Gravitationally Mediated Entanglement: Newtonian Fields, Gravitons and Black Holes

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Princeton University

ICTP: Witnessing Quantum Aspects of Gravity in a Lab

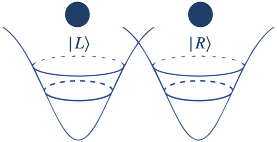
D. Danielson, G.S., & R.M. Wald Phys. Rev. D 105, 086001 (2022) [arXiv:2112.10798]

D. Danielson, G.S. & R.M. Wald [arXiv:2205.06279],[arXiv:2301.00026],[2407.02567]

see also: A. Belenchia, R. M. Wald, F. Giacomini, E. Castro-Ruiz, C. Brukner & M. Aspelmeyer [arXiv:1807.070105]
J. Wilson-Gerow, A. Dugad & Y. Chen [arXiv:2405.00804]
and A. Biggs & J. Maldacena [arXiv:2405.02227]

September 25, 2024

(Gedanken)experiments and Quantum Gravity



[Carney et al. 2019],[Bose et al. 2017], [Marletto et al. 2017], ...

"One should think about designing a gedankenexperiment which uses a gravitational link and at the same time shows quantum interference."

[Feynman, 1957 Chapel Hill Conference]

- To probe properties of quantum gravity, it's useful to consider situations where both quantum theory and gravity play an essential role.
- What aspects of quantum gravity can we learn from such (gedanken)experiments?

A Gedankenexperiment

- Consider an experimentalist Alice who controls a charged particle with spin. At some early time, she passes this particle through a Stern-Gerlach apparatus thus creating a quantum spatial superposition

$$\frac{1}{\sqrt{2}}(|A_1; \uparrow\rangle + |A_2; \downarrow\rangle)$$

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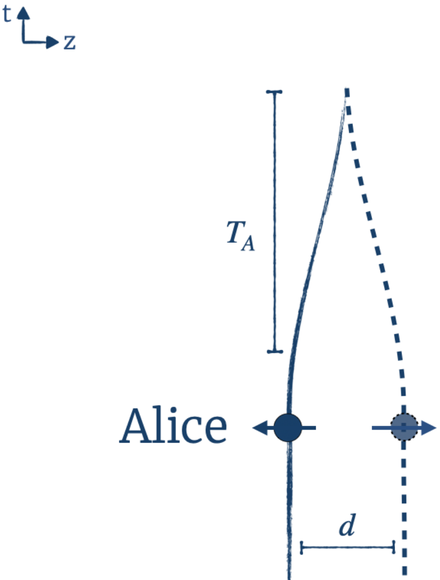

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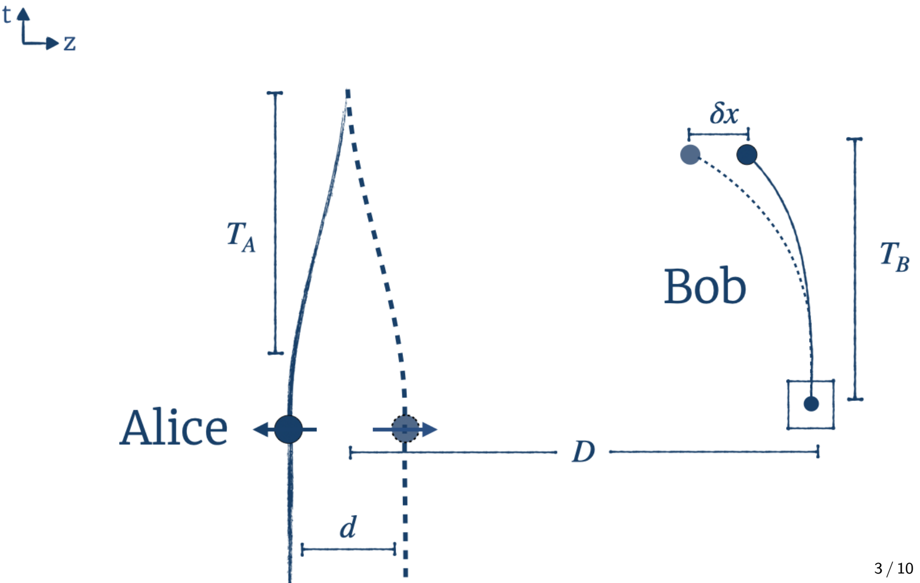

