Proposal for testing the Schrödinger-Newton equation

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Institute for Theoretical Physics
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In collaboration with
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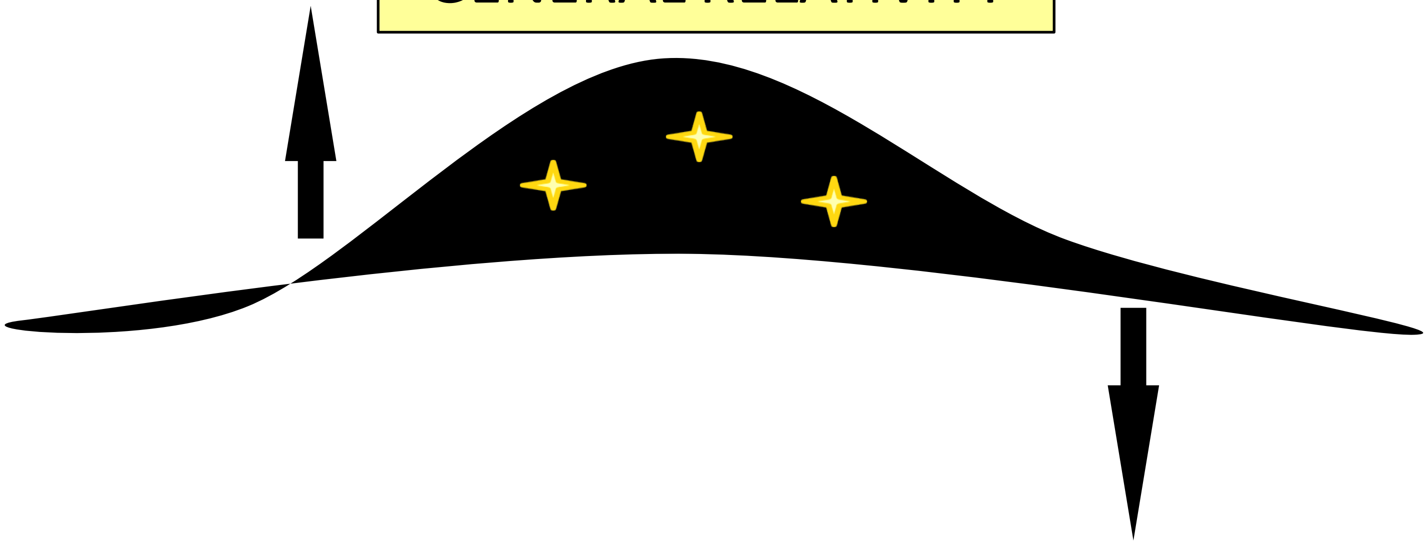
Scientific Motivation

