ICTP-SAIFR/Principia Institute

Witnessing Quantum Aspects of Gravity in a Lab

Underground test of gravity-related wave function collapse

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PART 1: Introduction to the DP model

THE MEASUREMENT PROBLEM

The Schrödinger equation:

- **Linear**
- **Deterministic**

$$
i\hbar\frac{d}{dt}\left|\Psi\left(t\right)\right\rangle =H\left|\Psi\left(t\right)\right\rangle
$$

The wave packet reduction postulate:

- Non Linear
- **Stochastic**

There are two different laws for the evolution of the state vectors but it is not clear when

to use which one.

PART 1: Introduction to the DP model

COLLAPSE MODELS: GENERAL FEATURES

A. Bassi and G.C. Ghirardi, Phys. Rep. **379**, 257 (2003).

IDEA: to merge the Schr**Ö**dinger evolution and the wave function collapse into a unified dynamics.

The new dynamics must be:

- 1) Close to Schrödinger equation for microscopic systems but collapsing efficiently macroscopic systems.
- 2) **Non linear**, otherwise there is no collapse;
- 3) **Stochastic**, otherwise there can be faster than light signaling. (N. Gisin, Helv. Phys. Acta **62.4**, 363-371 (1989)) For example the Schrӧdinger-Newton equation suffers of this problem. (M. Bahrami, A. Großardt, S. Donadi, A. Bassi, New J. Phys. **16(11)**, 115007 (2014)

PART 1: Introduction to the DP model

THE CSL MODEL

(CONTINUOUS SPONTANEOUS LOCALIZATIONS MODEL) G.C. Ghirardi, P. Pearle and A. Rimini, Phys. Rev. A **42**, 78 (1990).

$$
d|\psi_t\rangle = \left[\underbrace{\left(\frac{i}{\hbar} \hat{H} dt \right)}_{2m_0^2} \underbrace{\frac{\sqrt{\lambda}}{m_0} \int d\boldsymbol{x} \left(\hat{\mu}(\boldsymbol{x}) - \underbrace{\langle \hat{\mu}(\boldsymbol{x}) \rangle_{\psi_t} \rangle dW_t(\boldsymbol{x})}_{\text{Non linearity}} \underbrace{\text{Stochasticity}}_{\text{Non linearity}} \right. \\ \left. \underbrace{-\frac{\lambda}{2m_0^2} \int d\boldsymbol{x} \int d\boldsymbol{y} g(\boldsymbol{x} - \boldsymbol{y}) \left(\hat{\mu}(\boldsymbol{x}) - \underbrace{\langle \hat{\mu}(\boldsymbol{x}) \rangle_{\psi_t} \rangle \right)}_{\text{(A)}} \left(\hat{\mu}(\boldsymbol{y}) - \underbrace{\langle \hat{\mu}(\boldsymbol{y}) \rangle_{\psi_t} \right)} dt \right] | \psi_t \rangle
$$

$$
\mathbb{E}\left[dW_t(\mathbf{x})dW_t(\mathbf{y})\right] = g(\mathbf{x} - \mathbf{y})dt \qquad g(\mathbf{x} - \mathbf{y}) = e^{-(\mathbf{x} - \mathbf{y})^2/4r_C^2}
$$
\n• Localization in space; $\hat{\mu}(\mathbf{x}) = \text{mass density}$

• Amplification mechanism.

THE MASTER EQUATION

$$
\frac{d\hat{\rho}_t}{dt} = -\frac{i}{\hbar} \left[\hat{H}, \hat{\rho}_t \right] - \frac{\lambda}{2m_0^2} \int d\boldsymbol{x} \int d\boldsymbol{x}' e^{-\frac{(\boldsymbol{x} - \boldsymbol{x}')^2}{4r_C^2}} \left[\hat{\mu}(\boldsymbol{x}), \left[\hat{\mu}(\boldsymbol{x}'), \hat{\rho}_t \right] \right]
$$