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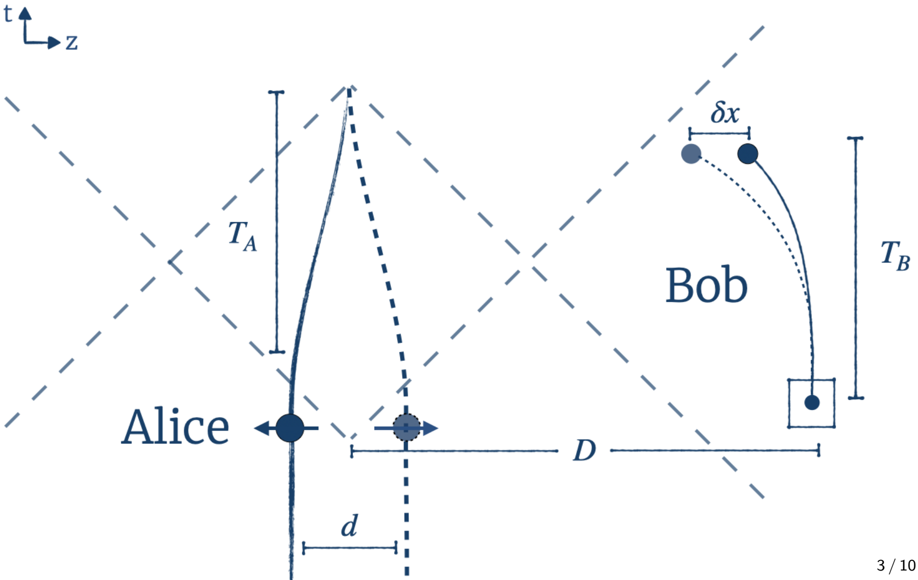
- Meanwhile Bob wants to destroy Alice's coherence. He releases a particle from a trap and attempts to entangle with Alice's particle via the superposed Coulomb field of Alice's particle.
- Bob performs this experiment at spacelike separation from Alice's recombination.


t
z
T_A
δx
T_B
Bob
Alice
D
d

Back-of-the-Envelope Analysis

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$$\mathscr{D} = 1 - |\langle\Psi_1|\Psi_2\rangle| = 1 - e^{-\frac{1}{2}\langle N\rangle_{\Psi_1 - \Psi_2}} \text{ where } \langle N\rangle_{\Psi_1-\Psi_2} \sim (q_A d/T_A)^2$$

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- Both inequalities cannot be satisfied for $T_A, T_B < D$. If Alice can maintain coherence, then Bob does not have enough time to get "which path" information. If Bob has enough time, then Alice decoheres herself be entangling radiation.

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  which cannot be simultaneously satisfied if $T_{\mathrm{A}}, T_{\mathrm{B}} < D$.
- The analysis and conclusions parallel the EM case with "dipole" $\leftrightarrow$ "quadrupole". Quantized radiation and vacuum fluctuations of the gravitational field are essential to avoid contradictions with causality and complementarity

Deficiencies of the Analysis

- While the above analysis indicate the properties of the gravitational field necessary in order to have a consistent description of a quantum superposition, these arguments were at only the back-of-the-envelope level. It isn't clear that Bob cannot get *any* "which-path" information.