Low-energy quantum gravity

GENERAL RELATIVITY

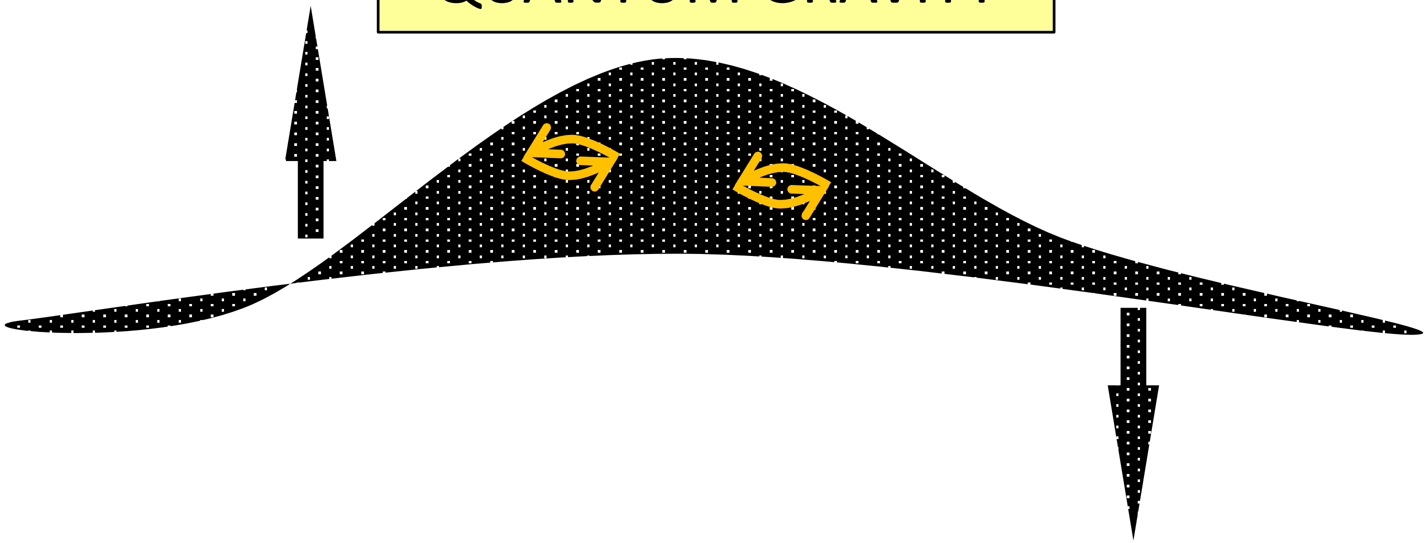


Spacetime and fields harmoniously evolved with classical rules

$$G_{ab}(g) = 8\pi T_{ab}(g, \Phi)$$
$$\nabla_a \nabla^a \Phi + m^2 \Phi = 0$$

Low-energy quantum gravity

QUANTUM GRAVITY



Spacetime and fields harmoniously evolved with quantum rules

?

Low-energy quantum gravity

QUANTUM FIELD THEORY

Fields evolved in a predefined spacetime with quantum rules

$$G_{ab}(g) = 8\pi T_{ab}(g, \Phi)$$

$$\nabla_a \nabla^a \Phi + m^2 \Phi = 0$$

$$\nabla_a \nabla^a \hat{\phi} = 0$$

Low-energy quantum gravity

SEMI-CLASSICAL GRAVITY

Backreaction of quantum fields in a classical spacetime

$$G_{ab}(g) = 8\pi T_{ab}(g, \Phi) + 8\pi\langle\, \hat{T}_{ab}\left(g, \hat{\phi}\right) \rangle$$

$$\nabla_a \nabla^a \Phi + m^2 \Phi = 0$$

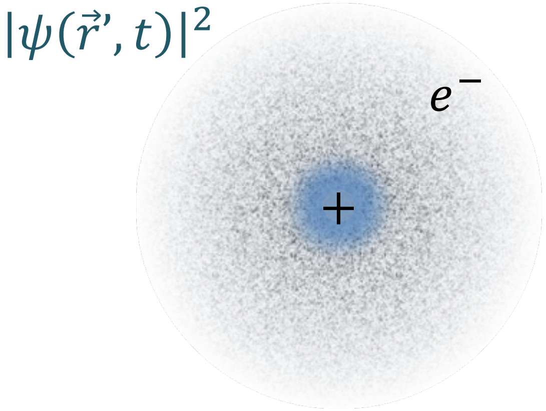
$$\nabla_a \nabla^a \hat{\phi} = 0$$

Schrödinger-Newton equation

Bagrami, Großardt, Donaldi, Bassi, NJP (2014)

$$i\hbar \frac{\partial}{\partial t}\psi(\vec{r},t) = -\frac{\hbar^2}{2\mu}\nabla^2\psi(\vec{r},t) + V_{\text{ext}}(\vec{r},t)\,\psi(\vec{r},t) + U_{\text{self}}^{\text{grav}}(\vec{r},t)\,\psi(\vec{r},t)$$

$$U_{\text{self}}^{\text{grav}}(\vec{r},t) = -Gm^2 \int d^3\vec{r}\,' \frac{|\psi(\vec{r}\,',t)|^2}{|\vec{r}-\vec{r}\,'|}$$



