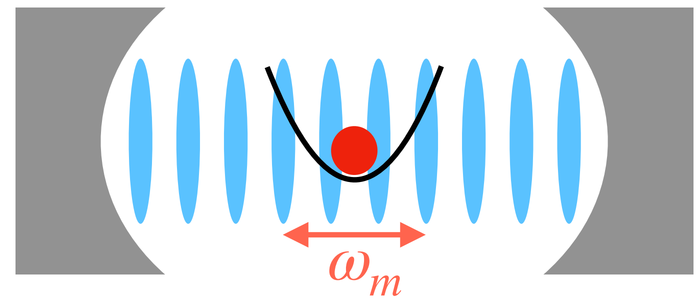


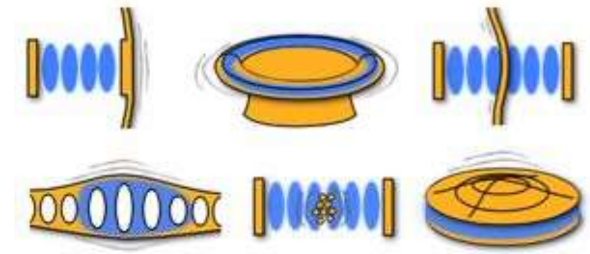
Trapped ions



Neutral atom tweezers

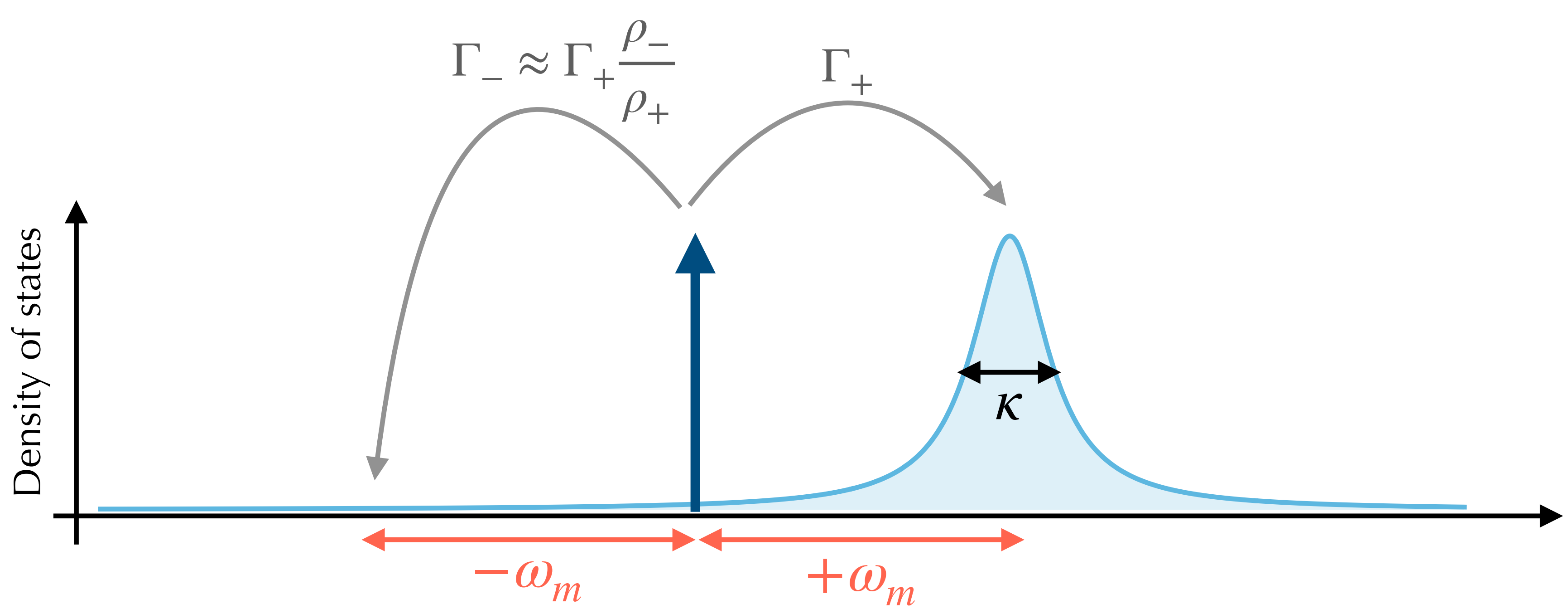






Micro-/nano-optomechanics

$$\hat{H}_{\text{int}} = \hbar g(\hat{a} + \hat{a}^\dagger)(\hat{b} + \hat{b}^\dagger)$$



Detailed balance: $$n_{\text{eff}} \approx \frac{n_m \Gamma_m}{\Gamma_+ - \Gamma_-} + \frac{n_s \Gamma_+}{\Gamma_+ - \Gamma_-} = \frac{n_m \Gamma_m / \Gamma_+}{1 - \rho_-/\rho_+} + \frac{n_s}{1 - \rho_-/\rho_+}$$

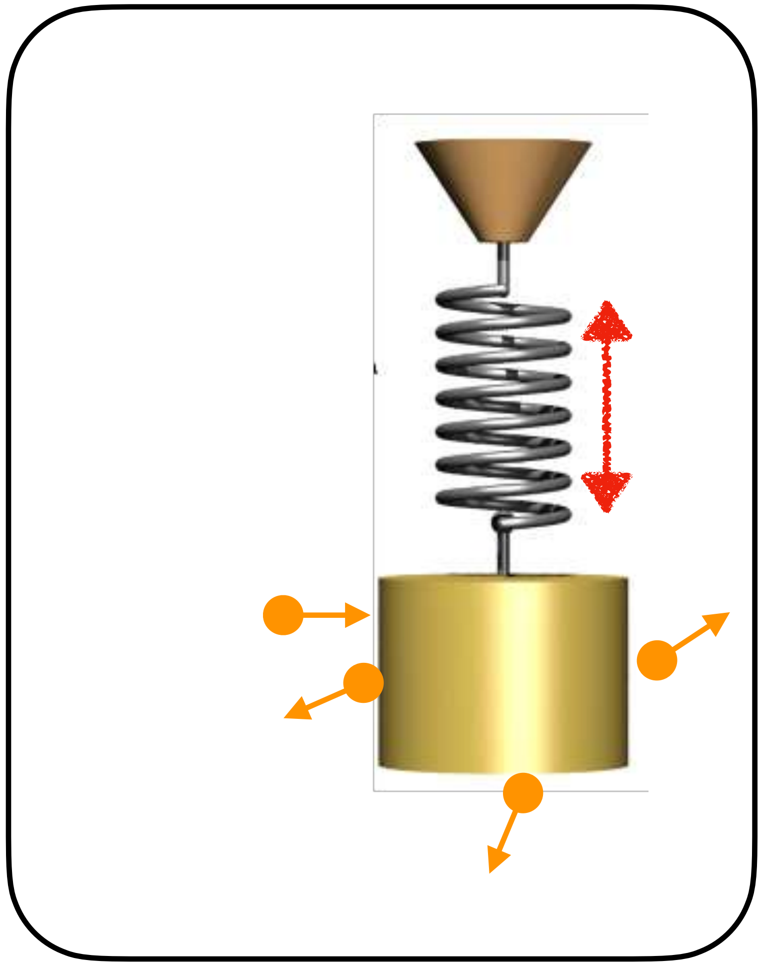
Insufficient if $\omega_m \ll \kappa$

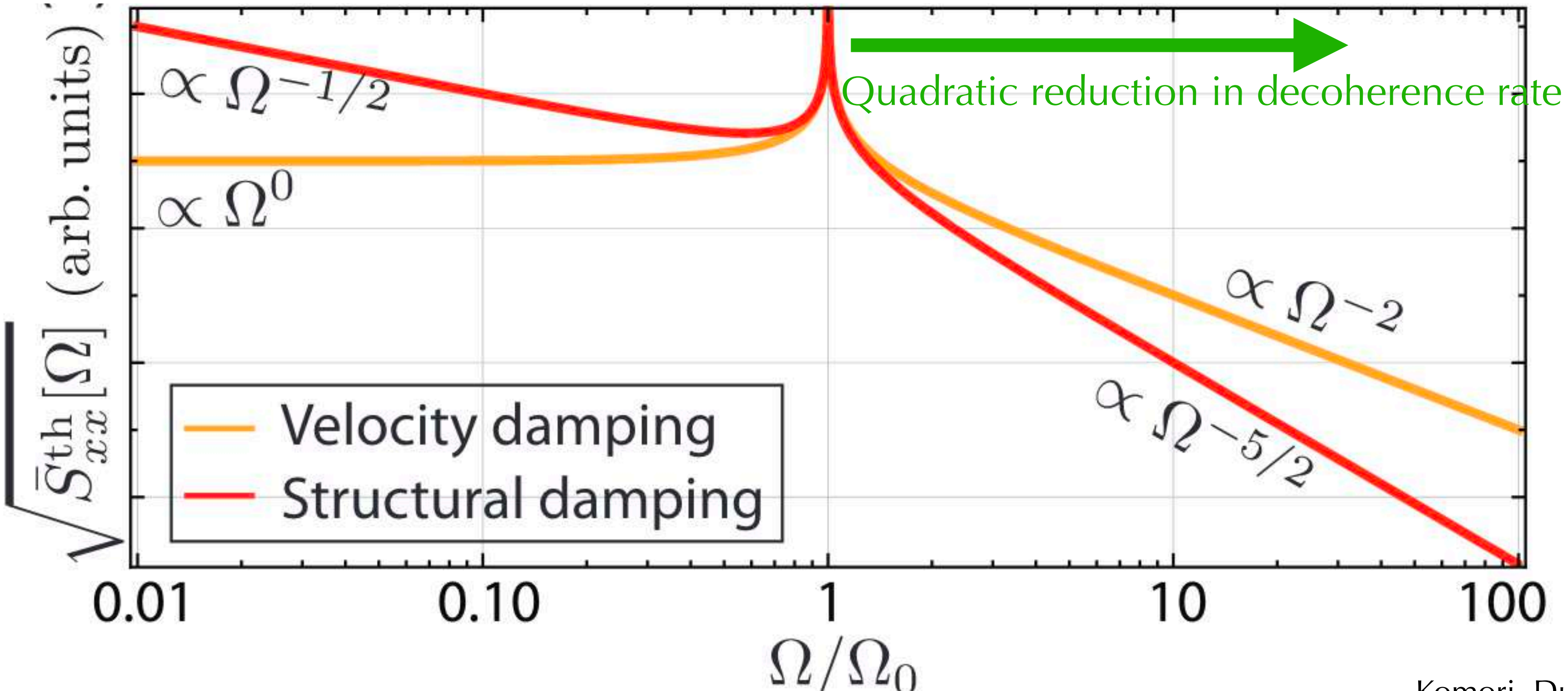
Velocity damping at high Kn:

$$\chi_{xF}[\Omega] = \frac{1}{m[-\Omega^2 + i\Omega\Gamma_0 + \Omega_0^2]}$$

"Structural" damping at low Kn:

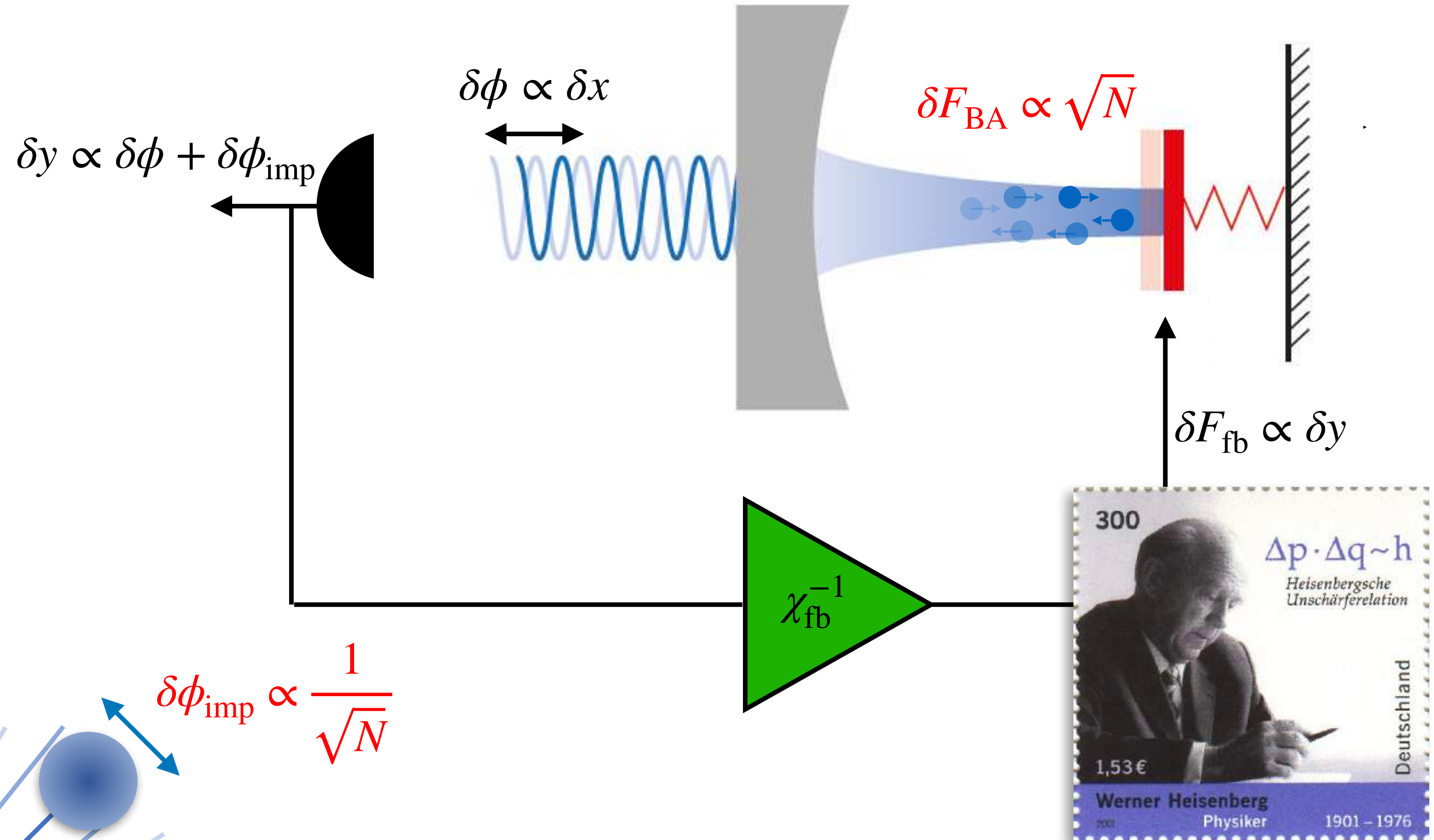
$$\chi_{xF}[\Omega] \approx \frac{1}{m[-\Omega^2 + \Omega_0^2(1 + i\phi)]}$$

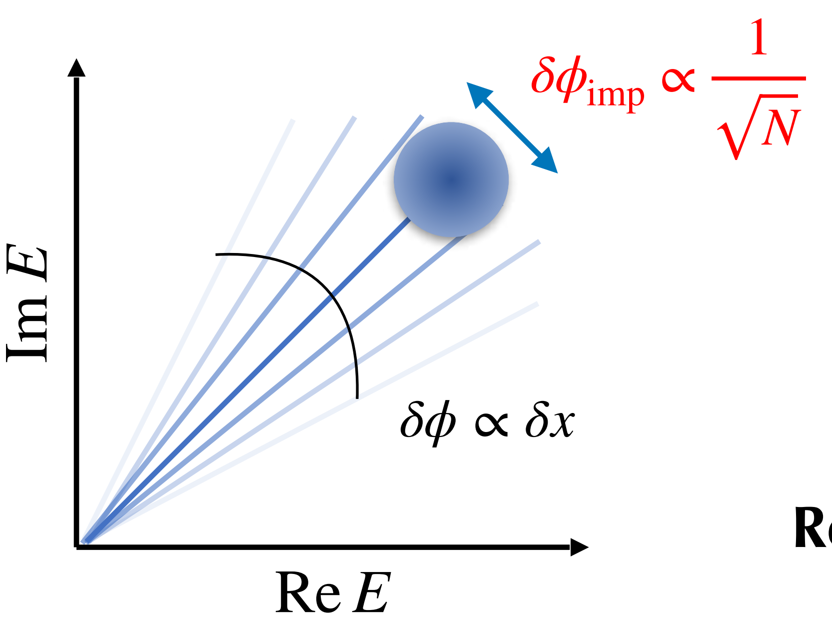




$$\Gamma_{\text{th}}[\Omega] = \frac{k_B T}{\hbar Q_0}\left(\frac{\Omega_0}{\Omega}\right)^2$$

Komori, Durovcikova, Sudhir, Phys. Rev. A 105, 043520 (2022)





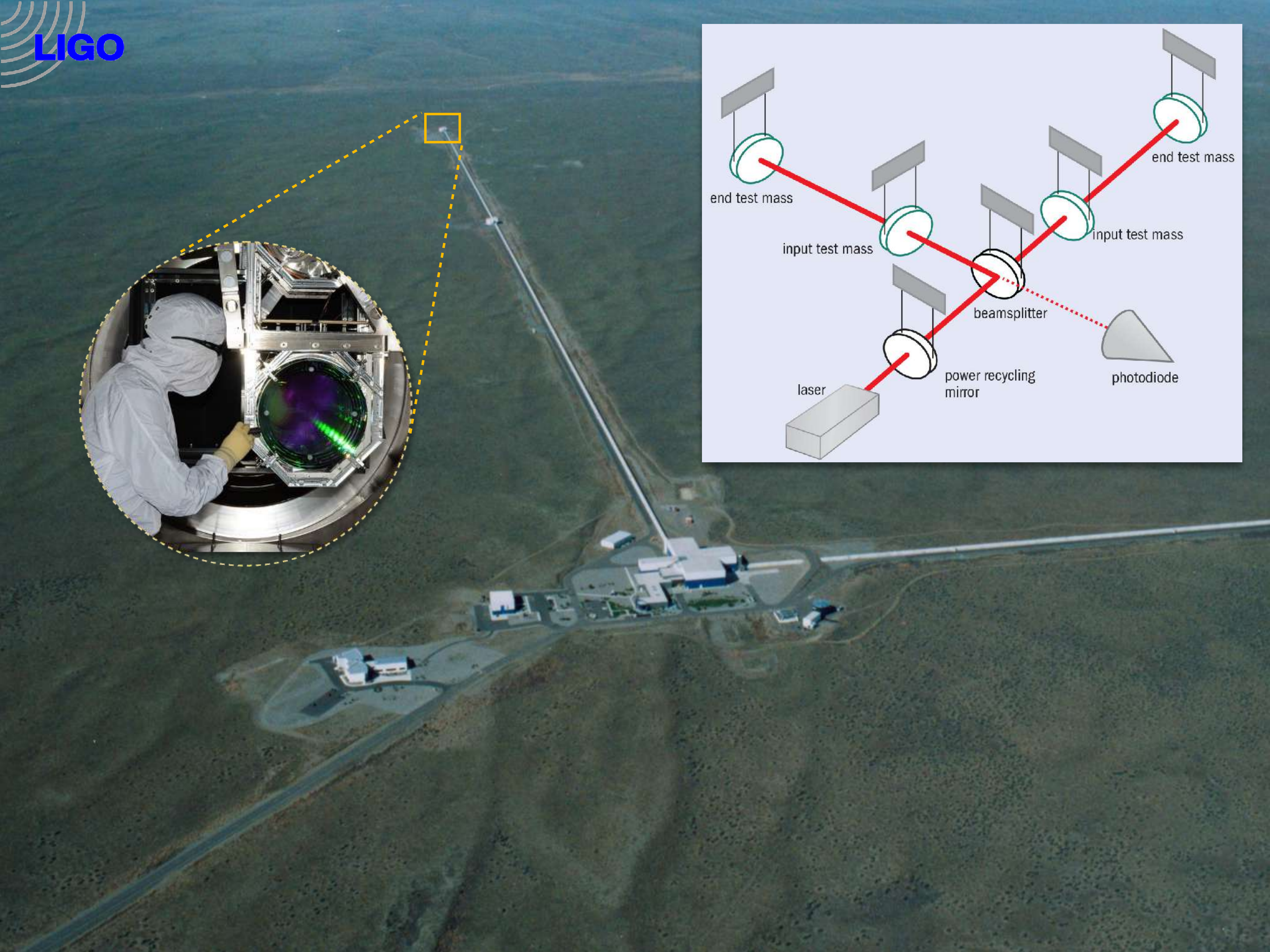
Requirements to prepare ground state of motion:

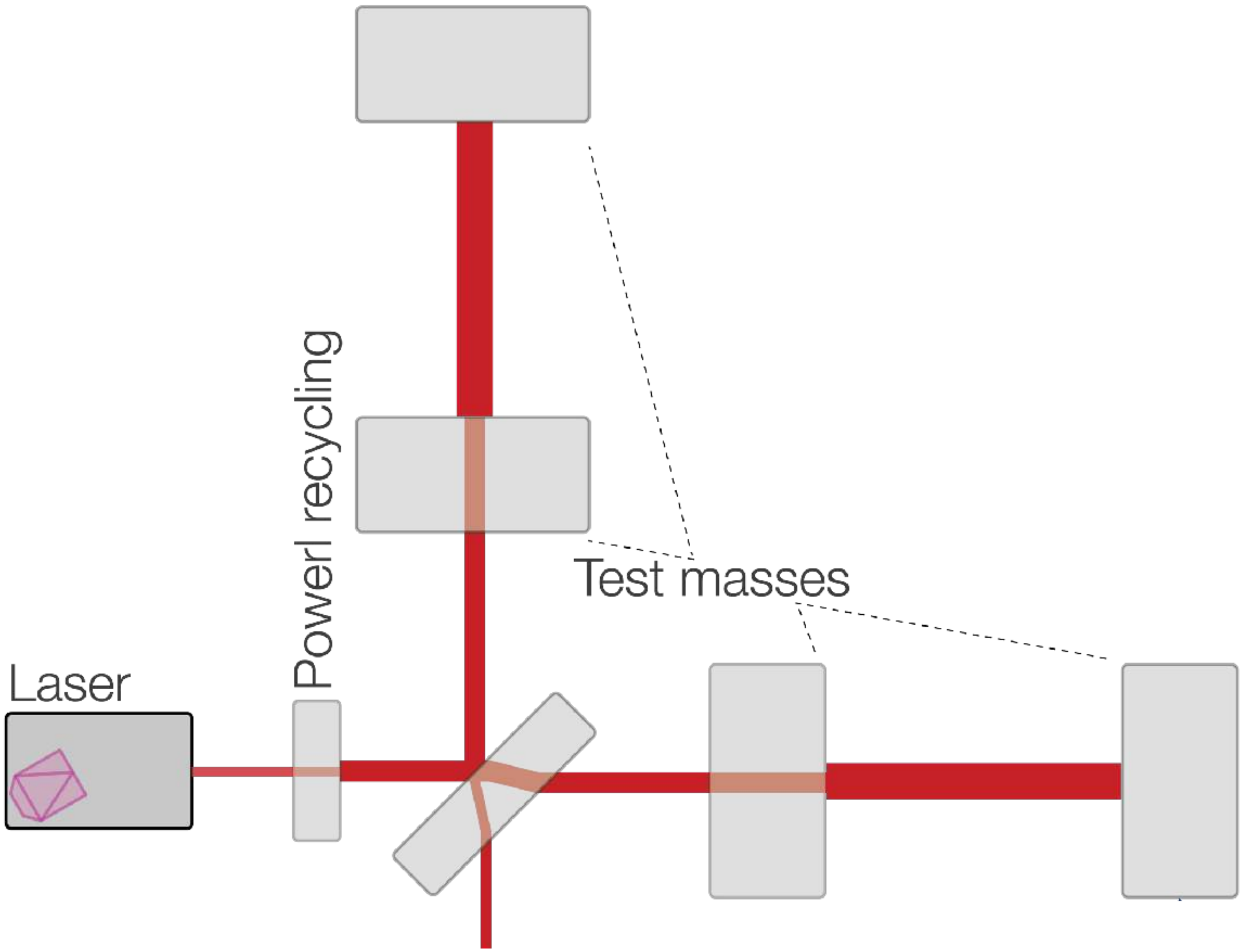
For viscous (Markovian) damping: $Q \gtrsim n_{th}, \quad S_{imp} \lesssim \frac{S_{zp}}{n_{th}}$