PART 1: Introduction to the DP model

$$
d|\psi_t\rangle=\left[\begin{array}{c} \text{Li. Diósi, Phys. Rev. A 40, 1165–1174 (1989).} \\ -\frac{i}{\hbar} \hat{H} \, dt + \sqrt{\frac{G}{\hbar}} \int d\boldsymbol{x} \left(\hat{\mu}(\boldsymbol{x}) - \langle \hat{\mu}(\boldsymbol{x}) \rangle \right) dW_t(\boldsymbol{x}) - \\ \frac{\text{Schrödinger}}{\text{Collapse}} \end{array}\right. \\ -\frac{G}{2\hbar} \int d\boldsymbol{x} d\boldsymbol{y} \frac{\left(\hat{\mu}(\boldsymbol{x}) - \langle \hat{\mu}(\boldsymbol{x}) \rangle \right)\left(\hat{\mu}(\boldsymbol{y}) - \langle \hat{\mu}(\boldsymbol{y}) \rangle \right)}{|\boldsymbol{x}-\boldsymbol{y}|} dt \right] |\psi_t\rangle
$$

$$
\frac{d\hat{\rho}(t)}{dt}=-\frac{i}{\hbar}\left[\hat{H},\hat{\rho}(t)\right]-\frac{G}{2\hbar}\int\frac{d\bm{x}d\bm{y}}{|\bm{x}-\bm{y}|}\left[\hat{\mu}(\bm{x}),\left[\hat{\mu}(\bm{y}),\hat{\rho}(t)\right]\right]
$$

$$
\langle \boldsymbol{a} | \hat{\rho}(t) | \boldsymbol{b} \rangle = \langle \boldsymbol{a} | \hat{\rho}(0) | \boldsymbol{b} \rangle e^{-t/\tau}
$$

$$
\tau^{-1} = \frac{G}{2\hbar} \int d\boldsymbol{x} d\boldsymbol{y} \frac{(\mu_a(\boldsymbol{x}) - \mu_b(\boldsymbol{x})) \left(\mu_a(\boldsymbol{y}) - \mu_b(\boldsymbol{y})\right)}{|\boldsymbol{x} - \boldsymbol{y}|}
$$

PART 1: Introduction to the DP model

 PENROSE PROPOSAL R. Penrose, Gen. Relativ. Gravit. **28**, 581–600 (1996). R. Penrose, Found. Phys. **44**, 557–575 (2014).

PART 2: experimental tests of the DP model

TESTING THE COLLAPSE MODELS

 S. L. Adler and A. Bassi, Science **325**, 275 (2009). M. Carlesso, S. Donadi, L. Ferialdi, M. Paternostro, H. Ulbricht, A. Bassi. Nat. Phys. 18, 243 (2022).

Modified Schrödinger dynamics \implies models can be tested against QM.

Two type of experiments:

- 1. *Interferometric* experiments: one searches for loss of coherences in spatial superposition;
- 2. *Non-Interferometric* experiments: collapse being random \implies diffusive effects on the system e.g. heating that can be measured in principle.

Non-interferometric experiments provide better bounds, they do **not require** to prepare the systems in spatial superposition.

PART 2: experimental tests of the DP model

TEST OF THE MODEL

Interferometric tests, there are interesting proposals:

- 1) Optomechanical devices (W. Marshall, et al. Phys. Rev. Lett. **91**, 130401 (2003)).
- 2) B.E.C. (R. Howl, R. Penrose, I. Fuentes, New J. Phys. **21**, 043047 (2019)).
- 3) Experiments in space: no gravity ---> more time (MAQRO, CAL, etc..). (A. Belenchia, et al. Nature **596**, 32–34 (2021), G. Gasbarri et al. Commun. Phys. **4**, 155 (2021))
- …but they are still hard to perform.

Non interferometric tests does not require to create large superpositions!

- 1. LISA Pathfinder (B. Helou, B. Slagmolen, D. E. McClelland, and Y. Chen, Phys. Rev. D **95**, 084054 (2017).)
- 2. Spontaneous heating (A. Vinante, H. Ulbricht., AVS Quantum Science **3,** 4 (2021));
- 3. When the system is charged, the master equation implies **radiation emission**, even when it is well localized! .