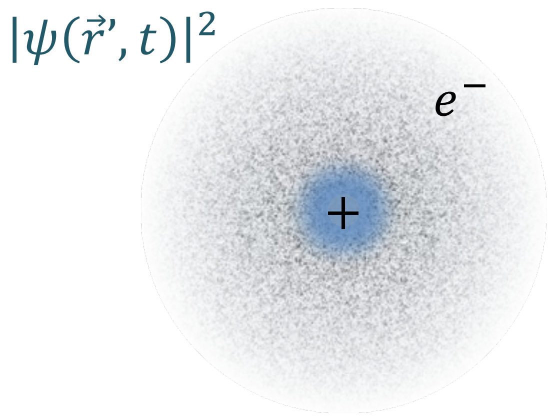
one-electron state

Schrödinger-Coulomb equation

da Silva, Aguiar, GEAM, PRA (2023)

$$i\hbar \frac{\partial}{\partial t}\psi(\vec{r},t) = -\frac{\hbar^2}{2\mu}\nabla^2 \psi(\vec{r},t) + V_{\text{ext}}(\vec{r},t)\,\psi(\vec{r},t) + U_{\text{self}}^{\text{elec}}(\vec{r},t)\,\psi(\vec{r},t)$$

$$U_{\text{self}}^{\text{elec}}(\vec{r},t) = k_e e^2 \int d^3\vec{r}' \frac{|\psi(\vec{r}',t)|^2}{|\vec{r} - \vec{r}'|}$$



one-electron state

LEVEL	Schrödinger equation	Schrödinger-Coulomb equation
n=1	−13.598 eV	−1.2561 eV

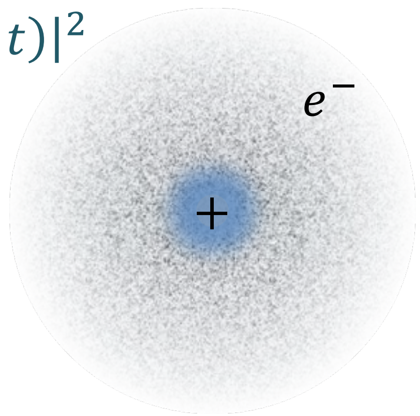
Schrödinger-Newton equation

da Silva, Aguiar, GEAM, PRA (2023)

$$i\hbar \frac{\partial}{\partial t} \psi(\vec{r}, t) = -\frac{\hbar^2}{2\mu} \nabla^2 \psi(\vec{r}, t) + V_{\text{ext}}(\vec{r}, t)\, \psi(\vec{r}, t) + U_{\text{self}}^{\text{grav}}(\vec{r}, t)\, \psi(\vec{r}, t)$$

$$U_{\text{self}}^{\text{grav}}(\vec{r}, t) = -Gm^2 \int d^3\vec{r}\,' \frac{|\psi(\vec{r}\,', t)|^2}{|\vec{r} - \vec{r}\,'|}$$

$$\frac{Gm_e^2}{k_e e^2} \sim 10^{-43}$$



one-electron state

Awfully difficult to test the Schrödinger-Newton equation with stationary states

Probing the Schrödinger-Newton equation in a Stern-Gerlach like experiment

Aguiar, GEAM, PRA (2024)

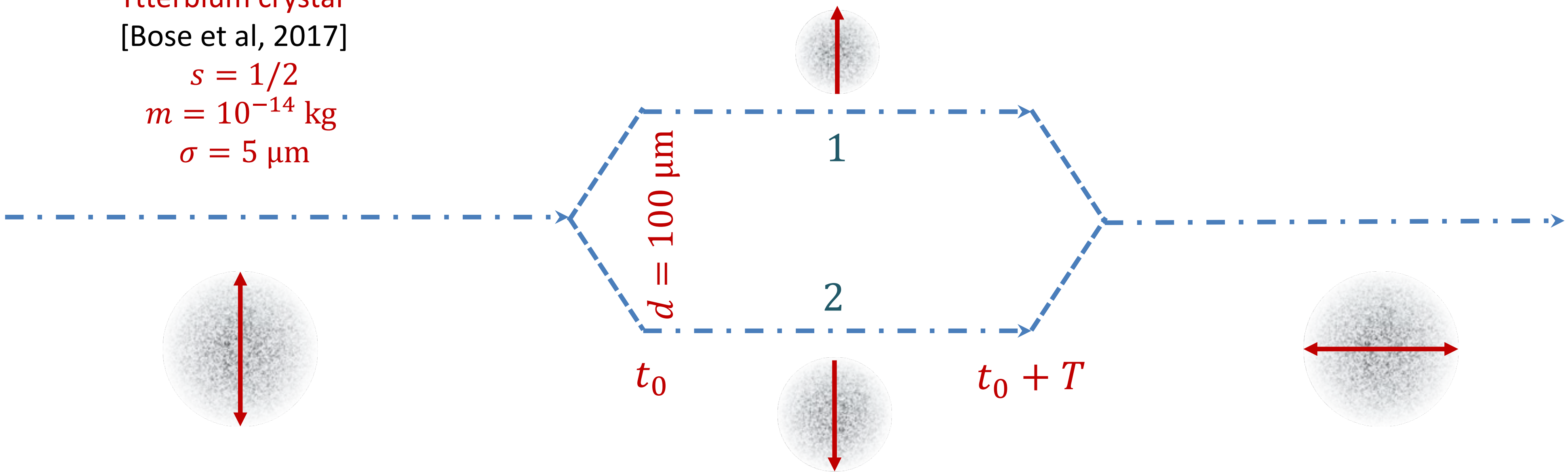
see also Großardt, PRD (2024)

