General Relativity and Quantum Field Theory

John F. Donoghue

1) GR makes a normal QFT

2) Emergent causality as a frontier of quantum physics

3) Quadratic gravity - the lab in the sky for causality

Newer work is with Gabriel Menezes

AMHERST CENTER FOR FUNDAMENTAL INTERACTIONS
Physics at the interface: Energy, Intensity, and Cosmic frontiers
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General Relativity and Quantum Physics go together naturally at ordinary scales

The modern day "laws of physics':

$$Z^{core} = \int [d\phi d\psi dA dg]_{\rm Limits} \exp\left[i \int d^4x \sqrt{-g} \left(-\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} F^a_{\mu\nu} F^a_{\alpha\beta} + \ldots \text{SM} \ldots - \Lambda_{cc} + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} \ldots \right) \right]$$

Metric must be included in the PI

- we know D.O.F. and interactions at ordinary scales

The covariant quantization of GR is well understood

- Feynman DeWitt and ghosts

“Limits” and Effective Field Theories

$$Z^{core} = \int [d\phi d\psi dA dg]_{\text{Limits}} \exp\left[i \int d^4x \sqrt{-g} \left(-\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} F^a_{\mu\nu} F^a_{\alpha\beta} + \ldots\text{SM}\ldots - \Lambda_{cc} + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} \ldots\right)\right]$$

Even SM will be an EFT

- need for new physics
- Landau pole for U(1) and Yukawas
- Higgs potential instability

SM is consistent, self-contained layer of reality

- not the ultimate theory

But we don’t need to know physics at all scales

- predictions using active DOF and interactions
- effective field theory

Effective Field Theory Techniques

Key ingredient is the Uncertainty Principle
- unknown physics from high energy is local

But quantum effects sample all energies!
- loops sensitive to wrong physics at high energy
- U.P. says that this “wrongness” will be local
- like the coefficient of a local Lagrangian – measure or match

Appelquist-Carrazzone theorem
- for renormalizable theories
- HE effects in renormalized parameters or suppressed by powers of the heavy scale

Higher Dimensional Local Lagrangians

General Relativity as an EFT

Normal QFT with attention paid to the energy scales involved

Low energy symmetry and fields

- general covariance and the metric as the active

Path Integral with limits

$$Z^{core} = \int [d\phi d\psi dA dg]_{\text{Limits}} \exp\left[i \int d^4x \sqrt{-g} \left(-\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} F^a_{\mu\nu} F^a_{\alpha\beta} + \ldots\text{SM}\ldots \right.\right.$$
$$\left.\left. - \Lambda_{cc} + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} \ldots \right)\right]$$

Can renormalize "non-renormalizable" theories

- divergences are local
- respect the symmetries of the theory (i.e. dim-reg.)

Comments:

R^2 terms needed for both SM and graviton loops

Stable at low energy (No Ostrogradsky instability)

Unitary in Minkowski

Veltman proof 1963

- no restriction on momentum dependence of interaction
- no use of Wick rotation
- no use of renormalizability
- only stable particles appear in unitarity sum

Usual causality structure in Minkowski

- when treated as EFT

GREFT is predictive

GR will need to be modified at high energy

- effects included in local action

Nonlocal effects are reliable

- only from low energy D.O.F. and interactions
- long distance propagation

In calculations near Minkowski:

- nonanalytic only from nonlocal

$$(q^2)^n \rightarrow \Box^n \delta(x)$$

$$\log(-q^2) \rightarrow L(x-y) = \langle x | \log \Box | y \rangle$$

Renormalization

One loop calculation: ‘t Hooft and Veltman

$$Z[\phi, J] = Tr ln D$$

Divergences are local:

$$\Delta\mathcal{L}_0^{(1)} = \frac{1}{8\pi^2}\frac{1}{\epsilon}\left\{\frac{1}{120}R^2 + \frac{7}{20}R_{\mu\nu}R^{\mu\nu}\right\} \qquad \epsilon = 4 - d$$

dim. reg.
preserves
symmetry

Renormalize parameters in general action:

$$c_1^{(r)} = c_1 + \frac{1}{960\pi^2\epsilon}$$
$$c_2^{(r)} = c_2 + \frac{7}{160\pi^2\epsilon}$$

Pure gravity
“one loop finite”
since $R_{\mu\nu} = 0$

Note: Two loop calculation known in pure gravity Goroff and Sagnotti

$$\Delta\mathcal{L}^{(2)} = \frac{209\,\kappa}{2880(16\pi^2)^2}\frac{1}{\epsilon}\sqrt{-g}R^{\alpha\beta}{}_{\gamma\delta}R^{\gamma\delta}{}_{\rho\sigma}R^{\rho\sigma}{}_{\alpha\beta}$$

Order of six derivatves

What are the quantum predictions?