PART 2: experimental tests of the DP model

THEORETICAL CALCULATIONS: AN OUTLINE

Radiation emission rate: S. Donadi, K. Piscicchia, C. Curceanu, L. Diósi, M. Laubenstein, and A. Bassi, Nat. Phys. **17**, 74 (2021).

$$
\frac{d\Gamma_t}{d\omega_k} = \frac{k^2}{c} \sum_{\nu} \int d\Omega_k \frac{d}{dt} \langle a_{\mathbf{k}\nu}^\dagger a_{\mathbf{k}\nu} \rangle_t
$$

Adjoint Master Equation:

$$
\frac{d}{dt}O(t) = \frac{i}{\hbar}[H, O(t)] + \int d\mathbf{Q} \sum_{k,k'} \tilde{\Gamma}_{k,k'}(\mathbf{Q}) \left(e^{-\frac{i}{\hbar}\mathbf{Q} \cdot \mathbf{x}_{k'}} O(t) e^{\frac{i}{\hbar}\mathbf{Q} \cdot \mathbf{x}_{k}} - \frac{1}{2} \left\{ O(t), e^{-\frac{i}{\hbar}\mathbf{Q} \cdot \mathbf{x}_{k'}} e^{\frac{i}{\hbar}\mathbf{Q} \cdot \mathbf{x}_{k}} \right\} \right)
$$
\n
$$
\tilde{\Gamma}_{n,n'}(\mathbf{Q}) = \frac{4G}{\pi \hbar^2} \frac{\tilde{\mu}_n(\mathbf{Q}) \tilde{\mu}_{n'}^*(\mathbf{Q})}{Q^2}.
$$
\n
$$
H = H_s + H_{\text{int}} + H_{\text{int}}
$$

Perturbative treatment of the collapse and the EM interaction, calculation ….. and more calculation….. and finally: $10^{-4} < 1 < 10^{-12}$

$$
\frac{d\Gamma_t}{d\omega_k} = \frac{2}{3}\frac{Ge^2N^2N_a}{\pi^{3/2}\varepsilon_0c^3R_0^3\omega_k}
$$

$$
10^{-4} \lesssim \lambda \lesssim 10^{-4} A
$$

N = atomic number; Na = number of atoms;

 R_0 = mass density size.

PART 2: experimental tests of the DP model

AN EASIER APPROACH

L. Diósi and B. Lukács, Phys. Lett. A **181**, 366–368 (1993); S. L. Adler. J. Phys. A **40**, 2935–2957 (2007). S. Donadi, K. Piscicchia, R. Del Grande, C. Curceanu, M. Laubenstein, and A. Bassi, Eur. Phys. J. C **81**, 773 (2021).

$$
i\hbar \frac{d|\psi_t\rangle}{dt} = \left[\hat{H} + V_{cm}(t)\right] |\psi_t\rangle
$$

$$
V_{CSL}(t) = -\frac{\hbar\sqrt{\lambda}}{m_0} \int d\mathbf{y}\mu(\mathbf{y})w(\mathbf{y},t) \qquad \mathbb{E}[w(\mathbf{y},t)w(\mathbf{y}',t')] = e^{-\frac{(\mathbf{y}-\mathbf{y}')^2}{4r_C^2}} \delta(t-t')
$$

$$
V_{DP}(t) = \sqrt{8\pi G\hbar} \int d\mathbf{y}\mu(\mathbf{y})\xi(\mathbf{y},t) \qquad \mathbb{E}[\xi(\mathbf{y},t)\xi(\mathbf{y}',t')] = \frac{\delta(t-t')}{|\mathbf{y}-\mathbf{y}'|}
$$

Lead to the same master equation of the collapse equations —> same radiation emission

$$
\boldsymbol{a}_{cm}(t)=-\frac{\nabla V_{cm}}{m}
$$

$$
P(t) = \frac{e^2}{6\pi\varepsilon_0 c^3} \mathbf{a}_{cm}^2(t) = \int_0^{+\infty} d\omega \ \hbar\omega \frac{d\Gamma(t)}{d\omega}
$$

