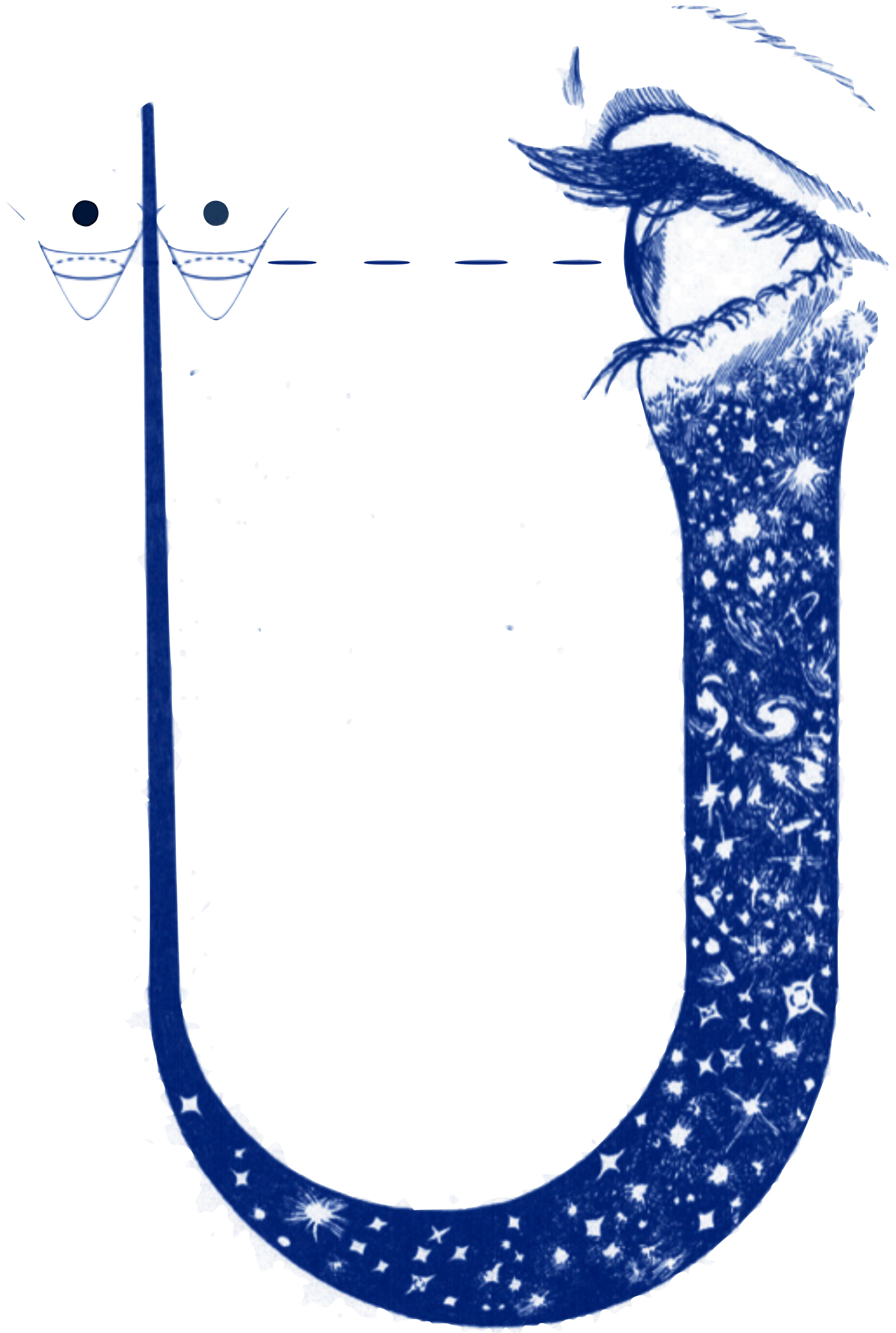
Black Holes and Cosmological Horizons Decohere Quantum ~~Superstitions~~ *Superpositions*

Daine L. Danielson[1], G. Satishchandran[2], R. M. Wald[1]

[1]The University of Chicago, [2]Princeton University

Witnessing Quantum Aspects of Gravity in a Lab, September 26, 2024



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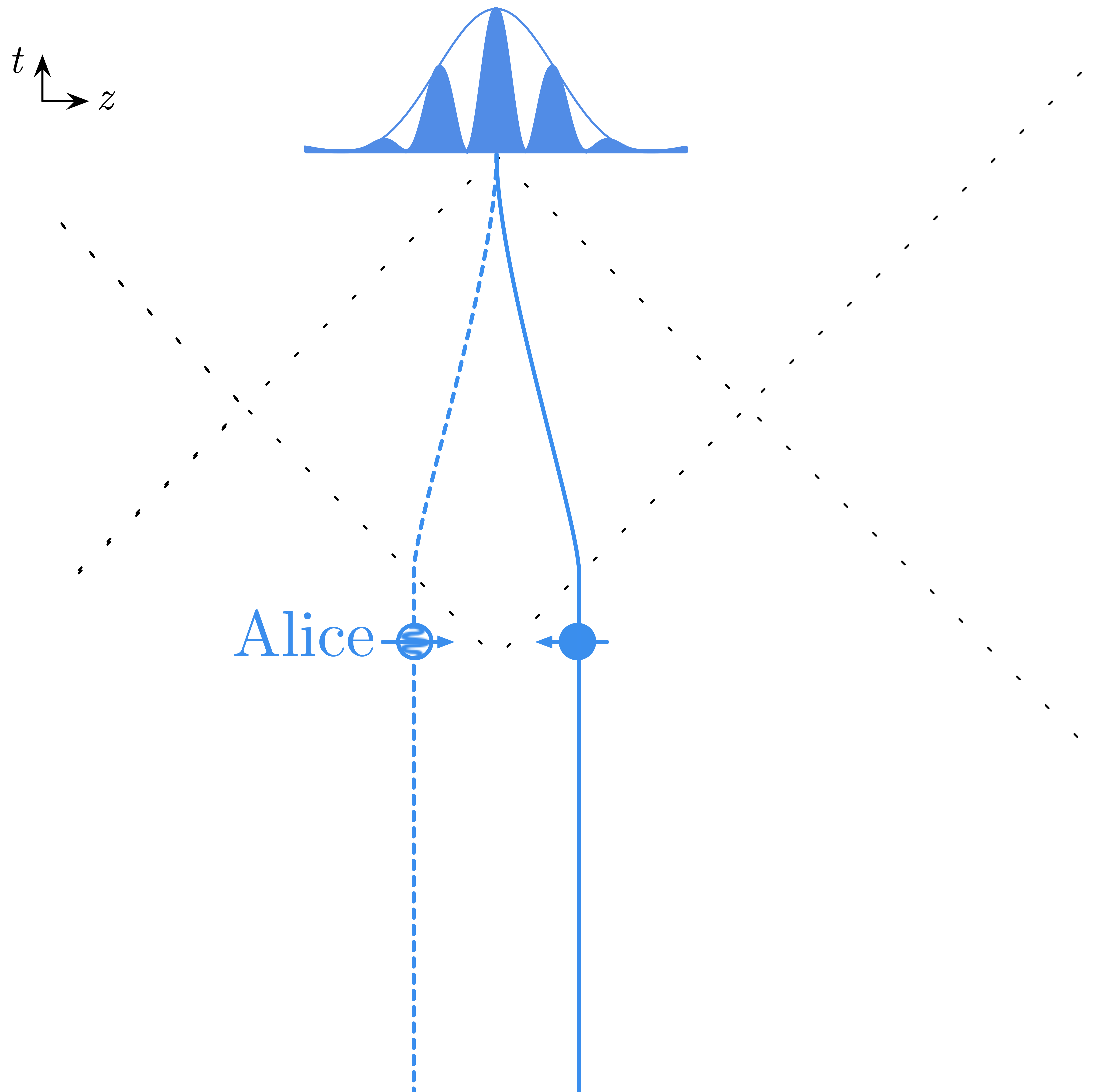
D.L.D., G. Satishchandran, R.M. Wald, *Int. J. Mod. Phys. D* 2241003 (2022), arXiv:2205.06279

Gravity Research Foundation Essay Competition, Third Award

D.L.D., G. Satishchandran R.M. Wald (2024), *Phys. Rev. D* 108, 025007 (2024), arXiv:2401.00026

D.L.D., G. Satishchandran, R.M. Wald, arXiv:2407.02567

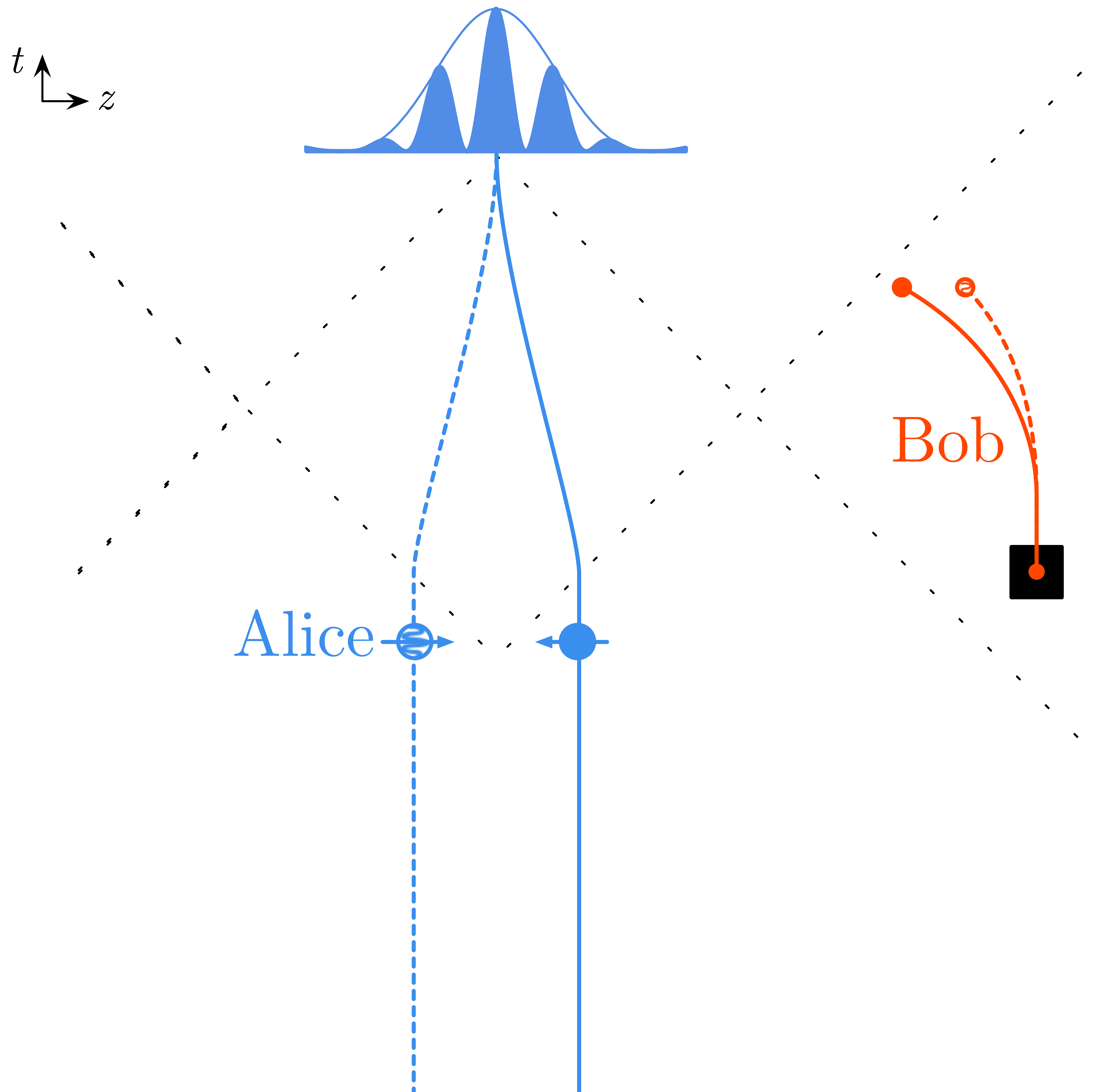
D.L.D., Jonah Kudler-Flam, G. Satishchandran (to appear)



In the past, Alice used a Stern-Gerlach apparatus to produce a **spatial superposition of a massive (or charged) body,** $\frac{1}{\sqrt{2}}\left(|\uparrow_z, A_1\rangle + |\downarrow_z, A_2\rangle\right)$

Later, she attempts an interference experiment and looks for signs of decoherence.