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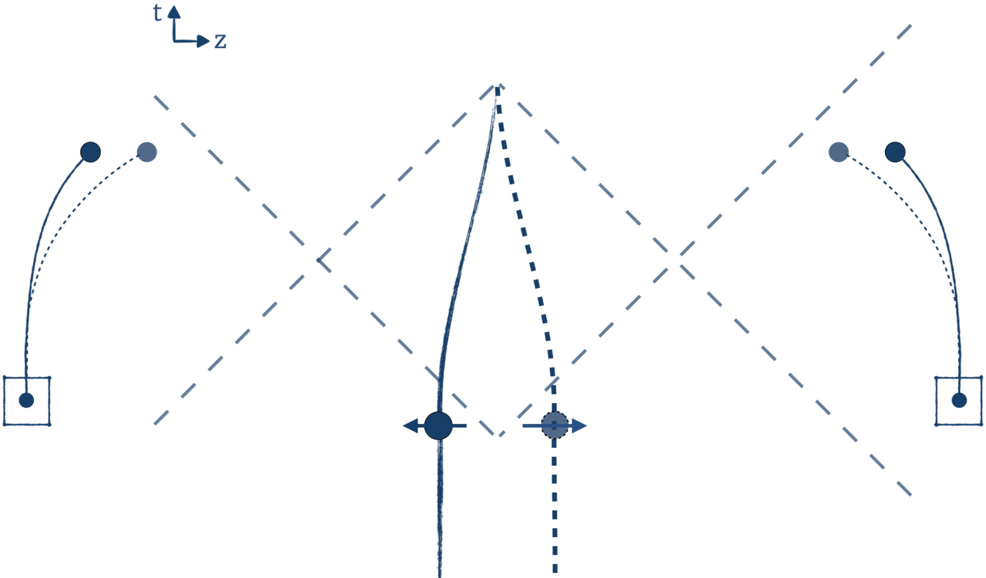

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To prove this we will obtain a precise relationship between causality and decoherence

Decoherence due to Alice

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- While the components of her particle are separated there is no unambiguous distinction between the Coulomb field of her particle and its radiation field [Unruh, '15].
- However, after the recombination, the components of her particle now share a *common* Coulomb field. On any Cauchy surface Σ_1 after the recombination, the total quantum state with the Coulomb field subtracted is

$$\frac{1}{\sqrt{2}}(|A_1;\uparrow\rangle \otimes |\Psi_1\rangle_{\Sigma_1} + |A_2;\downarrow\rangle \otimes |\Psi_2\rangle_{\Sigma_1})$$

where $|\Psi_1\rangle$ and $|\Psi_2\rangle$ are genuine radiation states which contain *all* of the "which path" information on Σ_1.

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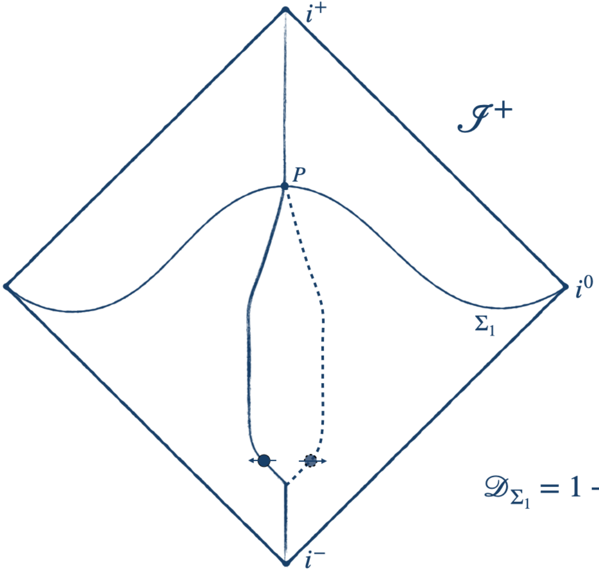

$$\mathscr{D}_{\Sigma_1} = 1 - |\langle \Psi_1 | \Psi_2 \rangle|$$

Decoherence due to Bob

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- The decoherence at any time Σ_2 where Bob's apparatus is interacting with the Coulomb/Newtonian field of Alice' particle