For structural (non-Markovian) damping: $Q \gtrsim n_{th}\left(\frac{\Omega_0}{\Omega_{SQL}}\right)^3, \quad S_{imp} \lesssim \frac{S_{zp}}{n_{th}^{2/3} Q_0^{1/3}}$

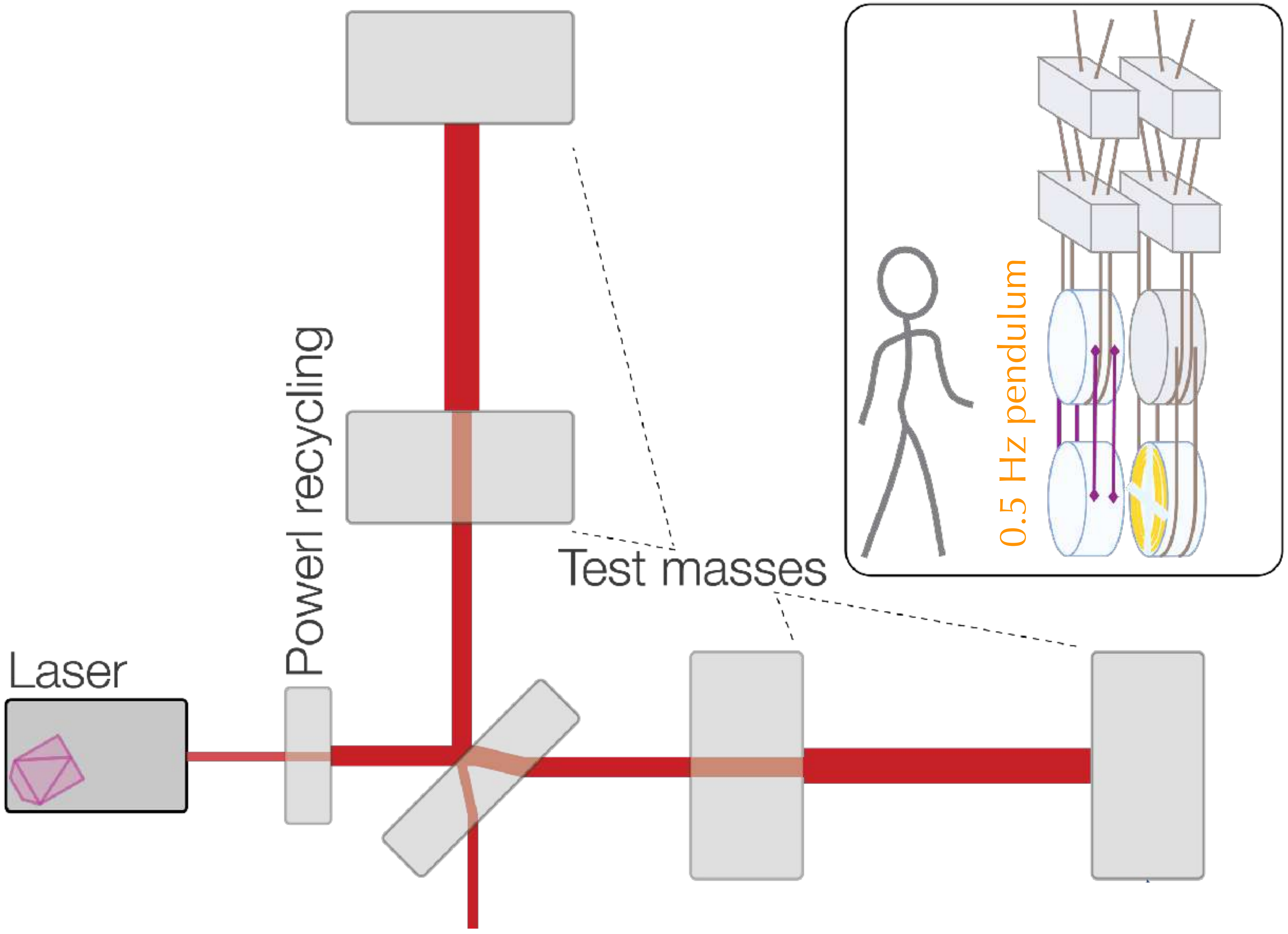
Relaxed requirements to trap and cool structurally-damped oscillators to ground state

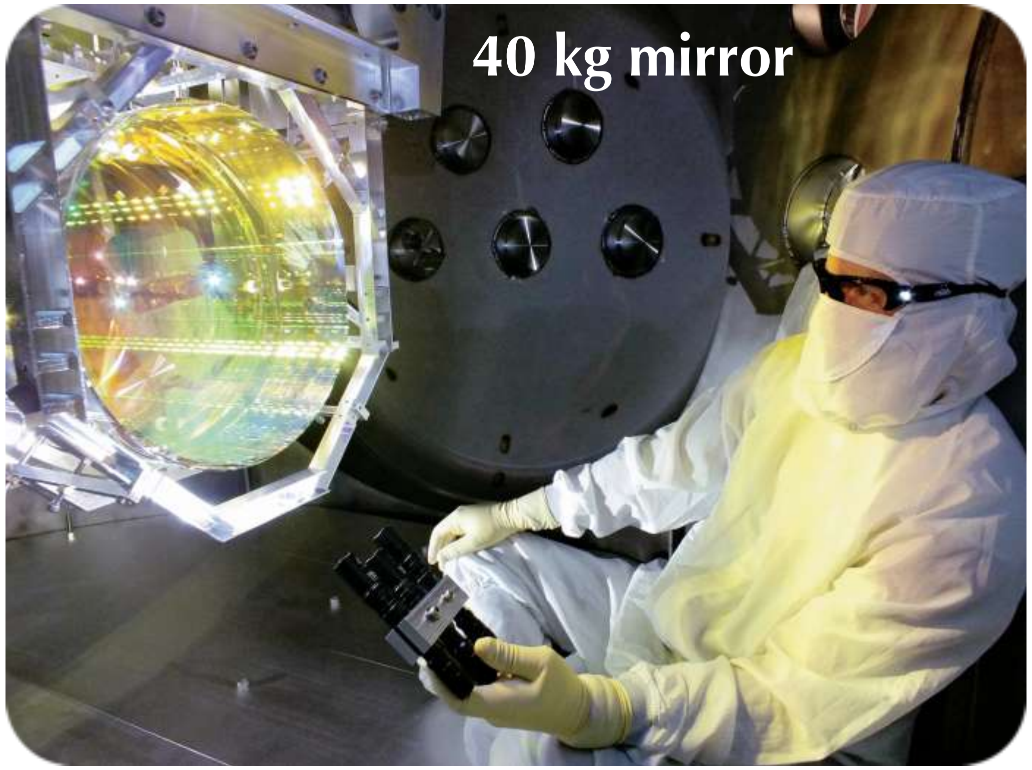
Komori, Durovcikova, Sudhir, Phys. Rev. A 105, 043520 (2022)


LIGO
end test mass
end test mass
input test mass
input test mass
beamsplitter
power recycling
mirror
laser
photodiode

