The predictive power of Asymptotically Safe Quantum Gravity: Can we test it?

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Introduction

What do we actually know about the fundamental interactions?

LHC, LIGO-VIRGO, EHT,…

General Relativity

Gravitational Interaction \leftrightarrow Spacetime Geometry

Energy&Matter deform spacetime geometry

GR predicts the existence of astonishing objects: Black Holes

$$
ds^{2} = -\left(1 - \frac{2GM}{c^{2}r}\right)c^{2}dt^{2} + \frac{dr^{2}}{\left(1 - \frac{2GM}{c^{2}r}\right)} + r^{2}d\Omega^{2}
$$

Singularity at $r = 0$

Initial singularity (*Big Bang)***?**

Standard Model of Particle Physics

- **• It is a quantum field theory!**
- **• Computations are typically divergent!**
- **• However… the theory is perturbatively renormalizable!**

Singularities in the SM as well…

The SM also has its own singularities

Perturbative renormalizability is neither necessary nor sufficient to ensure that a QFT is valid across *all* **energy scales**

- **• Patches of our observed world**
- **• Very successful descriptions but not complete**
- **• There are known missing pieces (neutrino masses, dark universe,…)**
- **• Structural problems: existence of singularities in both models**

Quantum fields deform spacetime

Spacetime can also fluctuate quantum mechanically

(?)

- **• (Bold HOPE) Can the resulting theory of quantum gravity-matter resolve the singularities of GR and the SM?**
- **• Immediate attempt: Implement a perturbative quantization of Gravity + SM**
- **• The resulting theory is** *perturbatively non-renormalizable!*
- **• This means that the theory requires infinitely many free parameters to absorb its divergences -** *loss of predictivity*

Introduce a ultraviolet cutoff and do explicit (quantum) computations

below the cutoff scale (as we do with the Standard Model!).

[See, e.g., works by Donoghue & Collaborators…]

Perturbatively non-renormalizable theories are perfectly valid

descriptions up to some scale!

[see, e.g., talks by John Donoghue and Alessandro Codello]

We *do* **have a theory of quantum gravity!**

Brief Comment on Scales

One of the biggest challenges in quantum gravity is to detect direct effects arising from quantum spacetime fluctuations

$$
\ell_p = \sqrt{\frac{\hbar G_N}{c^3}} \approx 10^{-35} m
$$

$$
M_p \approx 10^{19} \text{ GeV}
$$

$$
E_{ew} \approx 10^2 \text{ GeV}
$$

- **• If we take seriously such dimensional analysis, quantum gravity effects take place at very (ridiculously) short distances or high energies.**
- **• There is a huge gap between such a would-be quantum gravity scale and the most powerful colliders we have available** at the moment (it varies from $10^{15} - 10^{17}$ **orders of magnitude).**
- **• Common lore: microscopic physics is completely washed out at large distances.**

Is it meaningful to use Particle Physics knowledge to constrain quantum-gravity models?

Not *all* **microscopic information is washed out: An instructive analogy**

Bridging Scales

The mathematical tool that connects scales in quantum/statistical field theory is the renormalization group

• Lowering k establishes a sequence **of effective actions and hence a** *flow -* **the** *renormalization group flow*

Use the renormalization group as the bridge from quantum-gravity scales to Particle Physics - search for QG imprints

However…

We would like to have a quantum gravity-matter theory that is valid across all energy scales

Effective Quantum Gravity is valid below the Planck scale

- **• Aim: Construct a well-behaved theory of quantum gravity + matter at all energy scales**
- **• Should we departure from standard field-theoretic methods in order to build up such a theory?**

The Standard Model has its Landau poles at Transplanckian regimes

[Buttazzo et al. 2013]

If such a *fundamental* **quantum field theory of gravity + matter exists, how to test the quantum gravitational imprints using Particle Physics?**

A Logical Possibility

At the RG fixed point, the theory becomes scale-invariant and a continuum limit can be *safely* **taken**

Paradigmatic example: QCD

Coupling runs to zero at arbitrarily large energy scales: Asymptotic Freedom

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Another possibility…

Coupling runs to finite value at arbitrarily large energy scales: Asymptotic Safety

Could quantum gravity-matter system be asymptotically safe?