Schrödinger-Newton equation

$$i\hbar \frac{\partial}{\partial t}\psi_{\uparrow(\downarrow)}(t) \approx U_{\text{self}}^{\text{grav}}(\vec{r}_{\uparrow(\downarrow)}, t_0)\psi_{\uparrow(\downarrow)}(t)$$

$$U_{\text{self}}^{\text{grav}}(\vec{r}_{\uparrow(\downarrow)}, t) = -Gm^2 \int d^3\vec{r}' \frac{\left|\psi_{\uparrow(\downarrow)}(\vec{r}', t)\right|^2}{|\vec{r}_{\uparrow(\downarrow)} - \vec{r}'|}$$

One-particle state
Ytterbium crystal
[Bose et al, 2017]
$s = 1/2$
$m = 10^{-14}$ kg
$\sigma = 5\ \mu\text{m}$



Schrödinger-Newton equation

Aguiar, GEAM, PRA (2024)

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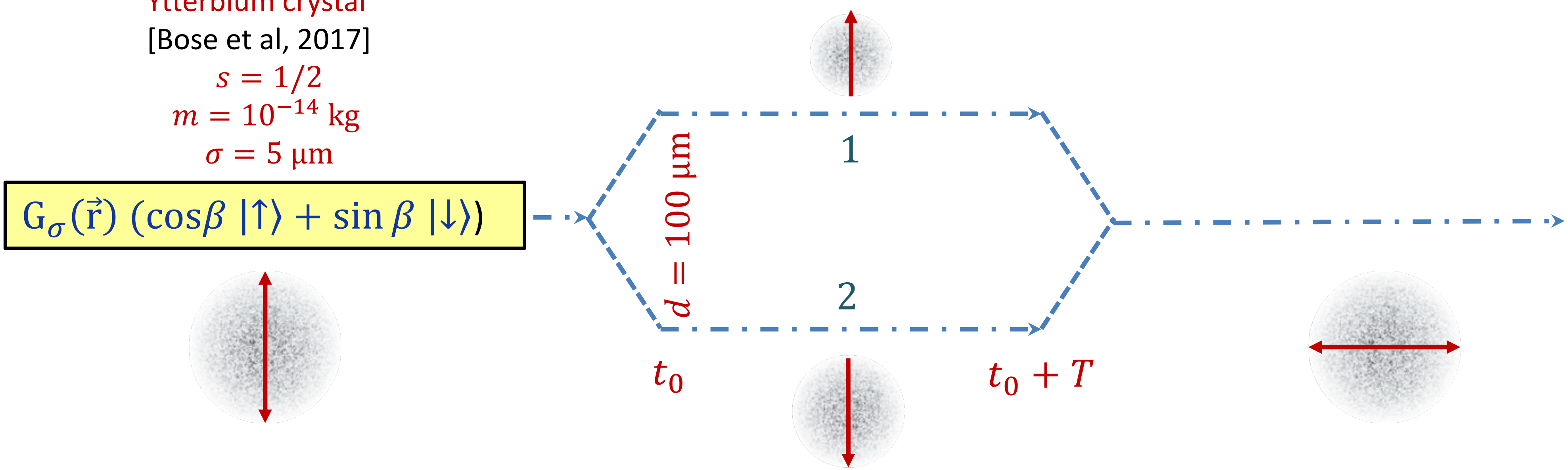
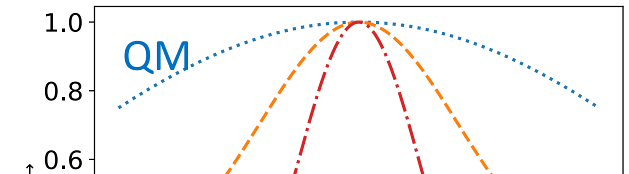