Not the divergences

- they come from the Planck scale
- unreliable part of theory

Not the parameters

- local terms in L
- we would have to measure them

Low energy propagation

- not the same as terms in the Lagrangian
- most always non-analytic dependence in momentum space
- can't be Taylor expanded – can't be part of a local Lagrangian
- long distance in coordinate space

$$Amp \sim q^2 \ln(-q^2) \quad , \quad \sqrt{-q^2}$$

Example 1: Corrections to the gravitational potential

Scattering potential

$$\begin{aligned} \langle f|T|i\rangle &\equiv (2\pi)^4\delta^{(4)}(p-p')(\mathcal{M}(q)) \\ &= -(2\pi)\delta(E-E')\langle f|\tilde{V}(\mathbf{q})|i\rangle \end{aligned}$$

Full result is the full scattering amplitude
NR Potential is a useful way of illustrating result

$$V(\mathbf{x}) = \frac{1}{2m_1}\frac{1}{2m_2}\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{x}}\mathcal{M}(\vec{q})$$

<u>What to expect:</u>

<u>Momentum space amplitudes:</u>

$$V(q^2) = \frac{GMm}{q^2}\left[1 + a'G(M+m)\sqrt{-q^2} + b'G\hbar q^2 \ln(-q^2) + c'Gq^2\right]$$

Non-analytic

-gives
delta function
potential
-local term
live here

<u>Relation to position space:</u>

$$\int \frac{d^3q}{(2\pi)^3}\, e^{i\mathbf{q}\cdot\mathbf{r}} \frac{1}{|\mathbf{q}|^2} = \frac{1}{4\pi r}$$

$$\int \frac{d^3q}{(2\pi)^3}\, e^{i\mathbf{q}\cdot\mathbf{r}} \frac{1}{|\mathbf{q}|} = \frac{1}{2\pi^2 r^2}$$

$$\int \frac{d^3q}{(2\pi)^3}\, e^{i\mathbf{q}\cdot\mathbf{r}} \ln(\mathbf{q}^2) = \frac{-1}{2\pi r^3}$$

<u>General expansion:</u>

$$V(r) = -\frac{GMm}{r}\left[1 + a\frac{G(M+m)}{rc^2} + b\frac{G\hbar}{r^2c^3}\right] + cG^2Mm\delta^3(r)$$

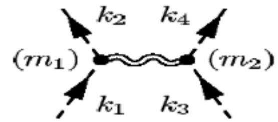
Classical quantum

Result:

:

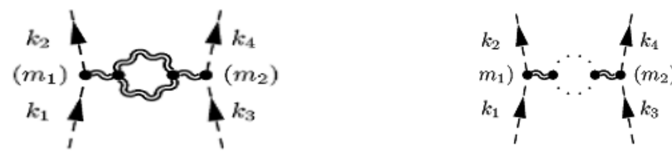
$$V(r) = -\frac{Gm_1m_2}{r}\left[1 + 3\frac{G(m_1+m_2)}{r} + \frac{41}{10\pi}\frac{G\hbar}{r^2}\right]$$

Lowest order:



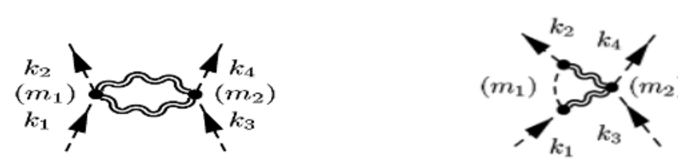
Vertex corrections:

Vacuum polarization:
(Duff 1974)

