QUANTUM GRAVITY LECTURE

P.C.E. STAMP

Sao Paulo ICTP, Sep 23-27, 2024

Physics & Astronomy
UBC
Vancouver

Pacific Institute
for
Theoretical Physics

HIGH_ENERGY VIEW: PROBLEM #1

SOME NUMBERS

$$h/2\pi = \hbar = 1.0546\times10^{-34}\ \mathrm{kg\ m^2\ sec^{-1}}$$
$$G_N = 6.672\times10^{-11}\mathrm{m^3\ kg^{-1}\ sec^{-2}}$$
$$c = 2.99792458\times10^{8}\ \mathrm{m/sec}$$

Planck Length: $L_P = (hG/2\pi c^3)^{1/2} = 1.616 \times 10^{-35}$ m

Planck Mass: $M_P = (hc/2\pi G)^{1/2} = 2.18 \times 10^{-8}$ kg

Planck Energy: $E_P = (hc^5/2\pi G)^{1/2} = 1.96 \times 10^{9}$ J $= 1.22 \times 10^{19}$ GeV $= 1.42 \times 10^{32}$ K

A v small grain of sand (0.4 mm diameter) has mass M_P
A Planck energy would raise 5 tons of water from 0°C to 100°C

EXPERIMENTS: **at Planck scales, one needs a Planck energy $E_P = M_P c^2$ to be deposited in a volume $V_P = L_P^3$, ie., for a "Planck energy density" $\rho_P = E_P/L_P^3$**

Planck Energy Density: $\rho_P = 2\pi c^7/hG^2 = 4.68 \times 10^{113}$ J/m³ $= 2.61 \times 10^{123}$ GeV/m³

This energy density is 10^{45} higher than current LHC !!

So – PROBLEM #1 is that a high-energy theory is utterly beyond the reach of any conceivable earth-based experiment.

Only viable testing ground is close to ~ t_P ~ 5.39×10^{-44} s of Big Bang

HIGH-ENERGY VIEW: PROBLEM #2

Field	Force F	Quanta
Maxwell (EM)	$F_{12} = k \frac{Q_1 Q_2}{R^2}$	photons
Weak	$F_{12} = g \frac{T_{12}}{R^2} e^{-M_W R}$	W bosons
Strong	$F_{12} \rightarrow$ Const (for $R \rightarrow \infty$)	gluons
Gravitational	$F_{12} = G \frac{M_1 M_2}{R^2}$	gravitons

The 1st three forces can be treated by conventional quantum field theory. They are "renormalizable", and the short-distance behaviour is OK.

But at short distances & high energies, Gravity blows up, because

$E = Mc^2$ so $M = E/c^2 = h/c\lambda$

$\sim h/cR$ & force becomes $F_{12} \rightarrow G \frac{h^2}{c^2} \frac{1}{R^4} = \frac{8\pi}{\kappa} \frac{L_P^4}{R^4}$

So - PROBLEM #2: Gravity is "perturbatively non-renormalizable".

Thus some new high-energy framework is required. This is hard

> It is not well-appreciated how hard it is to build a consistent theoretical framework to unify general relativity and quantum mechanics. Currently, superstring theory is the only credible theory to have achieved the unification. H. Ooguri (2021)

PROBLEM #3: Low-E INCOMPATIBILITY of QM & GR

Consider a 2-slit experiment. Ignoring gravity we write

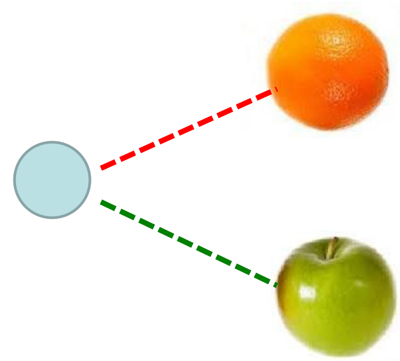
$$\Phi(\mathbf{r},t) \equiv \langle \mathbf{r}|\Phi(t)\rangle \;=\; a_1\Phi_1(\mathbf{r},t) + a_2\Phi(\mathbf{r},t)$$

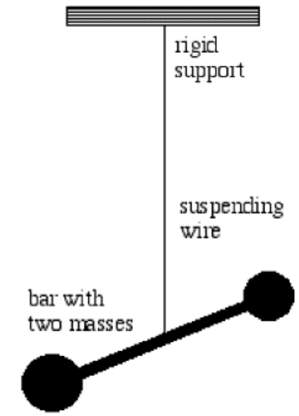
with interference term

$$\langle \Phi_1(t)|\Phi_2(t)\rangle \;=\; \int d^3r \;\langle \Phi_1^*(\mathbf{r},t)|\Phi_2(\mathbf{r},t)\rangle$$

However, with gravity included we must have

$$|\Psi\rangle \;=\; a_1|\Phi_1;\tilde{g}^{\mu\nu}_{(1)}(x)\rangle + a_1|\Phi_2;\tilde{g}^{\mu\nu}_{(2)}(x)\rangle$$





Several problems with this...

(i) There are 2 different coordinate systems, $(\mathbf{r}_j, t_j)$, defined by the 2 different metrics $\tilde{g}^{\mu\nu}_{(j)}(x)$ & in general we cannot relate these. The 2 metrics have different vacua.

(ii) All matter fields in QFT need the background spacetime to define causal relationships. Thus, eg., for a fermionic field we have

$$[\hat{\psi}(x),\, \hat{\psi}(x')] \equiv \hat{\psi}(x)\hat{\psi}(x') - \hat{\psi}(x')\hat{\psi}(x) = 0$$ **(spacelike separated – no causal relation)**

but non-zero for time-like separated intervals. If the metric field is quantized we then require $[\hat{g}_{ab}(x),\, \hat{g}_{cd}(x')] = 0$ **(for x-x' spacelike separated)**

But this eqtn. is meaningless: $s^2 = (x-x')^2$ is defined by $g_{\mu\nu}(x)$!! Then a quantum fluctuation in the metric can change matter field causal relations.

(iii) A "wave-function collapse" causes non-local changes; because the matter couples to the metric, this causes drastic unphysical changes in the metric.

So, PROBLEM #3: trying to superpose different spacetimes leads to apparently meaningless results. Causal relations in standard QFT require a specific background spacetime. This problem has nothing to do with high energies – it happens instead when we have "mass superpositions"

THE LOW-ENERGY ROAD

or

LOOKING for a FAILURE of QUANTUM MECHANICS

In this view, the key questions have nothing to do with the PLANCK SCALE.

WE'VE BEEN LOOKING IN THE WRONG PLACE…!!

WHY SO MANY PEOPLE HAVE PROBLEMS with QUANTUM MECHANICS

(i) The state of a quantum system is represented by a state vector $|\psi>$

But $|\psi>$ can't represent a real physical object - changes in $|\psi>$ happen non-locally (cf EPR paradox). But if $|\psi>$ only represents 'information', different observers can assign different $|\psi>$. We then lose all reference to the physical world.

(ii) In QM, $|\psi>$ "collapses" when a "measurement" is made.

We write $\langle M_j \rangle = Tr\{\hat{M}_j \hat{\rho}\}$ for a mmt. M_j; the projection operator is an EXTRANEOUS NON-QUANTUM AGENT. But measurements are physical operations, & are part of the world! This is a contradiction.

Either the whole world is quantum mechanical – in which case the foundations of QM are self-contradictory – or the "classical world" of measurements is different, and apparently revolves around external "classical" set-ups, whose defining properties are very ambiguous and seem to devolve on experimenters.

In the latter case QM is formulated completely <u>anthropocentrically</u> – this is a throwback to mediaeval times, & is scientifically implausible

(iii) If QM is generally true, we get "macroscopic superpositions of states".

If QM is generally true then clearly we can have macroscopic quantum states. Even measuring systems can then be in "Schrodinger's Cat" superpositions. But what does it mean for a macroscopic system to be in a superposition of states?

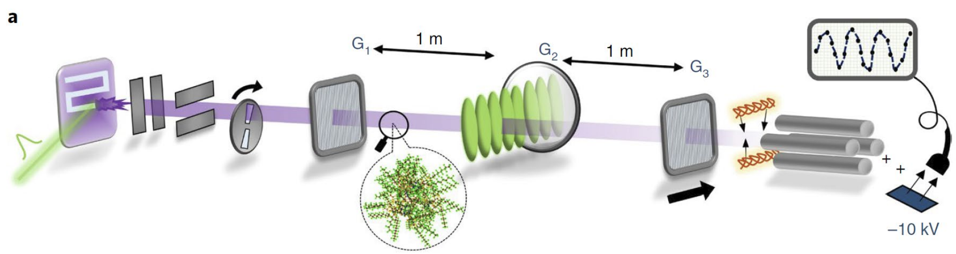
"….I think I can safely say that nobody understand Quantum Mechanics" **(R.P. Feynman, 1965)**

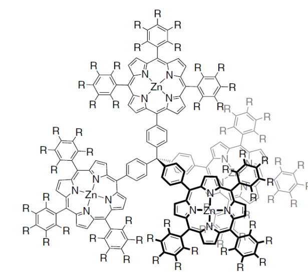
Question: HOW MACROSCOPIC is QUANTUM MECHANICS?