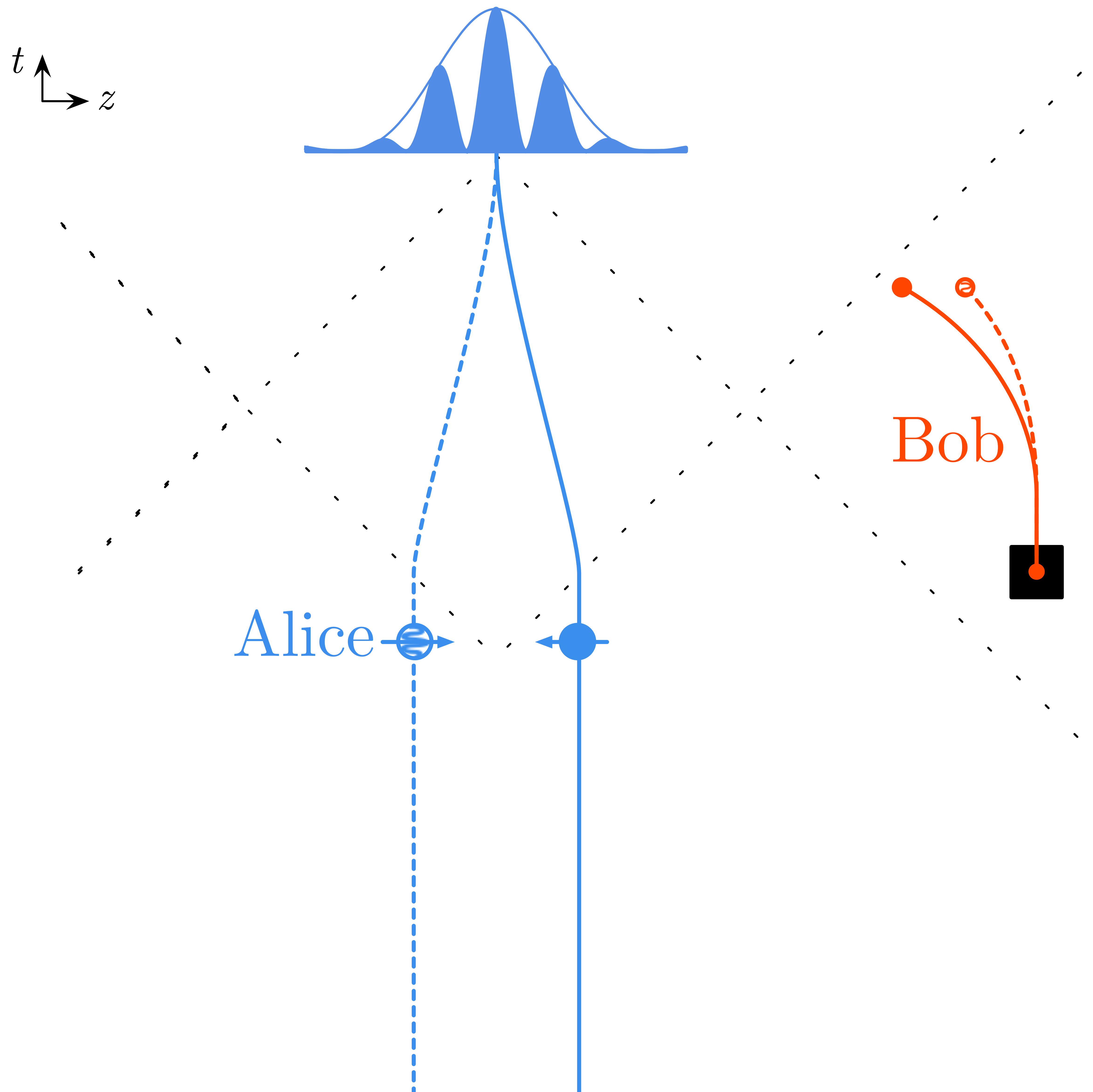
She can do this by, e.g., looking for coherent interference of the spin. She can measure spin along the *x*-axis. If she sees spin down even once, she knows her superposition has decohered.



Alice's particle is entangled with its own "Newtonian" field. Formally,

$$\frac{1}{\sqrt{2}}\left(|\uparrow, A_1\rangle \otimes |\psi_1\rangle + |\downarrow, A_2\rangle \otimes |\psi_2\rangle\right).$$

In a *spacelike-separated region*, Bob may attempt to measure Alice's superposed gravitational field by releasing a particle from a trap.



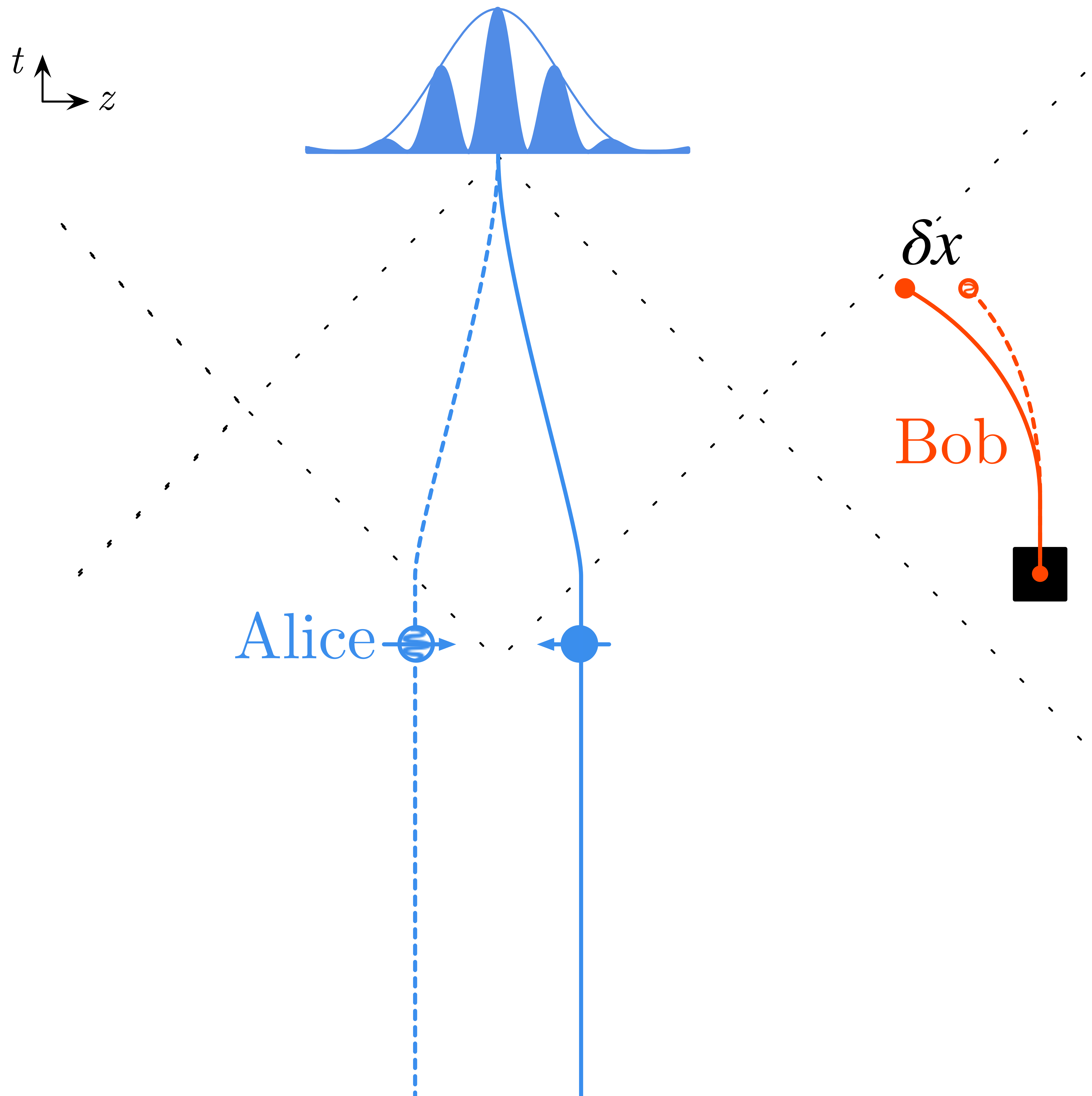
Bob can *choose* to measure, or not to measure, the field.

If Bob successfully measures the field, Alice's particle is decohered.

But Alice can tell whether her particle is decohered!

This seems paradoxical.

Paradox resolved...



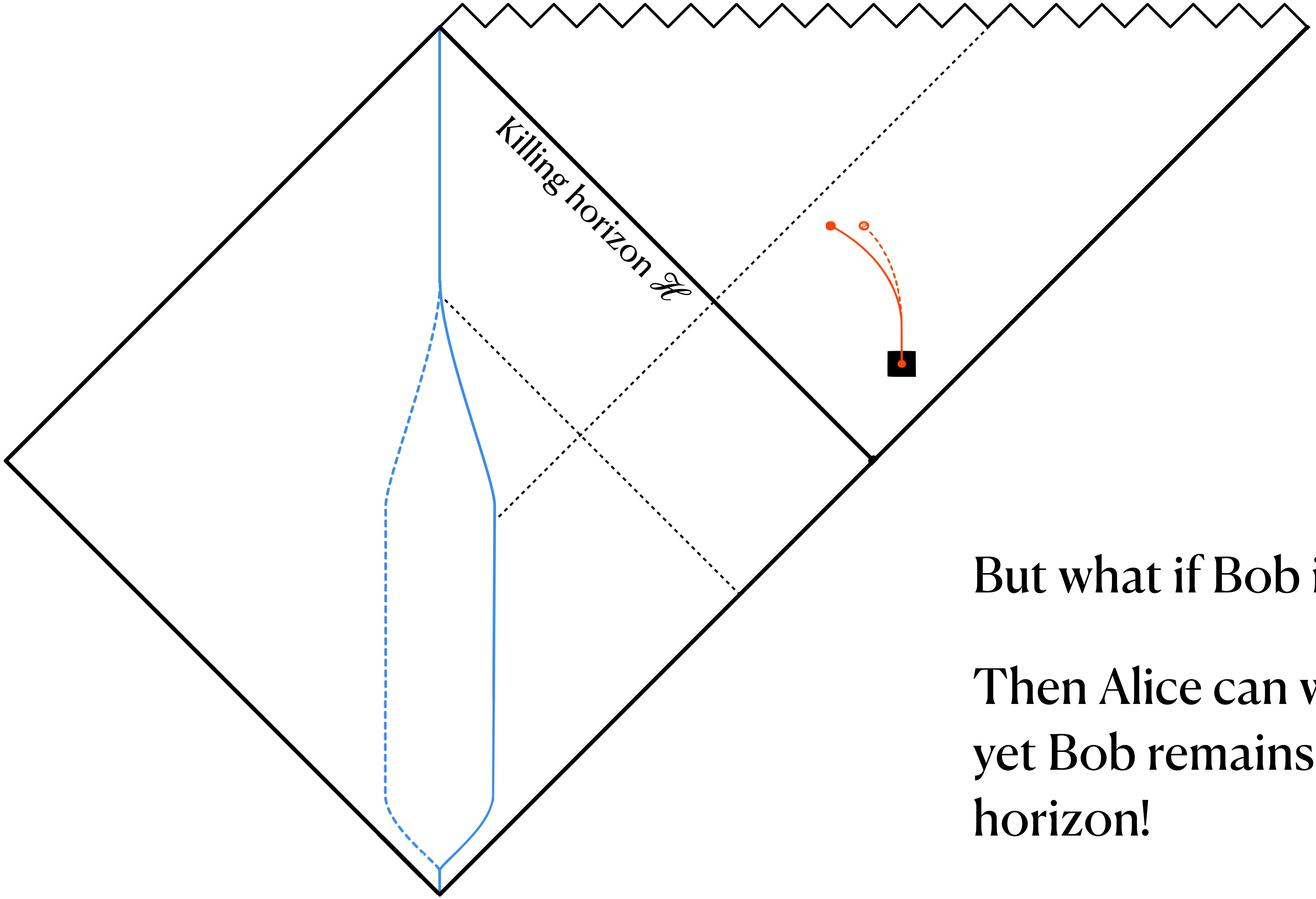
Bob's precision is limited by **vacuum fluctuations** of the metric, requiring $\delta x > \Delta x$. In QED, $\Delta x \sim q/m$. In gravity, $\Delta x \sim l_{\mathrm{P}}$.

Alice needs to recombine slowly to avoid producing **entangling radiation:** $\langle N \rangle \ll 1$. $2\,|\rho_{\mathrm{L,R}}| = |\langle h_{\mathrm{L}} | h_{\mathrm{R}} \rangle| = e^{-\frac{1}{2}\langle N \rangle}$.

If Alice goes slower, Bob must measure from farther away to remain spacelike. Thus he measures a weaker field, requiring more time.

[DLD, Satishchandran, Wald (2022). Belenchia *et al.* (2019).]

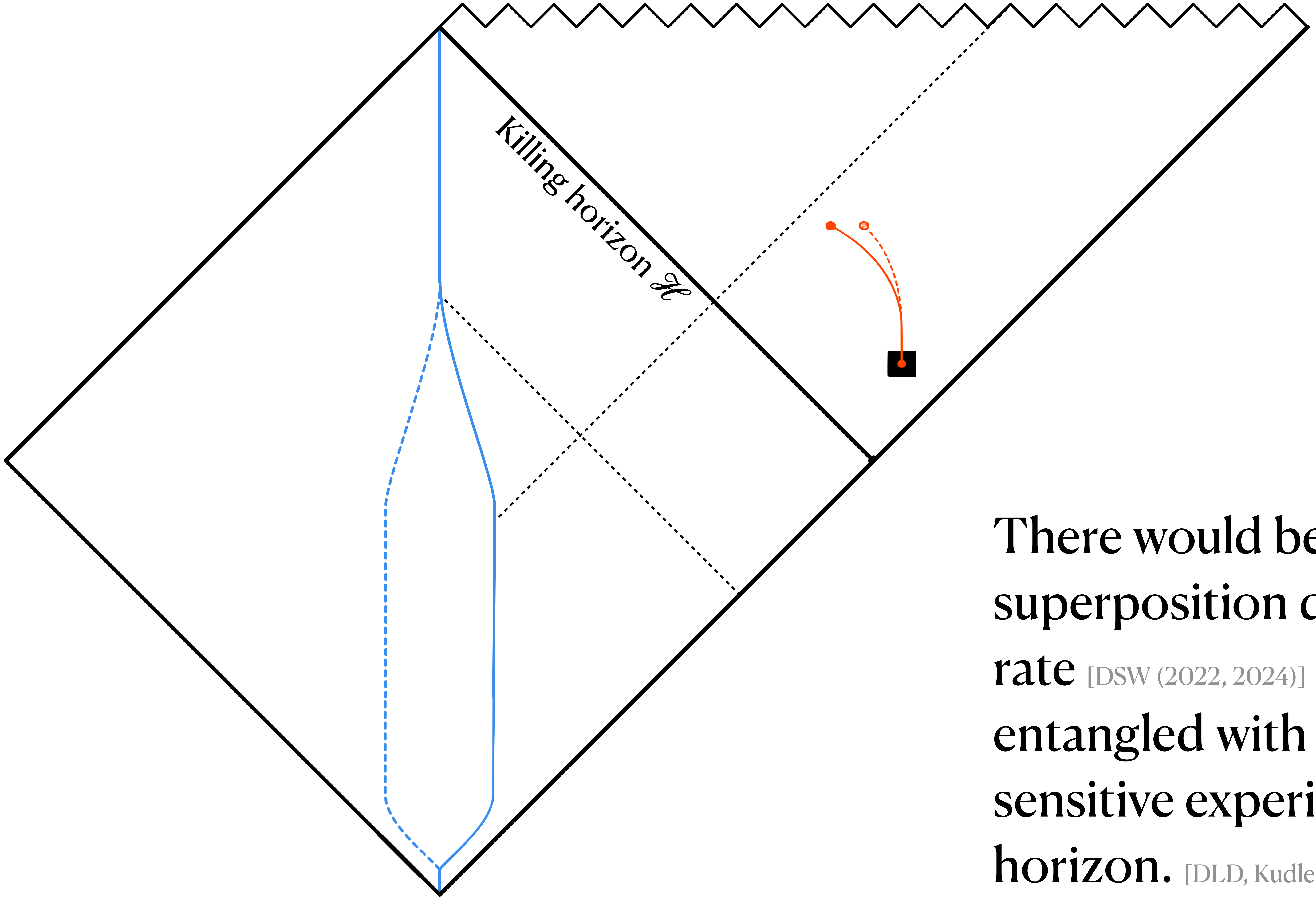
Paradox restored?