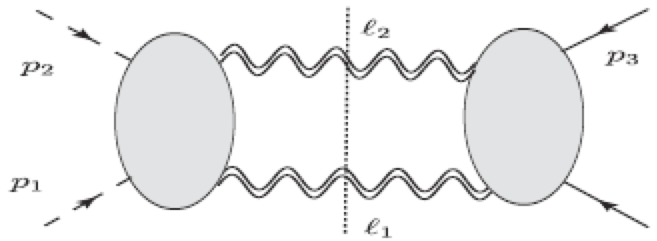


Box and crossed box

Others:



Example 2: Light bending at one loop



Using unitarity methods

$$
\begin{aligned}
i\mathcal{M}^{[\eta(p_1)\eta(p_2)]}_{[\phi(p_3)\phi(p_4)]} \simeq \frac{\mathcal{N}^\eta}{\hbar}(M\omega)^2\Bigg[&\frac{\kappa^2}{t} + \kappa^4\frac{15}{512}\frac{M}{\sqrt{-t}} + \hbar\kappa^4\frac{15}{512\pi^2} \\
&\times\log\left(\frac{-t}{M^2}\right) - \hbar\kappa^4\frac{bu^\eta}{(8\pi)^2}\log\left(\frac{-t}{\mu^2}\right) \\
&+ \hbar\kappa^4\frac{3}{128\pi^2}\log^2\left(\frac{-t}{\mu^2}\right) \\
&+ \kappa^4\frac{M\omega}{8\pi}\frac{i}{t}\log\left(\frac{-t}{M^2}\right)\Bigg], \qquad (11)
\end{aligned}
$$

Can convert amplitude to bending angle using eikonal method

Result different for scalars, photons and gravitons

$$\theta = \frac{4GM}{b} + \frac{15}{4}\frac{G^2M^2\pi}{b^2} + \frac{8\mathrm{bu}^\eta - 47 + 64\log\frac{2r_0}{b}}{\pi}\frac{G^2\hbar M}{b^3}$$

with

$\mathrm{bu}^\eta = (371/120,\ 113/120,\ -29/8)$ for (scalar photons gravitons)

<u>Classical physics from loops</u>

Folk theorem – the loop expansion is the $\hbar$ expansion

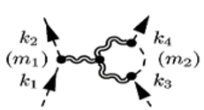
- not true
- classical physics also present in loop expansion
- hidden factors of hbar

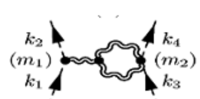
$$\mathcal{L} = \hbar\bar{\psi}\left(i\partial\!\!\!/ - \frac{m}{\hbar}\right)\psi$$

- at one loop, present in $\sqrt{q^2}$ non-analyticity

$$\sqrt{\frac{m^2}{-q^2}} \to \hbar\sqrt{\frac{m^2}{-\hbar^2 q^2}}$$

- both classical and quantum present in some diagrams





$$M_{5(a)+5(b)}(\vec{q}) \;=\; 2G^2 m_1 m_2\left(\frac{\pi^2(m_1+m_2)}{|\vec{q}|} + \frac{5}{3}\log\vec{q}^2\right)$$

$$M_{5(c)+5(d)}(\vec{q}) \;=\; -\frac{52}{3}G^2 m_1 m_2 \log\vec{q}^2$$

This has become a vibrant subfield

Trajectories of massless particles are not universal

Recall:

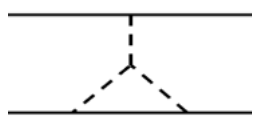
$$\theta = \frac{4GM}{b} + \frac{15}{4}\frac{G^2M^2\pi}{b^2} + \frac{8\mathrm{bu}^\eta - 47 + 64\log\frac{2r_0}{b}}{\pi}\frac{G^2\hbar M}{b^3}$$

with

$\mathrm{bu}^\eta = (371/120,\ 113/120,\ -29/8)$ for scalars, photons, gravitons

The quantum corrections amount to tidal forces

-long range propagation



- sample gravitational fields at more than one position

Not geodesic motion

Light cones etc likely uncontrolled approximations

Evident from bending calculations above

Corrections are tiny at low energy

But eventually become of order unity as EFT fails

Classical concepts seem to fail

- lightcones
- geodesics
- Penrose diagrams
- manifold structure
- **causality ?**