(1) SPIN & MASS SUPERPOSITION EXPTS: Expts show very large number of spins in identical superposed states (likewise for BEC). But these are not "Cat states", and do not involve macroscopic superpositions. One can also try to superpose a massive body in 2 different states. In reality one finds a maximum "degree of macroscopicity" of $\Delta N_{tot} \sim O(10^2 - 10^3)$ for spin systems, and mass superpositions $m \sim 10^5$ AMU $\sim 10^{-14}\, M_P$

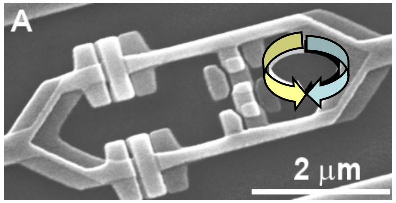
B Julsgaard et al., Nature 413, 400 (2001)
S Takahashi et al., Nature 476, 76 (2011)

M Arndt, K Hornberger, Nat Phys 10, 271 (2014)
T Juffmann et al., Rep Prog Phys. 76, 086402 (2013)





(2) PHASE SUPERPOSITION/ENTANGLEMENT: A famous example - SQUID macroscopic superposition experiment (Leggett). One finds the N-particle entanglement in expts:



Circulating current in Delft SQUID

		L	ΔI_p	$\Delta\mu$	ΔN_{tot}
SUNY	Nb	560 μm	2–3 μA	$5.5 - 8.3 \times 10^9 \mu_B$	3800–5750
Delft	Al	20 μm	900 nA	$2.4 \times 10^6 \mu_B$	42
Berkeley	Al	183 μm	292 nA	$4.23 \times 10^7 \mu_B$	124

Korsbakken et al., Phys Rev A75, 042106 (2007)
Korsbakken et al., Europhy Lett 89, 30003 (2010)
Volkoff & Whaley, Phys Rev A89, 012122 (2014)

SO – QM is very far from being demonstrated at macroscopic scale

The CWL THEORY

COLLABORATORS

Andrei Barvinsky (Lebedev, Moscow)
Dan Carney (Berkeley)
Yanbei Chen (Caltech)
Jordan Gerow-Wilson (UBC)

CWL THEORY

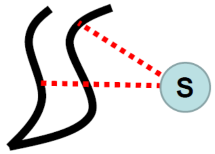
PCE Stamp, Phil Trans Roy Soc A370, 4429 (2012)
" , New J. Phys. 17, 06517 (2015)
AO Barvinsky, PCE Stamp, Phys Rev D98, 084052 (2018)
D Carney, PCE Stamp, J Taylor, Class Q Grav 36, 034001 (2019)
AO Barvinsky, J Wilson-Gerow, PCE Stamp, Phys Rev D103, 064028 (2021)
J Wilson-Gerow, PCE Stamp, Phys Rev D105, 084015 (2022)
J Wilson-Gerow, PCE Stamp, to be published
Y Chen, J Wilson-Gerow, PCE Stamp, Phys Rev D109, 064078 (2024)

CONVENTIONAL QGRAV & QFT

J Wilson-Gerow, C Delisle, PCE Stamp, Class Q Grav 35, 164001 (2018)
C Delisle, J Wilson-Gerow, PCE Stamp, /arXiv: 1905.05333
C DeLisle, J Wilson-Gerow, PCE Stamp, JHEP 03 (2021), 290
J Wilson-Gerow, PCE Stamp, Ann. Phys (NY) 442, 168898 (2022)
C. DeLisle, PCE Stamp, Phys Rev A110, 022223 (2024)

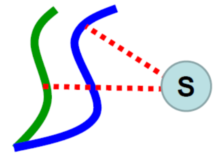
ONE KEY IDEA behind CWL THEORY: PATHS ARE FUNDAMENTAL

Gravity (the metric field $g^{\mu\nu}$) sees all fields the same – all it sees is the stress-energy tensor $T_{\mu\nu}$ But this means that it can't even distinguish multiple paths for a single particle from multiple paths for multiple particles!



2 paths for SINGLE object

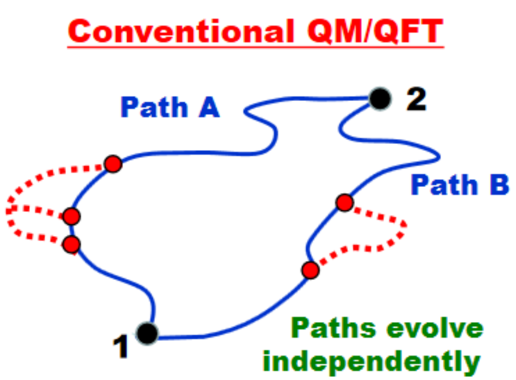
Gravity does not distinguish this from:

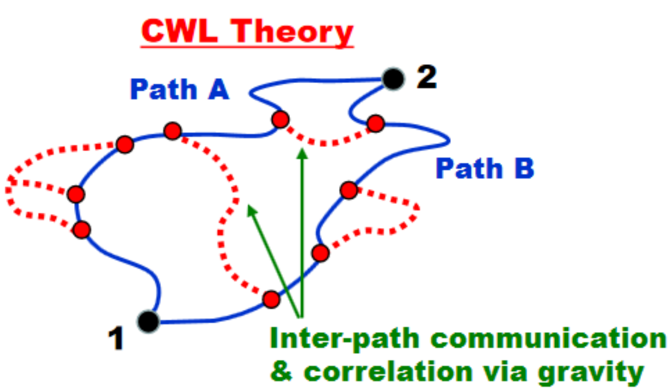


2 paths for TWO DIFFERENT objects

CONCLUSION: paths for a SINGLE OBJECT can interact via gravity

This implies a breakdown of the superposition principle





One finds that this gives

$$\mathcal{K}(2,1) = \lim_{N\to\infty} \left(\prod_{n=1}^{N} \mathcal{K}_n(2,1) \right)^{\alpha_N}$$

FORMAL STRUCTURE of CWL THEORY: GENERATING FUNCTIONAL

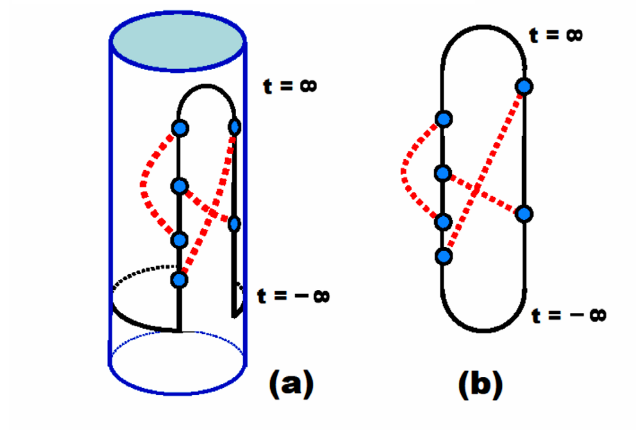
A scalar field has generating functional $Z_\phi[g,J] \;=\; \oint D\phi\, e^{i(S_\phi[g,\phi] \,+\, \int J\phi)}$

Conventional Quantum Gravity:

This has generating functional:

$$\mathcal{Z}[J] \;=\; \oint Dg\, e^{i(S_G[g]+\frac{1}{2}\chi^\mu c_{\mu\nu}\chi^\nu - i\mathrm{Tr}\ln\Xi)}\; Z_\phi[g,J]$$

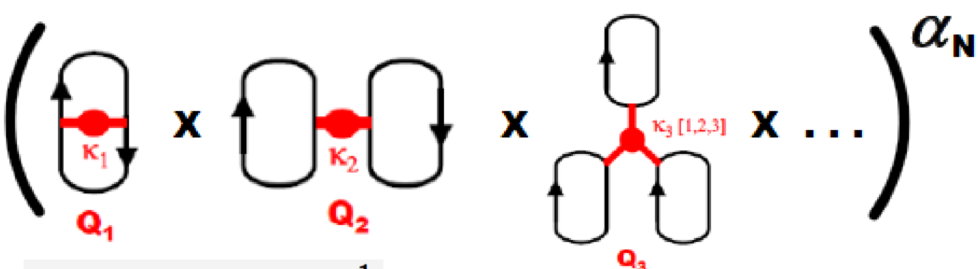
$$=\; \oint \mathcal{D}g\, e^{iS_G[g]}\Delta[g]\delta(\chi^\mu(g))\; Z_\phi[g,J]$$



CWL Theory: **This has generating functional**

$$\tilde{\mathcal{Q}}[J] \;=\; \lim_{N\to\infty}\left(\prod_{n=1}^{N}\mathcal{Q}_n[J]\right)^{\alpha_N} \;=\; \lim_{N\to\infty}\left[\prod_{n=1}^{N}\int \mathcal{D}g_n\, e^{inS_G[g_n]}\prod_{k=1}^{n}\int \mathcal{D}\phi_k^{(n)}\; e^{iS[g_n,\phi_k^{(n)}]+J\phi_k^{(n)}}\right]^{\alpha_N}$$

$$=\; \lim_{N\to\infty}$$



We note that log Q[J] is additive over these "tower" or "path" contributions, and α_N rescales things.