Weinberg

Go non-perturbative!

Steven Weinberg conjectured that Quantum Gravity could be an *asymptotically safe quantum field theory*

["Ultraviolet divergences in quantum theories of gravitation" - S. Weinberg] - *An Einstein centenary survey* **edited by Israel and Hawking, 1979**

> *"[…] this paper will be chiefly concerned with another possibility, that a quantum field theory which incorporates gravitation may satisfy a generalized version of the condition of renormalizability known as asymptotic safety."*

Asymptotic Safety Condition: the dimensionless counterparts of the *essential* **couplings should reach a UV renormalization group fixed point**

At the fixed point the couplings cease to run: *quantum scale invariance*

Different Strategies:

- **• Provide a lattice construction of quantum gravity-matter systems and perform Monte-Carlo simulations: Causal Dynamical Triangulations**
- **• Use (semi-)analytical tools such matrix/tensor models in order to define a discretized path integral over geometries**
- **• Look for a continuum limit!**
- **• Use non-perturbative methods in the continuum to search for a non-trivial fixed point**
- Functional Renormalization Group, Dyson-Schwinger equations, n-PI methods, ϵ **-expansion,…**
- **• Compute beta functions non-perturbatively**

Most of the progress in Asymptotically Safe Quantum Gravity was achieved by the use of the FRG. This is by now called the Asymptotic Safety program for quantum gravity

Functional Renormalization Group

^Z[*J*] ⁼ [∫]^Λ *^ϕ* ^e−*S*[*ϕ*]+[∫] ^d*^d ^x ^J*(*x*)*ϕ*(*x*) *Zk*[*J*] ⁼ [∫]^Λ *ϕ* e−*S*[*ϕ*]−Δ*Sk*[*ϕ*]+[∫] ^d*^d ^x ^J*(*x*)*ϕ*(*x*) **regulator "action":** Δ*Sk* = 1 ² [∫] ^d*dx ^ϕ*(*x*) ^ℛ*k*(−∂²)*ϕ*(*x*) Λ *k k* = 0 **gives a (large) mass to field modes with momentum lower than** *k* **in flat space: Fourier modes** *^ϕ*(*x*) ⁼ [∫]*^p* e*ix*⋅*^p ϕ*˜(*p*) ℛ*k*(*p*²) *k*2 , *p*² < *k*² 0 , *p*² > *k*² **essentially Euclidean**

Properties:

interpolates between full effective **action and the "classical" one**

satisfies an exact flow equation

$$
\partial_t \Gamma_k = \frac{1}{2} \text{STr} \left[\left(\Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} \partial_t \mathcal{R}_k \right] \qquad \qquad \partial_t \equiv k \partial_k
$$

Γ

 $k = 0$

(exact) flow equation - Wetterich equation

conversion of functional integral into functional differential equation

solving the flow equation = solving the functional integral

 Γ_k $S_k = \Lambda$

k

Theory Space

(Infinitely many)

Space of all (essential dimensionless couplings) functionals of the field which are compatible with the symmetries of the theory

Approximations are necessary, but we don't need to use a perturbative scheme!

Looking for fixed points:

$$
\beta_i(\mathbf{g}^*) = 0, \quad i = 1, \dots, \infty
$$

$$
\mathbf{g}^* = (g_1^*, \dots, g_\infty^*)
$$

Linearized flow around the fixed point:

$$
\partial_t (g_i - g_i^*) = \sum_j \frac{\partial \beta_i}{\partial g_j} (g_j - g_j^*)
$$

diagonalize

Predictivity requires that the number of relevant directions is finite

finite-dimensional

critical surface

Asymptotic Safety:

Existence of a renormalization-group fixed point;

Fixed point features finitely many relevant directions;