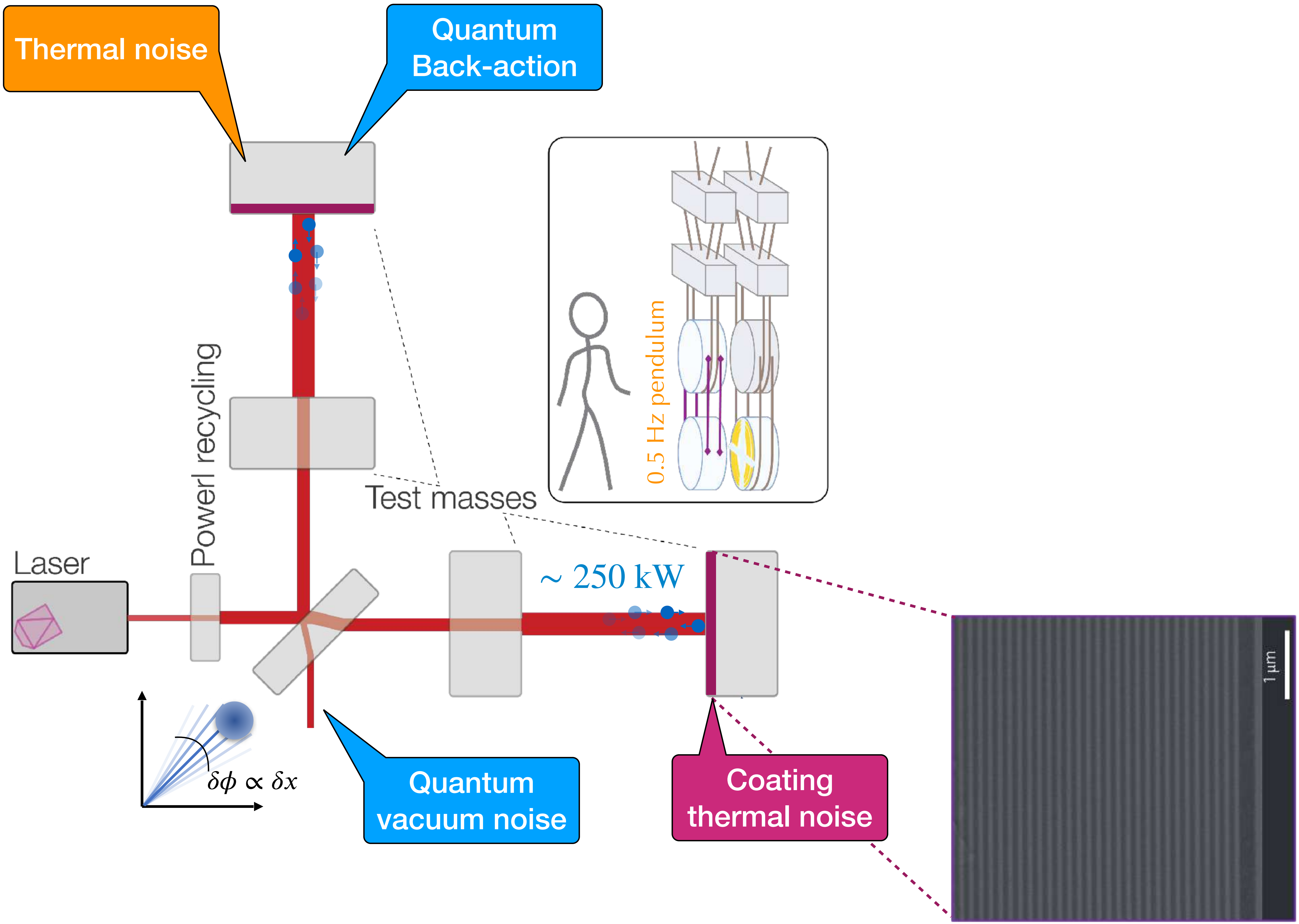

Laser
Powerl recycling
Test masses

Advanced LIGO as an optomechanical system

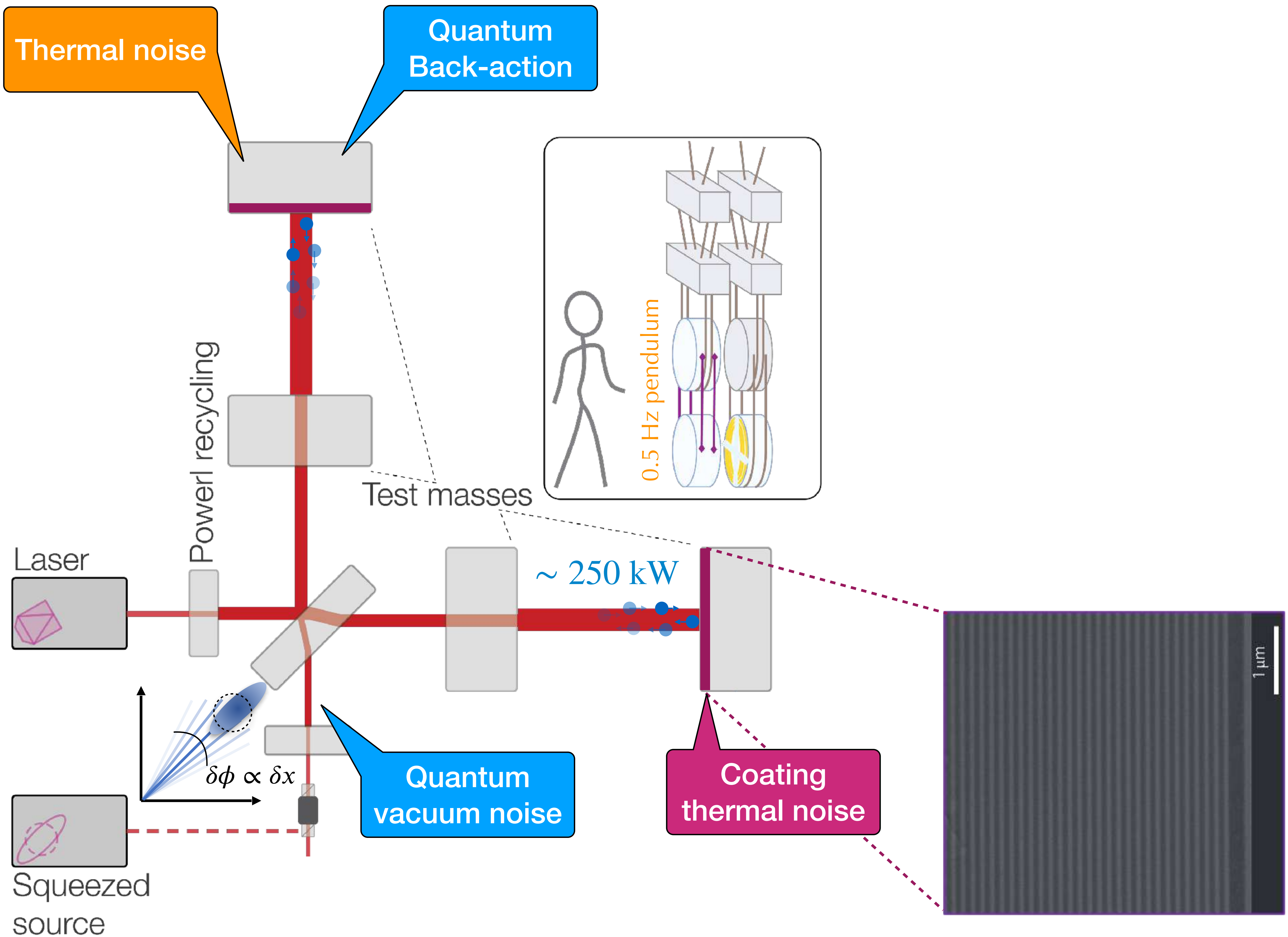




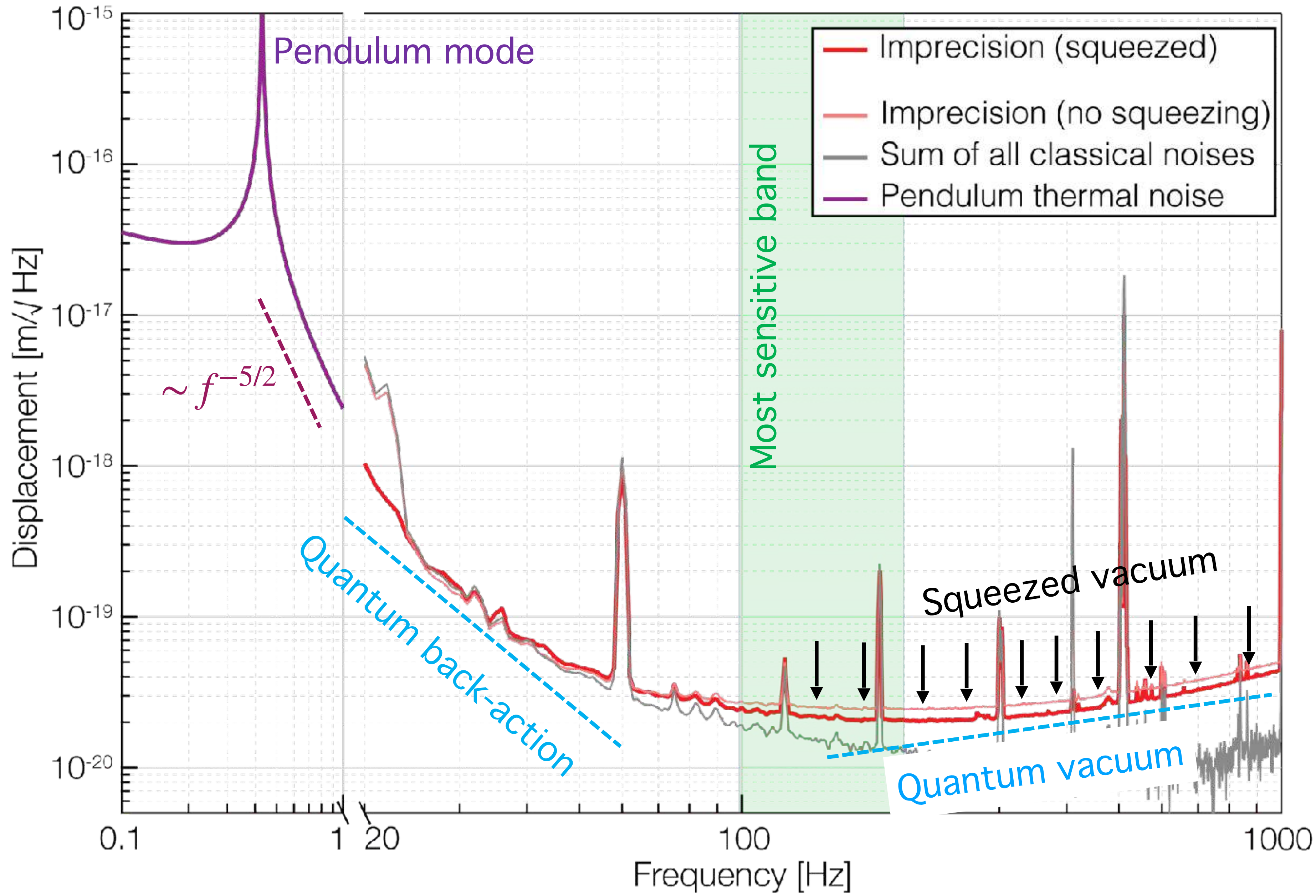
Advanced LIGO as an optomechanical system



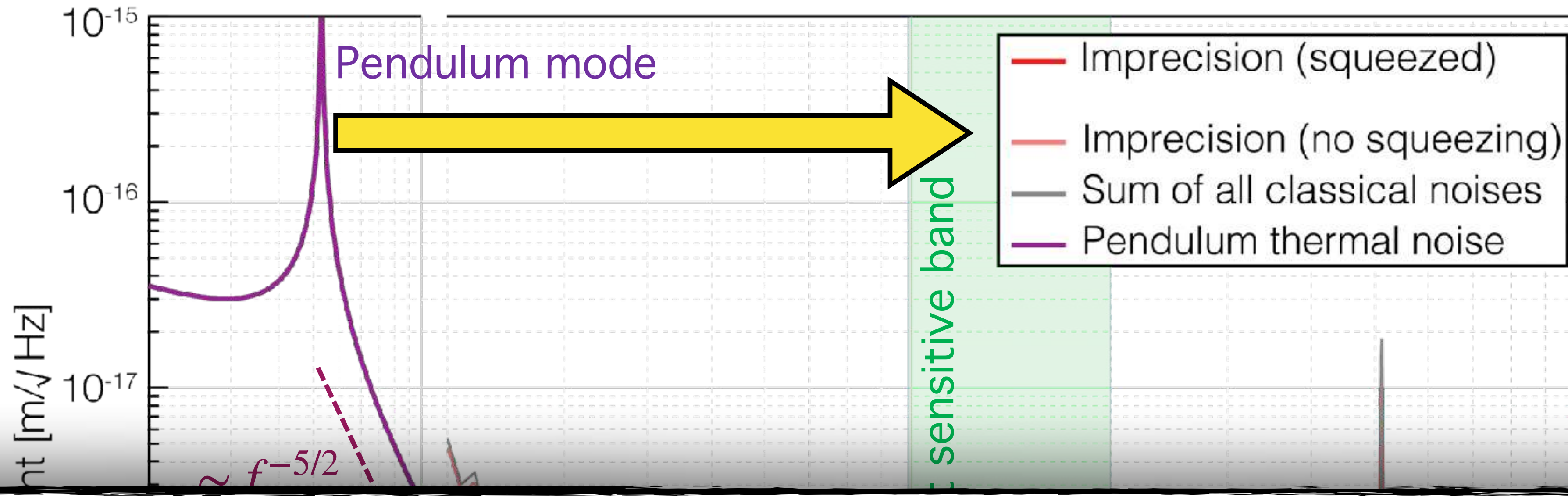
Advanced LIGO as an optomechanical system



Phys. Rev. Lett. 123, 231107 (2019)



Whittle … Sudhir, Science 372, 1333 (2021)



Measurement record

$$\delta y[\Omega] = \delta x[\Omega] + \delta x_{\text{imp}}[\Omega]$$

Intrinsic motion

$$\delta x[\Omega] = \chi_0[\Omega](\delta F_{\text{th}}[\Omega] + \delta F_{\text{BA}}[\Omega])$$