where $$\alpha_N \;=\; \left(\sum_{n=1}^{N} n\right)^{-1} \;=\; \frac{2}{N(N-1)}$$

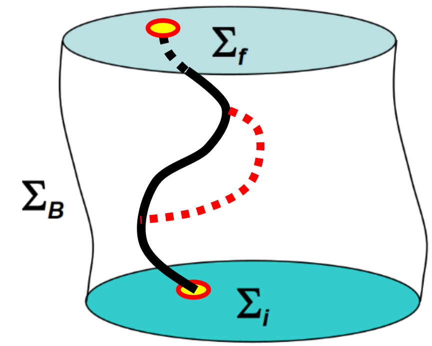
KEY RESULT: Following consistency requirements are obeyed: well-behaved *h* and l_P^2 expansions, classical limit, Ward identities

FORMAL STRUCTURE of CWL THEORY: PROPAGATORS

Conventional Quantum Gravity: **We write a propagator**

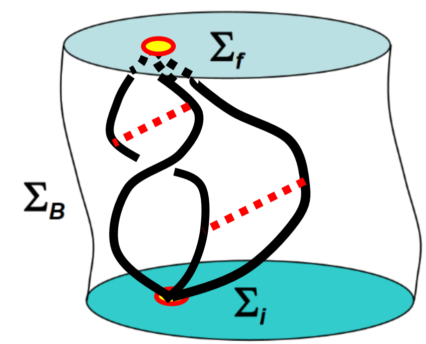
$$K(2,1) \equiv K(\Phi_2, \Phi_1; \mathfrak{h}_2^{ab}, \mathfrak{h}_1^{ab})$$
$$= \int_{\mathfrak{h}_1}^{\mathfrak{h}_2} \mathcal{D}g \, e^{\frac{i}{\hbar} S_G[g]} \Delta(g)\, \delta(\chi^\mu) \int_{\Phi_1}^{\Phi_2} \mathcal{D}\phi \; e^{iS_\phi[\phi, g]}$$

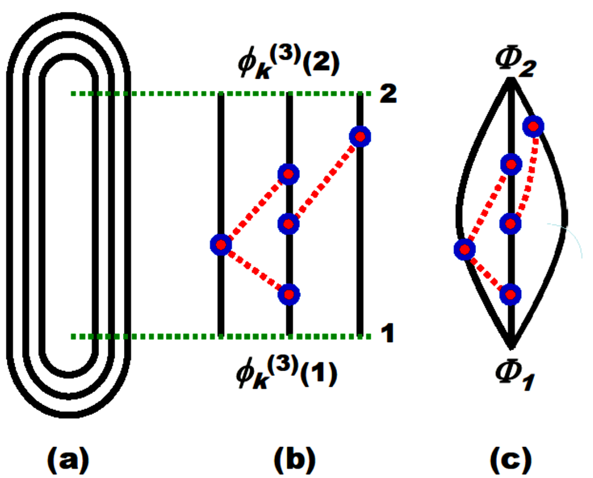
with metric & matter fields defined between hypersurfaces.



CWL Theory: **The matter propagator takes the form (suppressing FP factors, etc):**

$$\mathcal{K}(2,1) = \lim_{N\to\infty} \left(\prod_{n=1}^{N} \mathcal{K}_n(2,1) \right)^{\alpha_N}$$
$$= \lim_{N\to\infty} \left[\prod_{n=1}^{N} \mathcal{N}_n^{-1} \int \mathcal{D}g_n \, e^{inS_G[g_n]} \prod_{k=1}^{n} \int_{\Phi_1}^{\Phi_2} \mathcal{D}\phi_k^{(n)} \; e^{iS[\phi_k^{(n)}, g_n]} \right]^{\alpha_N}$$

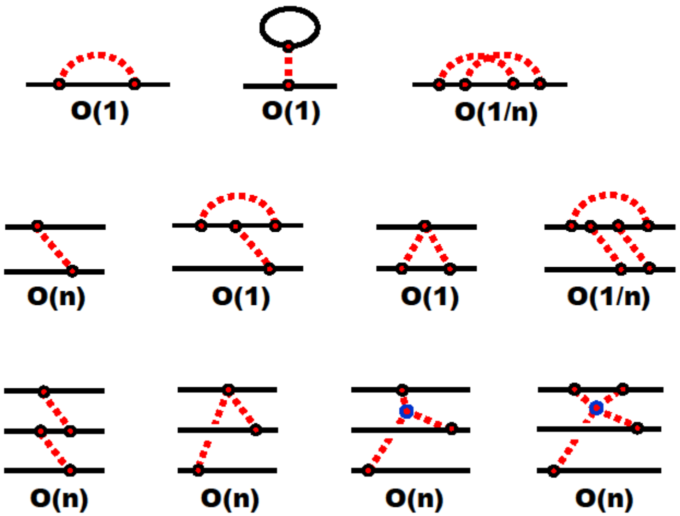




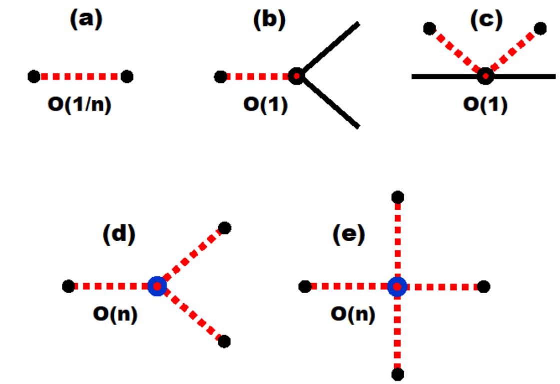
A perturbation expansion in powers of l_P^2 produces diagrams like those shown – these are generated by cutting diagrams for the generating functional Q (the diagram depicts a contribution from Q_3), ie., from the 3rd level, involving 3 different paths or "histories" for the field).

We cut the lines on the 2 hypersurfaces, & then "tether" them to the initial and final states.

PERTURBATION THEORY RESULTS



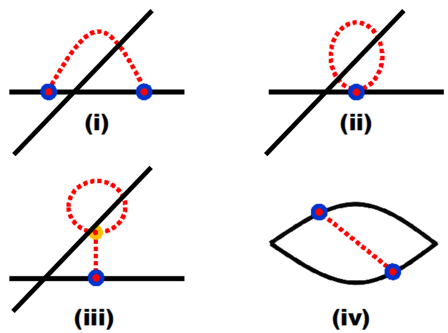
Consider "untethered graphs" for **K(2,1)** with *n* open matter lines. For large **n**, only those ~ **O(n)** survive. So, no loops containing gravitons survive – only "skeleton tree graphs.



Lowest-order Perturbation theory

The expansion parameter is

$$\ell_P^2 = 8Gh$$

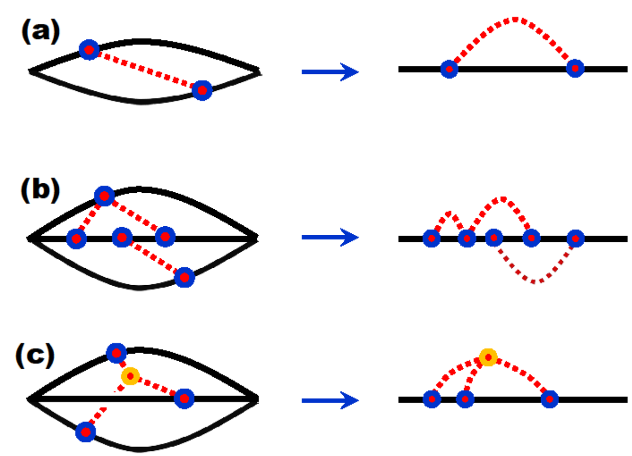


which is NOT dimensionless (this is related to the non-renormalizability of GR).

Only one graph survives (ie., graph (iv)). It describes the interaction between 2 paths of a single particle – we see the key feature of CWL here, that Q superposition has broken down.

LARGE MASSES

Particle lines collapse onto each other, reproducing classical mass dynamics (including radiation reaction, etc.)



A key physical question we will come to: what is the DYNAMICS of this collapse ?

