Deformations of AdS_5-Schwarzschild black branes: constraints from the viscosities of the quark-gluon plasma at LHC and RHIC

Roldão da Rocha

Witnessing Quantum Aspects of Gravity in a Lab, 27 Sept 2024,

ICTP-SAIFR/Principia Institute.

Outline

- Warm up: general relativity and AdS/CFT.

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- Response and transport coefficients.

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- Response and transport coefficients.
- Deformed black branes.
- Consistency with 2-loop corrections of quantum gravity.
- Applications to QCD (QGP).

Warm up: general relativity

- Einstein–Hilbert action, general relativity in 4D, vacuum:

$$S = \int d^4x \sqrt{-g}\, R, \qquad \text{where } g = \det g_{\mu\nu}$$

- $\delta S = 0 \Rightarrow \boxed{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 0}$ (Einstein's equations in the vacuum).

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- Schwarzschild:

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) dt^2 + \frac{1}{1 - \frac{2GM}{c^2 r}} dr^2 + r^2 d\Omega^2.$$

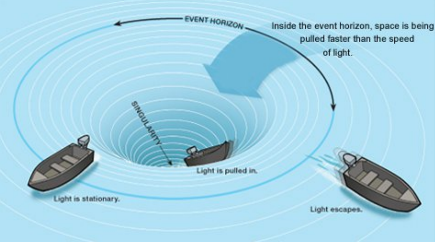
Warm up: general relativity

- **Schwarzschild in D dimensions**:

$$ds^2 = -\left(1 - \frac{2CG^{(D)}M}{r^{D-3}}\right)dt^2 + \frac{dr^2}{1 - \frac{2CG^{(D)}M}{r^{D-3}}}dr^2 + r^2 d\Omega^2_{D-2},$$

where

$$C = 2\pi\Gamma\left(\frac{D-1}{2}\right).$$



Lehner, Pretorius 2016

AdS_5–Schwarzschild black brane

$$D = 5$$

- Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda_5 g_{\mu\nu} = 0.$$

Curvature L, with $L^2 = -\frac{6}{\Lambda_5}$

- A solution: **AdS_5–Schwarzschild black brane**

$$ds^2 = \frac{r^2}{L^2}\left(-f(r)dt^2 + dx_i^2\right) + \frac{L^2}{r^2 f(r)} dr^2, \qquad i = 1, 2, 3,$$

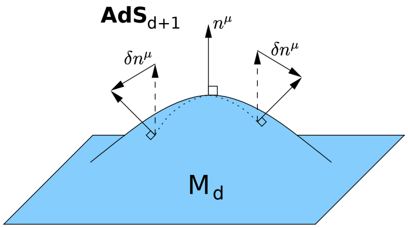
where

$$f(r) = 1 - \frac{r_0^4}{r^4}.$$

- R. Emparán, H. S. Reall, Living Rev. Rel., **11** (2008) 6.

"*The AdS–Schwarzschild black brane is the unique static, asymptotically AdS, solution in the vacuum*".

Embedding



- $\gamma_{\mu\nu}$ = AdS$_{d+1}$ metric;
- $g_{\mu\nu}$ = M_d **metric** induced by $\gamma_{\mu\nu}$:

$$g_{\mu\nu} = \gamma_{\mu\nu} + n_\mu n_\nu .$$

  (implements the projection onto M_d).
- Extrinsic curvature:

$$\begin{aligned} K_{\mu\nu} &= \frac{1}{2}\mathcal{L}_n g_{\mu\nu} \\ &= -g_\mu{}^\rho g_\nu{}^\sigma \nabla_\rho n_\sigma . \end{aligned}$$

- **Weyl tensor**:

$$C_{\mu\nu\sigma\rho} = R_{\mu\nu\sigma\rho} - \frac{1}{d-1}(g_{[\mu\sigma}R_{\nu]\rho} + g_{[\nu\rho}R_{\mu]\sigma}) + \frac{1}{d(d-1)}R\,g_{\mu[\sigma}g_{\nu\rho]},$$

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- **Weyl tensor electric component** (ADM):

$$E_{\mu\nu} = C_{\mu\nu\sigma\rho}n^\sigma n^\rho.$$

- T. Maeda, K. Sasaki, M. Shiromizu, Phys. Rev. D **62** (2000) 024012.

$$E_{\mu\nu} = -\frac{\Lambda_{d+1}}{d(d+1)}\gamma_{\mu\nu} - \partial_z K_{\mu\nu} + K_\mu{}^\rho K_{\rho\nu}.$$

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$$E_{\mu\nu} = -\frac{\Lambda_{d+1}}{d(d+1)}\gamma_{\mu\nu} - \partial_z K_{\mu\nu} + K_\mu{}^\rho K_{\rho\nu}.$$

- Weyl tensor: part of the curvature that **is not** locally determined by matter:

  Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda_d g_{\mu\nu} = T_{\mu\nu} + E_{\mu\nu}$$

- G. t' Hooft, Nucl. Phys. B **72** (1974) 461:

  S-matrix for string scatterings $\sim$ S-matrix in SU(N) Yang-Mills theory, $N \to \infty$.

AdS/CFT

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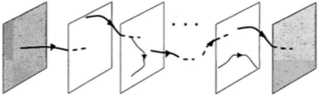
- • J. M. Maldacena, Adv. Theor. Math. Phys. **2** (1998) 231 [hep-th/9711200].
  • E. Witten, Adv. Theor. Math. Phys. **2** (1998) 253 [hep-th/9802150].
  • S. S. Gubser, I. R. Klebanov, A. M. Polyakov, Phys. Lett. B **428** (1998) 105 [hep-th/9802109].



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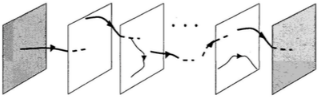

- Open strings ending on N branes $\Leftrightarrow$ SU(N) gauge fields.

  Conjecture: CFT 4D (**strongly-coupled**) is dual to gravity effectively in 5D (**weakly-coupled**).

D_3-branes

- Stack of N D_3-branes metric:

$$ds^2 = \left(1 + \frac{R^4}{r^4}\right)^{-1/2} (-dt^2 + dx^i dx_i) + \left(1 + \frac{R^4}{r^4}\right)^{1/2} (dr^2 + r^2 d\Omega_5^2),$$

where $R^4 = \frac{N}{2\pi^2 T_3}$, and T_3 is the D_3-brane tension.

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where $R^4 = \frac{N}{2\pi^2 T_3}$, and T_3 is the D_3-brane tension.

- Limit $r \ll R \Rightarrow \left(1 + \frac{R^4}{r^4}\right)^{-1/2} \approx \frac{R^2}{r^2}$:

$$ds^2 = \frac{r^2}{R^2}(-dt^2 + dx^i dx_i) + \frac{R^2}{r^2}dr^2 + \frac{R^2}{\not r^2}\not r^2 d\Omega_5^2.$$

$$\mathrm{AdS}_5 \times S^5.$$

- Poincaré coordinates, $z \equiv R^2/r$.

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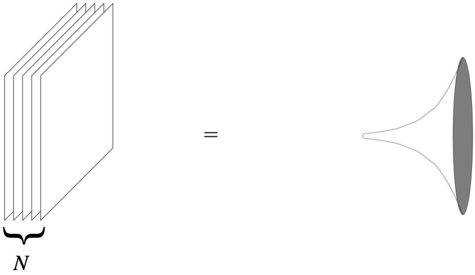
- For $z \to 0$,

$$ds^2 = \frac{R^2}{z^2}(-dt^2 + dx^i dx_i)$$

4D Minkowski = boundary of **$\mathbf{AdS}_5$**.

Stack of N D_3-branes: AdS/CFT

Effective geometry AdS_5 at low energies.



D. T. Son, AdS/CFT and Viscosity Bound, 2007; D. T. Son, A. O. Starinets, Ann. Rev. Nucl. Part. Sci. **57**, 95 (2007).

Should we worry about extra dimensions?

- $\mathrm{AdS}_5 \times S^5$ metric:

$$ds^2 = \frac{r^2}{R^2}\left(-dt^2 + dx^i dx_i\right) + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2. \tag{1}$$

- dt carries the factor $\frac{r}{R}$.

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- $\mathrm{AdS}_5 \times S^5$ metric:

$$ds^2 = \frac{r^2}{R^2}\left(-dt^2 + dx^i dx_i\right) + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2. \qquad (1)$$

- dt carries the factor $\frac{r}{R}$.
- Since

$$E = i\hbar\frac{\partial}{\partial t},$$

then (1) implies

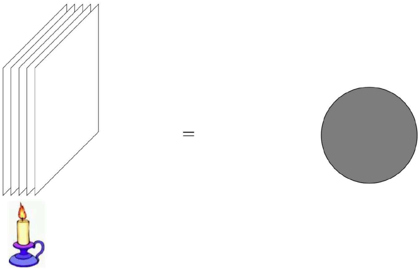
$$\frac{\partial}{\partial t} \mapsto \frac{R}{r}\frac{\partial}{\partial t} \Rightarrow \boxed{E \mapsto \frac{r}{R}E}$$

(Page 325, H. Nastase, *String Theory Methods for Condensed Matter Physics*, Cambridge, 2017).

- Additional dimension in AdS_5 = 4D energy scale. (AdS/QCD)

Finite temperature: AdS/CFT

- Finite temperature: effective geometry AdS_5–Schwarzschild.