PART 2: experimental tests of the DP model

EXPERIMENTAL PART: SETUP AND MEASURED SPECTRUM S. Donadi, K. Piscicchia, C. Curceanu, L. Diósi, M. Laubenstein, and A. Bassi, Nat. Phys. 17, 74 (2021).

m

1, germanium crystal; 2, electric contact; 3, plastic insulator; 4, copper cup; 5, copper end-cup; 6, copper block and plate; 7, inner copper shield; 8, lead shield.

$$
R_0 > 0.54 \times 10^{-10} \text{ m (95% c.l.)}
$$

$$
\mu_P \sim |\psi|^2 \gtrsim R_0 = 0.05 \times 10^{-10}
$$

$$
\langle u^2 \rangle = B/8\pi^2 \bigg\} R_0 = 0.05 \times 10^{-10}
$$

Data collected in 62 days: Grey line: measured, tot= 576 Green line: simulated, tot= 506

C I. J. Arnquist, et al. (Majorana Collaboration), Phys. Rev. Lett. **129**, 080401 (2022).

 RECENT DEVELOPMENTS ON RADIATION EMISSION K. Piscicchia, et al., Phys Rev. Lett. **132.25**, 250203 (2024)

When the wavelength of the emitted photon is larger than 0.1 Å (∼123 keV), the emission rate depends on the atomic structure in a non-trivial way. DP and CSL predictions become qualitatively different.

ARE ALL COLLAPSE DYNAMICS DIFFUSIVE?

- **Non-interferometric tests** provide a powerful way to test collapse theories.
- They are based on the **diffusive motion** induced by the noise responsible for collapse to systems.

TWO INTERESTING QUESTIONS

- Is this a feature of collapse models or something more general?
- Is it possible to have collapse without diffusion?

This is **interesting** from a **phenomenological** (non-interferometric tests) as well as from a **more fundamental** (diffusion is there or not?) point of view.

COLLAPSE DYNAMICS ARE DIFFUSIVE S. Donadi, L. Ferialdi, A. Bassi, Phys. Rev. Lett. **130**, 230202 (2023).

There is a dynamics describing the collapse. Under the assumptions:

1) No faster than light signalling. This implies there is a linear map for $\hat{\rho}$. (N. Gisin, Helv. Phys. Acta **62.4**, 363-371 (1989)).

2) The map describing the collapse being **completely positive** \rightarrow Kraus form:

$$
\varPhi[\hat{\rho}]=\sum_{k}\hat{A}_{k}\hat{\rho}\hat{A}_{k}^{\dagger}
$$

$$
\hat{A}_k = \hat{A}_k(\hat{\bm{q}},\hat{\bm{p}})
$$

3) The map being **space-translational covariance**:

$$
e^{-\frac{i}{\hbar}\hat{\boldsymbol{p}}\cdot \boldsymbol{x}}\,\boldsymbol{\varPhi}[\hat{\rho}]\,e^{\frac{i}{\hbar}\hat{\boldsymbol{p}}\cdot \boldsymbol{x}}=\boldsymbol{\varPhi}\left[e^{-\frac{i}{\hbar}\hat{\boldsymbol{p}}\cdot \boldsymbol{x}}\,\hat{\rho}\,e^{\frac{i}{\hbar}\hat{\boldsymbol{p}}\cdot \boldsymbol{x}}\right]
$$

Collapse in space \implies Diffusion in momentum

HOW EFFECTIVE IS COLLAPSE IN THE DP MODEL?