BEYOND PERTURBATION THEORY → EXACT THEORY

Let's write $K_0(\Phi_2,\Phi_1|g_0) = \int_{\Phi_1}^{\Phi_2} \mathcal{D}\phi\, e^{iS_\phi[\phi,g_0]} = e^{i\psi_0(\Phi_2,\Phi_1|g_0)}$ **on frozen metric g_0.**

and write full CWL propagator as $\mathcal{K}(2,1) \equiv \mathcal{K}(\Phi_2,\mathfrak{h}_2^{ab};\Phi_1,\mathfrak{h}_1^{ab}) \to e^{i\Psi(2,1)}$

But the CWL phase is $\Psi(2,1) = -i \lim_{N\to\infty}\left[\alpha_N \sum_{n=1}^{N} \log \mathcal{K}_n(2,1)\right].$

where $\mathcal{K}_n(2,1) = e^{in(S_G[\bar{g}_{21}]+\psi_0(2,1|\bar{g}_{21}))+\mathcal{O}(n^0)}$

& so we get $\mathcal{K}(2,1) = e^{i(S_G[\bar{g}_{21}]+\psi_0(2,1|\bar{g}_{21}))}$ **where** $\frac{\delta}{\delta g}(S_G[g]+\psi_0(2,1|g))|_{g=\bar{g}_{21}} = 0.$

Now $\frac{\delta}{\delta g^{\mu\nu}(x)}\psi_0(2,1|g) = \frac{\langle\Phi_2|T_{\mu\nu}[x|g]|\Phi_1\rangle}{\langle\Phi_2|\Phi_1\rangle} \equiv \chi^{\mathbb{T}}_{\mu\nu}(2,1|x,g)$ **COMPLEX !**

& so the new Einstein eqtn is $G_{\mu\nu}(\bar{g}_{21}(x)) = 8\pi G_N\, \chi^{\mathbb{T}}_{\mu\nu}(2,1|x,\bar{g}_{21})$ **COMPLEX !**

think of $\chi^{\mathbb{T}}_{\mu\nu}(2,1|x,g)$ as an expectation value of $T_{\mu\nu}(x)$, but one conditional on the preselection and postselection of states $|\Phi_1\rangle$ and $|\Phi_2\rangle$ respectively for $\phi(x)$. This quantity is in general complex.

(cf weak Mmt)

Hence one can also write $\mathcal{K}(2,1) = e^{iS_G[\bar{g}_{21}]}\int_{\Phi_1}^{\Phi_2}\mathcal{D}\phi\, e^{iS_\phi[\phi,\bar{g}_{21}]}$

One can do the same analysis for the generating functional

CWL THEORY: SUMMARY of EXACT RESULTS

CONSISTENCY

1. Consistent classical limit
2. Well-behaved *h* and l_P^2 expansions
3. All Ward identities obeyed
4. Renormalizable

Thus, (super)string theory is not the only consistent theory of quantum gravity!

MATTER PROPAGATOR

We switch off gravitational dynamics, & define the propagator in a background g:

$$K^0_\phi(\Phi_2, \Phi_1|g) \;=\; \int_{\Phi_1}^{\Phi_2} \mathcal{D}\phi \; e^{iS_\phi[\phi,g]} \;=\; e^{i\psi_0(\Phi_2,\Phi_1|g)}$$

Then the full CWL propagator is

$$\mathcal{K}(2,1) = e^{i(S_G(2,1|[\bar{g}]) + \psi_0(2,1|\bar{g}))}$$

with $$\frac{\delta}{\delta g}\left(S_G(2,1|[g]) + \psi_0(2,1|g)\right)\Big|_{g=\bar{g}} = 0$$

The functional derivative is

$$\frac{\delta}{\delta g^{\mu\nu}}\psi_0(2,1|g) \;=\; -\frac{1}{2}\frac{\langle \Phi_2|T_{\mu\nu}|\Phi_1\rangle}{\langle \Phi_2|\Phi_1\rangle}$$

So, the result is that a particle propagates in the Einstein field produced by all paths; but it still shows superposition. For small masses we get conventional QM

GENERATING FUNCTIONAL

We can find the generating functional exactly:

$$\mathbb{Q}[J] = e^{i(S_G[\bar{g}_J] + W_0[J|\bar{g}_J])}$$

whose solution is the semiclassical Einstein e.o.m.

$$G_{\mu\nu}(x|\bar{g}_J) = 8\pi G_N \langle T_{\mu\nu}[x|\bar{g}_J]\rangle_J$$

even in the quantum regime

So: the sum over the QUANTUM Paths tells the QUANTUM spacetime metric field how to move; & the paths interact with each other via distorted metric field, sourced by interacting quantum paths.

INTUITION from 2nd-ORDER PERTURBATION THEORY

Consider dynamics at 2nd-order, for a SINGLE PARTICLE or SINGLE FIELD. The field propagator is

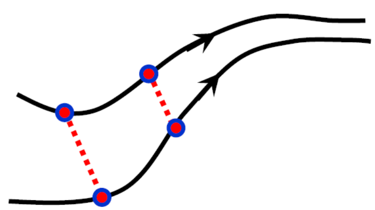
$$\mathcal{K}(2,1) \sim K_0^{-1}(2,1) \int_{\Phi_1}^{\Phi_2} \mathcal{D}\phi \int_{\Phi_1}^{\Phi_2} \mathcal{D}\phi' \, e^{i(S[\phi]+S[\phi'])} \, e^{iS_{CWL}[\phi,\phi']} \; + \; O(\ell_P^4)$$

where $$S_{CWL}[\phi,\phi'] \; = \; -\frac{\ell_P^2}{8} \int d^4x \int d^4x' D^{\mu\nu\alpha\beta}(x-x') T_{\mu\nu}(\phi(x)) T_{\alpha\beta}(\phi'(x'))$$

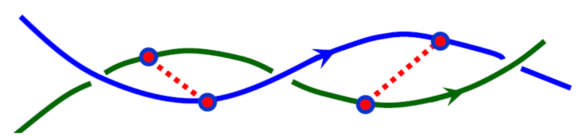
Newton

For a single SLOW PARTICLE we get: $$S_{CWL}[q,q'] \; \rightarrow \; \lim_{v \ll c} \; \frac{1}{8} \int_{t_1}^{t_2} dt \frac{Gm^2}{|\mathbf{r}(t) - \mathbf{r}'(t)|}$$

PATH-BUNCHING: **Consider a single particle. Naively the effect of the attractive interaction will be to cause different paths for the same particle to "bunch" together as time increases.**



However the actual motion, in the absence of dissipation, is more complex – we get oscillations in the Newtonian potential well between paths.



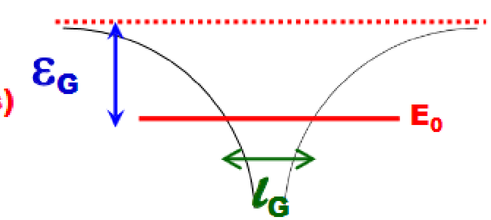
CHARACTERISTIC SCALES of POTENTIAL

$$l_G(m) = \left(\frac{M_p}{m}\right)^3 L_p$$ **Newton radius (gravitational analogue of the Bohr radius)**

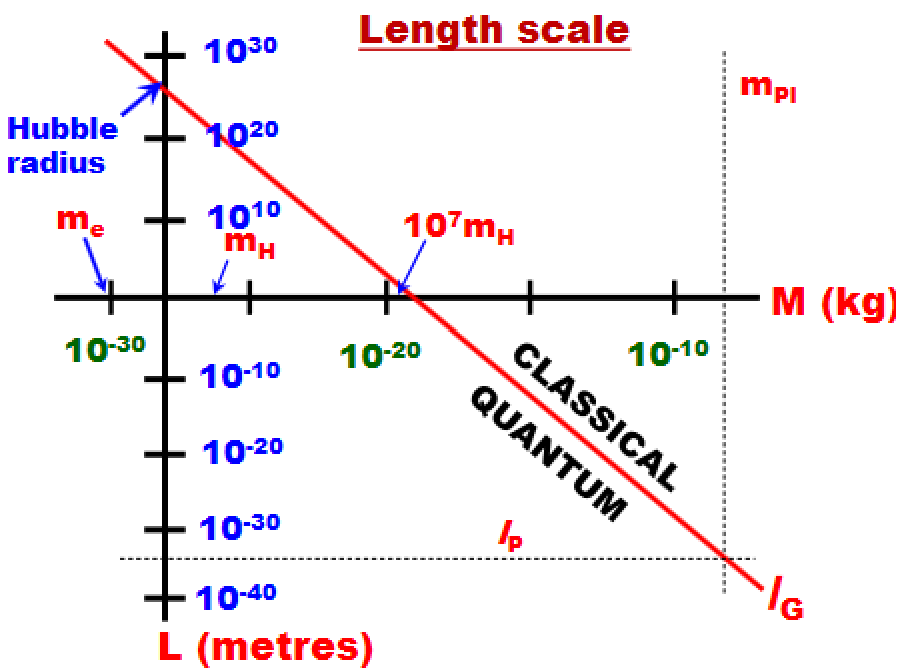
$$\epsilon_G(m) = G^2m^2/l_G(m) \equiv E_p(m/M_p)^5$$ **Mutual binding energy for paths**