*"...near-extremal D_3-**brane** is **dual to finite-temperature** $\mathcal{N} = 4$ supersymmetric $SU(N_c)$ **Yang–Mills theory**, in the limit of large N_c and large 't Hooft coupling..."*

• J. M. Maldacena, Adv. Theor. Math. Phys. **2** (1998) 231 [hep-th/9711200].

Fluid/gravity duality

- **Black branes** and hydrodynamical properties: viscosity, diffusion, and other response and transport coefficients.
- Dynamics in **AdS$_5$**: Einstein's equations

$$R_{MN} - \frac{1}{2} R g_{MN} + \Lambda_5 g_{MN} = 0.$$

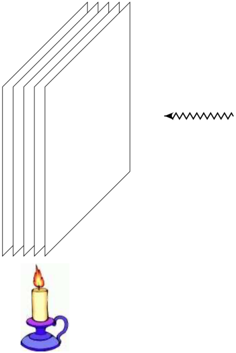
- Dynamics on the **AdS$_5$ boundary** (low energies)

$$\text{Navier–Stokes} \Leftrightarrow \nabla_\mu T^{\mu\nu} = 0.$$

• S. Bhattacharyya, V. E. Hubeny, S. Minwalla, M. Rangamani, JHEP **0802** (2008) 045.

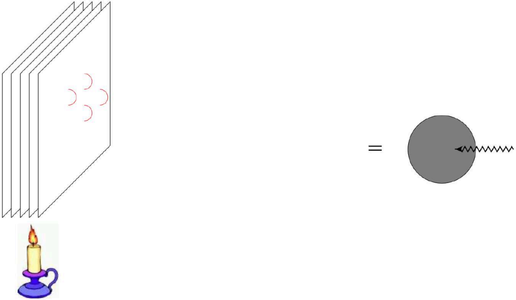
Viscosity and duality

- Interaction between the graviton and the stack of N D_3-branes:
  - P. Romatschke, D. T. Son, Phys. Rev. D **80** (2009) 065021.



D. T. Son, AdS/CFT and Viscosity Bound, 2007; D. T. Son, A. O. Starinets, Ann. Rev. Nucl. Part. Sci. **57**, 95 (2007).

Viscosity and duality



- Viscosity: absorption cross-section for gravitons at low energy $\propto$ black brane horizon area.
- P. Kovtun, D. M. Son, A. O. Starinets, JHEP **10** (2003) 064:

$$\eta = \lim_{\omega \to 0} \sigma_{\rm abs}(\omega) \quad = \quad - \lim_{\omega \to 0} \frac{1}{\omega} \int dt\, d\vec{x} e^{i\omega t} \left\langle \left[T_{xy}(t, \vec{x}), T_{xy}(0, \vec{0}) \right] \right\rangle .$$

Kovtun–Son–Starinets (KSS) result

- Kovtun, Son, Starinets, Phys. Rev. Lett. **94** (2005) 111601.

$$\boxed{\frac{\textbf{Shear viscosity}}{\textbf{Entropy density}} = \frac{\hbar}{4\,\pi k_B}\left[1+\frac{135\,\zeta(3)}{8(2g^2N_c)^{3/2}}+\cdots\right]},$$

where $\zeta(3)\approx 1.202056\ldots$ is the Apéry constant.

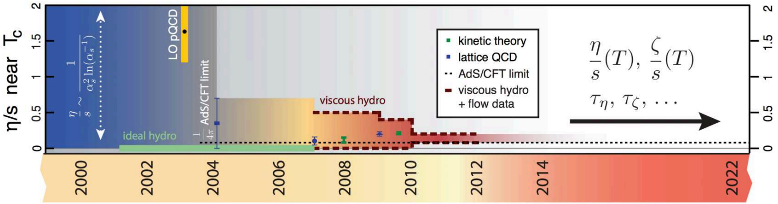
$$\boxed{\lim_{N_c\gg 1}\frac{\textbf{Shear viscosity}}{\textbf{Entropy density}} = \lim_{N_c\gg 1}\frac{\eta}{\mathbf{s}} \gtrsim \frac{\hbar}{4\,\pi\,k_{\mathrm{B}}} \simeq 6.08\times 10^{-13}\,\mathrm{K\,s}}$$

- Natural units: KSS limit

$$\boxed{\frac{\eta}{s}\gtrsim\frac{1}{4\pi}}$$

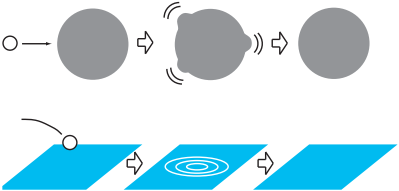
The KSS limit is universal

- C. Shen, U. Heinz, *The road to precision: Extraction of the specific shear viscosity of the quark-gluon plasma*, Nucl. Phys. News **25** (2015) 6.



η/s for the QGP is smaller than that of any known substance.

Kubo formula



Bulk	Boundary
⋆ Response properties at the horizon	⋆ transport coefficients [Kovtun, Son, Starinets (KSS)]
Einstein's equations	Navier-Stokes equations

M. Natsuume, Lect. Notes Phys. **903** (2015).

Perturbations $g_{\mu\nu} \mapsto g_{\mu\nu} + h_{\mu\nu}$

Fluid response + dissipation

- Energy-momentum tensor: 0^{th}-order = perfect fluid:

$$\left\langle T^{\mu\nu}_{(0)} \right\rangle = (\epsilon + P)\, u^\mu\, u^\nu + p\, g^{\mu\nu}\,.$$

• D. T. Son, A. O. Starinets, JHEP **0603** (2006) 052.

Fluid response + dissipation

- Energy-momentum tensor: $0^{\rm th}$-order = perfect fluid:

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• D. T. Son, A. O. Starinets, JHEP **0603** (2006) 052.

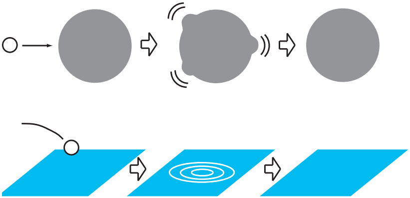
- $\Rightarrow$ $1^{\rm st}$-order (dissipation):

$$\left\langle T^{\mu\nu}_{(1)}\right\rangle = -P^{\mu\alpha}\, P^{\nu\beta}\left[\eta\left(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha - \frac{2}{3}\, g_{\alpha\beta}\,\nabla_\lambda u^\lambda\right) + \zeta\, g_{\alpha\beta}\nabla_\lambda u^\lambda\right]\,,$$

η: Shear viscosity,
ζ: Bulk viscosity,

$P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$: projection.

Measuring the viscosity



Theoretical physicist: perturb the system by gravitational waves
(M. Natsuume, Lect. Notes Phys. **903** (2015))

$$g^{(0)}_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & h_{xy}(t) & 0 \\ 0 & h_{xy}(t) & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

= perturbation on the boundary metric.

Shear viscosity

- Remember that for **viscous fluids**:

$$\langle T^{\mu\nu}\rangle = \overbrace{(\varepsilon + p)u^\mu u^\nu + pg^{\mu\nu}}^{\left\langle T^{\mu\nu}_{(0)}\right\rangle} + \left\langle T^{\mu\nu}_{(1)}\right\rangle.$$

$$\left\langle T^{\mu\nu}_{(1)}\right\rangle = -P^{\mu\alpha}P^{\nu\beta}\left[\eta\left(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha - \frac{2}{3}g_{\alpha\beta}\nabla_\lambda u^\lambda\right) + \zeta g_{\alpha\beta}\nabla_\lambda u^\lambda\right] ,$$

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- Non-vanishing contribution in the covariant derivative: Christoffel symbol:

$$\nabla_x u_y = \partial_x u_y - \Gamma^\alpha_{xy}u_\alpha = -\Gamma^0_{xy}u_0 = \Gamma^0_{xy} = \nabla_y u_x\ .$$

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- $1^{\rm st}$ order in $h_{\mu\nu}$:

$$\delta\left\langle T^{\mu\nu}_{(1)}\right\rangle \sim -\eta(\nabla_x u_y + \nabla_y u_x) \ .$$

- Christoffel symbol:

$$\Gamma^0_{xy} = \frac{1}{2}g^{00}(\partial_y g_{0x} + \partial_x g_{0y} - \partial_0 g_{xy}) = \frac{1}{2}\partial_0 h_{xy} \ .$$

Shear viscosity

- Therefore

$$\delta \left\langle T^{\mu\nu}_{(1)} \right\rangle = -2\eta\Gamma^{0}_{xy} = -\eta\partial_0 h_{xy} \ .$$

- Fourier transform

$$\boxed{\delta \left\langle T^{\mu\nu}_{(1)}(\omega, \vec{q} = 0) \right\rangle = i\omega\eta h_{xy}.}$$

Shear viscosity

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$$\boxed{\delta\left\langle T^{\mu\nu}_{(1)}(\omega,\vec{q}=0)\right\rangle = i\omega\eta h_{xy}.}$$

- Comparing to

$$\delta\left\langle T^{\mu\nu}_{(1)}\right\rangle = -G_R^{xy,xy}h_{xy},$$

one obtains the **Kubo formula** for the **shear viscosity**:

$$\eta = -\lim_{\omega\to 0}\frac{1}{\omega}\Im\, G_R^{xy,xy}(\omega,\vec{q}=0)\ .$$

- **Shear viscosity**: **Kubo formula**

$$\boxed{\eta = -\lim_{\substack{\omega \to 0 \\ q \to 0}} \frac{\Im\, G_R^{xy,xy}(\omega, \vec{q})}{\omega}}$$

- **Shear viscosity**: **Kubo formula**

$$\boxed{\eta = -\lim_{\substack{\omega\to 0\\ q\to 0}} \frac{\Im\, G_R^{xy,xy}(\omega,\vec{q})}{\omega}}$$

- **Bulk viscosity**: **Kubo formula**

$$\boxed{\zeta = \lim_{\substack{\omega\to 0\\ q\to 0}} \frac{1}{\omega}\Im G_R^{PP}(\omega,\vec{q})}$$

where

$$G_R^{PP}(\omega,\vec{q}) = \frac{k_i k_j k_m k_n}{k^4}\left[G_R^{ij,mn}(\omega,\vec{q}) + \frac{1}{3}\delta_{ab}T^{ab}\left(\delta^{im}\delta^{jn} + \delta^{in}\delta^{jm} - \delta^{ij}\delta^{mn}\right)\right] + \frac{1}{3}\delta_{ij}T^{ij} - \frac{4}{3}G_R^{xy,xy}(\omega,\vec{q}).$$

is the response to longitudinal fluctuations.