Asymptotically Safe Quantum Gravity

- the technical side -

- the technical side - Asymptotically Safe Quantum Gravity

Quantum Gravity and the Functional **Renormalization Group**

The Road towards Asymptotic Safety

MARTIN REUTER AND FRANK SAUERESSIG

CAMBRIDGE MONOGRAPHS ON MATHEMATICAL PHYSICS

Asymptotically Safe Quantum Gravity

- the technical side -

But see also…

- the technical side - Asymptotically Safe Quantum Gravity

No background to set a scale: *background field method*

$$
g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}
$$

Euclidean

The spectrum of the Laplacian of the background metric defines a scale

background independence is encoded in split symmetry $\bar{g}_{\mu\nu} \rightarrow \bar{g}_{\mu\nu} + \epsilon_{\mu\nu}$ $h_{\mu\nu} \rightarrow h_{\mu\nu} - \epsilon_{\mu\nu}$

The gravitational action is invariant under general coordinate transformations: *gauge invariance*

Introduction of a gauge fixing term:

Faddeev-Popov procedure

$$
Z_k[\mathcal{J}] = \int \mathcal{D}h_{\mu\nu} \mathcal{D}\bar{C}_\alpha \mathcal{D}C^\beta e^{-S[\bar{g}+h]-S_{\text{gf}}[\bar{g};h]-S_{\text{gh}}[\bar{g};h,\bar{C},C]-\Delta S_k[\bar{\Phi};\Phi] + \int d^dx \sqrt{\bar{g}} \mathcal{J} \cdot \Phi} \equiv e^{W_k[\mathcal{J}]}
$$

In complete analogy: construction of effective average action $\Gamma_k = \Gamma_k[\bar{\Phi}; \Phi]$

$$
\partial_t \Gamma_k[\bar{\Phi}, \Phi] = \frac{1}{2} \text{STr} \left[\left(\Gamma_k^{(0,2)}[\bar{\Phi}, \Phi] + \mathbb{R}_k \right)^{-1} \partial_t \mathbb{R}_k \right]
$$

The effective average action is a functional of two fields;

Integrating the flow and taking *k=0* **leads to an effective action that depends on two fields, but background independence is guaranteed by BRST symmetry;**

Choices…

Our starting point was a path integral over Riemannian metrics

Such a linear split of the metric might introduce many spurious configurations in the nonperturbative realm!

Alternative:

$$
g_{\mu\nu} = \bar{g}_{\mu\alpha} \left(e^{\bar{g}^{-1}h} \right)^{\alpha}_{\nu}
$$

avoid the previous problems + cover the space of Riemannian metrics

In the path integral, should we adopt different variables that lead to the same field equations **in the case of GR?**

Palatini
$$
(g_{\mu\nu}, \Gamma^{\alpha}_{\beta\sigma})
$$
 or pure e^a_μ or $(e^a_\mu, \omega^{bc}_\nu)$

No *a priori* **reason to choose one formulation instead of the other**

En route to the Reuter fixed point

With the FRG equation, we are ready to compute the beta functions beyond perturbative schemes and look for fixed points

Practical Strategy

- **• Choose an ansatz for the effective average action**
- **Compute the beta functions of the couplings present in the chosen truncation**
- **• Look for suitable fixed-point solutions**
- **• Enlarge the truncation following some ordering principle**
- **• Investigate a possible (apparent) onset of stability of the results**

Einstein-Hilbert Truncation

 $\Gamma_k =$ 1 $\left(\frac{1}{8\pi G_k}\right) d^d x \sqrt{g} (2\Lambda_k - R) +$ gauge − fixing sector

> In $d=4$, a suitable fixed point for the **dimensionless Newton constant and cosmological constant is found:** $g_k = k^2 G_k$ and $\lambda_k = k^{-2} \Lambda_k$.

The fixed point has two relevant directions.