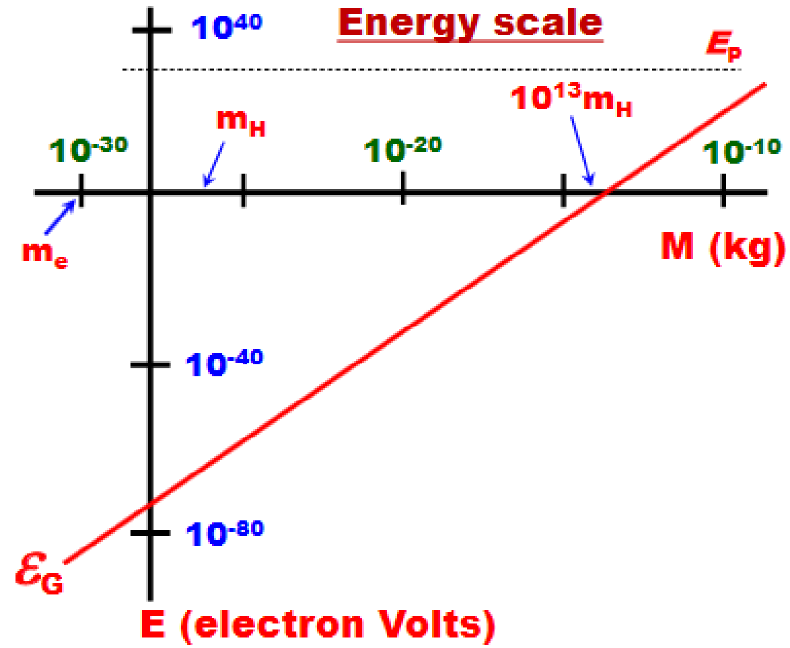
$$R_s \; = 2Gm/c^2$$ **Schwarzchild radius for the particle** **(Classical)**

Characteristic potential



SINGLE PARTICLE "TOY MODEL" : Variation of scales with MASS

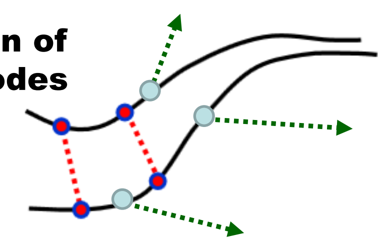


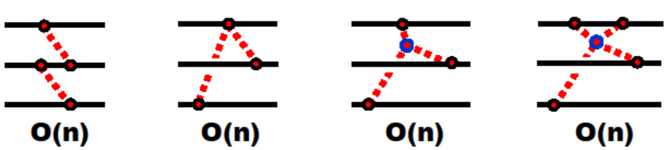


There are 3 things wrong with this 2nd-order perturbation theory

1. It does not describe an extended body
2. the centre of mass will couple to a "bath" of "environmental" degrees of freedom
3. It only describes interactions between pairs of paths

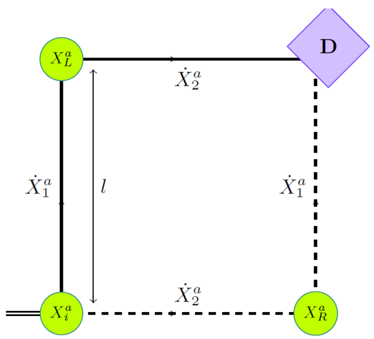
Emission of bath modes

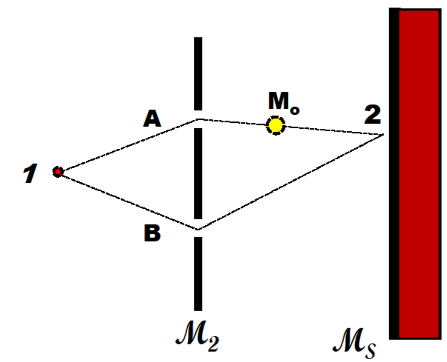


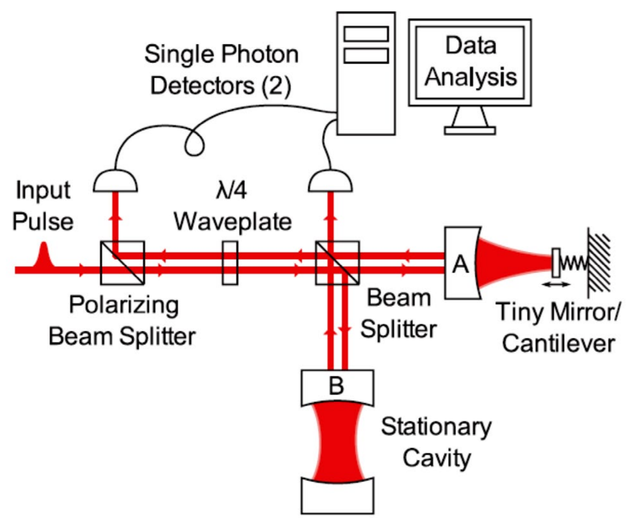


3-path graphs

2-PATH EXPERIMENTS

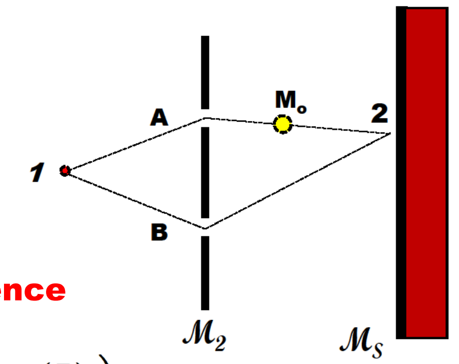






The 2-PATH EXPERIMENT in CWL THEORY

Only wimps specialize in the general case. Real scientists pursue examples.
MV Berry: Ann NY Acad Sci 755, 303 (1995)



If we ignore gravity, we just have

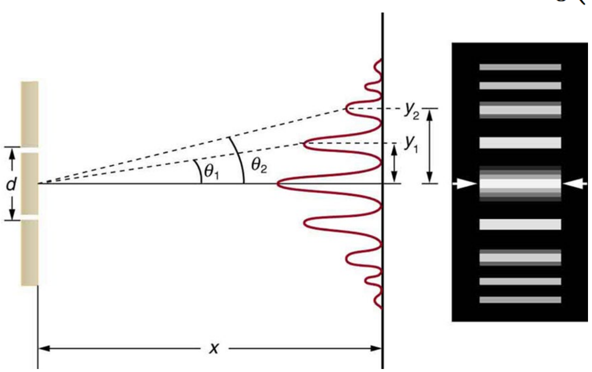
$$K_0(2,1) \;=\; \sum_{\alpha}^{A,B} \Omega_o^{(\alpha)}\, e^{iS^0_{21}[q^{(\alpha)}|\eta]}$$

Sum amplitudes over 2 paths

This then gives

$$K_0(2,1) \;=\; 2\,\Omega_o\, e^{i\bar{S}^0_{21}}\, \cos(\Delta S_{21})$$

interference



here

$$\bar{S}^0_{21} \;\equiv\; \tfrac{1}{2}\Big(S^0_{21}[q^{(A)}] + S^0_{21}[q^{(B)}]\Big)$$

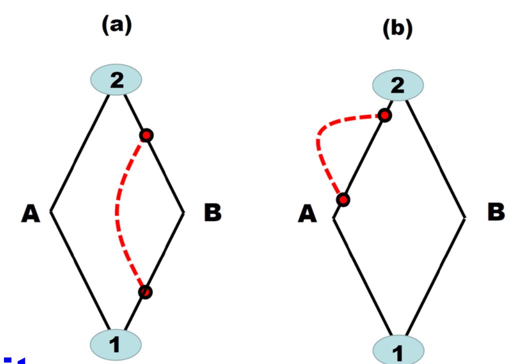
$$\Delta S_{21} \;\equiv\; \tfrac{1}{2}\Big(S^0_{21}[q^{(A)}] - S^0_{21}[q^{(B)}]\Big)$$

& the probability of arriving at point 2 on the screen is then $|K_o(2,1)|^2$

What happens when we add gravity?

1. CONVENTIONAL QUANTUM GRAVITY: Then we just get

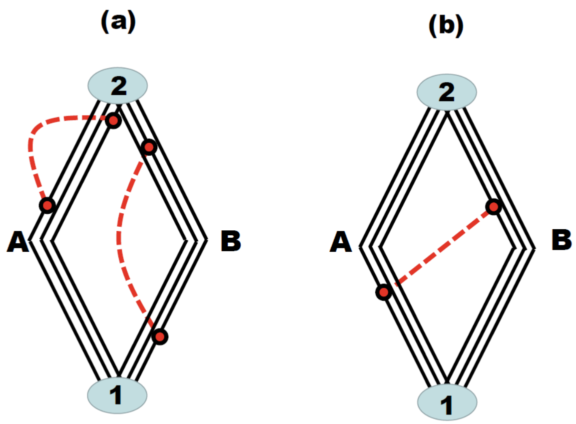
$$K(2,1) \;=\; \sum_{\alpha}^{A,B} K_0^{(\alpha)}(2,1)\; e^{\frac{i}{2}\int d^4x \int d^4x'\, T_{\mu\nu}(x|q^{(\alpha)})\, D^{\mu\nu\lambda\sigma}(x,x')\, T_{\lambda\sigma}(x'|q^{(\alpha)})}$$



Each path is renormalized SEPARATELY by gravitons. We never see these renormalizations, since we can't switch off gravity.