(M. Natsuume, Lect. Notes Phys. **903** (2015)).

- **AdS$_5$–Schwarzschild black brane**

$$ds^2 = \frac{r^2}{L^2}\left(-f(r)dt^2 + dx_i^2\right) + \frac{L^2}{r^2 f(r)}dr^2, \qquad i = 1, 2, 3,$$

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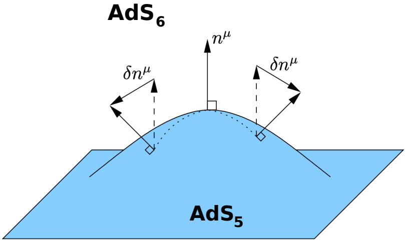
- Strongly-coupled CFT dual to the AdS$_5$–Schwarzschild black brane, at finite temperature.

  - R. A. Janik, R. B. Peschanski, Phys. Rev. D **73** (2006) 045013.
  - C. P. Herzog, A. Karch, P. Kovtun, C. Kozcaz, L. G. Yaffe, JHEP **07** (2006) 013.

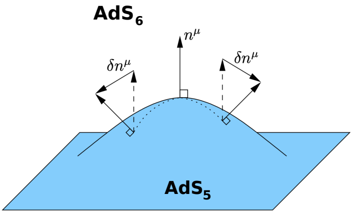


- **Gauss equation**:

$$^{(6)}R^{\mu}{}_{\nu\rho\sigma} = {}^{(5)}R^{\mu}{}_{\nu\rho\sigma} - K^{\mu}{}_{\rho}K_{\nu\sigma} + K^{\mu}{}_{\sigma}K_{\nu\rho}.$$

- Contracting with the induced metric $g_{\mu\nu}$ of AdS_5 and using Einstein's equations: **Hamiltonian constraint**.

$$\boxed{\mathcal{H} \equiv {}^{(5)}R + K^2 - K_{\mu\nu}K^{\mu\nu} - 16\pi n^\mu n^\nu T_{\mu\nu} = 0}$$



- **Codazzi equations**:

$$^{(6)}R_{\mu\nu\rho\sigma}n^{\sigma} = D_{\nu}K_{\mu\rho} - D_{\mu}K_{\nu\rho}$$

- Contracting with the induced metric $g_{\mu\nu}$ of AdS_5: **momentum constraint**.

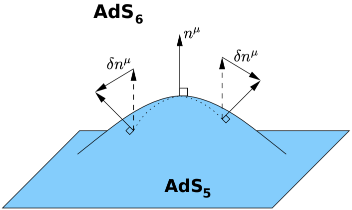
$$\boxed{\mathcal{M}^{\mu} \equiv D_{\nu}K^{\nu\mu} - D^{\mu}K - 8\pi g^{\mu\rho}n^{\sigma}T_{\rho\sigma} = 0}$$

- RdR, Phys. Rev. D **105** (2022) 026014 [arXiv:2111.01244 [hep-th]];
- A. Martins, P. Meert, RdR, Nucl. Phys. B **957** (2020) 115087 [1912.04837 [hep-th]];
- R. Casadio, R. Cavalcanti, RdR, Eur. Phys. J .C **76** (2016) 556 [1601.03222 [hep-th]].

- **Deformed black branes** (coordinate change $u = r_0/r$):

$$ds^2 = -\frac{r_0^2}{u^2}N(u)\mathrm{d}t^2 + \frac{1}{u^2 A(u)}\mathrm{d}u^2 + \frac{r_0^2}{u^2}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j,$$

where r_0 is the horizon radius.



Deformed black branes

- **AdS$_5$ deformed black branes**

$$ds^2 = -\frac{r_0^2}{u^2}N(u)\mathrm{d}t^2 + \frac{1}{u^2 A(u)}\mathrm{d}u^2 + \frac{r_0^2}{u^2}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j,$$

$\Rightarrow$ **Hamiltonian constraint** + **momentum constraint**:

$$2\frac{N''}{N} - \frac{N'^2}{N^2} + 2\frac{A''}{A} + \frac{A'^2}{A^2} - \frac{N'A'}{NA} + \frac{4}{r}\left(\frac{N'}{N} - \frac{A'}{A}\right) - 4\frac{A}{r^2} - f(r, r_0, \beta) = 0,$$

where $\beta \in \mathbb{R}$ and...

$$\ldots f(r, r_0, \beta) = -\frac{1}{r^{10}}\Bigg\{ -\left(10(\beta-1)+r^6-3r^2r_0^4\right)\left(\beta+r^6-r^2r_0^4-1\right)+\frac{4r^8\left(-2\beta+r^6+r^2r_0^4+2\right)^2}{\left(\beta+r^6-r^2r_0^4-1\right)^2}$$

$$+\frac{4r^8\left(4r^{12}+8(2-3\beta)r^8r_0^4+(20\beta-23)r^4r_0^8+3(4\beta-1)r_0^{12}\right)^2}{\left(2r^8-5r^4r_0^4+3r_0^8\right)^2\left(2r^4+(1-4\beta)r_0^4\right)^2}$$

$$-\frac{2r^8\left(8r^{16}-60r^{12}r_0^4+6(40\beta(2\beta-3)+67)r^8r_0^8+(4\beta-1)(20\beta+43)r^4r_0^{12}-9(1-4\beta)^2r_0^{16}\right)}{\left(2r^8-5r^4r_0^4+3r_0^8\right)\left(2r^4+(1-4\beta)r_0^4\right)^2}$$

$$+\frac{1}{2r^4+(1-4\beta)r_0^4}[r^2\left(2r^8+2r^6-5r^4r_0^4+(1-4\beta)r^2r_0^4+3r_0^8\right)\left(\beta+r^6-r^4-r^2r_0^4-1\right)]$$

$$+\frac{4r^8\left(r^6+r^2r_0^4+2-2\beta\right)\left(4r^{12}+8(2-3\beta)r^8r_0^4+3(4\beta-1)r_0^{12}\right)}{\left(2r^4-3r_0^4\right)\left(r^4-r_0^4\right)\left(2r^4+(1-4\beta)r_0^4\right)\left(\beta+r^6-r^2r_0^4-1\right)}$$

$$+2r^8\left(\frac{2r^8+5r^4r_0^4-9r_0^8}{2r^8-5r^4r_0^4+3r_0^8}-\frac{4r^4}{2r^4+(1-4\beta)r_0^4}+\frac{r^2\left(3r^4-r_0^4\right)}{\beta+r^6-r^2r_0^4-1}\right)\Bigg\}$$

Deformed black brane metric:

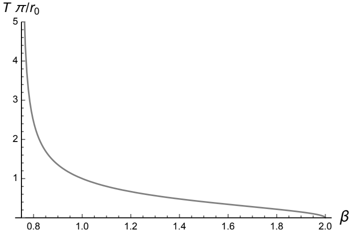
$$
\begin{aligned}
ds^2 &= -\frac{r_0^2}{u^2} N(u) \mathrm{d}t^2 + \frac{1}{u^2 A(u)} \mathrm{d}u^2 + \frac{r_0^2}{u^2} \delta_{ij} \mathrm{d}x^i \mathrm{d}x^j, \\
N(u) &= 1 - u^4 + (\beta - 1) u^6, \\
A(u) &= \left(1 - u^4\right) \left(\frac{2 - 3u^4}{2 - (4\beta - 1) u^4} \right),
\end{aligned}
$$

- Limit $\beta \to 1$: AdS_5–Schwarzschild black brane:

$$
ds^2 = -\frac{r_0^2}{u^2} \left(1 - u^4\right) \mathrm{d}t^2 + \frac{1}{u^2 \left(1 - u^4\right)} \mathrm{d}u^2 + \frac{r_0^2}{u^2} \delta_{ij} \mathrm{d}x^i \mathrm{d}x^j.
$$

- Hawking temperature at the horizon:

$$T = \frac{1}{4\pi}\sqrt{\lim_{u\to 1}\frac{g'_{tt}(u)}{g'_{rr}(u)}} = \frac{r_0}{\pi}\sqrt{\frac{\beta-2}{3-4\beta}}.$$



Deformed black brane temperature $\times$ β.

$\boxed{\Rightarrow \beta \in (0.75, 2)}$.