"Gravity is geometry" is a classical notion

- **not best for the quantum theory**
- **QG is QFT**

EFT fails at high energy

Two independent problems:

1) Increasing number of local parameters needed
   - matter loops at order R^2
   - graviton loops at all orders R^2, R^3 ....
2) Amplitudes grow with energy

$$\mathcal{M} = \mathcal{M}_0 \left[1 + Gq^2 + G^2 q^4 + ...\right]$$

Low energy physics points to Planck scale

But new physics could be anytime earlier

Summary of EFT section:

Phrasing issue as "QM incompatible with GR" is wrong

- misconceptions about limits of validity

(Say instead: "Our theory of quantum gravity is incomplete")

GR is a very normal quantum EFT

There are lessons about quantum gravity here

$$Z^{core} = \int [d\phi d\psi dA dg]_{\text{Limits}} \exp\left[i \int d^4x \sqrt{-g} \left(-\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} F^a_{\mu\nu} F^a_{\alpha\beta} + \ldots\text{SM}\ldots - \Lambda_{cc} + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} \ldots \right) \right]$$

There is no known conflict between GR and QFT in regions where both are expected to be valid

For a lot more detail:

[Submitted on 1 Feb 2017]

EPFL Lectures on General Relativity as a Quantum Field Theory

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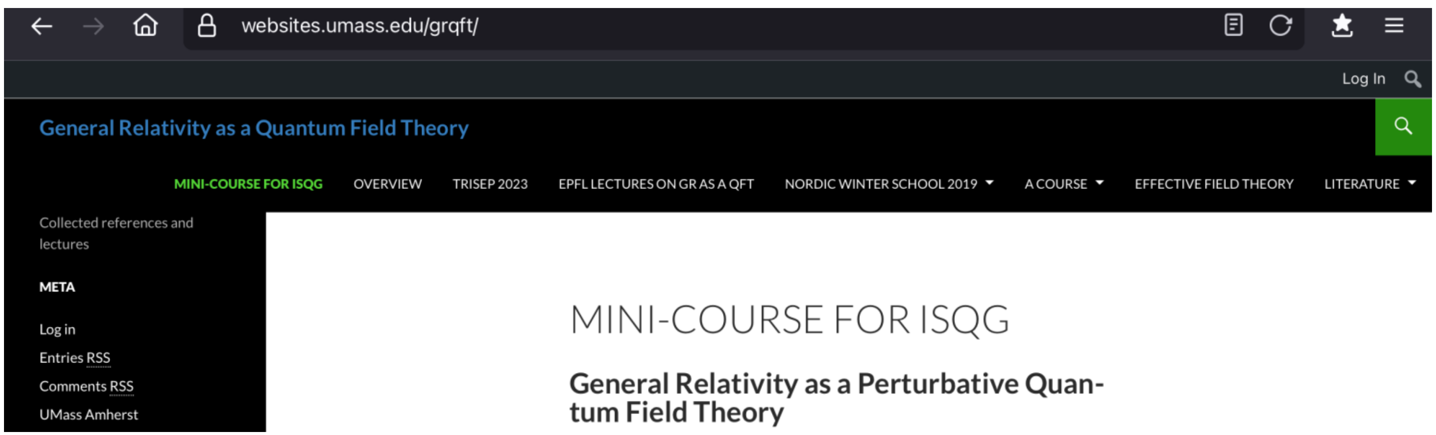
These notes are an introduction to General Relativity as a Quantum Effective Field Theory, following the material given in a short course on the subject at EPFL. The intent is to develop General Relativity starting from a quantum field theoretic viewpoint, and to introduce some of the techniques needed to understand the subject.