2. CWL THEORY: Then we get $\mathcal{K}(2,1) = K_0(2,1)\, e^{i\Theta_{21}}$ **where $K_o(2,1)$ is the QM result, and the COMPLEX phase is**

$$\Theta_{21} = \frac{G_N}{4}\int_{t_1}^{t_2} dt \int \frac{d^3r d^3r'}{|\mathbf{r}-\mathbf{r}'|}\left\{\left[(T_A T'_A + T_B T'_B)(1-\tan^2(\Delta S_{21})) + 2\frac{T_A T'_B}{\cos^2(\Delta S_{21})}\right] + 2i\,(T_A T'_A - T_B T'_B)\tan(\Delta S_{21})\right\}$$



Analysis of this expression shows that the CWL "inter-path" correlations (shown at left for triple path contributions) strongly affect the regions of destructive interference (the "dark" fringes). The divergence in the phase can be corrected by a non-perturbative analysis.

For masses < 10^{-14} kg (10^{13} amu, or 10^{-6} M_P), the CWL corrections are negligible, and QM is obeyed.

3. SEMICLASSICAL GRAVITY THEORY: Although this theory has been known since Kibble to be internally inconsistent, we can still calculate with it. We get

$$K_{sc}(2,1) = A^{(sc)}(2,1)e^{i\Phi_{21}} \quad \textbf{where} \quad A^{(sc)}(2,1) = 2\Omega_o \cos(\Delta S_{21})$$

and

$$\Phi_{21}^{(sc)} = \bar{S}_{21}^o + \frac{G_N}{4}\int_{t_1}^{t_2} dt \int \frac{d^3r d^3r'}{|\mathbf{r}-\mathbf{r}'|}\left[(T_A T'_A + T_B T'_B) + (T_A T'_B + T'_A T_B)\right]$$

3 different theories, 3 different predictions.....

SOME INTERPRETATION...

Let's write, in the weak field regime: $\mathcal{K}(2,1) = K_0(2,1)e^{i\Theta_{21}}$

where $$\Theta_{21} = \frac{1}{2}\int_1^2 d^4y \left.\frac{\delta\psi_0[g]}{\delta g_{\mu\nu}(y)}\right|_{g=\eta} h_{\mu\nu}(y)$$

$$\frac{1}{2}G_N \int_{t_1}^{t_2} dt \int d^3r d^3r' \frac{1}{|\mathbf{r}(t) - \mathbf{r}'(t)|} \quad \chi_{00}^{\mathbb{T}}(2,1|\mathbf{r},t)\chi_{00}^{\mathbb{T}}(2,1|\mathbf{r}',t)$$

NOT $T_{\mu\nu}$

& for a particle we have $$\chi_{\mu\nu}^{\mathbb{T}}(2,1|x) = \frac{\int_{x_1}^{x_2} \mathcal{D}q e^{iS[q]} T_{\mu\nu}(x)}{\int_{x_1}^{x_2} \mathcal{D}q e^{iS[q]}}$$

Then for the 2-slit system we have

$$\mathcal{K}(2,1) = e^{iS_G[\langle\bar{g}\rangle_{AB}]} \sum_{\alpha}^{A,B} e^{iS_M[q^{(\alpha)}|\langle\bar{g}\rangle_{AB}]}$$

in which the gravitational term is just the path integral for the classical action $S_G[\langle\bar{g}\rangle_{AB}]$, integrated along the classical path in configuration for a metric field $\langle\bar{g}\rangle_{AB}$ sourced by both matter paths; and the matter term sums over the two matter paths, in the presence of the same background metric field $\langle\bar{g}\rangle_{AB}$. Thus the matter is still propagating quantum mechanically.

For a 2-path system,

$$|\psi(t)\rangle \sim \frac{1}{\sqrt{2}}[\delta(\mathbf{r} - \mathbf{r}_A(t)) + \delta(\mathbf{r} - \mathbf{r}_B(t))]$$

so that

$$\langle T_{00}(x)\rangle = \frac{\langle\psi|T_{00}(x)|\psi\rangle}{\langle\psi|\psi\rangle}$$
$$= \frac{m}{2}[\delta(\mathbf{r} - \mathbf{r}_A(t)) + \delta(\mathbf{r} - \mathbf{r}_B(t))]$$

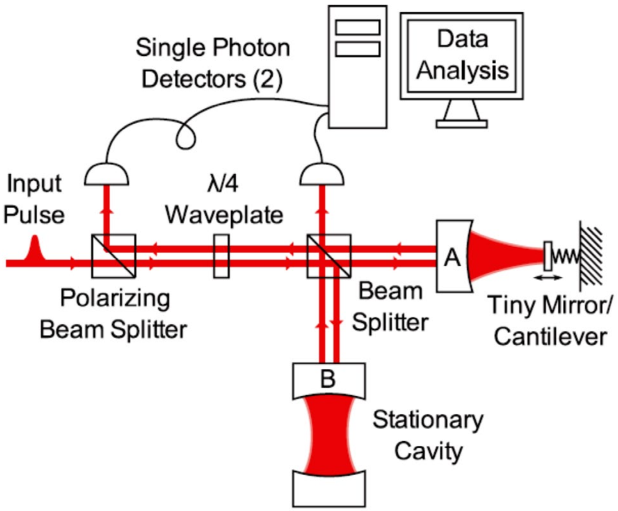
However in semiclassical theory we have

$$\langle T_{00}(x)\rangle = m\delta\left(\mathbf{r} - \frac{1}{2}[\mathbf{r}_A(t) + \mathbf{r}_B(t)]\right)$$

which is different

A 2-PATH CAVITY EXPERIMENT

One can look at interference between the 2 paths of an oscillating heavy mass. One way to do this is to entangle a photon with a heavy mirror, and then look for gravitational effects. Starting from a state, conventional QM gives:



$$|\psi(0)\rangle = \quad (1/\sqrt{2})(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B)|0\rangle_m$$

and we get

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} e^{-i\omega_c t}[|0\rangle_A|1\rangle_B|0\rangle$$

$$+ e^{i\kappa^2(\omega_m t - \sin\omega_m t)}|1\rangle_A|0\rangle_B\, |\kappa(1 - e^{-i\omega_m t})\rangle_m],$$

& we look at interference between the 2 branches. One can also look at interference between a 0-phonon and a 1-phonon state

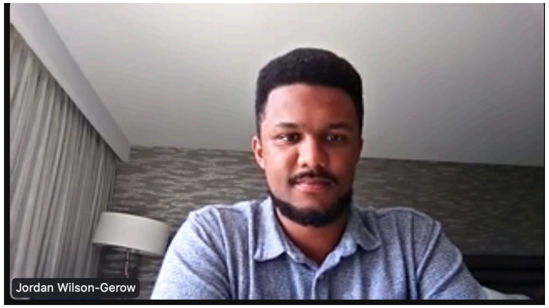
D Kleckner et al., N J Phys 10, 095020 (2008)

The experimental difficulty here is to reduce environmental decoherence effects – coming from the interaction with photons, or between, eg., charged defects in the system (or spin defects/nuclear spins) and EM fields.

The big theoretical challenge is to redo things for CWL theory

SO: HOW DO WE DO CWL THEORY
for
REAL CAVITY EXPERIMENTS ??

REAL WORLD CWL THEORY in OPTOMECHANICAL SYSTEMS



J. Wilson-Gerow
Y. Chen
P.C.E. Stamp

I pass with relief from the tossing sea of Cause and Theory to the firm ground of Result and Fact.

W. Churchill: *The Story of the Malakand Field Force: An Episode of Frontier War* (1898)

PULSED OPTOMECHANICS SETUP

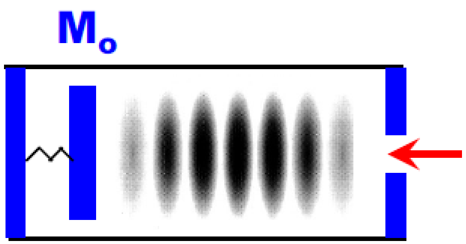
The basic Hamiltonian:

$$H = \frac{p^2}{2m} + \frac{m\omega_m^2}{2}q^2 + \omega_{\text{cav}} A^\dagger A - GqA^\dagger A - \sqrt{\kappa}(A^\dagger \alpha_{\text{in}} + A\alpha_{\text{in}}^*),$$

We want to generate resonant interactions between the laser cavity mode and the mirror

where $G = -\partial_q \omega_c(q)|_{q=0};$ **moving mirror/cavity coupling**

and $\alpha_{\text{in}}(t) = e^{-i\omega_L t}\omega_{\text{cav}}\frac{1}{\sqrt{\kappa}}\alpha(t)$ **incoming laser**



Then we have

$$H = \omega_m b^\dagger b + \omega_{\text{cav}} a^\dagger a - g(t)(b + b^\dagger)(ae^{i\omega_L t} + a^\dagger e^{-i\omega_L t})$$

Mirror operators

Cavity operators

Go to interaction representation $H_{int} = -g(t)(be^{-i\omega_m t} + b^\dagger e^{i\omega_m t})(ae^{i\Delta t} + a^\dagger e^{-i\Delta t})$

where we have defined $b \to e^{-i\omega_m t}b,\ a \to e^{-i\omega_{\text{cav}} t}a$

$$\Delta = \omega_L - \omega_{\text{cav}}.$$

Now the basic idea here is that energy sloshes back & forth between the mirror and the cavity field. Let us define the phase factor:

$$\boxed{\phi(t) = \int_0^t d\tau g(\tau)}$$

This gives the integrated effect of the coupling. We have

$\phi(t) = (n + \frac{1}{2})\pi/2$ **system swaps energy from one mode to the other**