L. Figurato, M. Dirindin, J. L. Gaona-Reyes, M. Carlesso, A. Bassi and S. Donadi, arXiv:2406.18494 (2024)

Consider a graphene plate in a superposition:

$$
\tau_{obs} = 0.01s \qquad L_{obs} = 25 \mu m
$$

$$
d = 4L
$$

RECENT DEVELOPMENTS ON RADIATION EMISSION

L. Figurato, M. Dirindin, J. L. Gaona-Reyes, M. Carlesso, A. Bassi and S. Donadi (coming soon)

Conclusions

CONCLUSIONS

- In the DP model the spontaneous collapse is related to gravity;
- Under general assumptions the spontaneous collapse implies diffusive effects, that leads to emission of radiation;
- The experiments based on radiation emission set the strongest lower bound on the parameter R_{0} (for markovian models). Moving to lower energies, the emission of the DP model is qualitatively different from that of other collapse models.
- The model does not collapse effectively some macroscopic systems, but the choice of these systems is arbitrary. Still, it seems reasonable to set an upper bound on $R_{0}^{\text{}}$ such that $4\times10^{-10}~\textrm{m} \lesssim R_0 \lesssim 10^{-2}~\textrm{m}$.

The Diósi-Penrose model and its experimental tests

Collaborators

COLLABORATORS

Radiation emission

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Theoretical bound

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The Diósi-Penrose model and its experimental tests

FINANCIAL SUPPORT

THE GROUP AT QUEEN'S UNIVERSITY

THANKS

PART 1: Introduction to the DP model

WHY DECOHERENCE IS NOT ENOUGH

A. Bassi and G.C. Ghirardi, Phys. Rep. **379**, 257 (2003).

Why changing the dynamics at the level of the state vectors:

$$
|\psi\rangle = \tfrac{|+\rangle + |-\rangle}{\sqrt{2}} \qquad \overset{\text{Collapse}}{\longrightarrow} \qquad \{50\% \; |+\rangle, 50\% \; |-\rangle \}
$$

The corresponding dynamics for the density matrix is:

$$
\rho = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}
$$
 College $\rho = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Is the diagonalization of the density matrix a sufficient condition? NO.

$$
\rho = \tfrac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \{50\% \mid + \rangle, 50\% \mid - \rangle \} \\ \{50\% \mid \xrightarrow{+} + \mid - \rangle, 50\% \mid \xrightarrow{+} \rangle - \mid - \rangle \\ \left\{50\% \mid \xrightarrow{\langle 2 \rangle} , 50\% \mid \xrightarrow{\langle 1 \rangle} - \rangle \right\}
$$

PART 3: Collapse ⇒ diffusion

Sandro Donadi

COLLAPSE IMPLIES DIFFUSION

Under the hypothesis:

- The probability the state collapses is Poissonian in time;
- **•** Translational covariance and momentum independence;
- **•** Time of decay is that given by Penrose;

$$
\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}[H,\rho(t)] + \mathcal{L}[\rho(t)]
$$

$$
\bullet \ \mathcal{L}[\rho(t)] = \int d\boldsymbol{Q} \tilde{\mathbb{I}}[\rho \boldsymbol{Q} t] \sum_{j=1}^{\infty} \Biggl[\Biggl\{ e^{\textstyle \int \hspace{-0.25cm} \int \hspace{-0.
$$

$$
\mathbf{a}|\mathcal{L}[\hat{\rho}(t)]|\boldsymbol{b}\rangle = \left[-\frac{1}{\tau_{\text{\tiny DP}}(\boldsymbol{a},\boldsymbol{b})}\right]\langle \boldsymbol{a}|\hat{\rho}(t)|\boldsymbol{b}\rangle \longrightarrow \tilde{\Gamma}(\boldsymbol{Q}) = \frac{4G}{\pi\hbar^2}\frac{|\tilde{\mu}(\boldsymbol{Q})|^2}{Q^2}
$$

$$
\frac{d\rho(t)}{dt}=-\frac{i}{\hbar}\left[H,\rho(t)\right]-\frac{4\pi G}{\hbar}\int d\bm{x}\int d\bm{y}\frac{1}{\left|\bm{x}-\bm{y}\right|}\left[\hat{\mu}(\bm{y}),\left[\hat{\mu}(\bm{x}),\rho(t)\right]\right].
$$

WHAT I WILL PRESENT

- 1. Motivations and introduction to the Diósi-Penrose (DP) model;
- 2. Experimental tests of the model.
- 3. Theoretical bounds on the DP model.