New black brane solutions

- Expand the action (near-horizon)

$$S_E = -\frac{1}{16\pi G}\overbrace{\int d^5x\sqrt{g}\,(R-2\Lambda_5)}^{I_{\rm bulk}} - \frac{1}{8\pi G}\overbrace{\lim_{u\to 0}\int d^4x\sqrt{h}K}^{I_{\text{Gibbons–Hawking}}} + I_{\rm c.t}\,,$$

- A. Martins, P. Meert, RdR, Nucl. Phys. B **957** (2020) 115087 [1912.04837 [hep-th]];
- R. Casadio, R. Cavalcanti, RdR, Eur. Phys. J .C **76** (2016) 556 [1601.03222 [hep-th]],

$$S_E = \frac{Vbr_0^4}{8\pi G}\left(\frac{11-15\beta+3\beta^2}{2}\right)$$

is the partition function in the dual field theory on the AdS_5 boundary (GKPW)

- S. S. Gubser, I. R. Klebanov, A. M. Polyakov, Phys. Lett. B **428** (1998) 105.
- E. Witten, Adv. Theor. Math. Phys. **2** (1998) 253.

- $S_E = bF$, where F = free energy.

Consistent with 2-loop quantum corrections to gravity

- Deformed black branes:

$$
\begin{aligned}
ds^2 &= -\frac{r_0^2}{u^2} N(u)\mathrm{d}t^2 + \frac{1}{u^2 A(u)}\mathrm{d}u^2 + \frac{r_0^2}{u^2}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j, \\
N(u) &= 1 - u^4 + (\beta - 1)\, u^6, \\
A(u) &= \left(1 - u^4\right)\left(\frac{2 - 3u^4}{2 - (4\beta - 1)\, u^4}\right),
\end{aligned}
$$

- **Gibbons–Hawking** term:

$$
\begin{aligned}
&-\frac{4}{u^4\left((1-4\beta)u^4+2\right)^2}\sqrt{-\frac{(3u^8-5u^4+2)\left((\beta-1)u^6-u^4+1\right)}{(4\beta-1)u^4-2}} \\
&\times u^4\left[-32\beta + u^2\left(-4\beta + u^2\left(56\beta + 9(\beta-1)(4\beta-1)u^{10} + (6-24\beta)u^8\right.\right.\right. \\
&\left.\left.\left.-5\left(4\beta^2+\beta-5\right)u^6 + 24u^4 - 8(\beta-4)(\beta-1)u^2 - 46\right) + 4\right) + 8\right].
\end{aligned}
$$

- **Counterterm**: $\sim u^{-4}\sqrt{N(u)A(u)}$.

New black brane solutions: thermodynamics

- **Free energy**:

$$F = \frac{\pi^3 V}{8G}\left(\frac{11-15\beta+3\beta^2}{2}\right)\left(\frac{3-4\beta}{\beta-2}\right)^2 T^4,$$

- Entropy density:

$$s = -\frac{1}{V}\frac{\partial F}{\partial T} = -\frac{\pi^3}{2G}\left(\frac{11-15\beta+3\beta^2}{2}\right)\left(\frac{3-4\beta}{\beta-2}\right)^2 T^3, .$$

- Pressure:

$$P = -\frac{\partial F}{\partial V} = -\frac{\pi^3}{8G}\left(\frac{11-15\beta+3\beta^2}{2}\right)\left(\frac{3-4\beta}{\beta-2}\right)^2 T^4 ,$$

- Energy density:

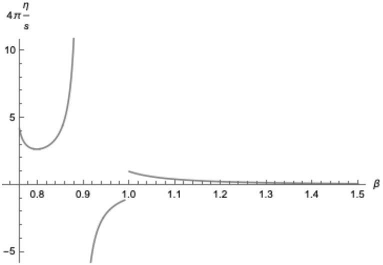
$$\varepsilon = \frac{F}{V} - Ts = \frac{5\pi^3}{8G}\left(\frac{11-15\beta+3\beta^2}{2}\right)\left(\frac{3-4\beta}{\beta-2}\right)^2 T^4$$

- **Specific heat**:

$$C_V = -\frac{3\pi^3}{2G}\left(\frac{11-15\beta+3\beta^2}{2}\right)\left(\frac{3-4\beta}{\beta-2}\right)^2 T^3.$$

- **Shear viscosity-to-entropy density**:

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(\frac{1}{11 - 15\beta + 3\beta^2}\right)\left(\frac{\beta - 2}{3 - 4\beta}\right)^{1/2}$$



$\frac{\eta}{s}$ as a function of β.

New black brane solutions

- Deformed black brane metric:

$$
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ds^2 &= -\frac{r_0^2}{u^2} N(u) \mathrm{d}t^2 + \frac{1}{u^2 A(u)} \mathrm{d}u^2 + \frac{r_0^2}{u^2} \delta_{ij} \mathrm{d}x^i \mathrm{d}x^j, \\
N(u) &= 1 - u^4 + (\beta - 1)\, u^6, \\
A(u) &= \left(1 - u^4\right) \left(\frac{2 - 3u^4}{2 - (4\beta - 1)\, u^4} \right),
\end{aligned}
$$

- For $\beta \to 1$, the KSS result for the AdS_5–Schwarzschild black brane is obtained:

$$
\frac{\eta}{s} = \frac{1}{4\pi}
$$

- **Bulk viscosity-to-entropy density ratio**:

$$\frac{\zeta}{s} = \beta^4(12\beta^2 - 2\beta + 7)\Pi\left(\frac{(12\beta^2 - \beta^3 + 9)}{(6-5\beta)^2}; \tanh^{-1}(\beta^2 - 3)\middle|\beta^2 - 1\right)$$
$$+(12\beta^2 - 2\beta + 7)F\left(\tanh^{-1}\left((\beta^2 - 3)\right), \frac{14\beta^2 - 6\beta + 9}{(\beta+1)^2}\right),$$

where Π and F are incomplete elliptic integrals.

- I. Kuntz, RdR, Nucl. Phys. B **993** (2023) 116258 [arXiv:2211.11913 [hep-th]].
- RdR, Phys. Rev. D **105** (2022) 026014 [arXiv:2111.01244 [hep-th]].
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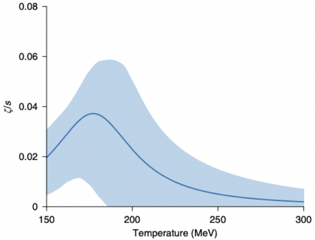
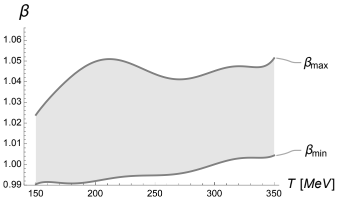
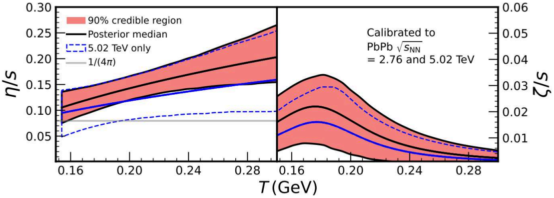


Duke group (J. E. Bernhard, J. S. Moreland, S. A. Bass, Nature Phys. **15** (2019) 1113).

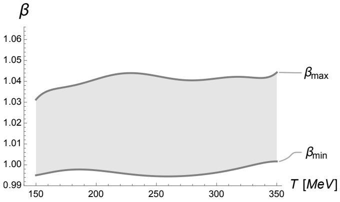


QGP: Duke group (**RdR, 2409.17325 [hep-th]**).

QGP and experiments: Jyväskylä-Helsinki-Munich

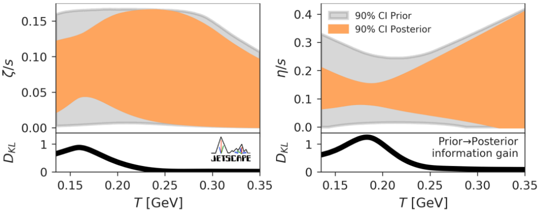


QGP: Jyväskylä-Helsinki-Munich group (J. E. Parkkila, A. Onnerstad, S. F. Taghavi, C. Mordasini, A. Bilandzic, M. Virta, D. J. Kim, Phys. Lett. B **835** (2022) 137485 .

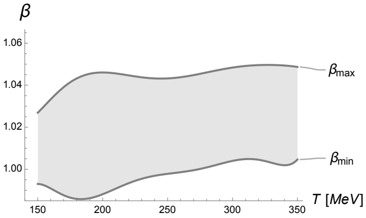


QGP at LHC: Jyväskylä-Helsinki-Munich group **(RdR, 2409.17325 [hep-th])**.

QGP and experiments: JETSCAPE Bayesian model



JETSCAPE Bayesian model (D. Everett *et al.* [JETSCAPE], Phys. Rev. Lett. **126** (2021) 242301).



RHIC + LHC; JETSCAPE Bayesian model **(RdR, 2409.17325 [hep-th])**.

New black brane solutions

- Deformed black brane metric in AdS_5, from embedding protocol:

$$
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- It implies that considering

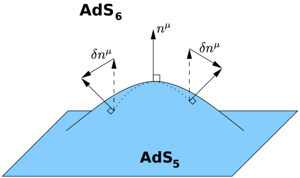
$$S = \int d^5x\sqrt{-g}\,[R - 2\Lambda_5]$$

is not enough! However, Kovtun, Son, Starinets, Phys. Rev. Lett. **94** (2005) 111601 does not consider embeddings.