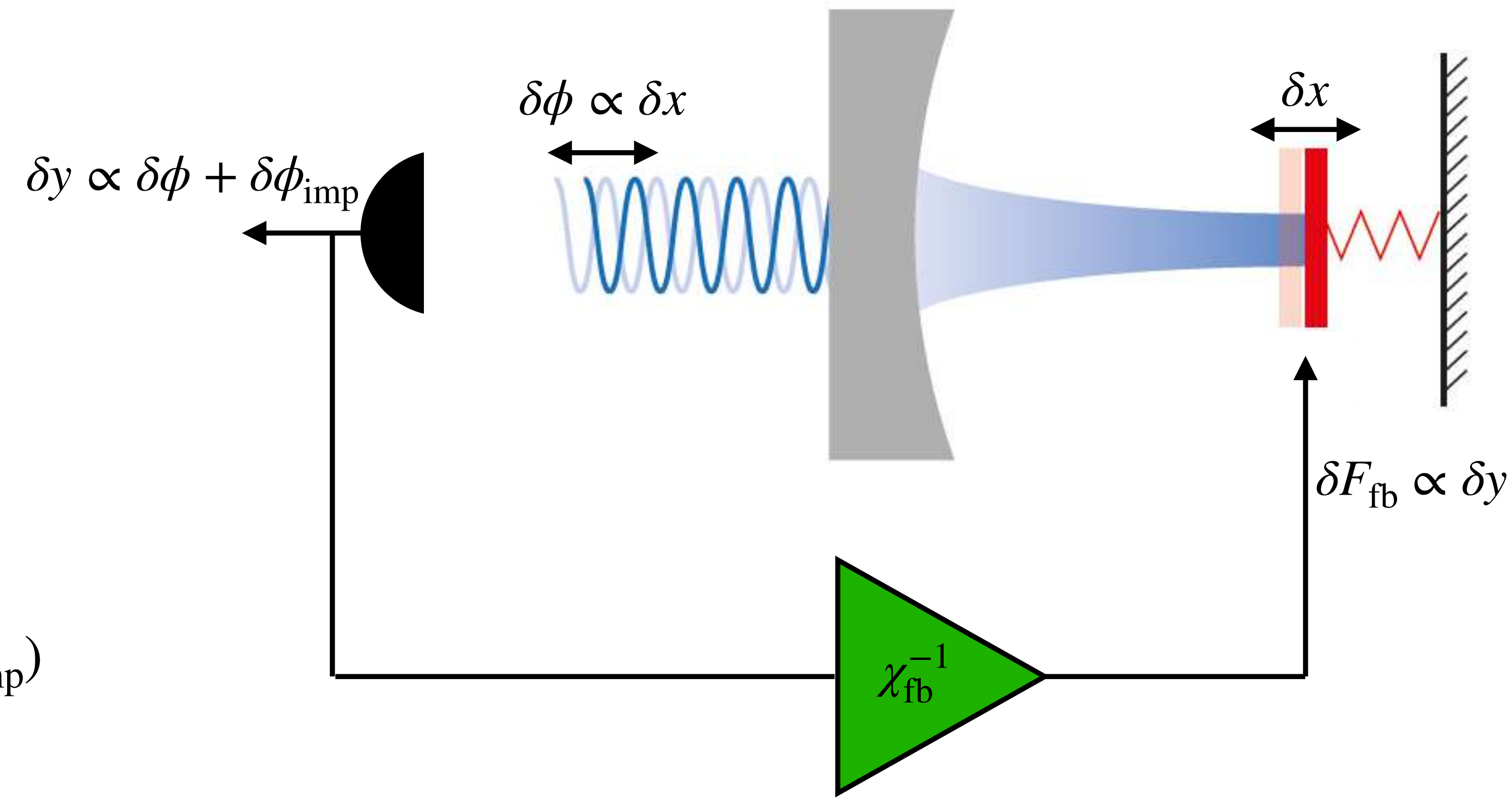
Modified motion

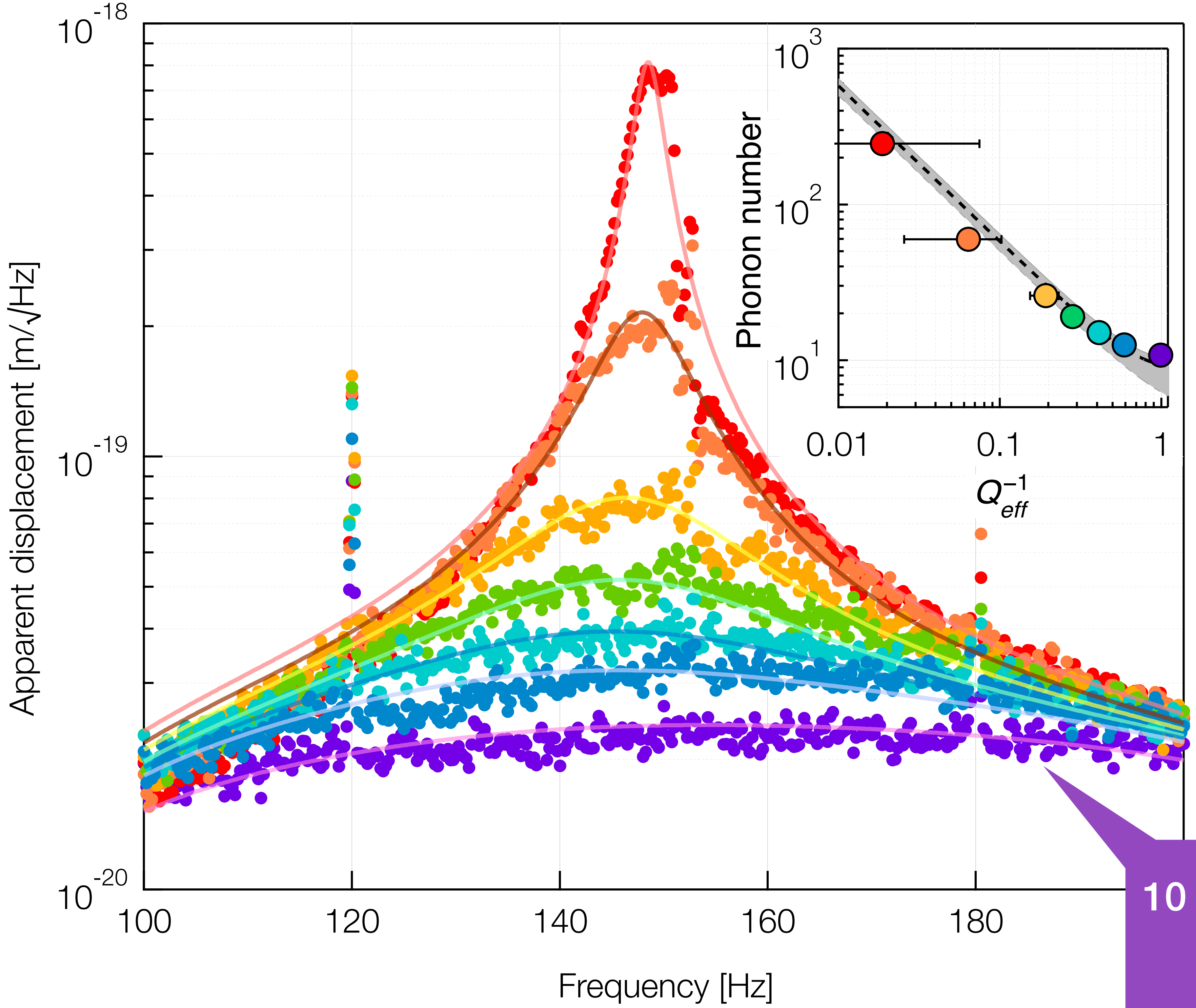
$$\chi_0^{-1}\delta x = \delta F_{\text{th}} + \delta F_{\text{BA}} + \delta F_{\text{fb}}$$

$$= \delta F_{\text{th}} + \delta F_{\text{BA}} + \chi_{\text{fb}}^{-1}(\delta x + \delta x_{\text{imp}})$$

$$(\chi_0^{-1} + \chi_{\text{fb}}^{-1})\delta x = \delta F_{\text{th}} + \delta F_{\text{BA}} + \chi_{\text{fb}}^{-1}\delta x_{\text{imp}}$$

Filter design   Quantum noise

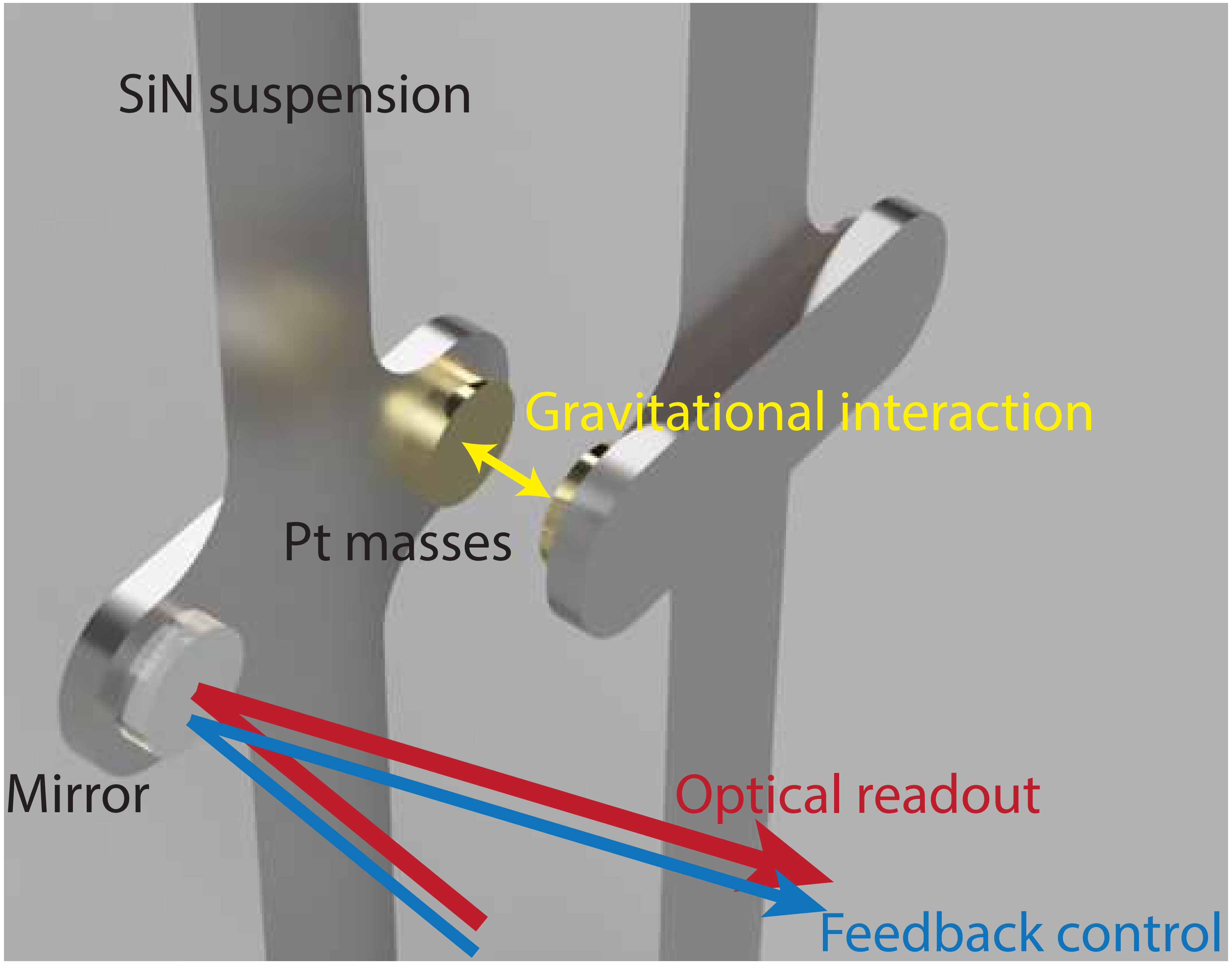




10 kg object near its ground state (~77 nK)

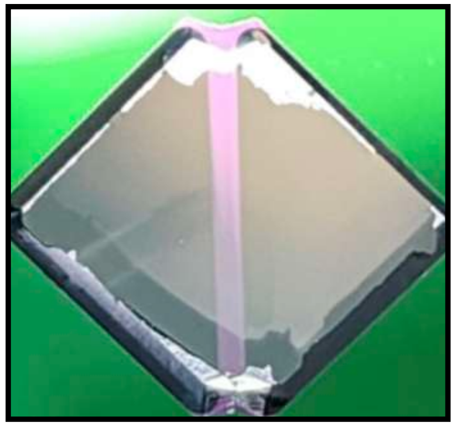
Whittle … Sudhir, Science 372, 1333 (2021)

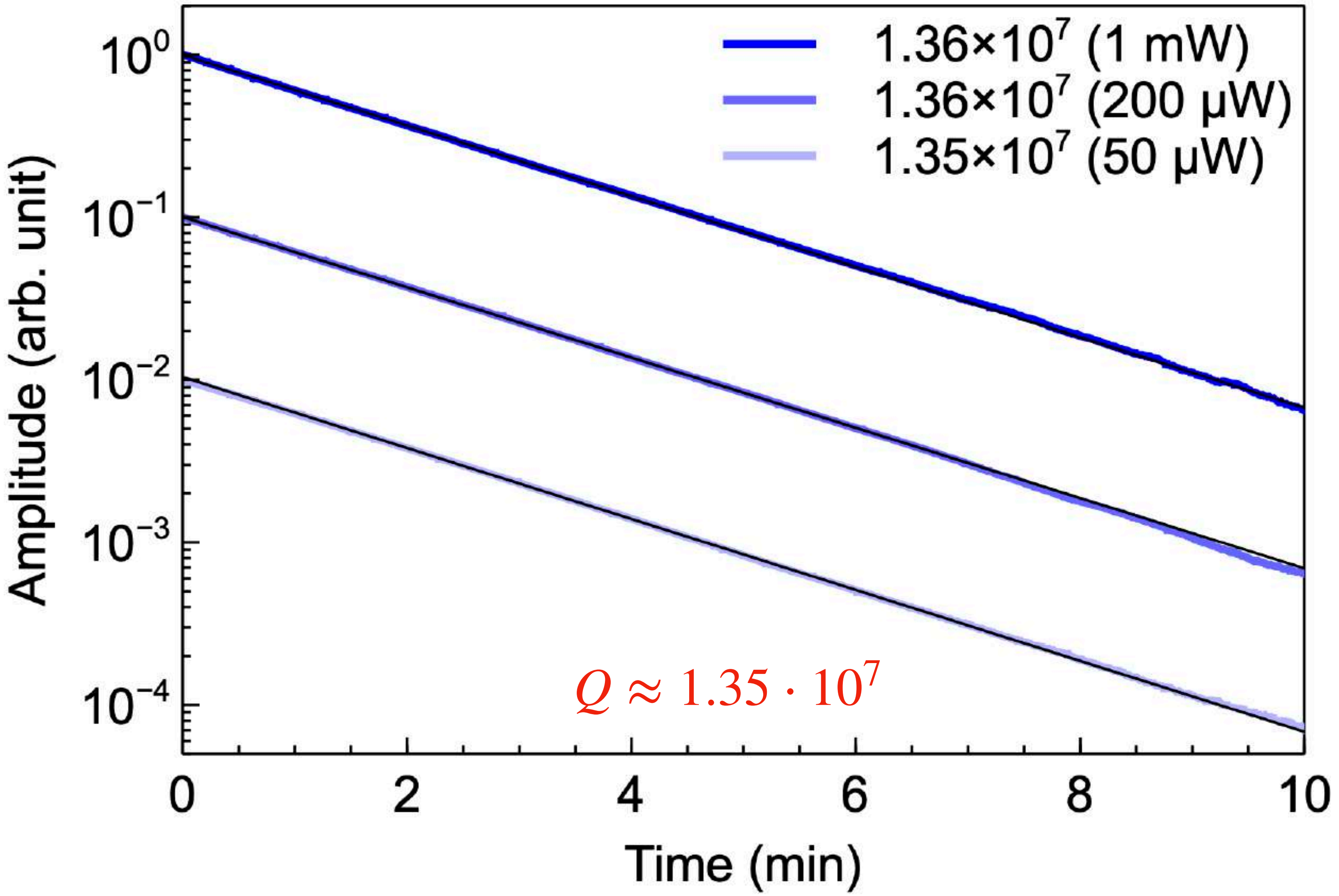
- High-Q mass-loaded (~ 10 mg) mechanical suspensions
- Quantum-noise-limited measurement and control at zero-point level
- Isolate gravitational interaction between oscillators