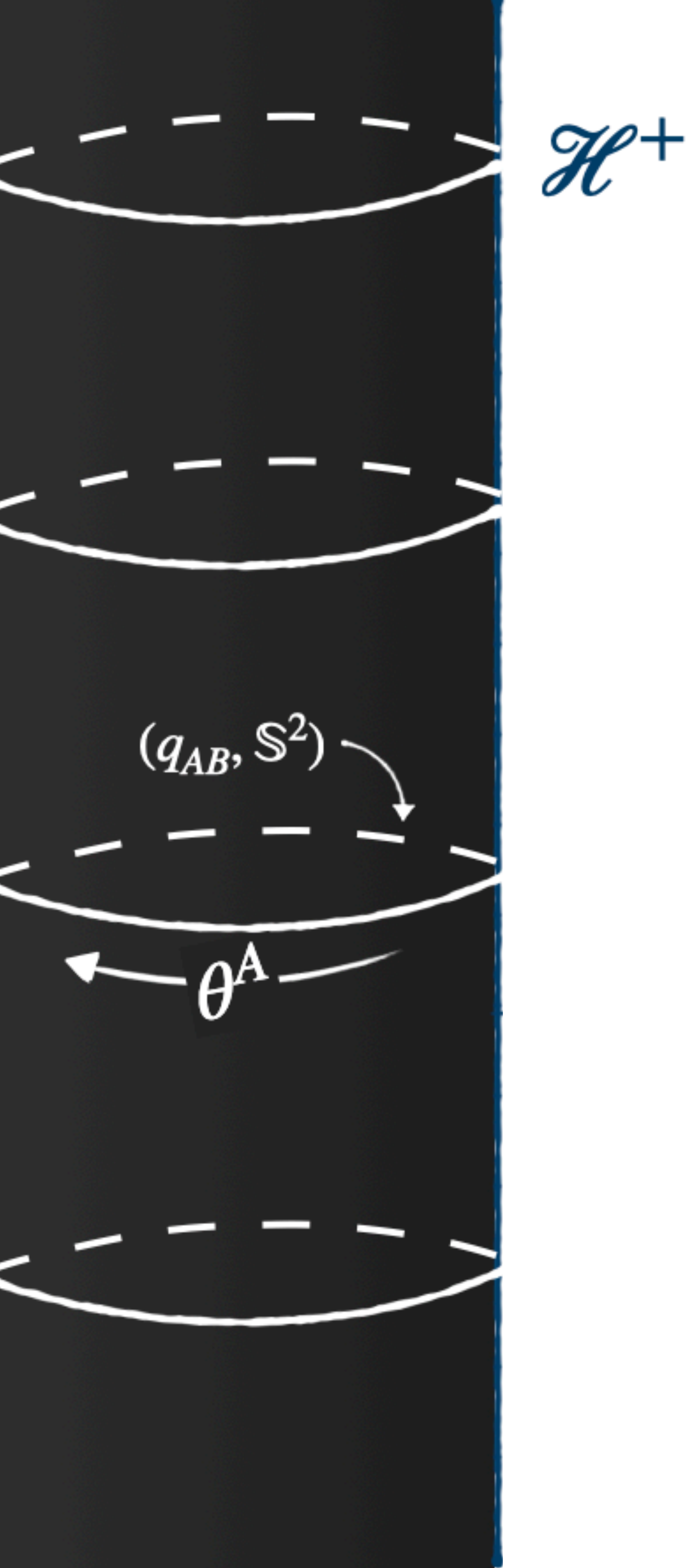
But what if Bob is behind a horizon?

Then Alice can work as slowly as she likes, yet Bob remains spacelike, just beyond the horizon!

Universal decoherence?

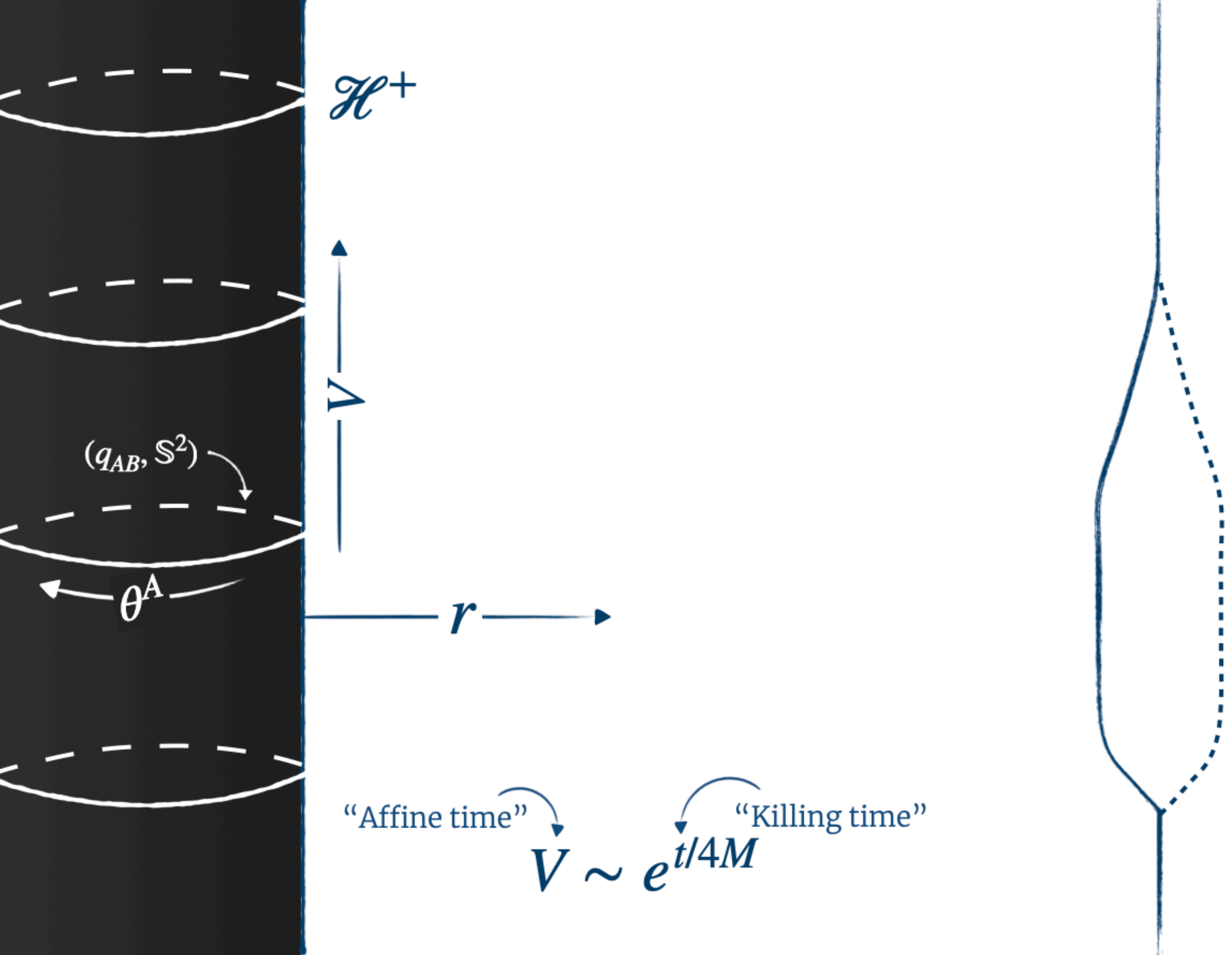


There would be no paradox if Alice's superposition decoheres at a constant rate [DSW (2022, 2024)] *as if* she were becoming entangled with any number of optimally-sensitive experiments hidden behind the horizon. [DLD, Kudler-Flam, Satishchandran, (to appear)].



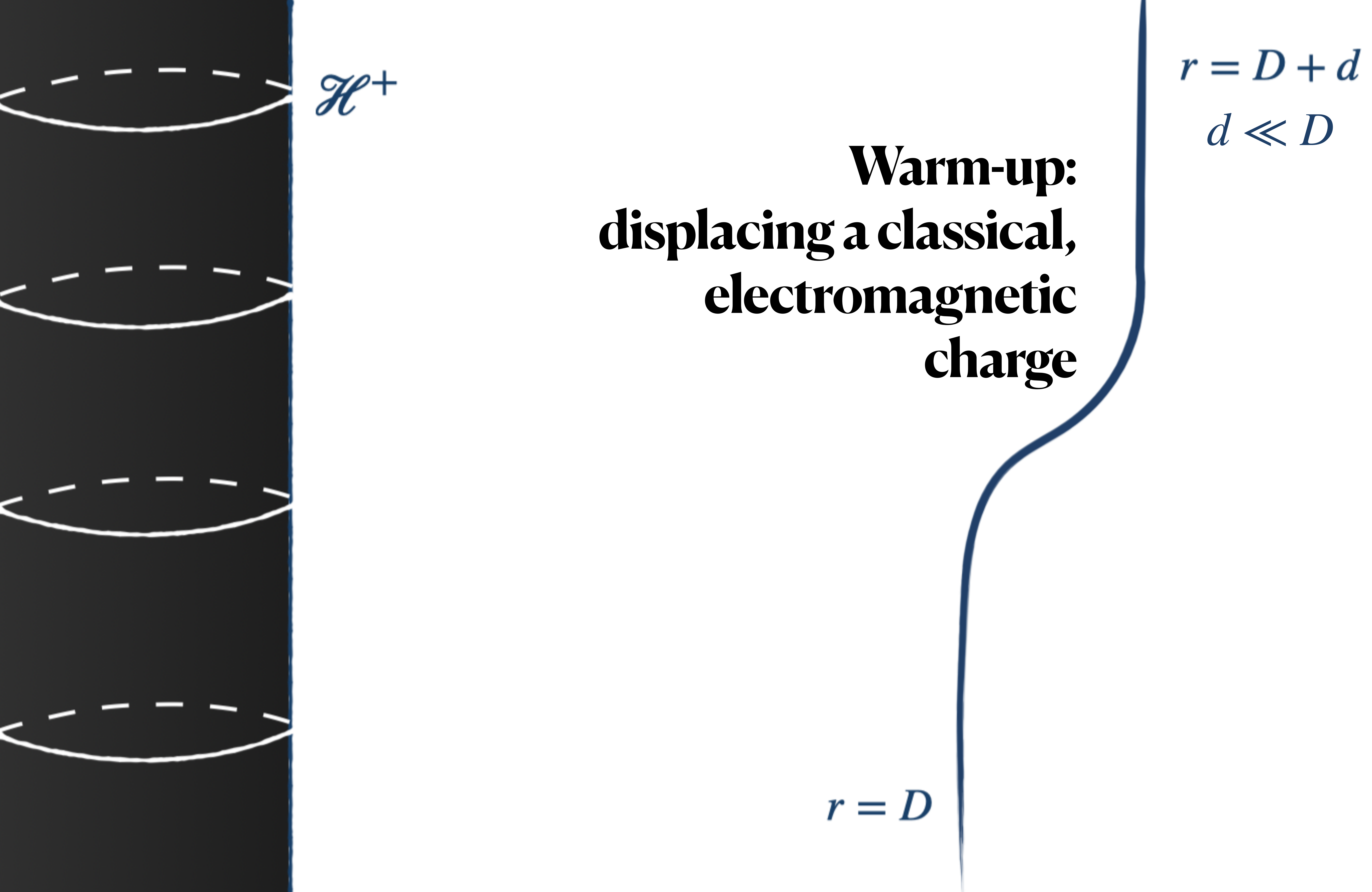
- Suppose Alice performs her experiment in a stationary laboratory outside a black hole.

- *Why should this be any different from flat spacetime?*



Daine Danielson, WQAGL, 2024

Warm-up: displacing a classical, electromagnetic charge



Unavoidable horizon radiation

V: affine time along the horizon

(angular indices on the horizon cross section)

$$\partial_V E_r = -\mathscr{D}^A E_A$$

Evolving Coulomb field on the horizon

Radiation into the black hole

$$E_A = -\partial_V A_A$$

$$\Delta E_r \neq 0 \implies \int_{-\infty}^{V} E_A \neq 0 \implies \Delta A_A \neq 0$$

Net change in the potential!