Deformed black branes

1$^{\rm st}$ construction: **deformed black branes** by embedding.

$$ds^2 = -\frac{r_0^2}{u^2}N(u)\mathrm{d}t^2 + \frac{1}{u^2 A(u)}\mathrm{d}u^2 + \frac{r_0^2}{u^2}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j,$$



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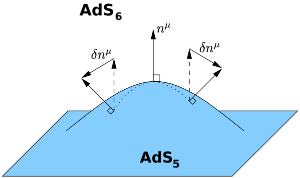

(Assuming AdS$_6$ might be somehow artificial, from the **top-down** point of view).

Consistent with 2-loop quantum corrections to gravity

- 2^{nd} construction: exact solution of the action **General relativity** + **Lee–Wick** + **Ricci cubic gravity** + **Einstein cubic gravity** + Gibbons–Hawking (GB) + counterterm (c.t).

$$
\begin{aligned}
S \;=\; & \int d^5x\sqrt{-g}\,[R-2\Lambda_5 \\
& +\beta_1 G_{\mu\nu}\Box R^{\mu\nu} \\
& +\beta_2\Big(-\frac{65}{324}R^3+\frac{29}{27}RR_{\mu\nu}R^{\mu\nu}-\frac{59}{81}R_\nu^\mu R_\rho^\nu R_\mu^\rho+14R_{\mu\nu}^{\ \ \rho\sigma}R_{\rho\sigma}^{\ \ \alpha\beta}R_{\alpha\beta}^{\ \ \rho\sigma} \\
& \quad -4R_{\mu\nu\rho\sigma}R^{\mu\nu\rho}_{\ \ \ \ \alpha}R^{\sigma\alpha}-\frac{7}{108}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}R+4R_{\mu\nu\rho\sigma}R^{\mu\rho}R^{\nu\sigma}\Big) \\
& +\beta_3\Big(\nabla_\mu R_{\rho\sigma}\nabla^\mu R^{\rho\sigma}+\nabla_\mu R_{\rho\sigma}\nabla^\sigma R^{\mu\rho}+\nabla_\mu R\nabla^\mu R+\nabla_\mu R_{\rho\sigma\tau\xi}\nabla^\mu R^{\rho\sigma\tau\xi} \\
& \quad -R^{\mu\nu}\Box R_{\mu\nu}+\frac{3}{8}R_{\mu\nu}\nabla^\mu\nabla^\nu R+\frac{7}{18}R\nabla^\mu\nabla^\nu R_{\mu\nu}\Big)\Big] \\
& +\overbrace{\lim_{u\to0}\int d^4x\sqrt{g}K}^{S_{GH}}+S_{\text{c.t}},
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$$

(**RdR, 2409.17325 [hep-th]**)

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& \qquad -4R_{\mu\nu\rho\sigma}R^{\mu\nu\rho}_{\ \ \ \ \alpha}R^{\sigma\alpha}-\frac{7}{108}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}R+4R_{\mu\nu\rho\sigma}R^{\mu\rho}R^{\nu\sigma}\Big) \\
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& \qquad -R^{\mu\nu}\Box R_{\mu\nu}+\frac{3}{8}R_{\mu\nu}\nabla^\mu\nabla^\nu R+\frac{7}{18}R\nabla^\mu\nabla^\nu R_{\mu\nu}\Big)\Big] \\
& +\overbrace{\lim_{u\to 0}\int d^4x\sqrt{g}K}^{S_{GH}}+S_{\rm c.t},
\end{aligned}
$$

(**RdR, 2409.17325 [hep-th]**)

- Consistent with **2-loop** quantum corrections to **5D gravity**:

  - M. H. Goroff and A. Sagnotti, Nucl. Phys. B **266** (1986) 709.

Applications to QCD

- Literature: QGP $\sim$ QFT dual to the AdS_5–Schwarzschild black brane.

  R. Baier, P. Romatschke, D. T. Son, A. O. Starinets, M. A. Stephanov, JHEP **0804** (2008) 100.

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- RdR, Phys. Rev. D **105** (2022) 026014 [arXiv:2111.01244 [hep-th]].

  Deformed black branes in Poincaré-like coordinates:

$$ds^2 = \frac{R^2 e^{cz^2/2}}{z^2}\left(-N(z)\mathrm{d}t^2 + \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j + \frac{1}{A(z)}\mathrm{d}z^2\right),$$

where

$$\begin{aligned} N(z) &= 1 - \frac{z^4}{z_0^4} + (\beta - 1)\,\frac{z^6}{z_0^6}, \\ A(z) &= \left(1 - \frac{z^4}{z_0^4}\right)\left(\frac{2 - \frac{3z^4}{z_0^4}}{2 - (4\beta - 1)\,\frac{z^4}{z_0^4}}\right). \end{aligned}$$

with event horizon z_0.

Applications to QCD

• RdR, Phys. Rev. D **105** (2022) 026014 [arXiv:2111.01244 [hep-th]]:

Hagedorn temperature, QGP:

$$T_c = \frac{1}{\pi}\sqrt{\frac{c(\beta-2)}{2(3-4\beta)}}.$$

Data from HotQCD Collaboration:

$T_c = 156.5 \pm 1.5$ **MeV** [A. Bazavov *et al.*, Phys. Lett. B **795** (2019) 15 [**HotQCD**]$\Rightarrow \beta = 1.025$

$T_c = 158.0 \pm 0.6$ **MeV** [S. Borsanyi *et al.*, Phys. Rev. Lett. **125** (2020) 052001] $\Rightarrow \beta = 1.021$.

Conclusions

- AdS-Schwarzschild black branes are robust, but it is allowed to (slightly) deform them, up to $\sim 2.5\%$.

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- AdS-Schwarzschild black branes are robust, but it is allowed to (slightly) deform them, up to $\sim 2.5\%$.
- Deformed black branes in AdS_5 are obtained by embedding or considering higher-order curvature terms.
- We used experimental data from the QGP to bound deformations of the AdS-Schwarzschild black brane.

The end

Thanks

Universidade Federal do ABC

CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

FAPESP

CAPES

AdS_4/CFT_3 Condensed Matter Theory

- Bulk: AdS_4; $\Leftrightarrow$ dual field theory = condensed matter

$$S = \int d^4x \sqrt{-g} \left(R - 2\Lambda_4 - \frac{L^2}{4} F_{\mu\nu} F^{\mu\nu} \right).$$

- Only regular solution, with electric flux on the AdS_4 boundary and Poincaré symmetry:
  Extreme AdS_4–Reissner–Nordström black brane, with planar horizon:

$$ds_4^2 = -\frac{r^2 f(r)}{L^2} dt^2 + \frac{L^2}{r^2 f(r)} dr^2 + \frac{r^2}{L^2} dx^2 + \frac{r^2}{L^2} dy^2,$$

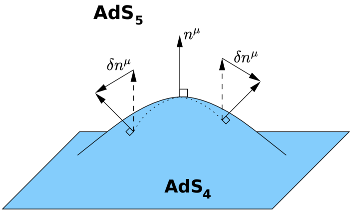
  with *blackening factor*:

$$f(r) = 1 - (1 + Q^2) \left(\frac{r_0}{r}\right)^3 + Q^2 \left(\frac{r_0}{r}\right)^4.$$

- A. J. Ferreira-Martins, P. Meert, RdR, Eur. Phys. J. C **79** (2019) 646 [arXiv:1904.01093 [hep-th]]
- RdR, Annals Phys. **465** (2024) 169663, 2310.07860 [hep-th]

Deformed black branes:

$$ds_4^2 = -\frac{r^2 f(r)}{L^2}dt^2 + n(r)\frac{L^2}{r^2}dr^2 + \frac{r^2}{L^2}dx^2 + \frac{r^2}{L^2}dy^2.$$



New black brane solutions em AdS_4

- A. J. Ferreira-Martins, P. Meert, RdR, Eur. Phys. J. C **79** (2019) 646 [arXiv:1904.01093 [hep-th]]
- RdR, Annals Phys. **465** (2024) 169663, arXiv:2310.07860 [hep-th]

AdS_4 deformed black brane:

$$
\begin{aligned}
ds_4^2 \quad = \quad & -\frac{r^2}{L^2}\left[1-(1+\mathfrak{q}^2)\left(\frac{r_0}{r}\right)^3+\mathfrak{q}^2\left(\frac{r_0}{r}\right)^4\right]dt^2 \\
& +\left\{\frac{1}{\left(1-(1+\mathfrak{q}^2)\left(\frac{r_0}{r}\right)^3+\mathfrak{q}^2\left(\frac{r_0}{r}\right)^4\right)}\left(\frac{1-\frac{r_0}{r}}{1-\frac{r_0}{r}\left[1+\frac{1}{3}(\beta-1)\right]}\right)\right\}\frac{L^2}{r^2}dr^2 \\
& +\frac{r^2}{L^2}dx^2+\frac{r^2}{L^2}dy^2.
\end{aligned}
$$

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& +\frac{r^2}{L^2}dx^2+\frac{r^2}{L^2}dy^2.
\end{aligned}
$$

- Limit $\beta \to 1$: **AdS_4–Reissner–Nordström black brane**.