Tensile stress => torsional dissipation dilution*

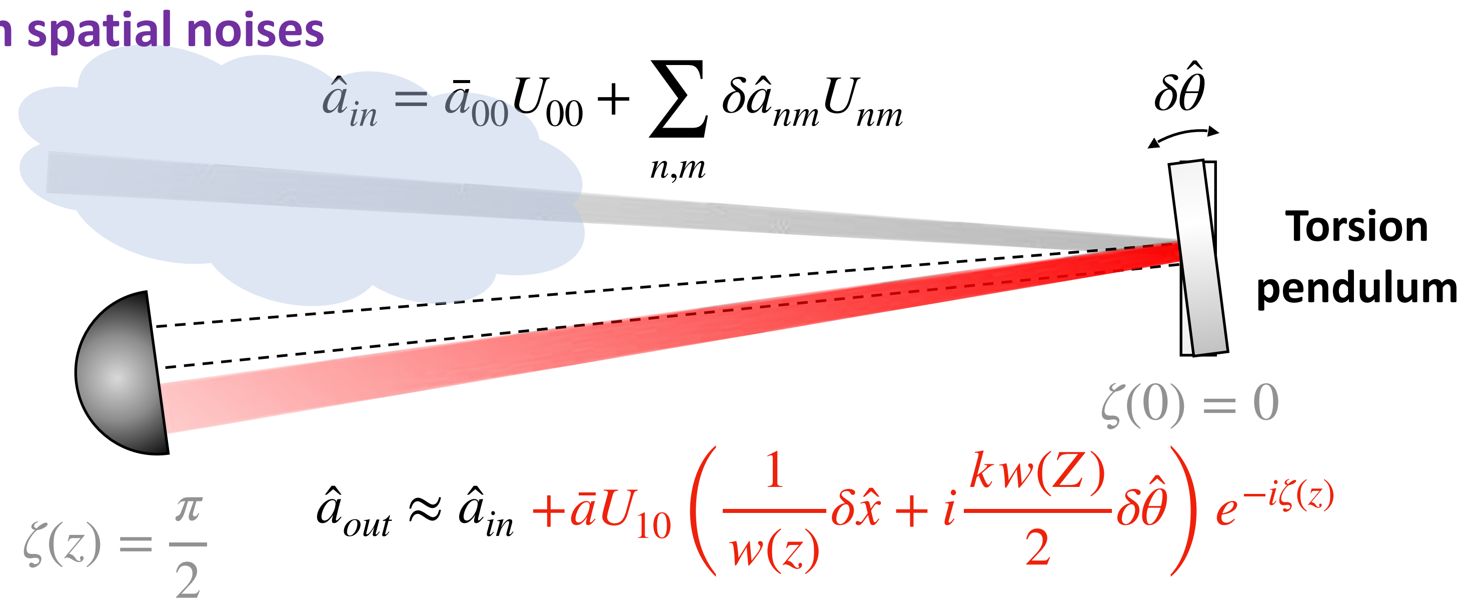
$$Q \approx Q_0 \frac{\sigma}{2E}\left(\frac{w}{h}\right)^2$$
$$\approx 10^7 \cdot \left(\frac{Q_0}{2500}\right)\left(\frac{\sigma}{0.8\,\mathrm{GPa}}\right)\left(\frac{w/h}{0.5\mathrm{mm}/400\mathrm{nm}}\right)^2$$

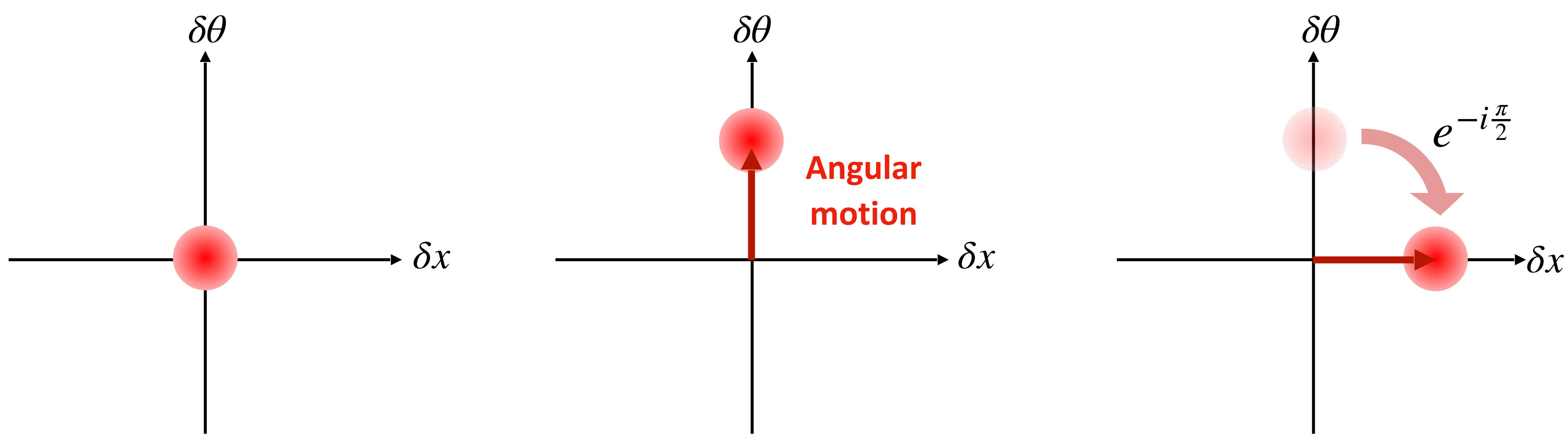




*Pratt et al., Phys. Rev. X 13, 011018 (2023)
Shen, Hayward,…, Sudhir arXiv:2409. (2024)