A "memory effect" on the black hole horizon

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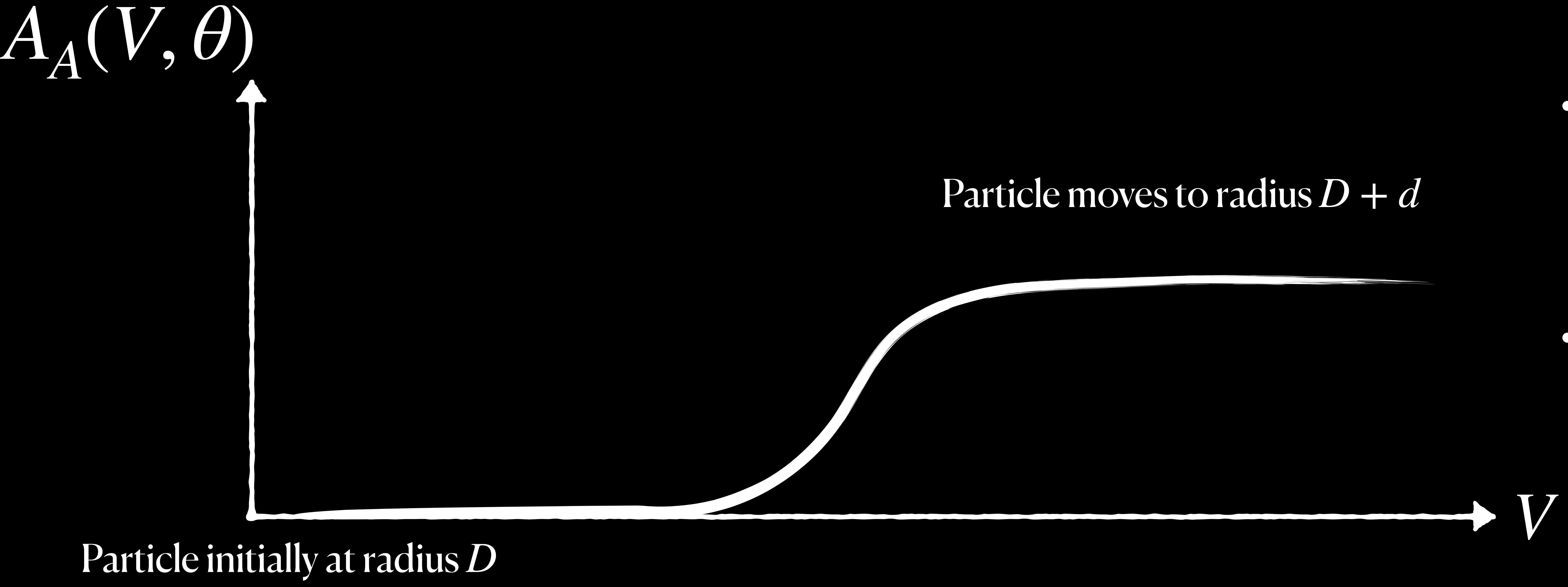


- A direct mathematical analog of the "memory effect," but on a black hole horizon.
- Unlike the memory effect at null infinity, this occurs on horizons in all dimensions in which radiation exists.

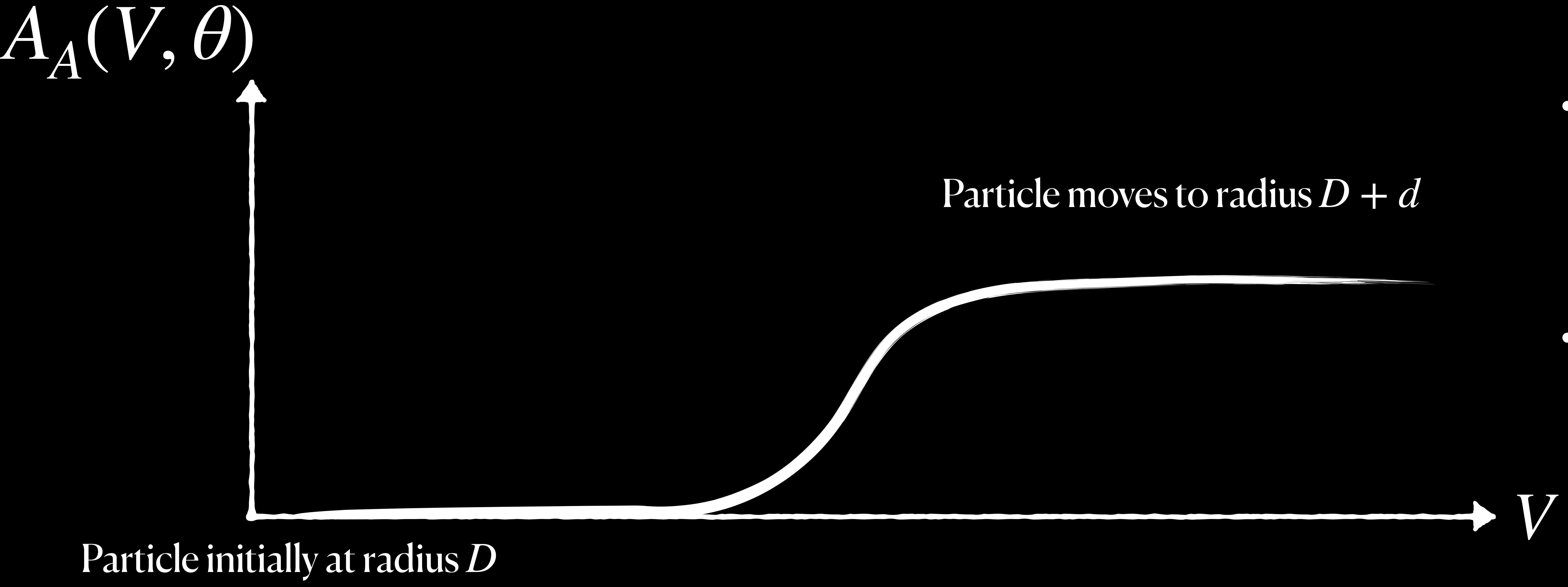
Soft photons on the black hole horizon

$$\hat{A}_A(\omega) \sim \frac{1}{\omega}$$

so large ΔV means arbitrarily low energy...

$$\langle N \rangle \propto \int_{S^2} d\Omega \int_0^\infty d\omega \;\, \omega \hat{A}^B \hat{A}_B \to \infty$$

but *many* (soft) photons!
For a permanently displaced particle, the permanently shifted Coulomb field produces an "infrared divergence."



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Consider one branch of Alice's experiment…

V:Affine time along the horizon

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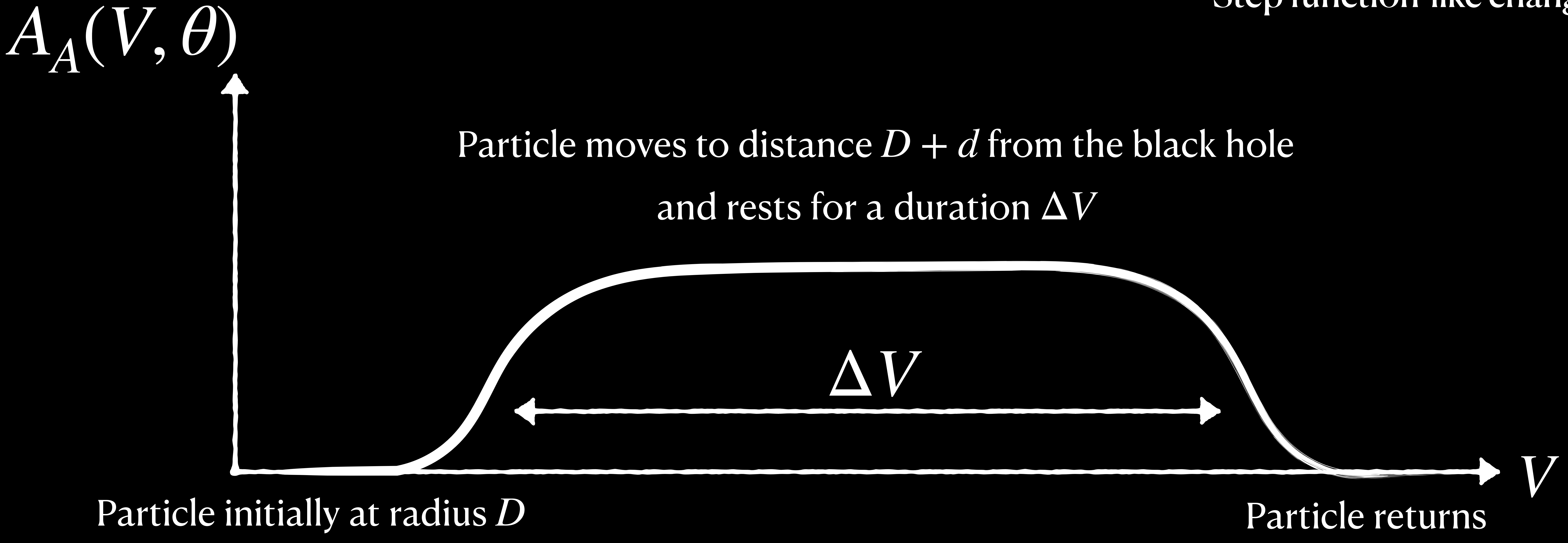
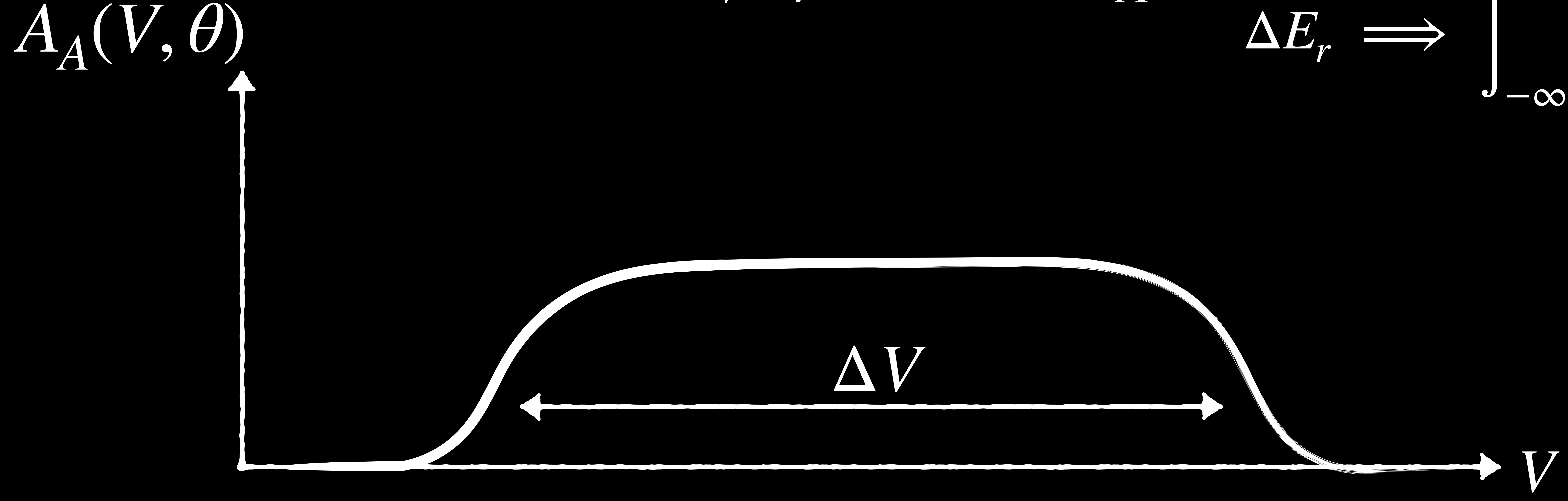
Step function-like change in the potential



$$E_A = -\,\partial_V A_A$$

$$\partial_V E_r = -\,\mathscr{D}^A E_A$$

$$\Delta E_r \implies \int_{-\infty}^{V} E_A \implies \Delta A_A$$



Fourier transform: for large ΔV, low-frequency behavior goes as $\hat{A}_A(\omega) \sim \dfrac{1}{\omega}$ so large ΔV means arbitrarily low energy...

$$\langle N \rangle \propto \int_{S^2} d\Omega \int_0^\infty d\omega\ \omega \overline{\hat{A}^B} \hat{A}_B \sim \ln \Delta V$$

but *many* (soft) photons!

Soft horizon photons are unavoidable

In Alice's proper time T, the difference between the branches of her experiment *necessarily* sources entangling photons $\langle N \rangle \sim \frac{G^3 M^3 q^2 d^2}{\hbar c^5 D^6} T$.

For long T, $\langle N \rangle \gtrsim 1$, and Alice's particle will be decohered *no matter what*.

NB: a horizonless object ($r_{\text{body}} > r_S$) would *not exhibit* this effect.

Black hole decoherence effect

for a charged particle

- Plugging in numbers, Alice's superposition decoheres after

$$T_D \sim \frac{\hbar c^6 D^6}{G^3 M^3 q^2 d^2} \sim 10^{43} \text{ years } \left(\frac{D}{\text{a.u.}}\right)^6 \cdot \left(\frac{M_\odot}{M}\right)^3 \cdot \left(\frac{e}{q}\right)^2 \cdot \left(\frac{\text{m}}{d}\right)^2 .$$

- If our Sun were a black hole, an electron on Earth superposed by a meter would decohere in 10^{32} times the age of the universe. But, if this experiment were done at the innermost stable orbit, then $T_D \sim 5$ minutes!

Black hole decoherence effect

for a massive particle

- The analysis proceeds exactly as before in (linearized) quantum gravity.
- *All objects* source gravity!

- Any superposed body will therefore be decohered by soft horizon gravitons after a time,

$$T_D^{\mathrm{GR}} \sim \frac{\hbar c^{10} D^{10}}{G^6 M^5 m^2 d^4} \sim 10\ \mu\mathrm{s} \left(\frac{\mathrm{D}}{\mathrm{a.u.}}\right)^{10} \cdot \left(\frac{\mathrm{M}_\odot}{\mathrm{M}}\right)^{5} \cdot \left(\frac{\mathrm{M}_{\mathrm{Earth}}}{\mathrm{m}}\right)^{2} \cdot \left(\frac{\mathrm{R}_{\mathrm{Earth}}}{\mathrm{d}}\right)^{4}.$$

- The effect is weak, but universal.