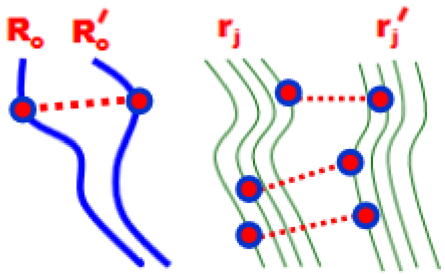
$\phi(t) = n\pi,$ **system returns to the original state**

REAL WORLD CWL THEORY

(1) Interpath potential - EXTENDED MASS

A solid extended body has action:

$$S_o[\mathbf{R}_o, \{\mathbf{r}_j\}] = \int d\tau \left[\frac{M_o}{2}\dot{\mathbf{R}}_o^2 + \sum_{j=1}^{N} \frac{m_j}{2}\dot{\mathbf{r}}_j^2 - \sum_{i<j}^{N} V(\mathbf{r}_i - \mathbf{r}_j) \right]$$



Assume

$$\omega^2_{\mathbf{Q}\mu} = \frac{1}{m}\sum_{i\neq j} V_{ij} e^{i\mathbf{Q}\cdot\mathbf{r}^{(o)}_{ij}} \quad \text{Phonon spectrum}$$

$$\langle u_i^\alpha(t_1) u_j^\beta(t_1)\rangle = \frac{1}{N}\sum_{\mathbf{Q}\mu} \frac{\hat{e}^\alpha_{\mathbf{Q}\mu}\hat{e}^\beta_{\mathbf{Q}\mu}}{2m\omega_{\mathbf{Q}\mu}} e^{i[\mathbf{Q}\cdot\mathbf{r}^{(o)}_{\mathbf{ij}} - \omega_{\mathbf{Q}\mu}(t_1-t_2)]} \quad \text{Phonon Correlator}$$

RESULTS: New interpath Potential has smooth and "spike" components

$$V_{smooth}(\vec{R}) = -\frac{GM^2}{|\vec{R}|}\mathrm{Erf}\left(\frac{\sqrt{\pi}}{2}\gamma\frac{|\vec{R}|}{a_0 L}\right) \quad \text{Smooth well}$$

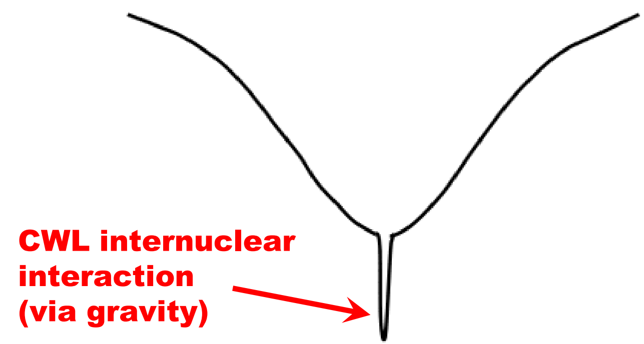
$$V_{spike}(|\vec{R}|) = -\frac{GMm}{|\vec{R}|}\mathrm{Erf}\left(\frac{|\vec{R}|}{\sqrt{2}\sigma}\right) \quad \text{Spike well}$$

with associated interpath oscillation frequencies

$$\omega_{eff} = \left(\frac{\pi}{6}\gamma^3 G\rho_{avg}\right)^{1/2} \quad \text{Smooth well}$$

$$\omega_{eff} = \left(\sqrt{\frac{2}{\pi}}\frac{Gm}{3\sigma^3}\right)^{\frac{1}{2}} \quad \text{Spike well}$$

FREQUENCIES INDEPENDENT of SIZE !!



Interpath Interaction potential for amorphous solid (cubic shape)

Example: Amorphous SiO_2

$$f^{SiO_2}_{spike} \approx 60.4\,\mathrm{mHz} \qquad \tau^{SiO_2}_{spike} \approx 16\,\mathrm{s},$$

$$f^{SiO_2}_{smooth} \approx 0.78\,\mathrm{mHz} \qquad \tau^{SiO_2}_{spike} \approx 21.2\,\mathrm{min.}$$

CWL THEORY: DYNAMICS of a HARMONIC OSCILLATOR

This problem can be solved exactly. Let's look first at a pair of paths, then at the full N-path problem

2-PATH PROBLEM: **We define sum and difference variables:**

$$Q = \tfrac{1}{2}(q_1 + q_2)$$
$$z = \tfrac{1}{2}(q_1 - q_2)$$

Then: $$\mathcal{K}(2,1|F) = K_0^{-1}(2,1|F) \int_1^2 \mathcal{D}Q \int_0^0 \mathcal{D}z \;\; e^{\frac{i}{\hbar}(S_0[Q+z|F]+S_0[Q-z|F])} \exp^{iS_{\mathrm{CWL}}[z]}$$

This factorizes: $$\mathcal{K}(2,1|F) = \frac{1}{K_0(2,1|F)} \int_1^2 \mathcal{D}Q e^{\frac{i}{\hbar}S_+[Q|F]} \int_0^0 \mathcal{D}z e^{\frac{i}{\hbar}S_-[z]} = K_0(2,1|F) K_C(0,0)$$

where the actions are

$$S_+[Q|F] = 2\int dt \left[\frac{1}{2}m(\dot{Q}^2 - \omega_m^2 Q^2) - F(t)Q(t)\right]$$

$$S_-[z] = 2\int dt \frac{1}{2}m(\dot{z}^2 - \omega_m^2 z^2) + \frac{Gm^2}{2}\int \frac{dt}{|z(t)|}$$

N-PATH PROBLEM: **Define sum and difference variables again:**

$$Q = N^{-1}\textstyle\sum_k q_k$$
$$z_k = q_k - Q$$

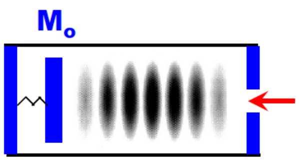
Then the CWL action is $$S_{\mathrm{eff}} = \sum_k S_o[q_k] + \sum_{k,k'} S_{\mathrm{CWL}}[q_k - q_{k'}]$$

with $$\sum_k S_o[q_k] = \frac{m}{2}\int dt N(\dot{Q}^2 - \omega_m^2 Q^2) + \sum_k \int dt(\dot{z}_k^2 - \omega_m^2 z_k^2)$$

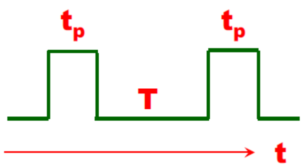
KEY RESULT: Again diagonalizable – the motion of the c.o.m. or standard QM coordinate is completely unaffected by CWL interactions! So – no experiment on a SHO can distinguish QM from CWL !!

PULSED OPTOMECHANICS EXPERIMENTS

Now, the basic idea of the experiment is to compare the "ground state" in standard QM with that in CWL theory. They are not the same.

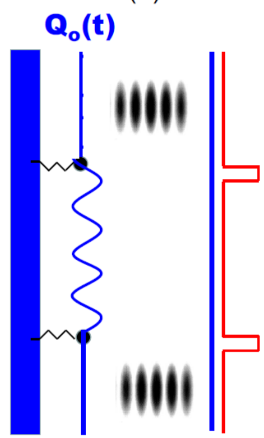


(a)



(b)

(c)



There are 3 ways to avoid the result just found:

(i) do non-linear experiments (eg., non-harmonic system)

(ii) use non-Gaussian states

(iii) pulse the system

Let's look at the 3rd option. We apply the pulse sequence shown in (b) at left (with the equivalent spacetime diagram shown in (c). Recall the key function here – the accumulated phase

$$\boxed{\phi(t) = \int_0^t d\tau g(\tau)}$$

The pulse sequence is $$g(t) = \frac{(2n+1)\pi}{2t_p}\left[\mathrm{rect}\left(\frac{t-\frac{1}{2}t_p}{t_p}\right) + \mathrm{rect}\left(\frac{t-\frac{1}{2}t_p-(T+t_p)}{t_p}\right)\right]$$

In ordinary QM, the system would return exactly to the starting ground state after each cycle.

Not in CWL theory. We expect to see deviations, which appear as a non-overlap of the 2 states after each cycle, ie., it looks like the SHO has been excited. However unfortunately the probability of excitation is

$$P(0) \sim \mathcal{O}\left(\left(\frac{\omega_{SN}}{\omega_m}\right)^4\right)$$

which is very small.

We need non-linear experiments !!

DISSIPATION, DECOHERENCE, & FINITE Temps

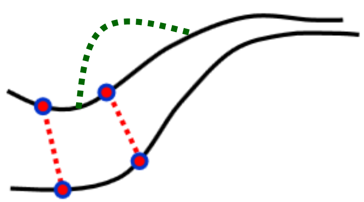
The effect of a finite T is to excite the phonons. The nuclei vibrate more and the result is that the spike potential gets wider. One finds

$$\xi_o^2(T) = \frac{\hbar^2}{2m} \int \frac{dE}{E} g(E) \left[1 + 2n(E,T) \right]$$

which for acoustic phonons gives $$\xi_o^2 = \frac{9\hbar^2}{m} \left[\frac{1}{4\theta_D} + \frac{1}{\theta_D^3} \int_0^{\theta_D} dE \frac{E}{e^{\beta\hbar E} - 1} \right]$$

Now, the effect of these phonons is also to cause decoherence and dissipation in the dynamics of the mirror. This is well-known physics, and is paramtrized for the mirror by a Q-factor; the dynamics is described by Caldeira-Leggett theory.