New black brane solutions in AdS_4

- Solution: **Deformed black brane metric in AdS_4**:

$$
\begin{aligned}
ds_4^2 \;=\;& -\frac{r^2}{L^2}\left[1-(1+\mathfrak{q}^2)\left(\frac{r_0}{r}\right)^3+\mathfrak{q}^2\left(\frac{r_0}{r}\right)^4\right]dt^2 \\
&+\left\{\frac{1}{\left(1-(1+\mathfrak{q}^2)\left(\frac{r_0}{r}\right)^3+\mathfrak{q}^2\left(\frac{r_0}{r}\right)^4\right)}\left(\frac{1-\frac{r_0}{r}}{1-\frac{r_0}{r}\left[1+\frac{1}{3}(\beta-1)\right]}\right)\right\}\frac{L^2}{r^2}dr^2 \\
&+\frac{r^2}{L^2}dx^2+\frac{r^2}{L^2}dy^2.
\end{aligned}
$$

- Limit $\lim_{r\to r_\beta} 1/n(r)=0$ implies an additional coordinate singularity:

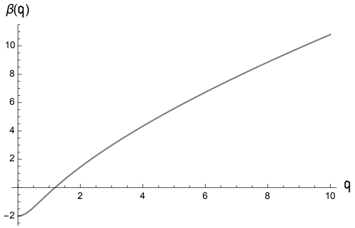
$$
r_\beta=\frac{r_0}{3}\left[2+\beta\right],
$$

- For r_β to be an event horizon = Killing horizon, either

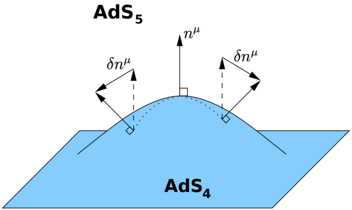
$$\beta = 1\,,$$

(corresponding to AdS_4–RN), or

$$\beta = \frac{2\sqrt[3]{2}}{\sqrt[3]{-7-27\mathfrak{q}^2+3\sqrt{3}\sqrt{3+14\mathfrak{q}^2+27\mathfrak{q}^4}}} + \frac{1}{\sqrt[3]{2}}\sqrt[3]{7+27\mathfrak{q}^2-3\sqrt{3}\sqrt{3+14\mathfrak{q}^2+27\mathfrak{q}^4}} - 3.$$



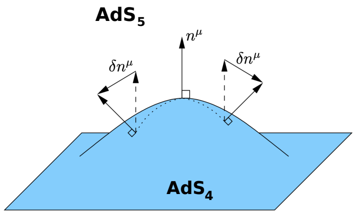
$\beta(\mathfrak{q}) \times \mathfrak{q}$.



- **Holographic supercondutors**:

$$S_{\text{BULK}} = \int \sqrt{-g}\,(R - 2\Lambda_4)\; d^4x + S_{\text{HS}}\ ,$$

$$S_{\text{HS}} = -\frac{1}{g_{\text{YM}}^2} \int \sqrt{-g} \left(\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + D^\mu \phi D_\mu \phi^* + m^2 |\phi|^2 \right)\; d^4x,$$



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$$S_{\text{BULK}} = \int \sqrt{-g}\,(R - 2\Lambda_4)\; d^4x + S_{\text{HS}}\ ,$$

$$S_{\text{HS}} = -\frac{1}{g_{\text{YM}}^2}\int \sqrt{-g}\left(\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + D^\mu\phi D_\mu\phi^* + m^2|\phi|^2\right)\, d^4x,$$

- Near-boundary limit, $u \to 0$, Higgs field in Ginzburg–Landau theory,

$$\phi(u) = \phi_1 u + \phi_2 u^2.$$

- Result for the **AdS$_4$–Reissner–Nordström black brane**:

$$ds_4^2 = -\frac{r^2}{L^2}\left[1-(1+q^2)\left(\frac{r_0}{r}\right)^3+q^2\left(\frac{r_0}{r}\right)^4\right]dt^2+\frac{1}{1-(1+q^2)\left(\frac{r_0}{r}\right)^3+q^2\left(\frac{r_0}{r}\right)^4}\frac{L^2}{r^2}dr^2$$
$$+\frac{r^2}{L^2}dx^2+\frac{r^2}{L^2}dy^2.$$

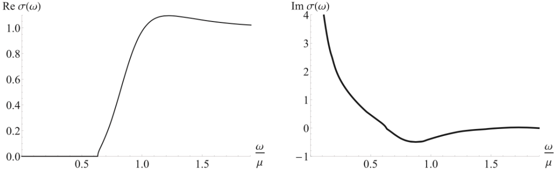


Figure 15.6 Real and imaginary parts of the conductivity $\sigma(\omega)$ at vanishing temperature $T = 0$ for the holographic superfluid with condensate $\langle \mathcal{O}_2 \rangle$. The real part of the conductivity displays a superconducting gap at low frequencies.

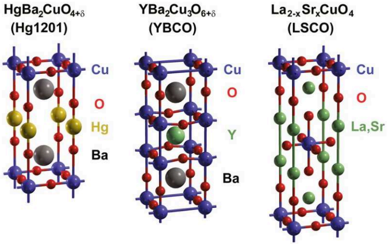
(M. Ammon and J. Erdmenger, *Gauge/gravity duality: Foundations and applications*, Cambridge University Press, 2015).

Holographic supercondutors

- ∼ 2000 types of **doped cuprates**

(Chin. Phys. Lett. **39** (2022) 077403; Physica C **514** (2015) 290).

• N. Barisic, M. Chan, Yuan Li, G. Yu, X. Zhao, M. Dressel, A. Smontara, M. Greven, Proc. Nat. Acad. Sci. **110** (30) 12235



Deformed black branes in AdS_4

- RdR, Annals Phys. **465** (2024) 169663, 2310.07860 [hep-th]

$$
\begin{aligned}
ds_4^2 \;=\;& -\frac{r^2}{L^2}\left[1-(1+\mathfrak{q}^2)\left(\frac{r_0}{r}\right)^3+\mathfrak{q}^2\left(\frac{r_0}{r}\right)^4\right]dt^2 \\
&+\left\{\frac{1}{\left(1-(1+\mathfrak{q}^2)\left(\frac{r_0}{r}\right)^3+\mathfrak{q}^2\left(\frac{r_0}{r}\right)^4\right)}\left(\frac{1-\frac{r_0}{r}}{1-\frac{r_0}{r}\left[1+\frac{1}{3}(\beta-1)\right]}\right)\right\}\frac{L^2}{r^2}dr^2 \\
&+\frac{r^2}{L^2}dx^2+\frac{r^2}{L^2}dy^2.
\end{aligned}
$$

- Maxwell equations:

$$u^4\partial_u^2 A_i + \left(2u^3 + \frac{(f'(u)n(u) + n'(u)f(u))}{2\sqrt{f(u)n(u)}}\right)\partial_u A_i + \left(\frac{\omega^2}{f^2(u)} + \frac{m^2\phi^2(u)}{\sqrt{f(u)n(u)}}\right)A_i = 0.$$

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- **Solution**: asymptotic behavior, for $u \to 0$:

$$\delta A_i = \delta A_i^{[0]} + A_i^{[1]}u + \mathcal{O}(u^2),$$

for $\delta A_i^{[0]} \sim A_i$, $A_i^{[1]} \sim \langle J^x \rangle$, whereas $E_i = \lim_{u\to 0}\partial_t\left(\delta A_i\right) = i\omega\delta A_i^{[0]}$.

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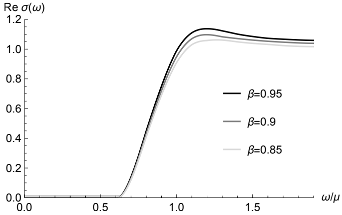
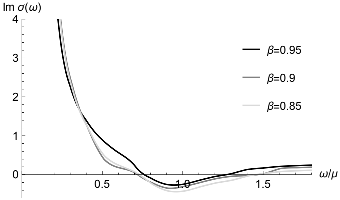
- The term $-A_i^{[0]}$ can be interpreted as the **superfluid velocity** whereas $A_i^{[1]}$ is the **supercurrent**.
- **Conductivity: Ohm's law**:

$$\sigma(\omega) = \frac{\langle J^i \rangle}{E^i} = -i\frac{\delta A_i^{[1]}}{\omega\delta A_i^{[0]}}.$$

Deformed black branes in AdS_4

- RdR, Annals Phys. 465 (2024) 169663 (2310.07860 [hep-th])





DC conductivity for holographic supercondutors $\times \omega/\mu$ for $T = 0$.

Holographic supercondutors

- $\sim$ 2000 types of **doped cuprates**

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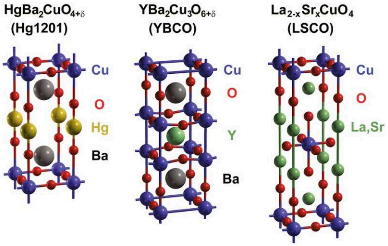

Bulk: AdS_4; $\Leftrightarrow$ dual field theory = condensed matter

$$S = \int d^4x\sqrt{-g}\left(f_3\left(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma}\right) - \frac{L^2}{4}F_{\mu\nu}F^{\mu\nu}\right),$$

where

$$\begin{aligned}
f_3 \quad = \quad & (R-2\Lambda_4) + \beta_1 G_{\mu\nu}\Box R^{\mu\nu} \\
& +\beta_2\left(-\frac{7}{20}R^3 + \frac{7}{5}RR_{\mu\nu}R^{\mu\nu} - \frac{7}{3}R_\nu^{\ \mu}R_\rho^{\ \nu}R_\mu^{\ \rho} + 14R_{\mu\nu}^{\ \ \rho\sigma}R_{\rho\sigma}^{\ \ \alpha\beta}R_{\alpha\beta}^{\ \ \rho\sigma}\right. \\
& \qquad \left. -4R_{\mu\nu\rho\sigma}R^{\mu\nu\rho}_{\ \ \ \ \alpha}R^{\sigma\alpha} - \frac{7}{20}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}R + 4R_{\mu\nu\rho\sigma}R^{\mu\rho}R^{\nu\sigma}\right) \\
& +\beta_3\left(\nabla_\mu R_{\rho\sigma}\nabla^\mu R^{\rho\sigma} + \nabla_\mu R_{\rho\sigma}\nabla^\sigma R^{\mu\rho} + \nabla_\mu R\nabla^\mu R + \nabla_\mu R_{\rho\sigma\tau\xi}\nabla^\mu R^{\rho\sigma\tau\xi}\right) \\
& \qquad \left.\left. -R^{\mu\nu}\Box R_{\mu\nu} + \frac{1}{6}R_{\mu\nu}\nabla^\mu\nabla^\nu R + \frac{3}{5}R\nabla^\mu\nabla^\nu R_{\mu\nu}\right)\right]
\end{aligned}$$

Lee–Wick + cubic gravity: renormalizability

- Lee–Wick: renormalizability and finite in $D = 5$
  - L. Modesto, Nuc. Phys. B **909** (2016) 584.
  - I. L. Shapiro, Phys. Lett. B **744** (2015) 67 [arXiv:1502.00106 [hep-th]].