$$\mathscr{D}_{\Sigma_2} = 1 - |\langle B_1|B_2\rangle|_{\Sigma_2}$$

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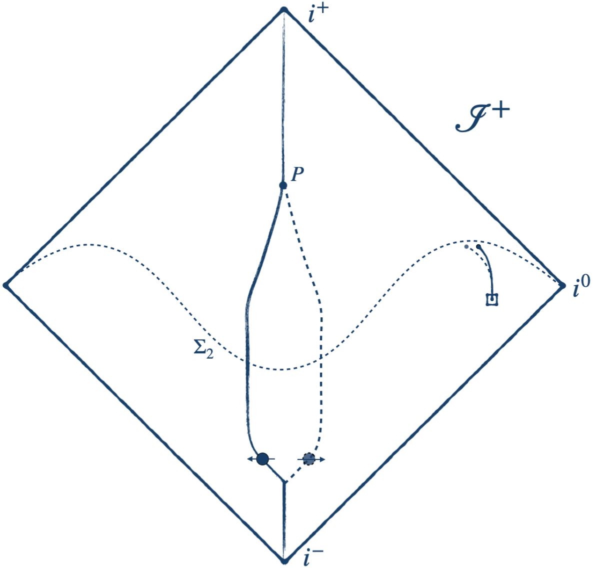

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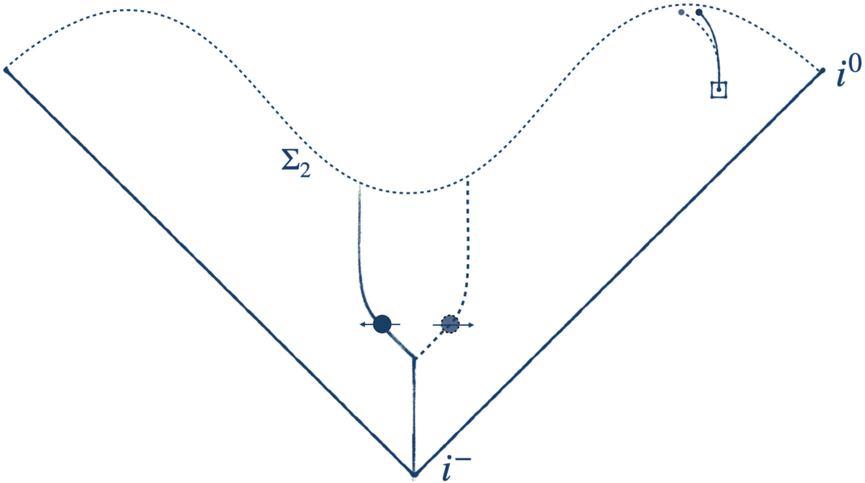

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Viewpoints (1) and (2) are equivalent!

Causality Bounds Quantum Coherence

- Consider the joint evolution of the Alice + Bob + radiation quantum state from Σ_1

$$\frac{1}{\sqrt{2}}\left(|A_1;\uparrow\rangle \otimes |\Psi_1\rangle_{\Sigma_1} + |A_2;\uparrow\rangle \otimes |\Psi_2\rangle_{\Sigma_1}\right) \otimes |B_0\rangle$$

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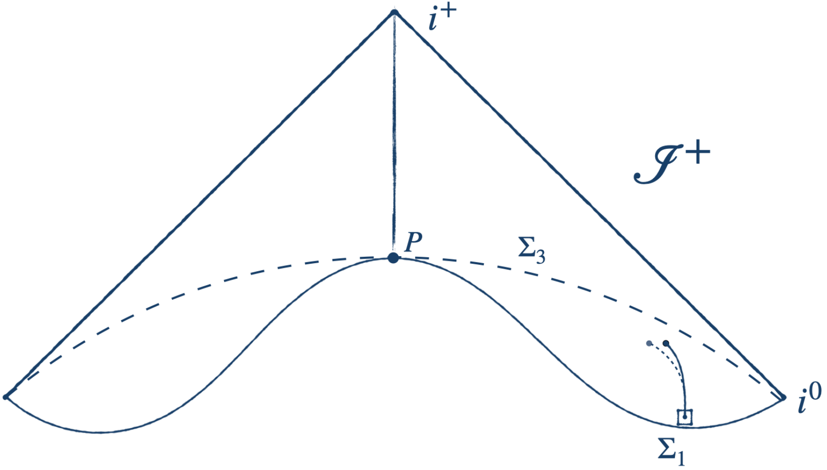