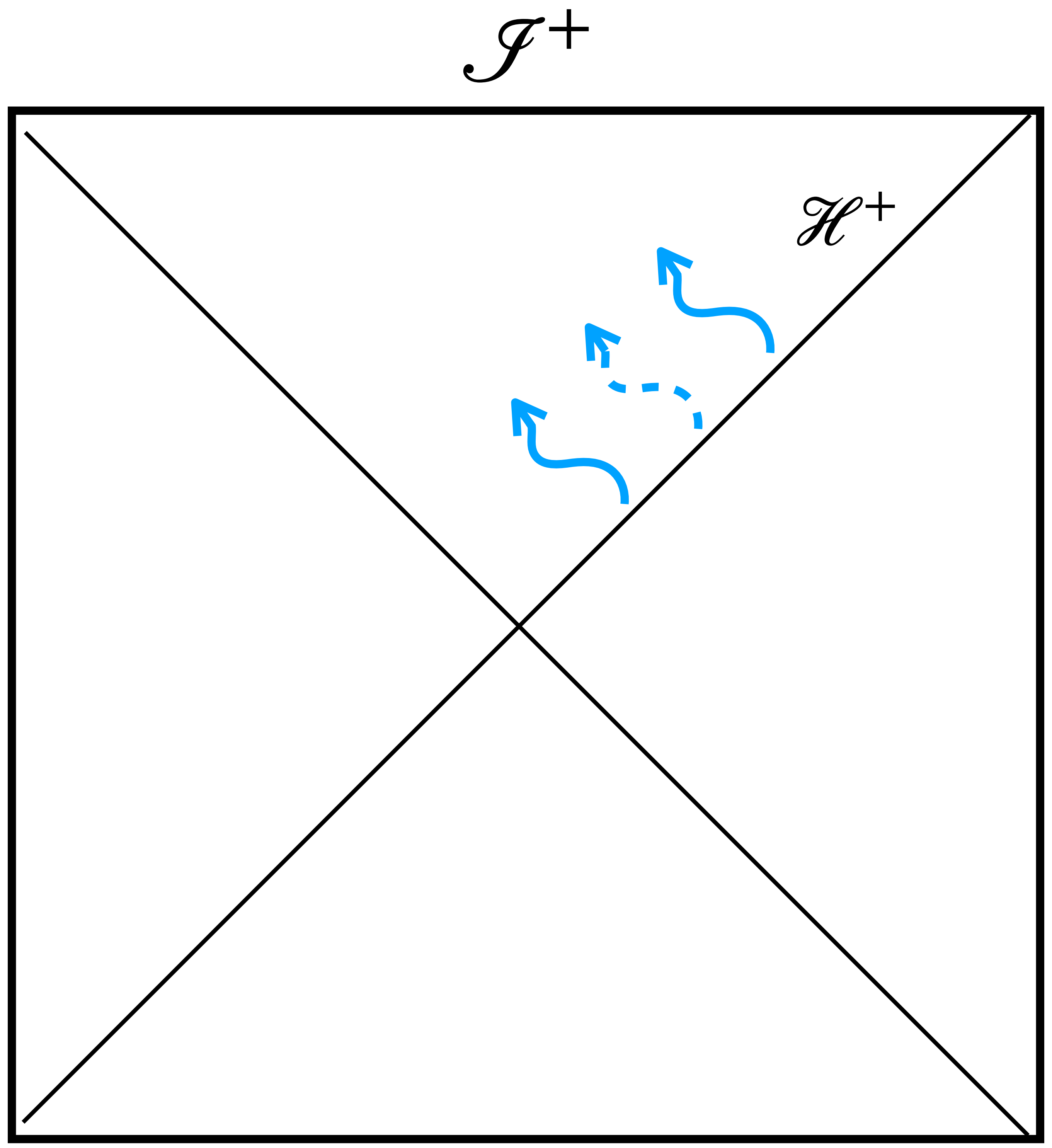
Local Description: Stimulated Emission

- Wilson-Gerow, Dugad, and Chen ('24, arXiv:2405.00804), DSW ('24, arXiv:2407.02567):
  - At low frequencies, the vacuum in Rindler spacetime or outside a black hole is populated by a low-energy population of modes down to zero frequency. This gives rise to *stimulated emission* of soft radiation from Alice's superposition into the horizon!
- Outside a black hole, the vacuum exhibits multipole fluctuations that whose spectrum approaches a *constant* at low frequencies. I.e.,
  - $\Delta|\vec{P}_{\text{EM}}|(\omega) \sim \frac{\sqrt{\epsilon_0 \hbar} G^{3/2} M^{3/2}}{c^3} \sim 10 \frac{\text{e} \cdot \text{m}}{\sqrt{\text{Hz}}} \left(\frac{M}{M_\odot}\right)^{3/2},$
  - $\Delta|Q_{\text{GR}}|(\omega) \sim \frac{\sqrt{\hbar} G^2 M^{5/2}}{c^5} \sim 10^{-1} \frac{\text{g} \cdot \text{m}^2}{\sqrt{\text{Hz}}} \left(\frac{M}{M_\odot}\right)^{5/2}$

Generalizations

- Wei and Gralla (arXiv:2311.11461): the effect also arises in the presence of a rotating black hole, now also depending on the angular momentum.
    - Also see for discussion of extremal black holes, involving *black hole Meisner effect*.
- Cosmological horizons…

Cosmological Horizons



- In de Sitter spacetime with a horizon radius R_H, the electromagnetic decoherence time is $T_D^{\text{EM}} \sim \dfrac{\hbar \epsilon_0 R_H^3}{q^2 d^2}$.
- The quantum gravitational decoherence time is $T_D^{\text{GR}} \sim \dfrac{\hbar R_H^5}{G m^2 d^4}$.
- Since $d \ll R_H$, the decoherence time will be much larger than the Hubble time R_H/c unless q is extremely large relative to the Planck charge $q_P \equiv \sqrt{\epsilon_0 \hbar c} \sim 11e$, or m much larger than the Planck mass $m \gg m_P \sim 10\mu\text{g}$.
- Nevertheless, we see that decoherence does occur despite the fact that Alice's lab is *inertial* in this case.

A black hole, or cosmological horizon,
will eventually decohere any quantum superposition.

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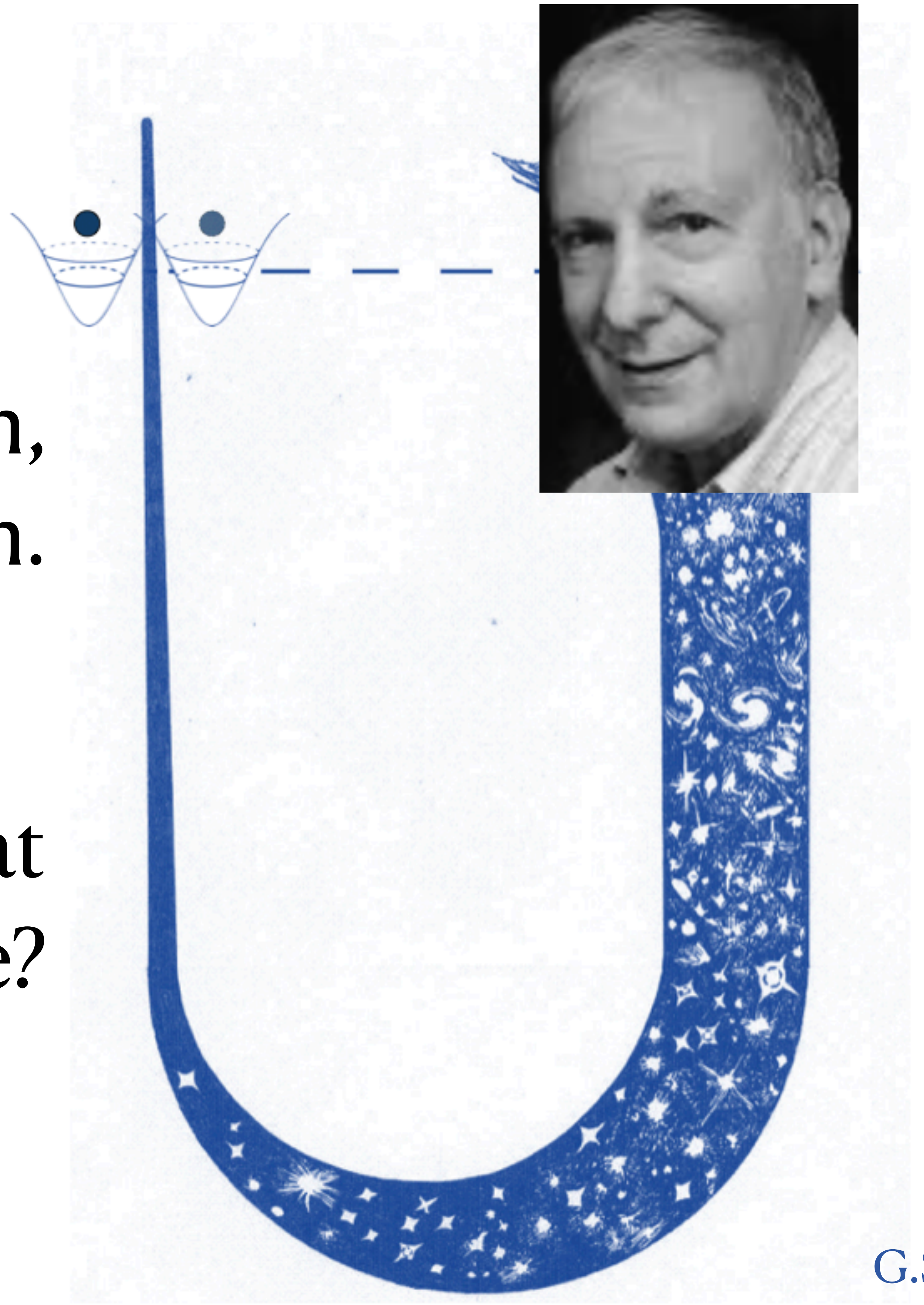
A black hole, or cosmological horizon, will eventually decohere any quantum superposition.

But what about the “Bobs” in the interior that motivated this whole adventure?



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Excursion into Quantum Information Theory

- We can think of the black hole as a quantum channel $\mathcal{N}$, acting on an *arbitrary* quantum experiment carried out by Alice.

> Decoherence of Alice's system due to the black hole is given by
> $$D(\mathcal{N}) := 1 - \frac{1}{1 - 1/\mathrm{Tr}\mathbf{1}}\left(\sup_{\mathscr{D}} F(\mathscr{D} \circ \mathcal{N}, \mathrm{Id})) - 1/\mathrm{Tr}\mathbf{1}\right)$$

- F is the "channel fidelity," a measure on distinguishability of channels. The above can be explicitly calculated in Tomita-Takesaki theory, when suitably generalized to include soft modes (to appear).

- $\mathscr{D}$ is an optimal recovery channel which Alice applies after her experiment, attempting to recover some coherence.

Plain English: $D(\mathcal{N})$ is the decoherence of Alice's degrees of freedom due to the black hole.
In the previous examples, this reduces to the familiar $D(\mathcal{N}) = 1 - |\langle\Psi_1|\Psi_2\rangle|$.

[DLD, Kudler-Flam, Satishchandran (to appear)]

How well can Bob really do?

- The “which path information available to Bob” can be generalized.

  Suppose we fill the black hole interior with any number of degrees of freedom, arranged so as to perform an optimal experiment to distinguish Alice’s field states.

How well can Bob really do?

- **The "which path information available to Bob" can be generalized. Suppose we fill the black hole interior with any number of degrees of freedom, arranged so as to perform an optimal experiment to distinguish Alice's field states.**

- The effect of Alice's experiment on the interior is captured by the "complementary channel" $\mathcal{N}^c$.

The distinguishability of the resulting interior states is determined by the **information content** of the complementary channel:

$$\mathcal{I}(\mathcal{N}^c) := 1 - \frac{1}{1 - 1/\mathrm{Tr}\mathbf{1}}\left(\mathcal{F}(\mathcal{N}^c) - 1/\mathrm{Tr}\mathbf{1}\right)$$

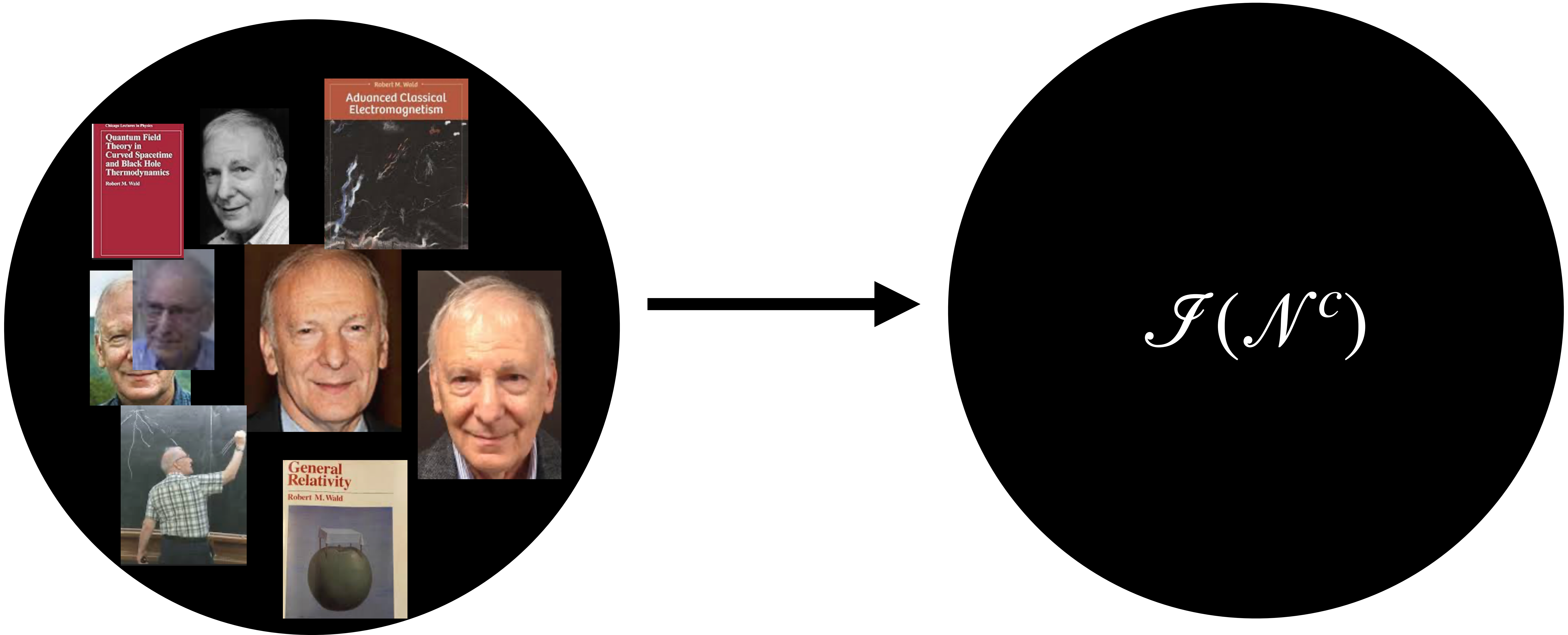
  - $\mathcal{F}(\mathcal{N}^c) = F(\mathcal{N}^c, R)$ where R is a channel that throws away all information about what Alice did.