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- A QFT is considered to be **finite** if the corresponding renormalization constants evaluated in the dimensional regularization scheme are free from divergences in all orders of perturbation theory.

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- Einsteinian cubic gravity:

  A. De Felice, S. Tsujikawa, Phys. Lett. B **843** (2023) 138047 [arXiv:2305.07217 [gr-qc]]:

  for $D = 4$ and $g_{tt} = -g_{rr}$, there is a ghost propagation mode.

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  for $D = 4$ and $g_{tt} = -g_{rr}$, there is a ghost propagation mode.
- Here $g_{tt} = -g_{rr}$ and it is (seems to be) ghost-free em 5D:
  - Y. Z. Li, H. Lu and J. B. Wu, *Causality and a-theorem Constraints on Ricci Polynomial and Riemann Cubic Gravities*, Phys. Rev. D **97** (2018) 024023 [arXiv:1711.03650 [hep-th]].

Graphene

- **Wiedemann–Franz law** (1853): for metals (**= Fermi liquid**),

$$\frac{\text{thermal conductivity}}{\text{electrical conductivity}} = \frac{\kappa}{\sigma} = LT \text{ (temperature)},$$

where

$$L = \frac{\pi^2}{3}\left(\frac{k_b}{e}\right)^2 = 2.44 \times 10^{-8} W\Omega/K^2$$

is the Lorenz number.

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  is the Lorenz number.

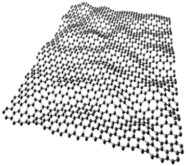
- Graphene **is not** a Fermi liquid [J. Crossno et al., Science **351** (2016) 1058]:
  Wiedemann–Franz law violation: graphene ($T \sim 10 - 100$ K):

$$\frac{\kappa}{\sigma} \approx \mathbf{20} \times LT,$$

- Thermal conductivity:

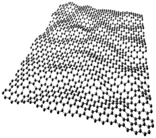
$$\kappa(\omega) = \frac{i}{\omega}(\epsilon + p - 2\mu\rho) + \mu^2\sigma(\omega).$$

Graphene



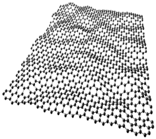
- ● A. Lucas, J. Crossno, K. Fong, P. Kim, S. Sachdev. Phys. Rev. B **93** (2016) 075426.

  Dirac fluids: electron-hole relativistic plasma in **graphene** = strongly-coupled quantum critical system (**strange metal**).



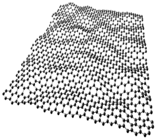
- Graphene and Dirac fluids.

Graphene



- Graphene and Dirac fluids.
- Dilaton (ϕ) + gauge fields (A_μ, B_μ), with $F = dA$ and $G = dB$; dissipation fields (χ_1, χ_2).

Graphene



- Graphene and Dirac fluids.
- Dilaton (ϕ) + gauge fields (A_μ, B_μ), with $F = dA$ and $G = dB$; dissipation fields (χ_1, χ_2).
- Action: (Y. Seo, G. Song, P. Kim, S. Sachdev, PRL **118** (2017) 036601)

$$\begin{aligned}\mathcal{L} &= R - \frac{1}{2}\left[\nabla_\mu\phi\nabla^\mu\phi + \Phi(\phi)\left(\nabla_\mu\chi_1\nabla^\mu\chi_1 + \nabla_\mu\chi_2\nabla^\mu\chi_2\right)\right] - V(\phi) \\ &\quad - \frac{Z(\phi)}{4}F^2 - \frac{W(\phi)}{4}G^2,\end{aligned}$$

$$\boxed{\left(\frac{\kappa}{\sigma}\right)_{\text{THEORY}} \sim 0.9 \times \left(\frac{\kappa}{\sigma}\right)_{\text{EXPERIMENTAL}}}$$

Graphene

- AdS_4 Deformed black branes: graphene.

- AdS$_4$ Deformed black branes: graphene.
- Action:

$$
\begin{aligned}
\mathcal{L} \quad = \quad & (R - 2\Lambda_4) + \beta_1 G_{\mu\nu}\Box R^{\mu\nu} \\
& + \beta_2 \left(-\frac{7}{20} R^3 + \frac{7}{5} R R_{\mu\nu} R^{\mu\nu} - \frac{7}{3} R^{\mu}_{\nu} R^{\nu}_{\rho} R^{\rho}_{\mu} + 14 R_{\mu\nu}^{\ \ \rho\sigma} R_{\rho\sigma}^{\ \ \alpha\beta} R_{\alpha\beta}^{\ \ \mu\nu} \right. \\
& \quad \left. - 4 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho}_{\ \ \ \alpha} R^{\sigma\alpha} - \frac{7}{20} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} R + 4 R_{\mu\nu\rho\sigma} R^{\mu\rho} R^{\nu\sigma} \right) \\
& + \beta_3 \left(\nabla_\mu R_{\rho\sigma} \nabla^\mu R^{\rho\sigma} + \nabla_\mu R_{\rho\sigma} \nabla^\sigma R^{\mu\rho} + \nabla_\mu R \nabla^\mu R + \nabla_\mu R_{\rho\sigma\tau\xi} \nabla^\mu R^{\rho\sigma\tau\xi} \right) \\
& \quad - \frac{1}{2} \left[\nabla_\mu \phi \nabla^\mu \phi + \Phi(\phi) \left(\nabla_\mu \chi_1 \nabla^\mu \chi_1 + \nabla_\mu \chi_2 \nabla^\mu \chi_2 \right) \right] \\
& \quad - V(\phi) - \frac{Z(\phi)}{4} F^2 - \frac{W(\phi)}{4} G^2 .
\end{aligned}
$$

Graphene

- AdS$_4$ Deformed black branes: graphene.
- Action:

$$
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&\qquad\left.-4R_{\mu\nu\rho\sigma}R^{\mu\nu\rho}_{\ \ \ \ \alpha}R^{\sigma\alpha}-\frac{7}{20}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}R+4R_{\mu\nu\rho\sigma}R^{\mu\rho}R^{\nu\sigma}\right) \\
&+\beta_3\left(\nabla_\mu R_{\rho\sigma}\nabla^\mu R^{\rho\sigma}+\nabla_\mu R_{\rho\sigma}\nabla^\sigma R^{\mu\rho}+\nabla_\mu R\nabla^\mu R+\nabla_\mu R_{\rho\sigma\tau\xi}\nabla^\mu R^{\rho\sigma\tau\xi}\right) \\
&\qquad-\frac{1}{2}\left[\nabla_\mu\phi\nabla^\mu\phi+\Phi(\phi)\left(\nabla_\mu\chi_1\nabla^\mu\chi_1+\nabla_\mu\chi_2\nabla^\mu\chi_2\right)\right] \\
&\qquad-V(\phi)-\frac{Z(\phi)}{4}F^2-\frac{W(\phi)}{4}G^2.
\end{aligned}
$$

- $$\Rightarrow \boxed{\left(\frac{\kappa}{\sigma}\right)_{\text{THEORY}} = 0.986\times\left(\frac{\kappa}{\sigma}\right)_{\text{EXPERIMENTAL}}}$$

- **Electromagnetic potential**: solution to Maxwell equations $\partial_\mu\left(\sqrt{-g}F^{\mu\nu}\right)=0$:

$$A(r)=\alpha(\beta,r)\mathfrak{q}\left(2\sqrt{r-r_0}\sqrt{\beta+2}\sqrt{3r-r_0\,(\beta+2)}+r\,(\beta-1)\left[\pi-2\tan^{-1}\left(\frac{\sqrt{r-r_0(\beta+2)}}{\sqrt{3r-r_0\,(\beta+2)}}\right)\right]\right)$$

where

$$\alpha(\beta,r)=\frac{1}{2\sqrt{3}r_0\sqrt{\beta+2}\,r}.$$

New black brane solutions in AdS_4

- **Electromagnetic potential**: solution to Maxwell equations $\partial_\mu\left(\sqrt{-g}F^{\mu\nu}\right)=0$:

$$A(r)=\alpha(\beta,r)\mathfrak{q}\left(2\sqrt{r-r_0}\sqrt{\beta+2}\sqrt{3r-r_0\,(\beta+2)}+r\,(\beta-1)\left[\pi-2\tan^{-1}\left(\frac{\sqrt{r-r_0(\beta+2)}}{\sqrt{3r-r_0\,(\beta+2)}}\right)\right]\right)$$

where

$$\alpha(\beta,r)=\frac{1}{2\sqrt{3}r_0\sqrt{\beta+2}\,r}.$$

- AdS_4–RN setup when $\beta\to 1$:

$$\lim_{\beta\to 1}A(r)=\mathfrak{q}\left(\frac{1}{r}-\frac{1}{r_0}\right).$$

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- **Chemical potential** of the CFT_3 boundary:

$$\mu=-\frac{\mathfrak{q}}{6r_0}\left[6+\frac{\sqrt{3}\,(\beta-1)}{\sqrt{2+\beta}}\arctan\left(\sqrt{\frac{\beta+2}{3}}\right)\right].$$