Schrödinger-Newton equation

Aguiar, GEAM, PRA (2024)

$$\Delta\phi = 2\times10^{14}\left(\frac{m}{m_P}\right)^2 \frac{(T/1\mathrm{s})}{(\sigma/1\mu\mathrm{m})}$$



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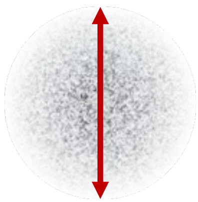
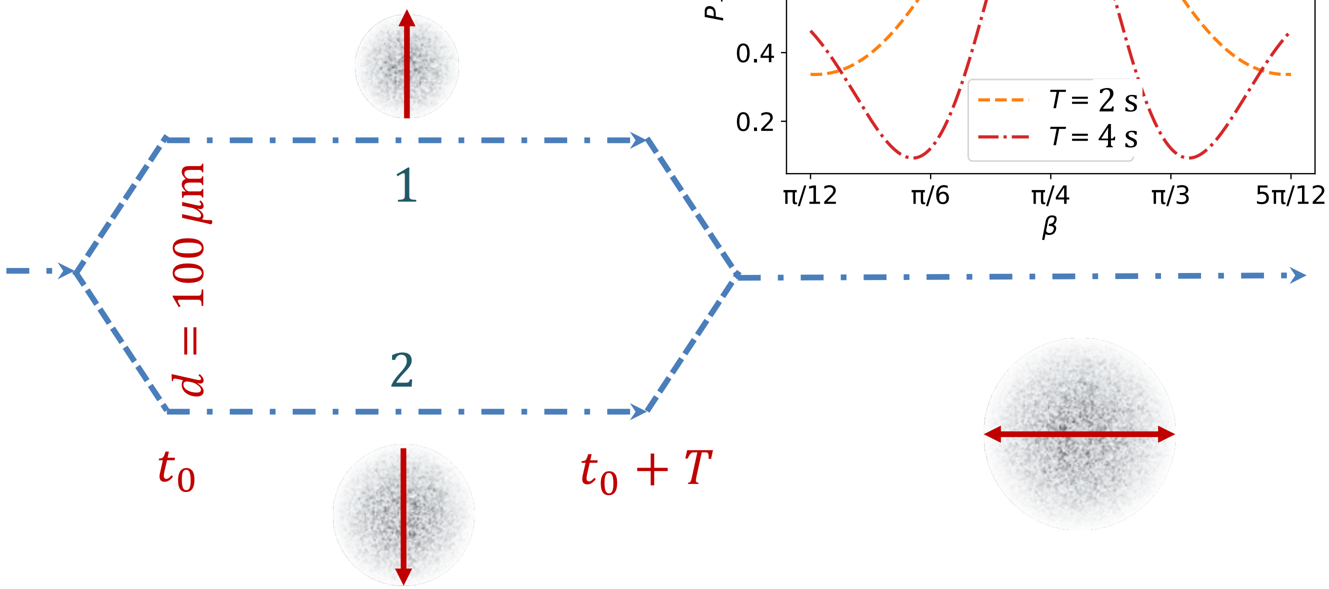
$\sigma = 5\ \mu$m

$$G_\sigma(\vec{r})\ (\cos\beta\ |\uparrow\rangle + \sin\beta\ |\downarrow\rangle)$$





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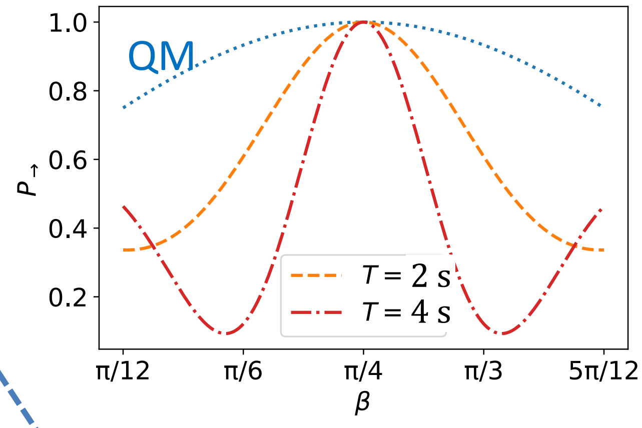