- **Plain English: $\mathcal{I}(\mathcal{N}^c) \iff$ what is the probability of correctly determining the correct internal state given an optimal measurement?**

- The distinguishability of the resulting interior states is determined by the "channel information"

$$\mathscr{I}(\mathscr{N}^c) := 1 - \frac{1}{1 - 1/\mathrm{Tr}\mathbf{1}} \left(\mathscr{F}(\mathscr{N}^c) - 1/\mathrm{Tr}\mathbf{1} \right)$$

- This is equivalent to the distinguishability of the black hole interior states themselves. I.e., from an exterior perspective, the "degrees of freedom" may as well be the black hole itself, without mention of Bob...

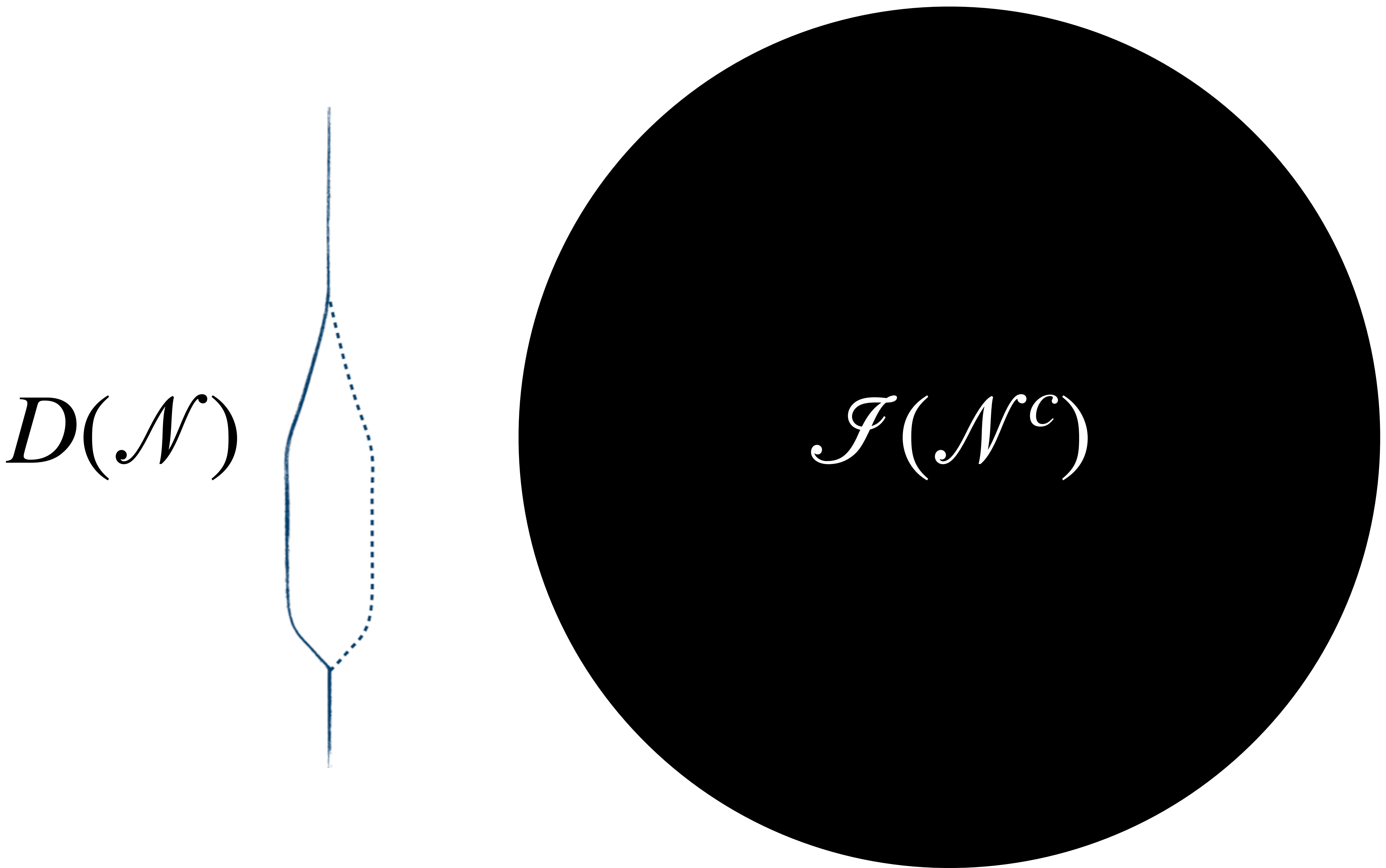


Theorem: $D(\mathcal{N}) = \mathcal{I}(\mathcal{N}^c)$

[DLD, Kudler-Flam, Satishchandran (to appear)]

- **The decoherence $D(\mathcal{N})$ of Alice experiment is equal to the information about Alice's experiment $\mathcal{I}(\mathcal{N}^c)$ available to Bob and his assistants in the interior, performing optimal experiments.**

- Equivalently, if we pretend the black hole is just some "quantum degrees of freedom," then the decoherence of Alice is precisely equal to the distinguishability of the resulting states on hypothetical "quantum degrees of freedom" of the black hole itself.



“Degrees of freedom…” Possible analogs?

- DSW (2024, arXiv:2407.02567): No such effect for…
    - Thermal state in spacetime of a star.
    - Global thermal state of Minkowski (Also see Wilson-Gerow, Dugad, and Chen [arXiv:2405.00804])
- **What if we try to develop a matter model with analogous “internal degrees of freedom”?**
    - Biggs and Maldacena (2024, arXiv:2405.02227):
        - EM case: it is relatively easy to construct a realistic matter model that induces similar decoherence on electromagnetically charged bodies in its exterior.
        - **GR case: the mimicking of black hole decoherence effects by an ordinary body *of the same physical size* as the black hole appears to require *extraordinary* properties of the matter!**

Summary and Conclusions

- In the vicinity of a Killing horizon, any quantum superposition generates long-range fields that register on the horizon.
- This necessitates the emission of soft, entangling gravitons across the horizon, by which the horizon harvests quantum information about the superposition. **Because all objects source gravity, this is a universal effect.**
- As seen from inside the black hole, this decoherence can be understood as being due to "optimal" experiments / degrees of freedom behind the horizon.

Questions?

$$T^{\rm BH}_{\rm Deco.} \sim \frac{\hbar c^{10} D^{10}}{G^6 M^5 m^2 d^4}$$

$$T^{\rm dS}_{\rm Deco.} \sim \frac{\hbar R_H^5}{G m^2 d^4}$$

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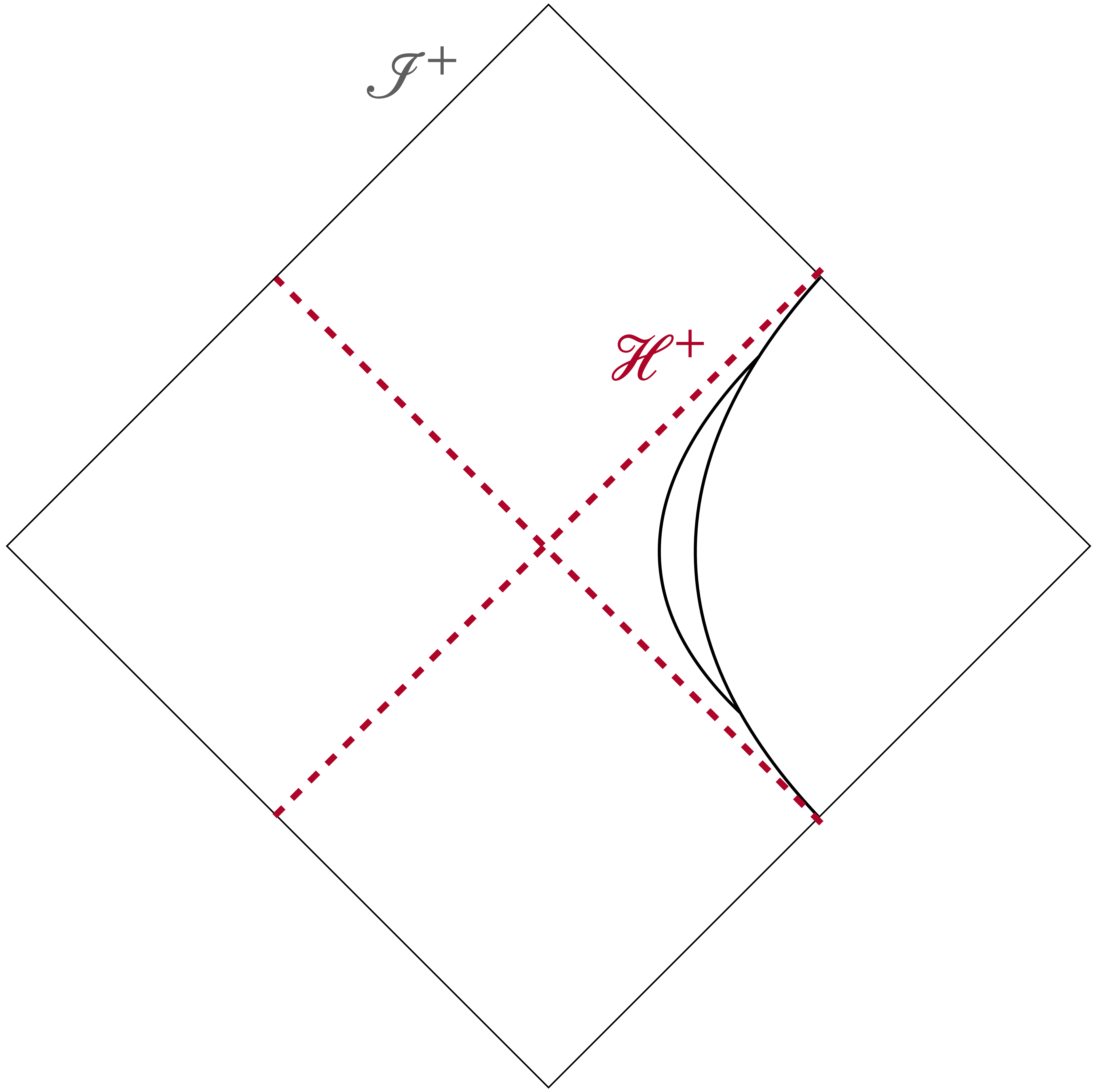
What about thermal radiation?