Main Take-Away

- A) These are rival models to Quantum Mechanics, not reinterpretations: they can be tested with experiments.
- B) Several kind of experiments, have been considered to set bounds on the parameters of the models.

COLLAPSE DYNAMICS ARE DIFFUSIVE

S. Donadi, L. Ferialdi, A. Bassi, Phys. Rev. Lett. **130**, 230202 (2023).

- We require that collapse cannot be used to do signalling in EPR-like setups. This implies the map must be **linear** (N. Gisin, Helv. Phys. Acta **62.4**, 363-371 (1989)).
- We also require the map implementing the collapse to be **completely positive** → Kraus form:

$$
\Phi[\hat{\rho}] = \sum_{k} \hat{A}_{k} \hat{\rho} \hat{A}_{k}^{\dagger}
$$

$$
\hat{A}_{k} = \hat{A}_{k}(\hat{\boldsymbol{q}}, \hat{\boldsymbol{p}})
$$

$$
\sum_{k} \hat{A}_{k}^{\dagger} \hat{A}_{k} = 1
$$

We require the map to be **space-translational covariant**:

$$
e^{-\frac{i}{\hbar}\hat{\boldsymbol{p}}\cdot\boldsymbol{x}}\varPhi[\hat{\rho}]\,e^{\frac{i}{\hbar}\hat{\boldsymbol{p}}\cdot\boldsymbol{x}}=\varPhi\left[e^{-\frac{i}{\hbar}\hat{\boldsymbol{p}}\cdot\boldsymbol{x}}\,\hat{\rho}\,e^{\frac{i}{\hbar}\hat{\boldsymbol{p}}\cdot\boldsymbol{x}}\right]
$$

COLLAPSE DYNAMICS ARE DIFFUSIVE

- 1) The map changes the average momentum: just measure it.
- 2) The map does not change the average momentum: we focus on the diffusion along the *j* direction is quantified by:

$$
\Delta_j(\hat{\rho}) = \text{Tr} \left[\hat{p}_j^2 \Phi[\hat{\rho}] \right] - \text{Tr} \left[\hat{p}_j^2 \hat{\rho} \right]
$$

If for all $\hat{\rho}$ we have $\Delta_i(\hat{\rho}) = 0$

$$
\Phi[\hat{\rho}]=\sum_{k}\hat{A}_{k}(\hat{\bm{p}})\hat{\rho}\hat{A}_{k}^{\dagger}(\hat{\bm{p}})
$$

This map cannot collapse in position.

No diffusion \Rightarrow No collapse \Rightarrow Yes Collapse \Rightarrow Yes diffusion

Moreover, if the map collapse in space plane waves which then experience diffusion, then all states experience diffusion.

Conclusion: Collapse in space comes together with diffusion in momentum.

PART 3: recent developments

THEORETICAL BOUND, SOME EQUATIONS

$$
\Delta E(\boldsymbol{d}) = 8\pi G m^2 \sum_{i,j=1}^N f(\boldsymbol{r}_{ij}, R_0, \boldsymbol{d})
$$

$$
f(\boldsymbol{r}_{ij}, R_0, \boldsymbol{d}) := \frac{\mathrm{erf}\left(\frac{r_{ij}}{2R_{\mathrm{eff}}}\right)}{r_{ij}} - \frac{\mathrm{erf}\left(\frac{|\boldsymbol{d}-\boldsymbol{r}_{ij}|}{2R_{\mathrm{eff}}}\right)}{|\boldsymbol{d}-\boldsymbol{r}_{ij}|}
$$

Collapse models and their experimental tests

PART 2: The CSL model and its experimental tests

Collapse models and their experimental tests

PART 2: CSL model and its experimental tests

TESTING COLLAPSE MODELS IN SPACE

G. Gasbarri et. al, Commun. Phys. **4**, 155 (2021); A. Belenchia, et al.. Nature **596**, 32–34 (2021).

Collapse is weak, it takes time for its effects to build up. In space a system can be let free for times of order of 100 s, not possible on Earth.

- MAQRO: http://maqro-mission.org/
- COST action QTSpace: http://www.qtspace.eu/