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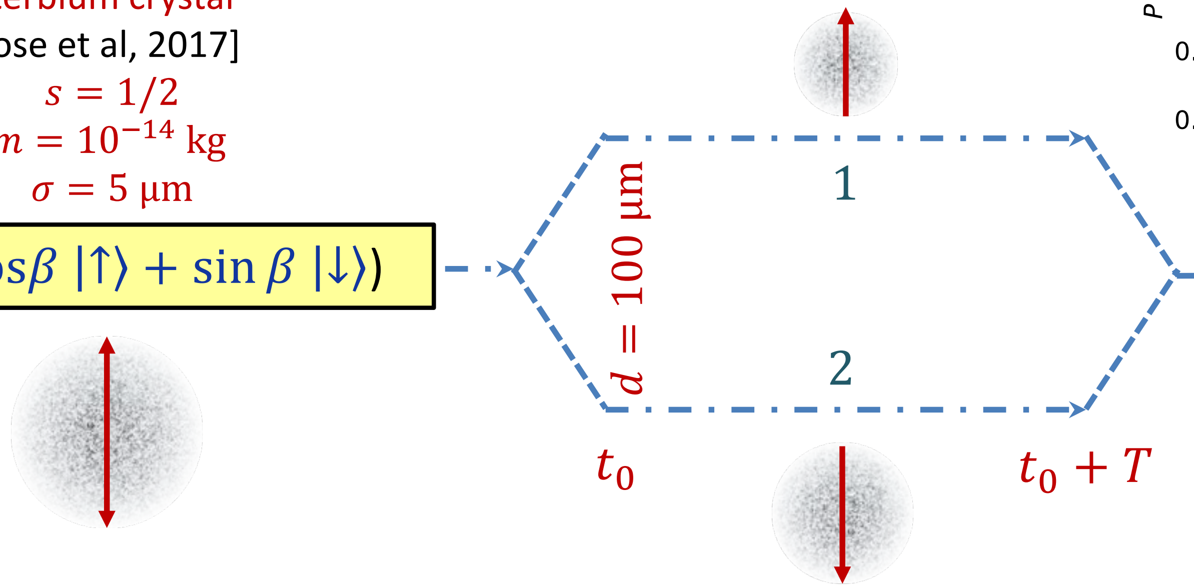
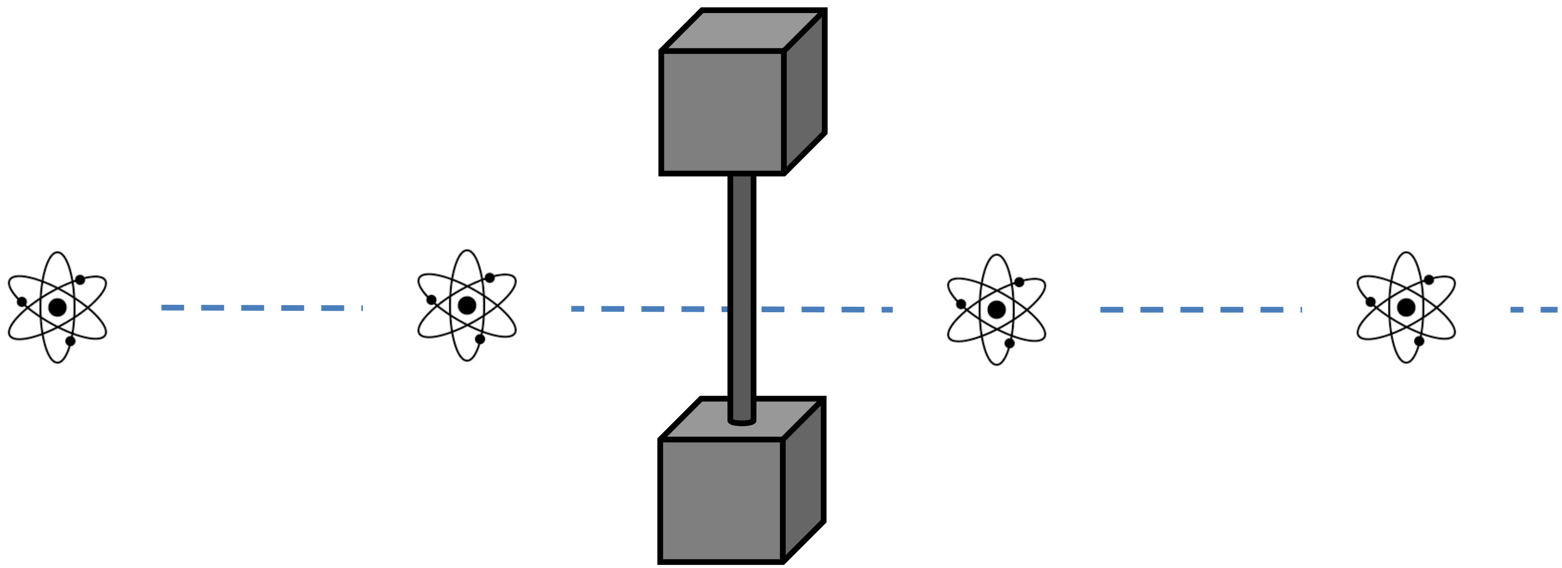