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- Alice's particle does not change under this evolution and Bob's apparatus simply becomes entangled with the radiation states emitted by Alice

$$\frac{1}{\sqrt{2}}(|A_1;\uparrow\rangle \otimes |\Psi_1'\rangle_{\Sigma_3} \otimes |B_1\rangle + |A_2;\uparrow\rangle \otimes |\Psi_2'\rangle_{\Sigma_3} \otimes |B_2\rangle)$$

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$$\frac{1}{\sqrt{2}}(|A_1;\uparrow\rangle \otimes |\Psi'_1\rangle_{\Sigma_3} \otimes |B_1\rangle + |A_2;\uparrow\rangle \otimes |\Psi'_2\rangle_{\Sigma_3} \otimes |B_2\rangle)$$

- Unitarity of the evolution implies

$$\langle\Psi'_1|\Psi'_2\rangle_{\Sigma_3}\langle B_1|B_2\rangle = \langle\Psi_1|\Psi_2\rangle_{\Sigma_1} \implies |\langle B_1|B_2\rangle| \geq |\langle\Psi_1|\Psi_2\rangle|_{\Sigma_1} \implies \mathscr{D}_{\text{Alice}} \geq \mathscr{D}_{\text{Bob}}$$

Summary and Conclusions

- If both Alice and Bob follow their protocols, then *both* (1) Newtonian-mediated entanglement and (2) on-shell graviton entanglement are *equivalent* veiwpoints
- It is *essential* that both (1) and (2) — or, alternatively, neither (1) nor (2) — are valid descriptions of the entanglement in order to provide a consistent description of a quantum spatial superposition and avoid contradictions with causality and complementarity.
- Additionally, we have shown that $\mathscr{D}_{\rm Alice} \geq \mathscr{D}_{\rm Bob}$, generalizing the analysis of [Belenchia et al, 2018]. However, read in the other direction, this implies that any quantum spatial superposition must be at least as decohered as any Bob(s) at spacelike separation.

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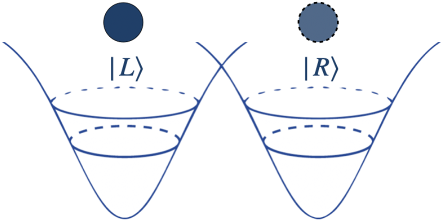

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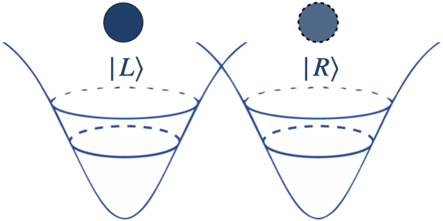

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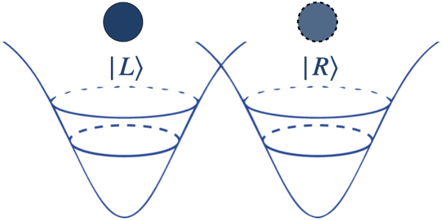

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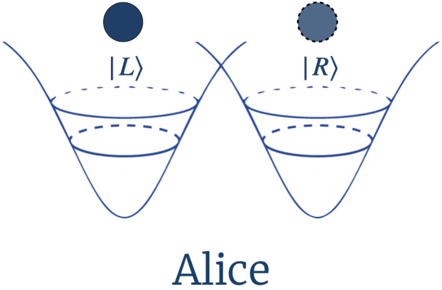

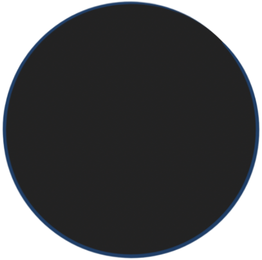

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- Additionally, we have shown that $\mathscr{D}_{\text{Alice}} \geq \mathscr{D}_{\text{Bob}}$ — generalizing the analysis of [Belenchia et al, 2018]. However, read in the other direction, this implies that any quantum spatial superposition must be at least as decohered as any Bob(s) at spacelike separation.

- Any physical body would decohere the superposition by gravitationally interacting with its internal degrees of freedom. As Daine will explain tomorrow, a black hole decoheres quantum superpositions *as if* it contains internal degrees of freedom.