[Reuter, Saueressig 02]

En route to the Reuter fixed point

With the FRG equation, we are ready to compute the beta functions beyond perturbative schemes and look for fixed points

Enlarging the truncation

$$
\begin{array}{cccc}\n1 & & & \\
\sqrt{g}R & & & \\
\sqrt{g}R^2 & \sqrt{g}R_{\mu\nu}R^{\mu\nu} & \sqrt{g}R_{\mu\nu}F_{\text{Ric}}(\Delta)R^{\mu\nu} & \sqrt{g}R F_{\text{R}}(\Delta)R & \cdots \\
\sqrt{g}R^3 & \sqrt{g}RR_{\mu\nu}R^{\mu\nu} & \cdots & \\
\vdots & \vdots & \ddots & \vdots\n\end{array}
$$

By now, many different directions of the theory space have been explored. A suitable fixed **point with rather stable properties persists against truncations enlargement.**

Example: truncation of the *f*(*R*) **form [Falls, Litim, Schroeder 19]**

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Towards phenomenology

In order to extract the effects of quantum-gravity fluctuations, one has to solve the "fully quantum equations of motion"

["Matter matters program": Dona, Eichhorn, Percacci '14]

The gravitational fixed point should be consistent with matter coupling

[G. P. de Brito, *ADP***, A. F. Vieira '20]**

Matter-fluctuations impact the running of gravitational couplings

Proof of principle example:

$$
\Gamma_k = \Gamma_k^{\text{grav}} + \Gamma_k^{\text{matter}} + \Gamma_k^{\text{gf}}
$$

$$
\Gamma_k^{\text{grav}} = \frac{1}{16\pi G_{\text{N},k}} \int d^d x \sqrt{g} f_k(R, R_{\mu\nu}^2)
$$

$$
\Gamma_k^{\text{matter}} = \frac{1}{2} \sum_{i=1}^{N_\phi} \int d^d x \sqrt{g} g^{\mu\nu} \partial_\mu \phi_i \partial_\nu \phi_i + \sum_{i=1}^{N_\psi} \int d^d x \sqrt{g} i \overline{\psi}_i \gamma^\mu \nabla_\mu \psi_i + \frac{1}{4} \sum_{i=1}^{N_A} \int d^d x \sqrt{g} g^{\mu\alpha} g^{\nu\beta} F_{i,\mu\nu} F_{i,\alpha\beta}
$$

[G. P. de Brito, *ADP***, A. F. Vieira '20]**

Matter-fluctuations impact the running of gravitational couplings

Proof of principle example:

 $\int d^dx \sqrt{g} f_k(R, R_{\mu\nu}^2)$

Faculty member at UNESP - Guaratinguetá

$$
\Gamma_k^{\text{matter}} = \frac{1}{2} \sum_{i=1}^{N_\phi} \int \mathrm{d}^d x \sqrt{g} \, g^{\mu\nu} \partial_\mu \phi_i \partial_\nu \phi_i + \sum_{i=1}^{N_\psi} \int \mathrm{d}^d x \sqrt{g} \, i \overline{\psi}_i \gamma^\mu \nabla_\mu \psi_i + \frac{1}{4} \sum_{i=1}^{N_A} \int \mathrm{d}^d x \sqrt{g} \, g^{\mu\alpha} g^{\nu\beta} F_{i,\mu\nu} F_{i,\alpha\beta}
$$

[G. P. de Brito, *ADP***, A. F. Vieira '20]**

Matter content seems to matter!

Detection of matter fields that are incompatible with the fixed-point structure would correspond to inconsistency with the Asymptotic Safety scenario

- **• Quantum-gravity fluctuations can impact the running of matter couplings**
- **• In the Standard Model: Abelian hypercharge and Higgs-Yukawa sectors feature a Landau pole**
- **• Can quantum-gravity fluctuations "cure" such singularities?**
- Consider a matter coupling g_i . In general, quantum-gravity contributions **to the beta functions take the following form:**

$$
\beta_{g_i}|_{\text{grav}} = -f_{g_i}g_i + \dots
$$

- The function f_{g_i} is a function of the gravitational couplings
- The sign of f_{g_i} determines if the corresponding coupling features a fixed **point (free or non-trivial) & its (ir)relevance**

- **• Choose a symmetry group and define the operators compatible with such symmetries in the gravitational sector (within truncations)**
- **• Choose the matter action (within truncations)**

• Compute explicitly the values of f_{g_i}

• Suitable values of f_{g_i} can render a UV complete theory of gravity + matter

- **• This is not (necessarily) a unified theory in the grand unification sense**
- **• Marginal couplings that are pushed towards irrelevance due to quantum gravitational contributions become predictions**