THANK YOU!

Features of Semiclassical Gravity

GENERAL RELATIVITY



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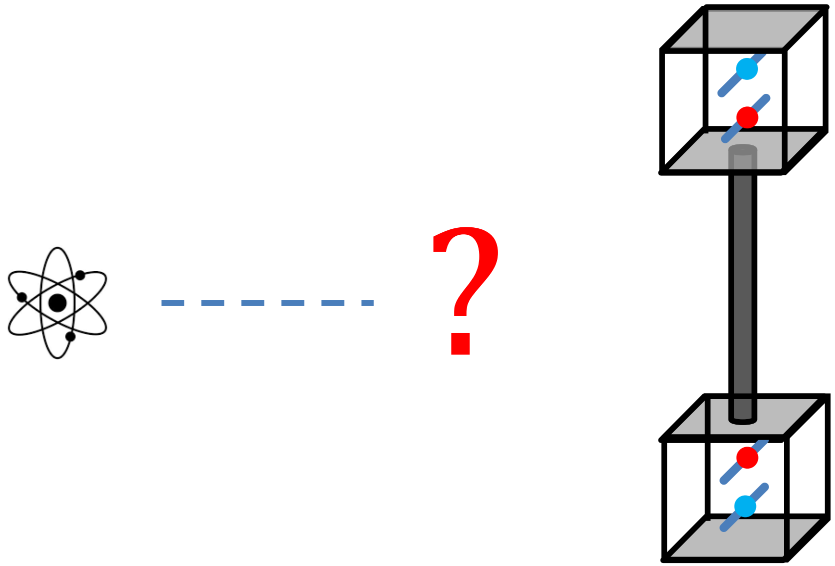
QUANTUM GRAVITY