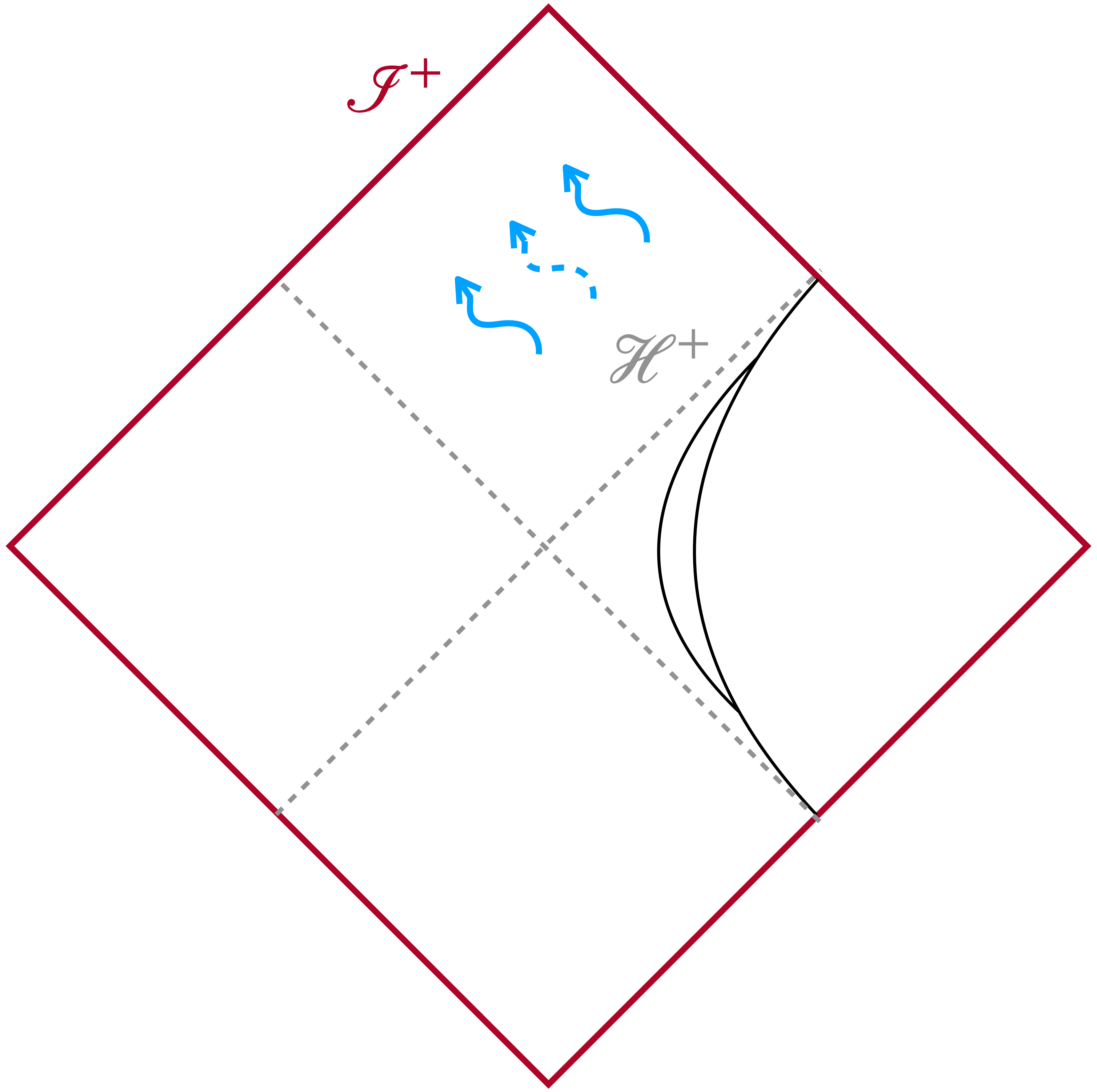
- Evidently it is not acceleration which causes the effect. But in every case we've considered, there is a *thermal bath* present in Alice's laboratory (Hawking radiation / Unruh radiation / Bunch-Davies quanta). Could this be the cause of the effect?
- Perhaps surprisingly, no! The effect of decoherence due to collisions with the appropriate thermal quanta is totally independent of our effect. For instance, the Unruh bath gives a thermal decoherence time of $T^{\mathrm{EM}}_{\mathrm{therm.}} = \dfrac{m^2}{q^4 d^2 a^5}$. Contrast with our effect, which goes as $T^{\mathrm{EM}}_{\mathrm{D}} \sim 1/(q^2 d^2 a^3)$.
- This collisional decoherence is negligible relative to our effect so long as the superpositions is smaller than the thermal wavelength $d \ll \lambda_T$.

Analog in flat spacetime: Rindler horizons



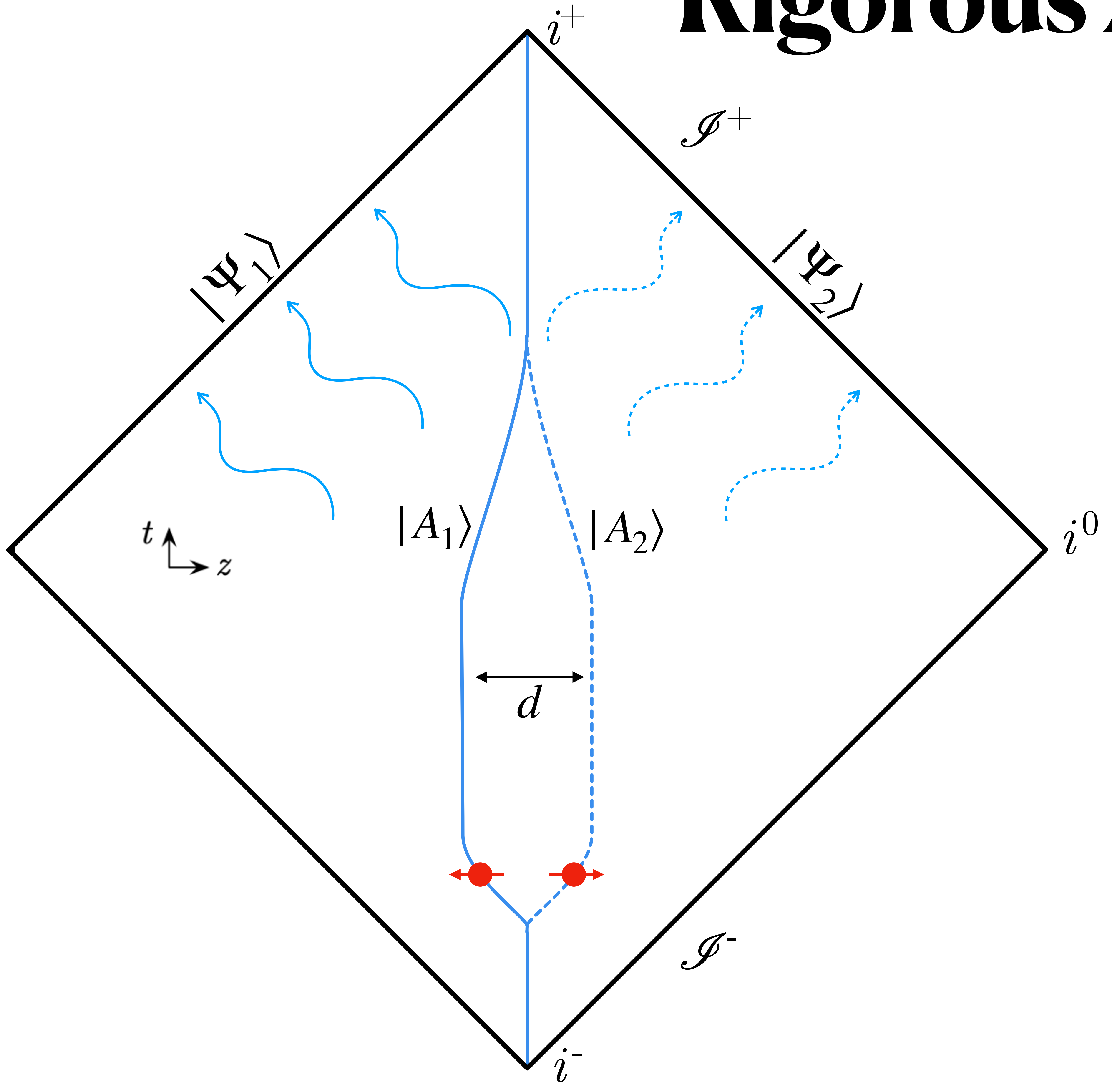
- Let's consider an analogous situation in flat spacetime, to build intuition. Here we can understand the effect from two distinct points of view: an accelerating frame, and the internal one.
- Suppose Alice performs her experiment in an accelerating laboratory. Then, a similar analysis applies on the Rindler horizon of her laboratory.

Analog in flat spacetime: Rindler horizons



- On what would an *inertial* observer blame Alice's decoherence?
- $\mathscr{D} = 1 - \langle \Psi_1 | \Psi_2 \rangle_{\mathscr{I}^+}$, using the Liénard-Wiechert solutions for the respective superposed trajectories of Alice's charged particle.
- Again we obtain $\langle N \rangle_{\mathscr{I}^+} \sim q^2 d^2 a^3 T \sim \langle N \rangle_{\mathscr{H}^+}$.
  - This had better be the case, since these are freely propagating photons, so their number is conserved.
- Previously, "soft" photons with Fourier spectrum $\omega_{\text{Rind.}} \sim ae^{-aT}$ on the horizon.
- Become "hard" photons with $\omega_{\text{inert.}} \sim ae^{aT}$ relative to inertial time.
- Due to the large blueshift from Rindler time to inertial time, an arbitrarily "soft" process in the Rindler frame can source "hard" radiation beyond the horizon.

Rigorous Analysis of Alice

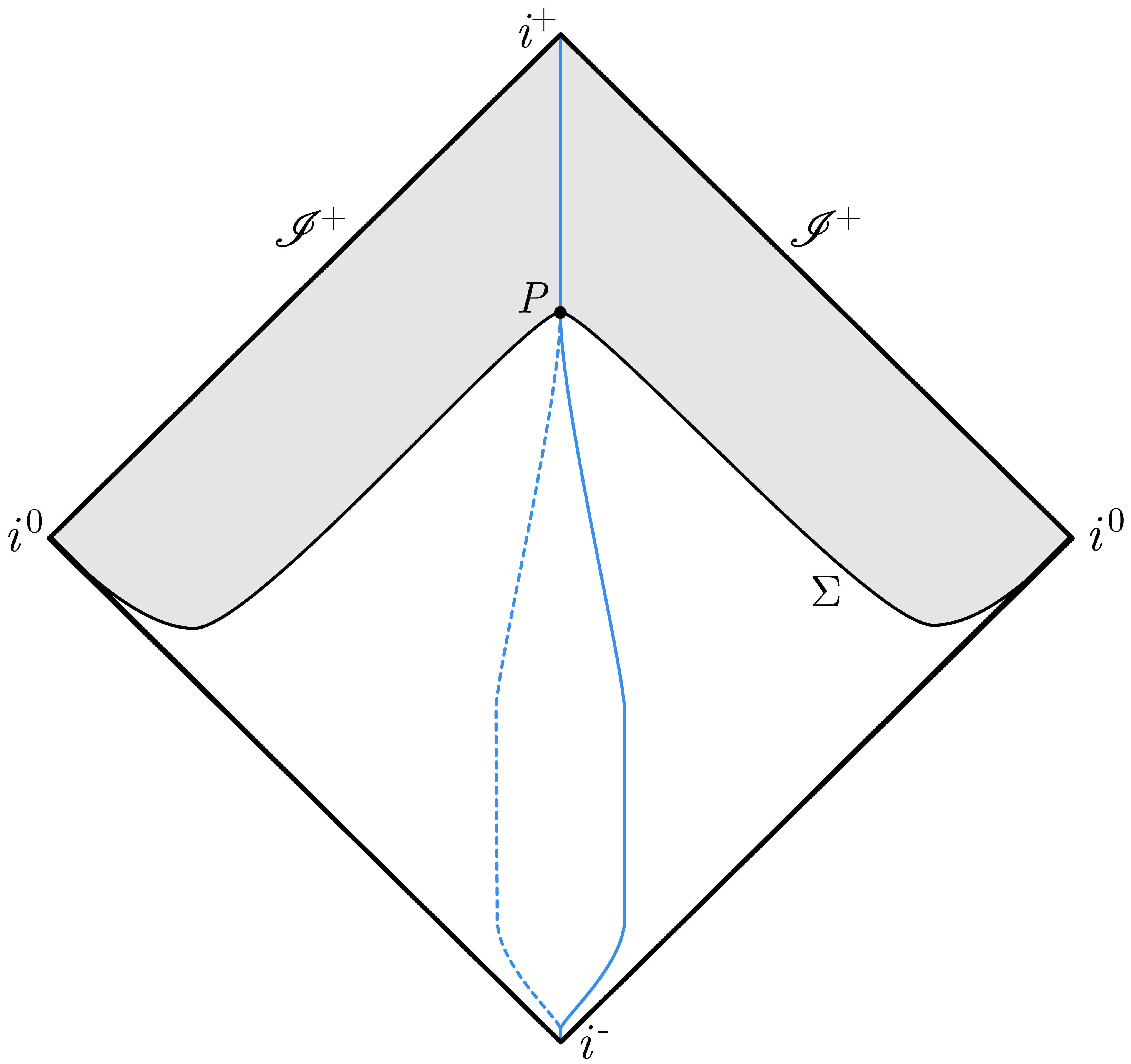


The decoherence due to Alice's radiation in the infinite time limit is,

$$\mathscr{D}_{\text{Alice}} = 1 - |\langle\Psi_1|\Psi_2\rangle_{\mathscr{I}^+}|.$$

How can we pull this expression back to a finite time surface, Σ?

Decoherence due to Alice's radiation



Alice interferes her particle at P. The monopole field after P contains no which-path information, so we can subtract it away.