However, what about the effect on the CWL "relative path" dynamics? This can be written in terms of a CWL influence functional" given by



$$\mathcal{F}[u,u'] = \exp \int d\tau_1 \int d\tau_2 (u(\tau_2) - u(\tau_1))$$
$$\times \left[\mathcal{D}(\tau_1 - \tau_2) u(\tau_2) - \mathcal{D}^*(\tau_1 - \tau_2) u'(\tau_2) \right]$$

Where $u=(Q-Q')/2$ is the "relative" or "difference coordinate" between paths. The finite T coupling function is

$$\mathcal{D}(\tau) = \sum_q \frac{1}{2m_q\omega_q^2} \left(\frac{\partial F_q(u)}{\partial u} \right)^2 \Bigg|_{u=(Q-Q')/2} \times \left[e^{i\omega_q\tau} + 2\frac{\cos\omega_q\tau}{e^{\beta\omega_q} - 1} \right]$$

We can also look at the effects of the <u>SPIN BATH</u> (describing here defects in the mirror coating); this is actually the most important source of decoherence

FORMAL ASPECTS of ENVIRONMENTAL EFFECTS

There are 2 kinds of environment, viz., the spin bath and oscillator bath

Oscillator bath

$$H_{\text{eff}}^{\text{osc}} = H_0 + H_{\text{int}} + H_{\text{env}}^{\text{osc}}$$

Bath: $$H_{osc} = \sum_{q=1}^{N_o} \left(\frac{p_q^2}{m_q} + m_q \omega_q^2 x_q^2 \right)$$

Int: $$H_{\text{int}}^{\text{osc}} = \sum_{q=1}^{N} [F_q(Q) x_q + G_q(P) p_q]$$

Spin bath

$$H_{\text{eff}}^{\text{sp}}(\Omega_0) = H_0 + H_{\text{int}}^{\text{sp}} + H_{\text{env}}^{\text{sp}}$$

Bath: $$H_{\text{env}}^{\text{sp}} = \sum_{k}^{N_s} \mathbf{h}_k \cdot \boldsymbol{\sigma}_k + \sum_{k.k'}^{N_s} V_{kk'}^{\alpha\beta} \sigma_k^{\alpha} \sigma_{k'}^{\beta}$$

Interaction: $$H_{\text{int}}^{\text{sp}} = \sum_{k}^{N_s} \boldsymbol{F}_k(P, Q) \cdot \boldsymbol{\sigma}_k$$

(1) Easy for oscillator baths (it is how Feynman set up quantum field theory); we integrate out a set of harmonic oscillators, to get:

$$\mathcal{F}[Q, Q'] = \prod_q^{N_o} \int \mathcal{D}x_q(\tau) \int \mathcal{D}x_q(\tau') \exp\left[\frac{i}{\hbar} \int d\tau \frac{m_q}{2} [\dot{x}_q^2 - \dot{x}_q'^2 + \omega_q^2 (x_q^2 - x_q'^2)] + [F_q(Q) x_q - F_q(Q') x_q'] \right]$$

Bilinear coupling →

$$F[q, q'] = \exp\left[-\frac{1}{\hbar} \int_{t_o}^{t} d\tau_1 \int_{t_o}^{\tau_1} d\tau_2 [q(\tau_1) - q'(\tau_2)][\mathcal{D}(\tau_1 - \tau_2) q(\tau_2) - \mathcal{D}^*((\tau_1 - \tau_2) q'(\tau_2)] \right]$$

Bath propagator

(2) For spin baths it is more subtle:

$$\mathcal{F}[Q, Q'] = \prod_k^{N_s} \int \mathcal{D}\boldsymbol{\sigma}_k(\tau) \int \mathcal{D}\boldsymbol{\sigma}_k(\tau') \exp\left[\frac{i}{\hbar} (S_{int}[Q, \boldsymbol{\sigma}_k] - S_{int}[Q', \boldsymbol{\sigma}_k'] + S_E[\boldsymbol{\sigma}_k] - S_E[\boldsymbol{\sigma}_k']) \right]$$

$$S_{int}^{sp}(Q, \boldsymbol{\sigma}_k) = - \int d\tau \sum_k^{N_s} \boldsymbol{F}_k(P, Q) \cdot \boldsymbol{\sigma}_k$$

$$S_{env}^{sp} = \int d\tau \left[\sum_k^{N_s} (\mathcal{A}_k \cdot \frac{d\boldsymbol{\sigma}_k}{dt} - \mathbf{h}_k \cdot \boldsymbol{\sigma}_k) - \sum_{k,k'}^{N_s} V_{kk'}^{\alpha\beta} \sigma_k^{\alpha} \sigma_{k'}^{\beta} \right]$$

Vector coupling

Berry phase coupling

(2) DISSIPATION, DECOHERENCE, & FINITE Temps

In standard QM, we have a density matrix propagator

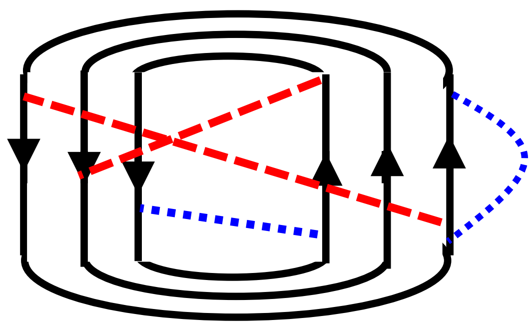
$$K(Q_2, Q_2'; Q_1, Q_1'; t, t') = \int_{Q_1}^{Q_2} \mathscr{D}q \int_{Q_1'}^{Q_2'} \mathscr{D}q' \, e^{-i/\hbar(S_0[q]-S_0[q'])} \mathscr{F}[q, q'],$$

with $$\mathcal{F}[Q, Q'] = \prod \langle \hat{U}_k(Q, t) \hat{U}_k^\dagger(Q', t) \rangle$$

Here the unitary operator $\hat{U}_k(Q, t)$ describes the evolution of the kth environmental mode, given that the central system follows the path $Q(t)$ on its 'outward' voyage, and $Q'(t)$ on its 'return' voyage; and $\mathcal{F}[Q, Q']$ acts as a weighting function, over different possible paths $(Q(t), Q'(t'))$.

CWL Theory – SUMMARY of ENVIRONMENTAL EFFECTS

The essential result is in the specimen graph shown



We have simultaneous interactions involving both the bath and the CWL gravitons.

The bath causes dissipation & decoherence – but it also causes path bunching

Now path bunching dynamics is controlled by dissipative coupling to environment. If dissipation can be parametrized by a Q-factor, the path-bunching time will be

$$\tau_{PB} \sim Q/\omega_{eff}$$

& so depends on system state preparation (NB: for LIGO, can have $Q \sim 10^{10}$)

"A theory is not a theory until it produces a number"
R.P. Feynman (Lectures on Physics, 1965)

HOWEVER....

"If it disagrees with Experiment, it's wrong. In that simple statement, is the key to Science. It doesn't make a difference how beautiful your theory is, it doesn't make a difference how smart you are, who made the guess, or what his name is – if it disagrees with experiment, it's wrong. That's all there is to it."

R.P Feynman (1965 Messenger Lectures)

RP Feynman (1920-1987)

The CWL THEORY

COLLABORATORS

Andrei Barvinsky (Lebedev, Moscow)
Dan Carney (Berkeley)
Yanbei Chen (Caltech)
Jordan Gerow-Wilson (UBC)

CWL THEORY

PCE Stamp, Phil Trans Roy Soc A370, 4429 (2012)
" , New J. Phys. 17, 06517 (2015)
AO Barvinsky, PCE Stamp, Phys Rev D98, 084052 (2018)
D Carney, PCE Stamp, J Taylor, Class Q Grav 36, 034001 (2019)
AO Barvinsky, J Wilson-Gerow, PCE Stamp, Phys Rev D103, 064028 (2021)
J Wilson-Gerow, PCE Stamp, Phys Rev D105, 084015 (2022)
J Wilson-Gerow, PCE Stamp, to be published
Y Chen, J Wilson-Gerow, PCE Stamp, Phys Rev D109, 064078 (2024)

CONVENTIONAL QGRAV & QFT

J Wilson-Gerow, C Delisle, PCE Stamp, Class Q Grav 35, 164001 (2018)
C Delisle, J Wilson-Gerow, PCE Stamp, /arXiv: 1905.05333
C DeLisle, J Wilson-Gerow, PCE Stamp, JHEP 03 (2021), 290
J Wilson-Gerow, PCE Stamp, Ann. Phys (NY) 442, 168898 (2022)
C. DeLisle, PCE Stamp, Phys Rev A110, 022223 (2024)