<u>But Physics is an experimental science</u>

Either or both GR and QFT could be modified

- **<u>outside our experimental understanding</u>**

GR almost certainly needs modification

a) metric and GCI with new interactions/particles, or

b) totally new DOF

<u>Quantum Physics also has frontiers</u>

- macroscopic domain is a low energy frontier
- for all interactions
- but gravity could be leading indicator
- probably as important as anything at LHC

Microcausality as high energy frontier of QFT

Gravity may be the leading indicator here also

Causality and analyticity – e.g. Källen-Lehman relation

$$D(q) = \frac{1}{\pi}\int_0^\infty ds \frac{\rho(s)}{q^2 - s + i\epsilon}$$

$\rho(s) > 0$

- all fields carry common $+i\epsilon$
- propagators can't fall faster than $1/q^2$

My thesis here:

1) Higher derivative theories have only emergent causality
2) We may have such a theory in Nature
3) Quadratic gravity as consistent self-contained layer of reality
   - with **emergent causality**

Talking about $i\varepsilon$ physics

A) "Laws of Physics" are not invariant under Time Reversal

- but **"covariant"**

Recall that *T* is **anti-unitary**

Neglecting *T*-violating phases, Lagrangian/Action is **invariant**

$$T^\dagger \mathcal{L} T = \mathcal{L} \quad , \qquad T^\dagger S T = S$$

But the "Laws of Physics" are more than the Lagrangian

- also need to include quantization rules

The path integral (or canonical quantization) is **not** invariant

$$T^\dagger Z_+ T = T^\dagger \int [d\phi]\, e^{+iS}\, T \to Z_- = \int [d\phi] e^{-iS}$$

We will see that this distinction is meaningful

B) <u>Requirements for Causality</u>

i) Operators commute at spacelike separations
ii) All fields share a common definition of past and future lightcones

The second is less commonly stated, but it implied
- past lightcone can propagate influences, future lightcone cannot

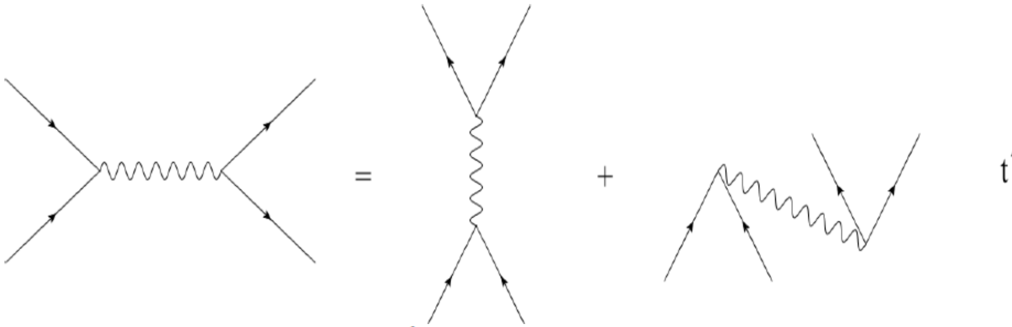
This is enforced by the $i\varepsilon$ prescription. Standard choice is $+i\varepsilon$

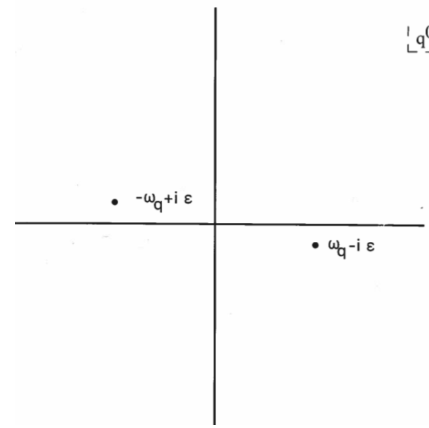
All field share a common $+i\varepsilon$ prescription in propagators

If not, causality violation.
Calculations due to Lee &Wick; Coleman;
and Grinstein, O'Connell, Wise

C) The $i\varepsilon$ prescription defines a time direction

- The forward light cone





$$iD_F(q) = \frac{i}{q^2 - m^2 + i\epsilon}$$

Decompose into time orderings:

$$iD_F(x) = D_F^{\rm for}(x)\theta(t) + D_F^{\rm back}(x)\theta(-t)$$

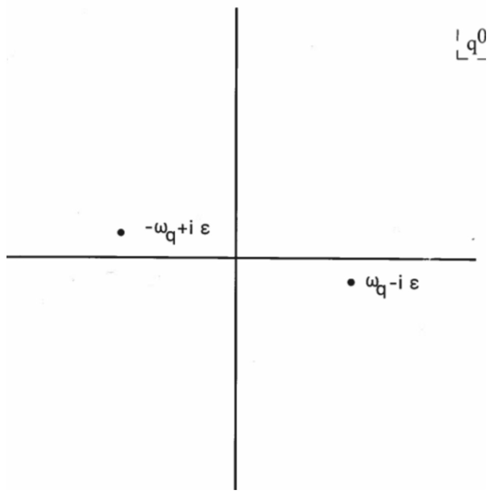
Positive energies propagate forward in time