Some exciting results were obtained in the Asymptotic Safety literature over the past two decades:

Prediction of the Higgs mass:

Asymptotic safety of gravity and the Higgs boson mass

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ABSTRACT

There are indications that gravity is asymptotically safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that this is indeed the case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we address the question of whether the mass of the Higgs boson m_H can be predicted. For a positive gravity induced anomalous dimension $A_{\lambda} > 0$ the running of the quartic scalar self-interaction λ at scales beyond the Planck mass is determined by a fixed point at zero. This results in $m_H = m_{min} = 126$ GeV, with only a few GeV uncertainty. This prediction is independent of the details of the short distance running and holds for a wide class of extensions of the SM as well. For A_{λ} < 0 one finds m_H in the interval m_{min} < m_H < m_{max} \simeq 174 GeV, now sensitive to A_{λ} and other properties of the short distance running. The case $A_{\lambda} > 0$ is favored by explicit computations existing in the literature.

Some exciting results were obtained in the Asymptotic Safety literature over the past two decades:

Retrodiction of top mass:

Top mass from asymptotic safety

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ABSTRACT

We discover that asymptotically safe quantum gravity could predict the top-quark mass. For a broad range of microscopic gravitational couplings, quantum gravity could provide an ultraviolet completion for the Standard Model by triggering asymptotic freedom in the gauge couplings and bottom Yukawa and asymptotic safety in the top-Yukawa and Higgs-quartic coupling. We find that in a part of this range, a difference of the top and bottom mass of approximately 170 GeV is generated and the Higgs mass is determined in terms of the top mass. Assuming no new physics below the Planck scale, we construct explicit Renormalization Group trajectories for Standard Model and gravitational couplings which link the transplanckian regime to the electroweak scale and yield a top pole mass of $M_{\text{t,pole}} \approx 171 \,\text{GeV}$. © 2017 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY ficense $(http://creativecommons.org/licenses/by/4.0/">$. Funded by SCOAP³.

Some exciting results were obtained in the Asymptotic Safety literature over the past two decades:

En route to test the compatibility of different mechanisms for neutrino masses in Asymptotic Safety [JHEP 08 (2019) 142]

On the impact of Majorana masses in gravity-matter systems

Gustavo P. de Brito, a,b Yuta Hamada, c Antonio D. Pereira d,b and Masatoshi Yamada b

Some exciting results were obtained in the Asymptotic Safety literature over the past two decades:

Asymptotically safe Standard Model + quantum gravity [SciPost Phys. 15, 105 (2023)]

The Asymptotically Safe Standard Model: From quantum gravity to dynamical chiral symmetry breaking

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Some exciting results were obtained in the Asymptotic Safety literature over the past two decades:

Asymptotically safe Standard Model + quantum gravity [SciPost Phys. 15, 105 (2023)]

Some exciting results were obtained in the Asymptotic Safety literature over the past two decades:

No ALPs is Asymptotically Safe Quantum Gravity [JHEP 06 (2022) 013]

Are there ALPs in the asymptotically safe landscape?

Gustavo P. de Brito, Astrid Eichhorn and Rafael R. Lino dos Santos

Several other results were obtained by this interplay between quantumgravity fluctuations and matter. See, e.g.,

[Submitted on 14 Dec 2022] Asymptotic safety of gravity with

matter

Astrid Eichhorn, Marc Schiffer

Several other results were obtained by this interplay between quantumgravity fluctuations and matter. See, e.g.,

Final words

It seems that we cannot exclude the possibility of quantum gravity to be described by an asymptotically safe standard QFT

Non-perturbative tools are mandatory in this case - Big Challenge!

We have indications that the theory coupled to matter can produce consistency checks and even predictions

Many challenges ahead:

How to transport everything that we have learnt so far to the Lorentzian setting?

Is the theory unitary?

Do we have a complete RG-trajectory that emanates from the UV to our IR?

How to connect the results obtained with the FRG and other non-perturbative schemes?

Many of those questions have been under investigation over the past few years. Little time to tell details.

Thank you

Predictive Power

The existence of the UV-fixed point imposes severe constraints on the RG-flow