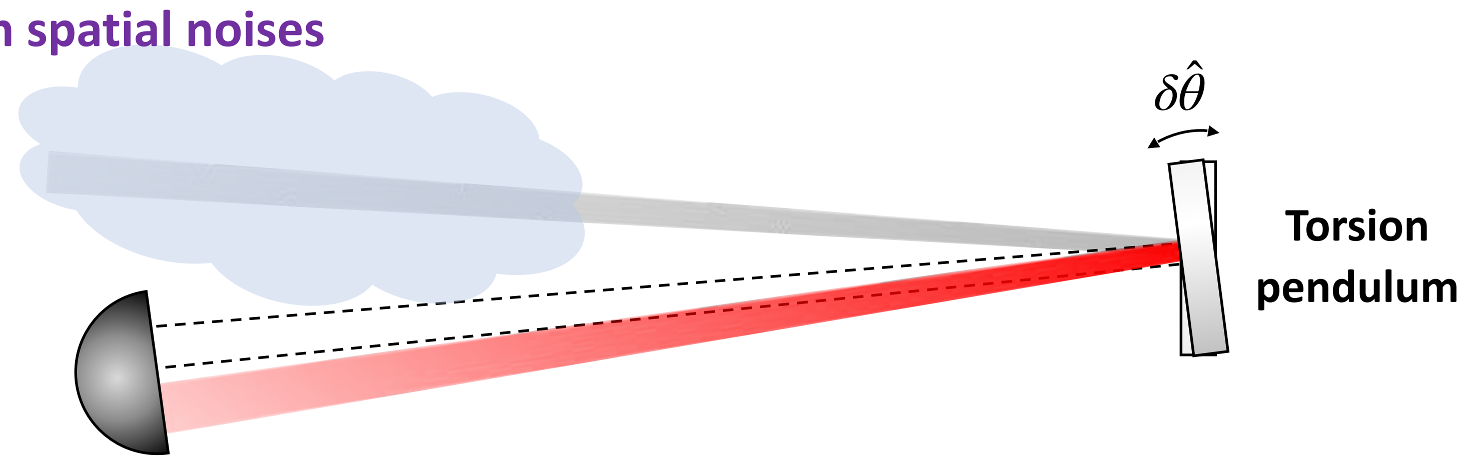
Quantum-limited optical lever

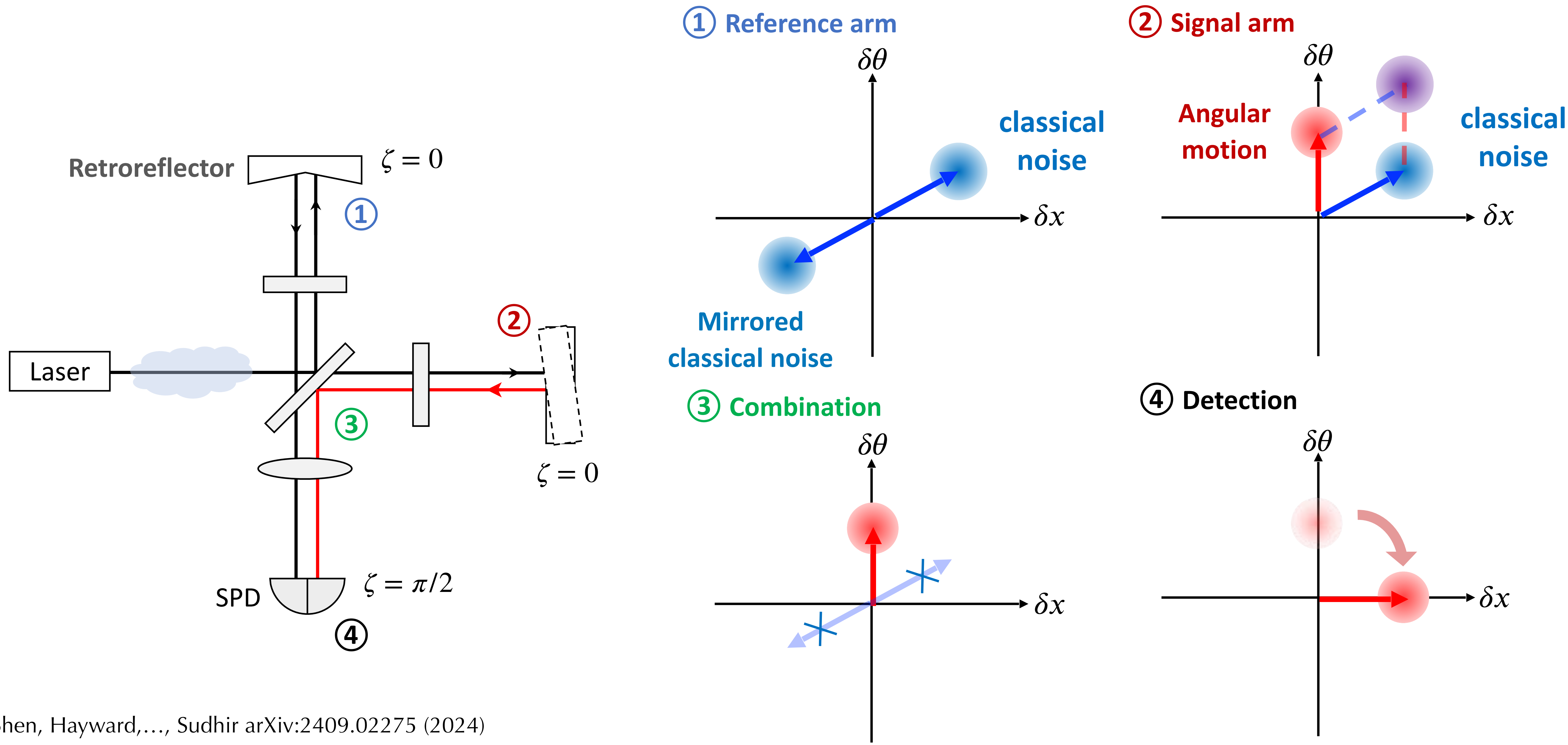




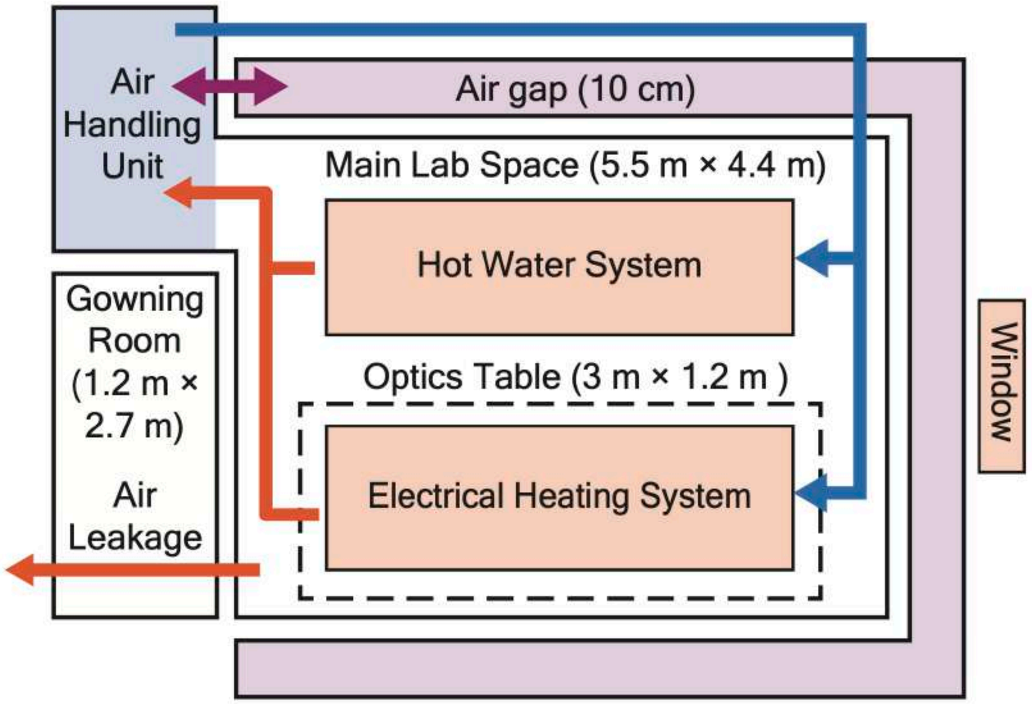
Shen, Hayward,…, Sudhir arXiv:2409.02275 (2024)

Quantum-limited optical lever

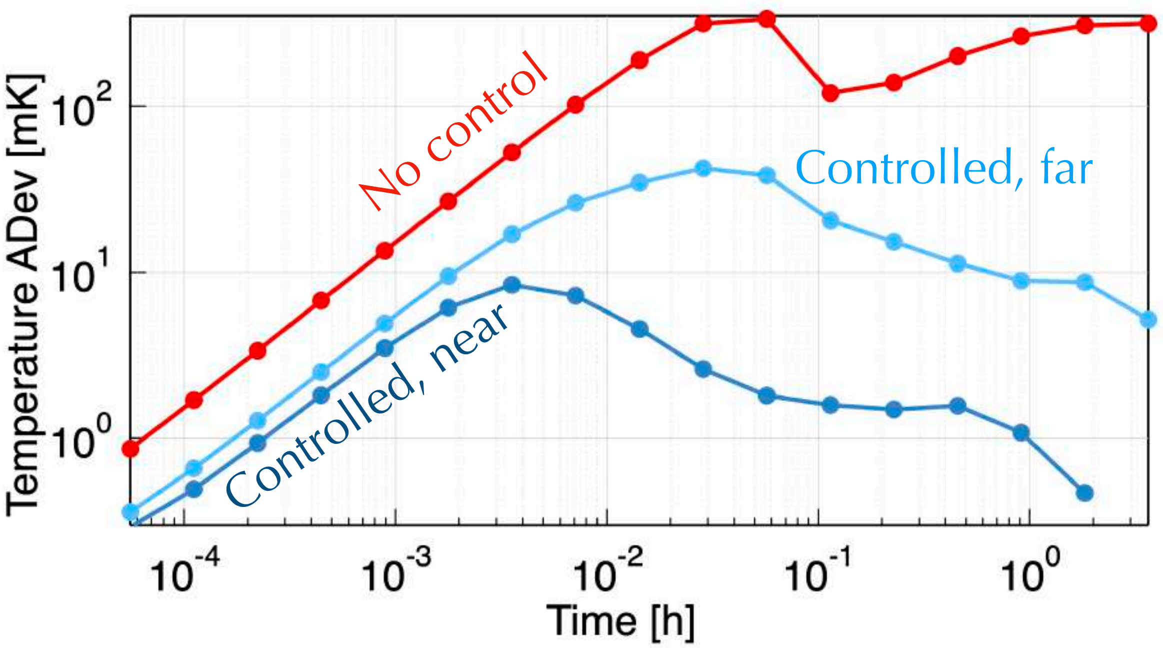


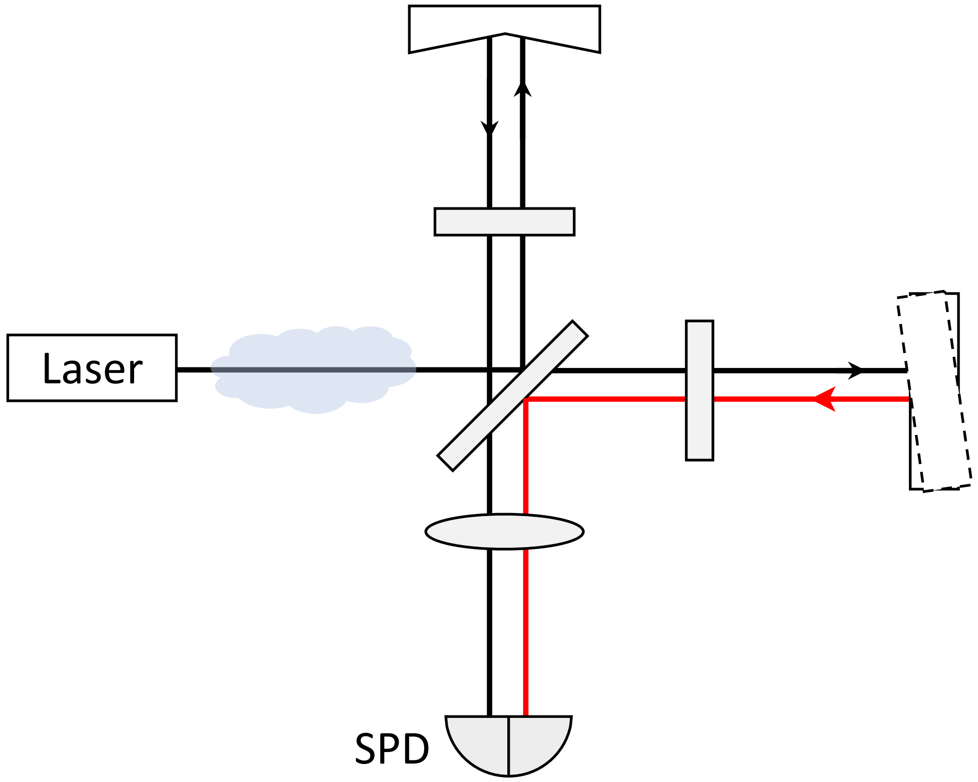


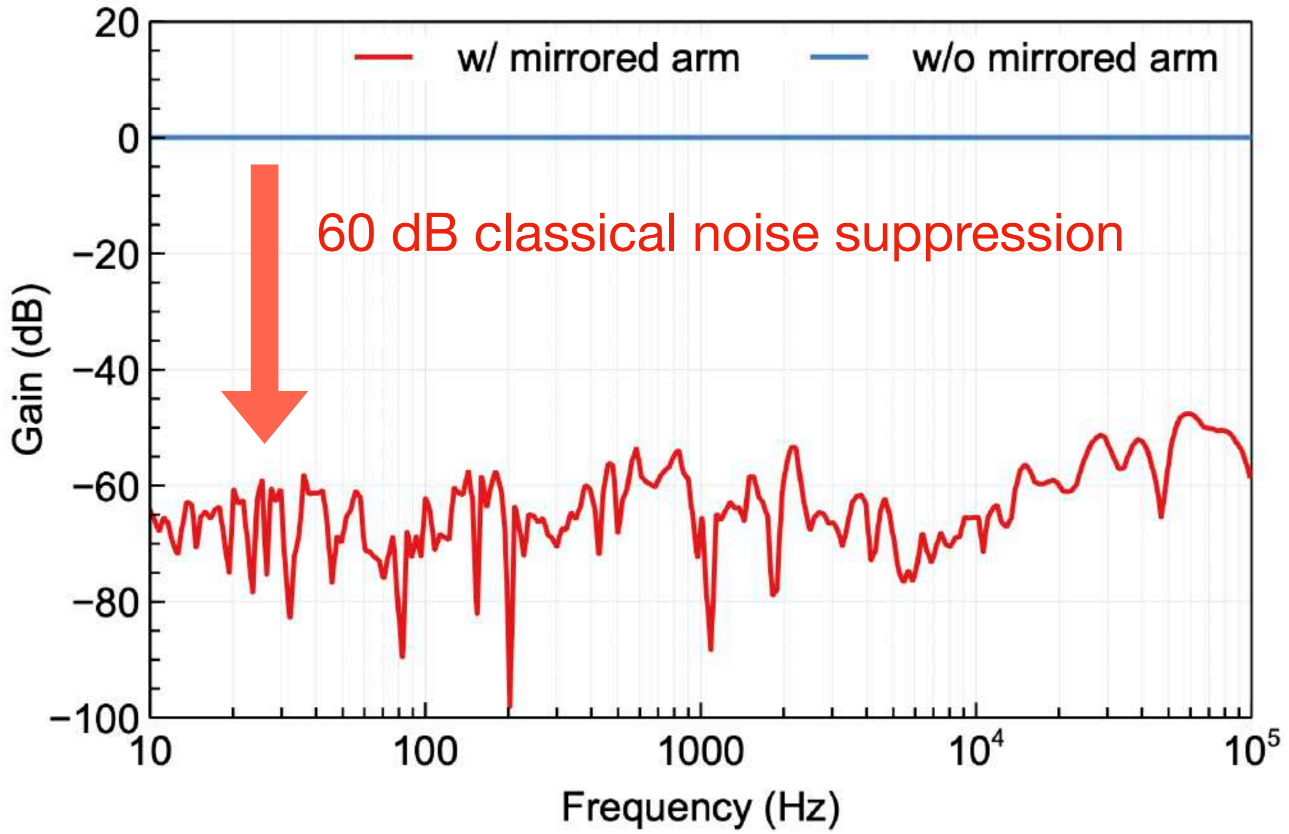
Shen, Hayward,…, Sudhir arXiv:2409.02275 (2024)