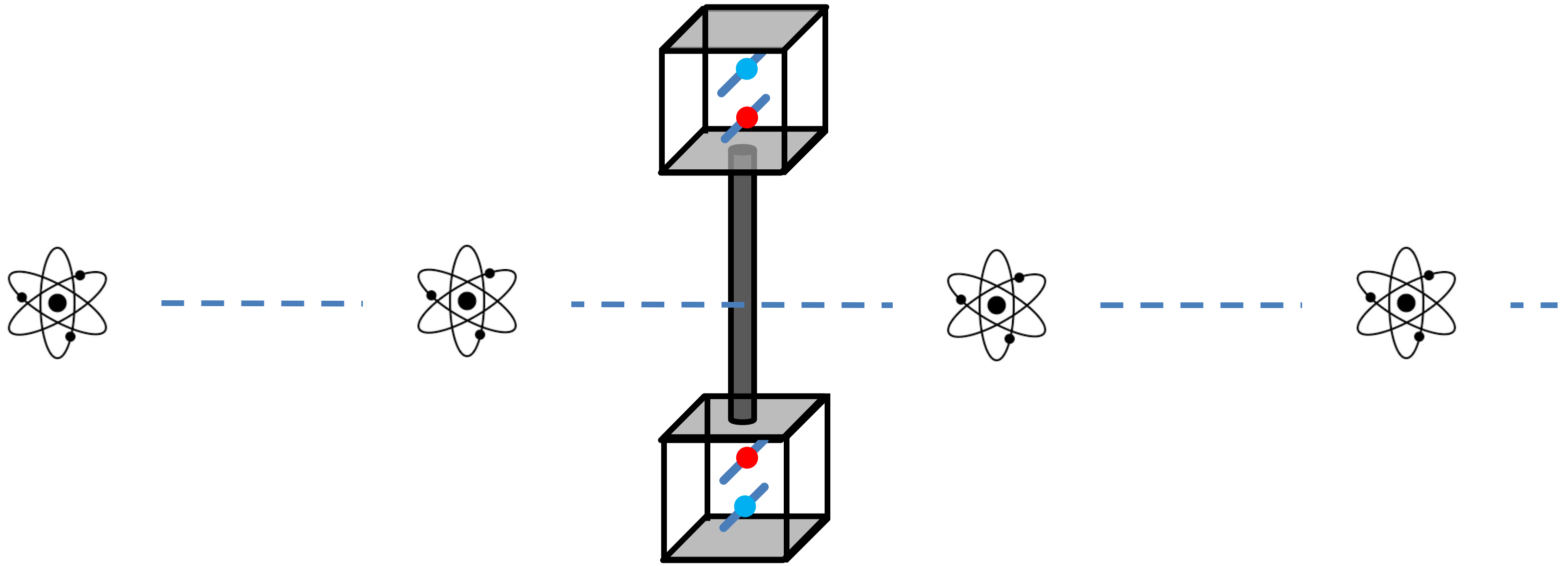


$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|E_0, E\rangle + |E, E_0\rangle \right)$$

$$\langle E_0|E_0\rangle = \langle E|E\rangle = 1, \qquad \langle E_0|E\rangle = 0$$

Features of Semiclassical Gravity

SEMI-CLASSICAL GRAVITY



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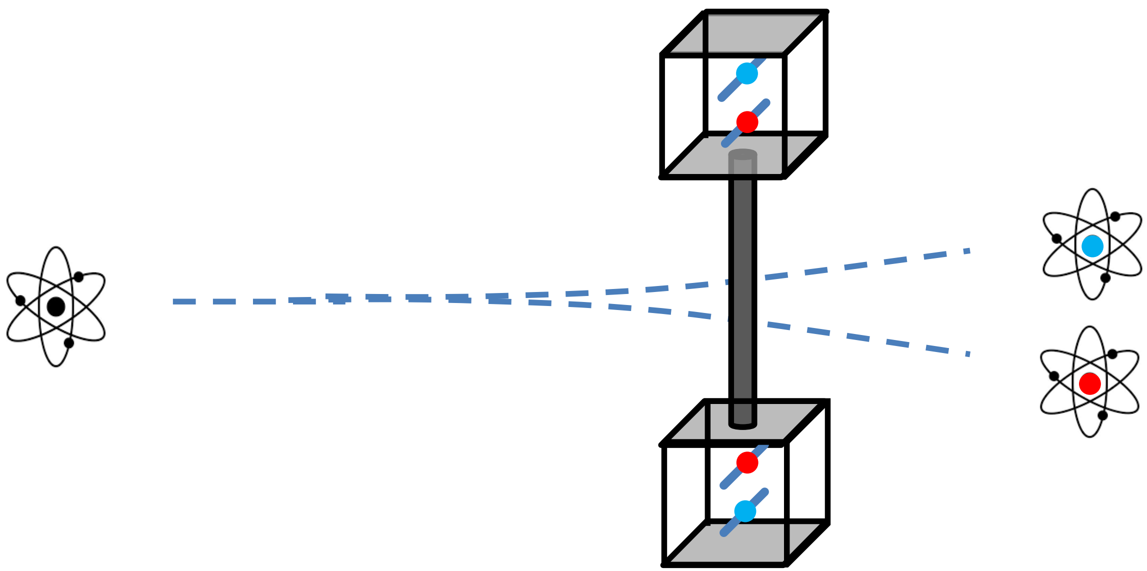

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