Violation of the KSS limit

- R. Cai, Z. Nie, N. Ohta, Y. W. Sun, Phys. Rev. D **79** (2009) 066004.
  M. Brigante, H. Liu, R. Myers, S. Shenker, Phys. Rev. D **77** (2008) 126006.
- Gauss–Bonnet + dilaton

$$S = -\frac{1}{16\pi G}\int d^5x\sqrt{-g}\left(R - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2 - 2\Lambda_5\right).$$

- Violation:

$$\frac{\eta}{s} = \frac{16}{25}\frac{1}{4\pi} < \frac{1}{4\pi}.$$

g(*tt*) and g(*rr*)

- T. Jacobson, *When is g(tt) g(rr) = -1*," Class. Quant. Grav. **24** (2007) 5717 [arXiv:0707.3222 [gr-qc]].
  M. Salgado, *A Simple theorem to generate exact black hole solutions*, Class. Quant. Grav. **20** (2003) 4551 [arXiv:gr-qc/0304010 [gr-qc]].
- Metrics with $g_{tt}g_{rr} = -1$ have Ricci tensors (and stress-energy tensor) with vanishing radial null-null components (or, equivalently, if the restriction of $R_{\mu\nu}|_{t\,-\,r\text{ subspace}} \propto g_{\mu\nu}$ (which implies that the radial pressure is equal to minus the energy density).
- $g_{tt}g_{rr} \neq -1$ the Morris–Thorne traversable wormhole, the Damour–Solodukhin wormhole, the Joshi-Malafarina-Narayan singularity, the naked singularity surrounded by a thin shell of matter, the BH in Clifton-Barrow f(R) gravity, the Sen BH, the Einstein–Maxwell–dilaton-1 BH, the BH in Loop Quantum Gravity, the DST BH, the BH in bumblebee gravity, and the Casimir wormhole (footnote 8, S. Vagnozzi, R. Roy, Y. D. Tsai, L. Visinelli, M. Afrin, A. Allahyari, P. Bambhaniya, D. Dey, S. G. Ghosh and P. S. Joshi, *et al.* Class. Quant. Grav. **40** (2023) 165007 [arXiv:2205.07787 [gr-qc]].)

- Only $F_{rt} \neq 0$:

$$T_{tt} = F_t{}^r F_{rt} - \frac{1}{4} g_{tt} F^2, \qquad T_{rr} = F_r{}^t F_{tr} - \frac{1}{4} g_{rr} F^2, \qquad T_{xx} = -\frac{1}{4} g_{xx} F^2 = T_{yy}.$$

- Equivalently, $A = A(r)\, dt$, and

$$A(r) = \mu - \frac{\mathrm{Q}}{r}, \tag{3}$$

with limit $\lim_{r \to r_0} A(r) = 0$ and $\mu = \frac{\mathrm{Q}}{r_0}$.

Transporte termoelétrico

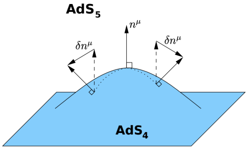
- Q = heat current; α = thermoelectric conductivity:

$$\begin{aligned} \vec{J} &= \sigma\vec{E} - \alpha\nabla T, \\ \vec{Q} &= \alpha T\vec{E} - \kappa\nabla T. \end{aligned}$$

- Thermal conductivity:

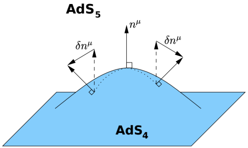
$$\begin{aligned} \kappa &= -\frac{i\omega}{T} G_R^{\vec{Q}\vec{Q}} \\ &= \frac{i}{\omega}(\epsilon + p - 2\mu\rho) + \mu^2\sigma(\omega), \end{aligned} \tag{4}$$

where $\rho = T_{00}$, $p = T_{11}$, and μ = chemical potential (Erdmenger, Eq. (15.47), p. 470.)



$$ds_4^2 = -\frac{r^2 f(r)}{L^2}dt^2 + n(r)\frac{L^2}{r^2}dr^2 + \frac{r^2}{L^2}dx^2 + \frac{r^2}{L^2}dy^2,$$

$f(r)$ fixed (AdS_4-RN).



$$ds_4^2 = -\frac{r^2 f(r)}{L^2} dt^2 + n(r)\frac{L^2}{r^2} dr^2 + \frac{r^2}{L^2} dx^2 + \frac{r^2}{L^2} dy^2,$$

$f(r)$ fixed (AdS$_4$-RN).

- **Hamiltonian + momentum constraints**:

$$n(r)\left[\frac{(r-r_0)n'(r)}{L^4}\left(-\mathfrak{q}^2 r_0^3 + r^3 + r^2 r_0 + r r_0^2\right)\left(\left(\left(\mathfrak{q}^2+1\right) r r_0^3 - 2\mathfrak{q}^2 r_0^4 + 2r^4\right)(r-(2\beta+7)r_0)^3 + 4r^3\right]\right.$$
$$+\frac{4r^4 n'(r)}{n(r)} - \frac{r^3}{\left[\left(\mathfrak{q}^2+1\right) r r_0^3 - \mathfrak{q}^2 r_0^4 - r^4\right]^2}\left[4\mathfrak{q}^4 r_0^8 - 24\left(\mathfrak{q}^2+1\right) r^5 r_0^3 + 32\mathfrak{q}^2 r^4 r_0^4\left[6\beta + (4\beta+59)\mathfrak{q}^2 + 21\right]\right.$$
$$\left. +3\left(\mathfrak{q}^2+1\right)^2 r^2 r_0^6 + \left(2\beta - 27\mathfrak{q}^2 - 29\right) r^2 r_0^2 - 8\mathfrak{q}^2\left(\mathfrak{q}^2+1\right) r r_0^7 + 12 r^8\right] = 0,$$

for $\beta \in \mathbb{R}$.

New black brane solutions em AdS_4

- $$\langle T^{\mu}_{\mu}\rangle = \left[\left(4\mathfrak{q}^2 - 3\left(\mathfrak{q}^2+1\right)\right)\left(\mathfrak{q}^2-3\right)^2((3\beta+2)-3)^2\right]\left(\frac{3+\beta}{4+3\beta}\right)^2. \qquad (5)$$

  It indicates a conformal anomaly owing to quantum corrections induced by $\beta \neq 1$.

- $$\begin{aligned}\langle T^{\mu}{}_{\mu}\rangle_{\rm CFT} &= \frac{c}{16\pi^2}\left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 2R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}R^2\right)\\ &\quad - \frac{a}{16\pi^2}\left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right), \qquad (6)\end{aligned}$$

  where the terms in parentheses are, respectively, the Euler density and the square of the Weyl curvature. On $u \to 0$, the Weyl curvature be expanded as

$$\begin{aligned} N^2\Bigg[&\frac{40}{3} + \frac{64}{9}(3\beta-1)u + \frac{2}{27}\left(315\beta^2+6\beta-37\right)u^2 + \frac{8}{81}u^3\left(27\beta(\beta(7\beta+4)-1)+189\mathfrak{q}^2+179\right)\\ &+\frac{1}{243}u^4\left((3\beta-1)(36\beta(3\beta(5\beta+9)+16)+1219)+27(123\beta-137)\mathfrak{q}^2\right)\Bigg] + \mathcal{O}\left(u^5\right) \qquad (7)\end{aligned}$$

- and the Euler density as

$$N^2\left[120+64(3\beta-1)u+24\left(9\beta^2-1\right)u^2+u^3\left(32\beta\left(6\beta^2+3\beta-1\right)+168\mathfrak{q}^2+\frac{1448}{9}\right)\right.$$
$$\left.+u^4\left(\frac{2}{27}(3\beta-1)(36\beta(3\beta(5\beta+7)+8)+853)+2(93\beta-79)\mathfrak{q}^2\right)\right]+\mathcal{O}\left(u^5\right), \qquad (8)$$

where $N^2=\pi L^3/2G_4$. Therefore the CFT boundary $u\to 0$ limit implies that

$$\langle T^\mu_{\ \mu}\rangle_{\rm CFT}=\frac{400N^2}{3}, \qquad (9)$$

Black brane

- N. Bilic, J. Fabris, JHEP **11** (2022) 013.

  Sec. 2.1, *AdS Planar black hole*:

  As pointed by Witten (Adv. Theor. Math. Phys. **2***, 505 (1998) [hep-th/9803131], in the limit of large AdS Schwarzschild BH the topology goes from* $S^1 \times S^{d-1}$ *to* $S^1 \times \mathbb{R}^{d-1}$*, where a Schwarzschild BH is approximated by a planar BH with a translationally invariant horizon. To see this, . . .*