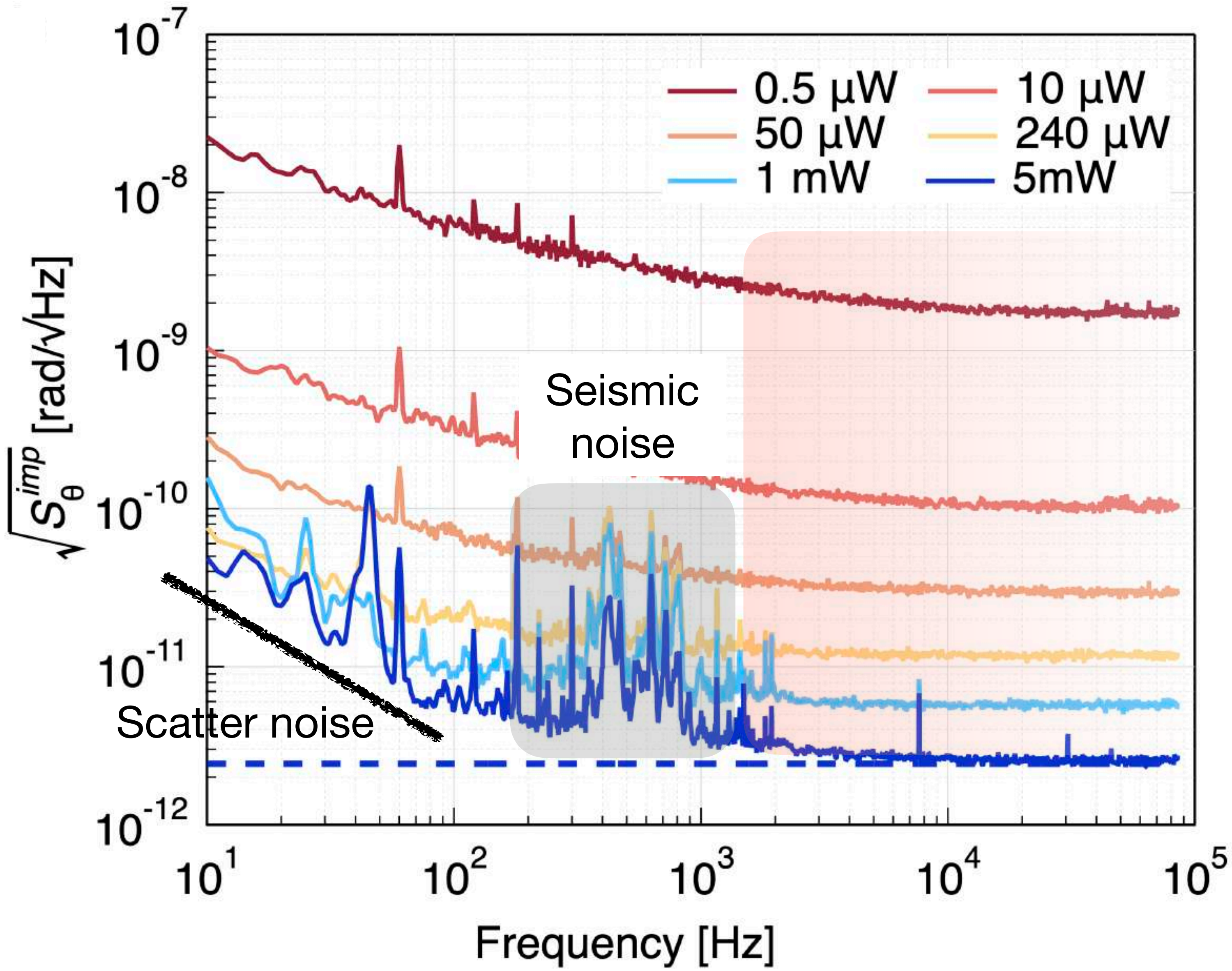
Fife, Shen, Sudhir Rev. Sci. Instru. (2024)





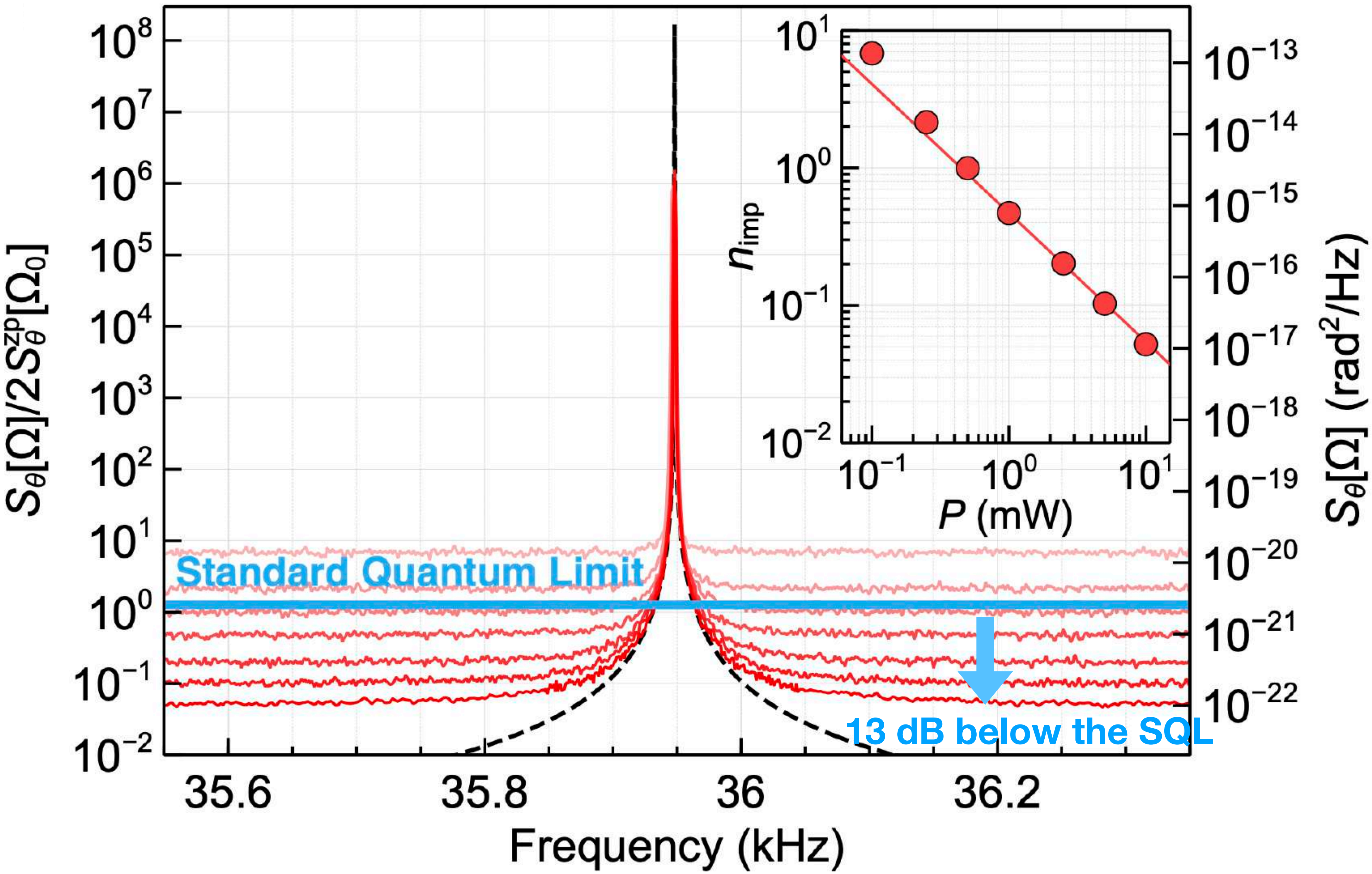


Shen, Hayward,..., Sudhir arXiv:2409.02275 (2024)



Quantum-noise-limited sensitivity of $10^{-12}\,\mathrm{rad}/\sqrt{\mathrm{Hz}}$

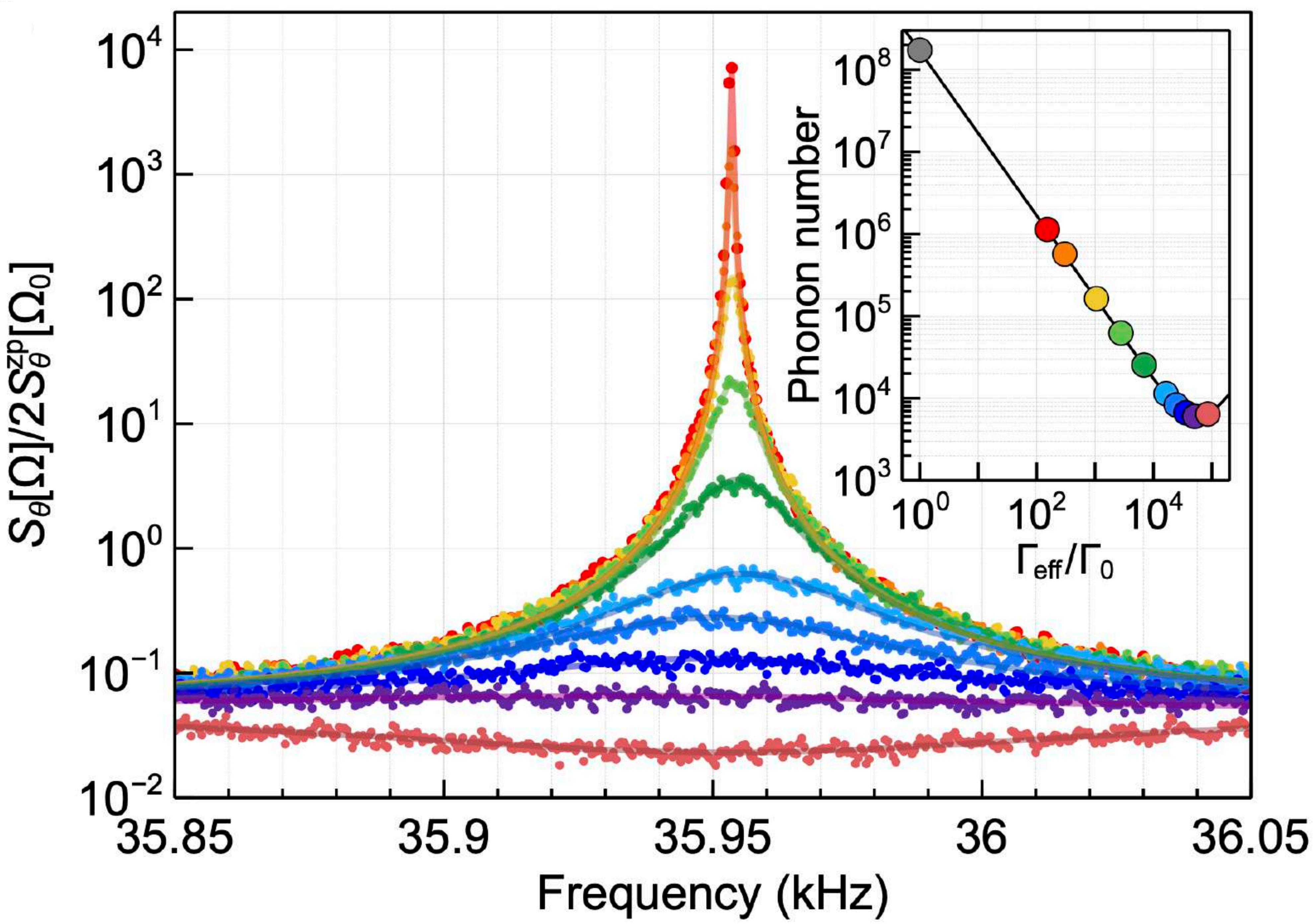
Shen, Hayward,…, Sudhir arXiv:2409.02275 (2024)



$$S_\theta = S_\theta^{th} + S_\theta^{zp} + S_\theta^{imp} + S_\theta^{ba}$$

$$\geq S_\theta^{zp} + 2\sqrt{S_\theta^{imp} S_\theta^{ba}} = (1+\sqrt{\pi})S_\theta^{zp}$$

Shen, Hayward,..., Sudhir arXiv:2409.02275 (2024)



Shen, Hayward,…, Sudhir arXiv:2409.02275 (2024)

precision.mit.edu