Unitary free-field evolution from Σ to $\mathscr{I}^+$ preserves the Fock inner product:

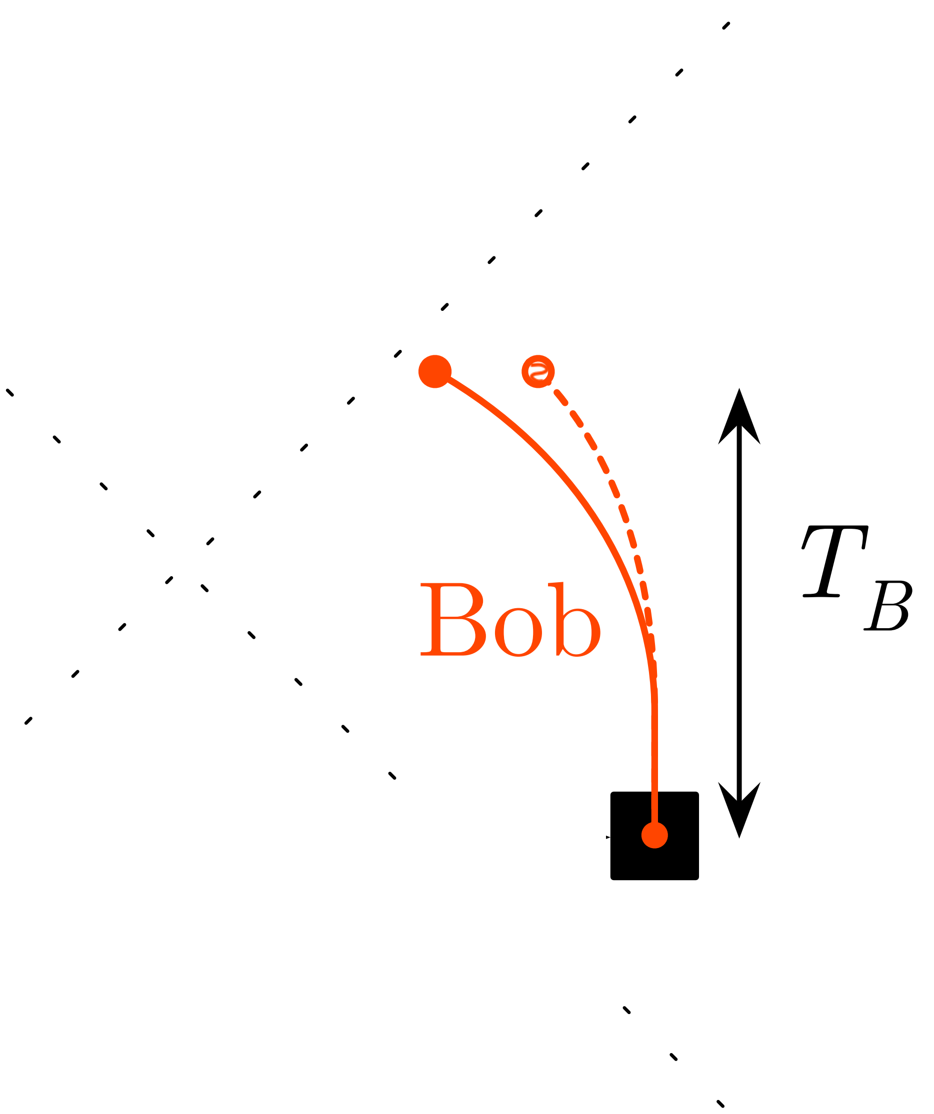
$$1 - |\langle \Psi_1 | \Psi_2 \rangle_{\mathscr{I}^+}| = \mathscr{D}_{\text{Alice}} = 1 - |\langle \Psi_1 | \Psi_2 \rangle_{\Sigma}|$$

Decoherence due to Bob's experiment

At any time after Alice has produced her superposition but prior to the beginning of Bob's experiment, we have the formal state,

$$\frac{1}{\sqrt{2}}\left(|\uparrow, A_1\rangle \otimes |\psi_1\rangle + |\downarrow, A_2\rangle \otimes |\psi_2\rangle\right) \otimes |B_0\rangle$$

Decoherence due to Bob



The final state of the matter degrees of freedom, in the infinite time limit, is $\frac{1}{\sqrt{2}}\left(|\uparrow, A_1\rangle_{i^+} \otimes |B_1\rangle_{i^+} + |\downarrow, A_2\rangle_{i^+} \otimes |B_2\rangle_{i^+}\right)$. The corresponding decoherence of Alice's particle due to Bob's experiment is,

$\mathscr{D}_{\text{Bob}} = 1 - |\langle B_1 | B_2\rangle_{i^+}|$. However, since Bob's apparatus decouples from the electromagnetic field after his experiment,

$1 - |\langle B_1 | B_2\rangle_{i^+}| = \mathscr{D}_{\text{Bob}} = 1 - |\langle B_1 | B_2\rangle_{T_B}|$.

The decoherence due to Bob's experiment at infinite time, i^+, is equal to the decoherence due to Bob at the time T_B, when he completes his experiment.

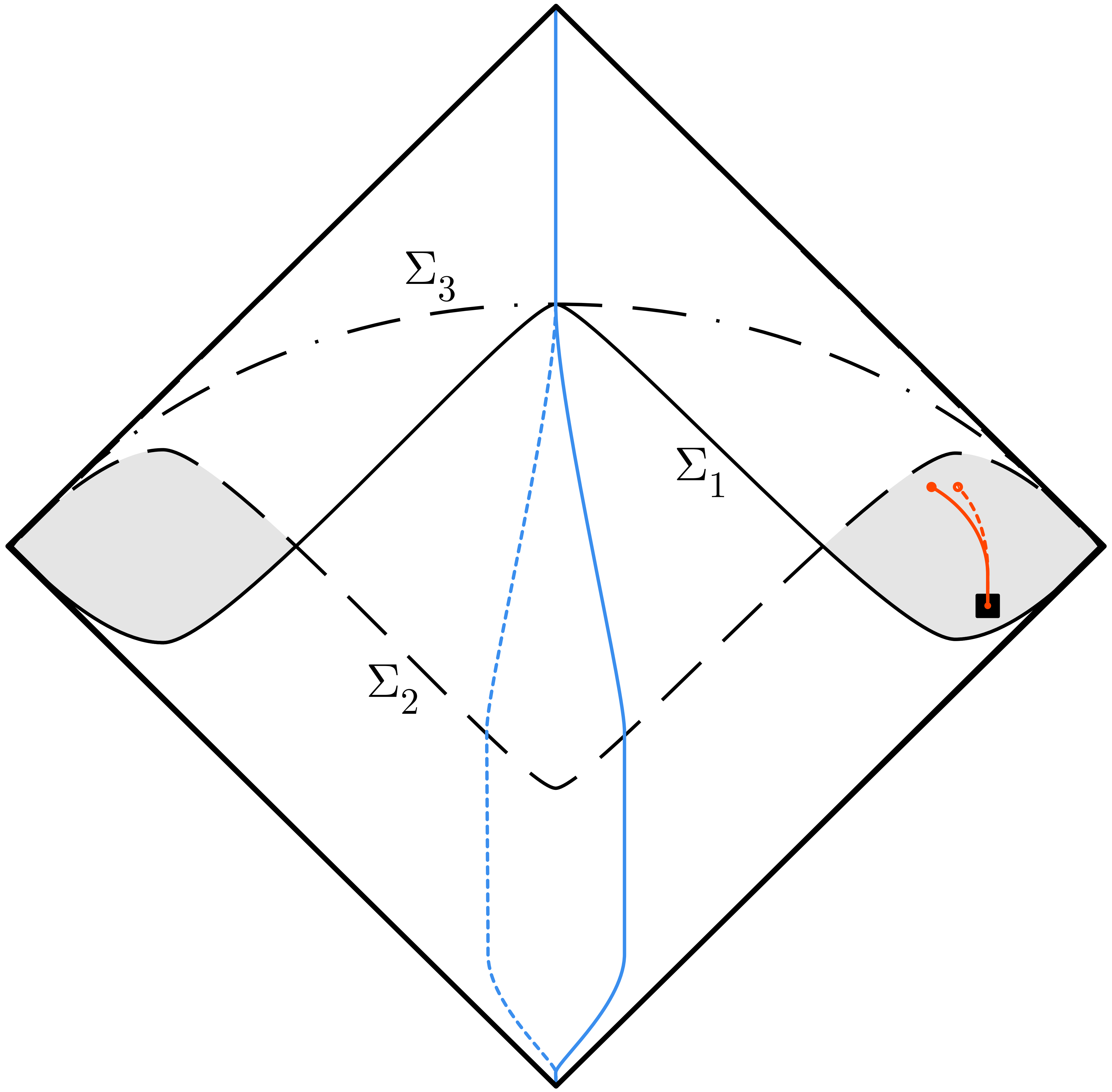
Reanalysis of the paradox

We will drop the T_B subscript, and denote $\mathcal{D}_{\text{Bob}} = 1 - |\langle B_1 | B_2 \rangle|$ the total decoherence associated with Bob's experiment, after that experiment has been completed.

It follows that there would be a paradox if

$|\langle B_1 | B_2 \rangle| < |\langle \Psi_1 | \Psi_2 \rangle_\Sigma|$. Then, *Bob could decohere Alice by an amount greater than she would be decohered by her own radiation.*

Bob's experiment



Now, allow Bob to perform any arbitrary experiment in the shaded region. Subtracting off the common Coulomb field, the state on Σ_1 is

$$\frac{1}{\sqrt{2}}\Big(|\uparrow, A_1\rangle \otimes |\Psi_1\rangle_{\Sigma_1} |\downarrow, A_2\rangle \otimes |\Psi_2\rangle_{\Sigma_1} \Big) \otimes |B_0\rangle$$

Time evolution gives the state on Σ_3. E.g,

$$\frac{1}{\sqrt{2}}\Big(|\uparrow, A_1\rangle \otimes |\Psi'_1\rangle_{\Sigma_3} \otimes |B_1\rangle + |\downarrow, A_2\rangle \otimes |\Psi'_2\rangle_{\Sigma_3} \otimes |B_2\rangle \Big)$$

By unitarity,

$$\langle \Psi'_1 | \Psi'_2 \rangle_{\Sigma_3} \langle B_1 | B_2 \rangle = \langle \Psi_1 | \Psi_2 \rangle_{\Sigma_1} \langle B_0 | B_0 \rangle = \langle \Psi_1 | \Psi_2 \rangle_{\Sigma_1}$$

We've now shown that $\langle \Psi_1' | \Psi_2' \rangle_{\Sigma_3} \langle B_1 | B_2 \rangle = \langle \Psi_1 | \Psi_2 \rangle_{\Sigma_1}$.

The decoherence associated with Bob's experiment is bounded by the decoherence Alice inflicts on herself by radiation:

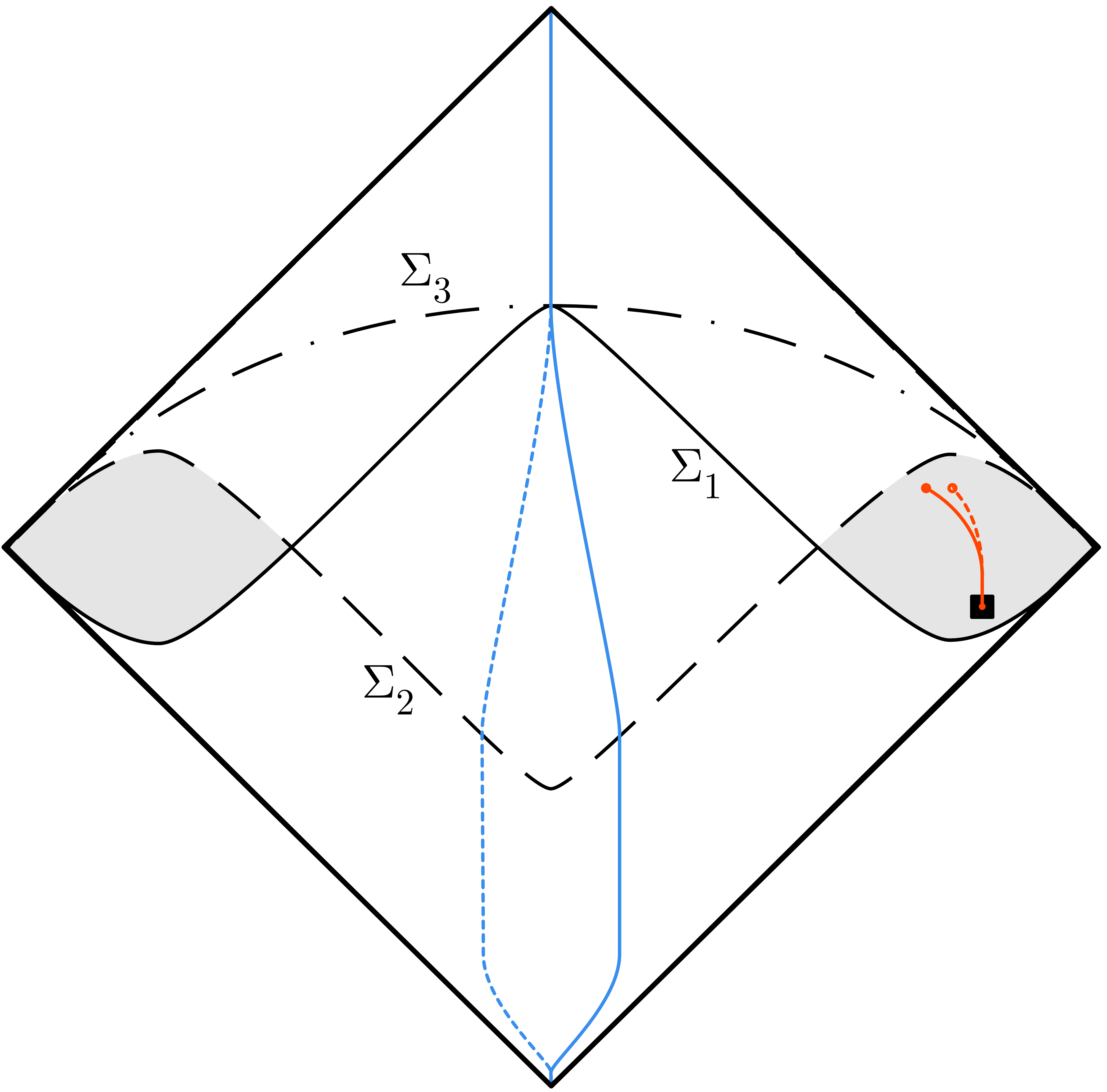
$$1 - |\langle \Psi_1 | \Psi_2 \rangle_{\Sigma_1}| \geq 1 - |\langle B_1 | B_2 \rangle|$$

$$\mathscr{D}_A \geq \mathscr{D}_B$$

Thus Bob can never decohere Alice by an amount greater than she would decohere herself in his absence.

The field states have distinct expected electric fields; the fact that they are not orthogonal is the result of *vacuum fluctuations* of the field.

General Resolution of the Paradox (in flat spacetime)



$$\langle \Psi_1' | \Psi_2' \rangle_{\Sigma_3} \langle B_1 | B_2 \rangle = \langle \Psi_1 | \Psi_2 \rangle_{\Sigma_1}$$

Indeed, the decoherence Alice observes if Bob performs a experiment is always *precisely equal* to the decoherence attributable to her own radiation, before Bob has even begun to measure!

Therefore, no paradox can ever arise.