$$D_F^{\rm for}(x) = \int \frac{d^3q}{(2\pi)^3 2E_q} e^{-i(E_q t - \vec{q}\cdot\vec{x})}$$

$$D_F^{\rm back}(x) = (D_F^{\rm for}(x))^*$$

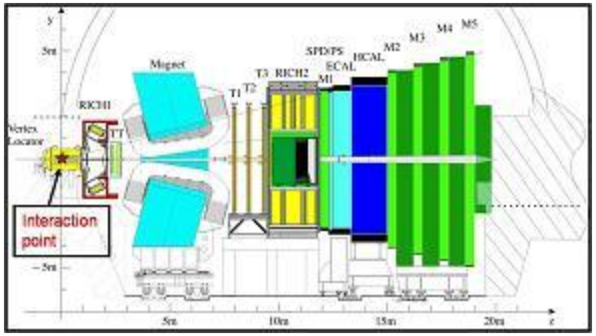
This is the arrow of causality

Enforced by analyticity properties



$$iD_F(q) = \frac{i}{q^2 - m^2 + i\epsilon}$$

Example: long-lived resonance production
- production A+B → R
- decay R → C+D
- decay always happens later
  - this is the arrow of causality



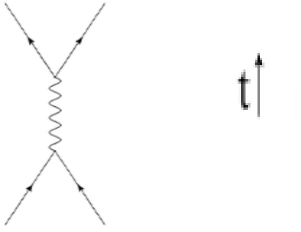
LHCb

Note: Time reversal relates A+B → C+D and C+D → A+B
- but experiment runs both reactions forward in time

Recall:

"Cause before effect" is not enough

- leads to effects outside light cone



Causality also requires "effect before cause"

- negative energy / antiparticles

D) The $e^{\pm iS}$ and $\pm i\varepsilon$ choices are connected

Consider generating functions:

$$Z_\pm[J] = \int [d\phi] e^{\pm iS(\phi,J)}$$
$$= \int [d\phi] e^{\pm i \int d^4x [\frac{1}{2}(\partial_\mu \phi \partial^\mu \phi - m^2\phi^2) + J\phi]}\ .$$

Need to make this better defined – add

$$e^{-\epsilon \int d^4x \phi^2/2}$$

Solved by completing the square:

$$Z_\pm[J] = Z[0] \exp\left\{ -\frac{1}{2} \int d^4x d^4y J(x)\ iD_{\pm F}(x-y) J(y) \right\}$$

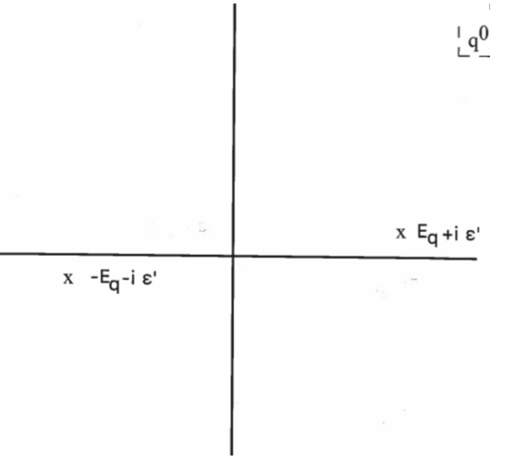
Yield propagator with specific analyticity structure

$$iD_{\pm F}(x-y) = \int \frac{d^4q}{(2\pi)^4} e^{-iq\cdot(x-y)} \frac{\pm i}{q^2 - m^2 \pm i\epsilon}$$

Using e^{-iS} results in time-reversed propagator

$$iD_{-F}(x) = D^{\rm for}_{-F}(x)\theta(t) + D^{\rm back}_{-F}(x)\theta(-t)$$

Positive energy propagates backwards in time



$$D^{\rm back}_{-F}(x) = \int \frac{d^3q}{(2\pi)^3 2E_q} e^{-i(E_q t - \vec{q}\cdot\vec{x})}$$

Use of this generating functional yields **time reversed scattering processes**

Opposite arrow of causality

Mini-summary

Under T, laws of physics are not invariant, but covariant
- transform into similar laws with opposite flow of time

But also, quantum physics carries a **single** preferred direction
- positive energy reactions propagate in this direction
- arrow of causality - analyticity
- determined by factors of *i* in the quantization condition

Quantum physics is unidirectional
- classical physics is bidirectional

Arrow of causality → arrow of thermodynamics

Theories with emergent causality

Higher derivative scalar

$$\mathcal{L}_{hd} = \frac{1}{2}\left[\partial_\mu\phi\partial^\mu\phi - m^2\phi\phi - \frac{1}{M^2}\Box\phi\Box\phi\right]$$

Interacting with normal matter

$$\mathcal{L} = \mathcal{L}_\chi + \mathcal{L}_\phi - g\phi\chi^\dagger\chi$$

$$\mathcal{L}_\chi \;=\; \partial_\mu\chi^\dagger\partial^\mu\chi - m_\chi^2\chi^\dagger\chi - \lambda(\chi^\dagger\chi)^2$$

For our purposes, consider *M* to be very large

I will pretend that renormalized $m \to 0$

Violation of microcausality

$$Z_\phi[\chi] = \int [d\phi] e^{i\int d^4x \left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2M^2}\Box\phi\Box\phi - g\phi\chi^\dagger\chi\right]}$$

Rewrite exactly using auxiliary field to remove higher derivative

$$Z_\phi[\chi] = \int [d\phi][d\eta] e^{i\int d^4x [\mathcal{L}(\phi,\eta)]}$$

$$\mathcal{L}(\phi,\eta) = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \eta\Box\phi + \frac{1}{2}M^2\eta^2 - g\phi\chi^\dagger\chi$$

Redefine field variables using $\phi(x) = a(x) - \eta(x)$

$$\begin{aligned} Z_\phi[\chi] &= \int [da] e^{i\int d^4x \left[\frac{1}{2}\partial_\mu a\partial^\mu a - g a\chi^\dagger\chi\right]} \\ &\times \int [d\eta] e^{-i\int d^4x \left[\frac{1}{2}\partial_\mu\eta\partial^\mu\eta - \frac{1}{2}M^2\eta^2 - g\eta\chi^\dagger\chi\right]} \quad \longleftarrow \quad ** \\ &= Z_a \times Z_\eta \end{aligned}$$

The three forms are exactly equivalent

The η field has the wrong $i\epsilon$ and will lead to causality violation

Emergent Causality

Integrate out the heavy η field

-usual gaussian integral – seen above

$$Z_\eta = N e^{\int d^4x d^4y \frac{1}{2} g \chi^\dagger(x)\chi(x)\; iD_{-F}(x-y)\; g\chi^\dagger(y)\chi(y)}$$

At low energy, this becomes a contact interaction

$$Z_\eta = N e^{i\int d^4x \frac{g^2}{2M^2}[\chi^\dagger(x)\chi(x)]^2}$$

The result is just a shift in λ in the χ interaction

$$\lambda \to \lambda' = \lambda - \frac{g^2}{2M^2}$$

The low energy limit has the usual causality structure

- just a normal QFT

You can see similar result directly in propagator

$$iD(q) = \frac{i}{q^2 - \frac{q^4}{M^2} + i\epsilon}$$

1) Avoid spacelike poles (tachyons)

   - requires $\frac{1}{M^2} > 0$

2) Poles at $q^2 = 0 - i\epsilon$ and $q^2 = M^2 + i\epsilon$

$$iD(q) = \frac{i}{q^2 + i\epsilon} - \frac{i}{q^2 - M^2 - i\epsilon}$$

Massive pole has opposite arrow of causality

Massive mode decays to light particles

- $M \rightarrow \chi\bar{\chi}$ - positive energy resonance
- this is important – **massive mode not an asymptotic state**

$$iD(q^2) = \frac{i}{q^2 - m^2 + i\epsilon - \frac{q^4}{M^2} + \Sigma(q)} \quad .$$

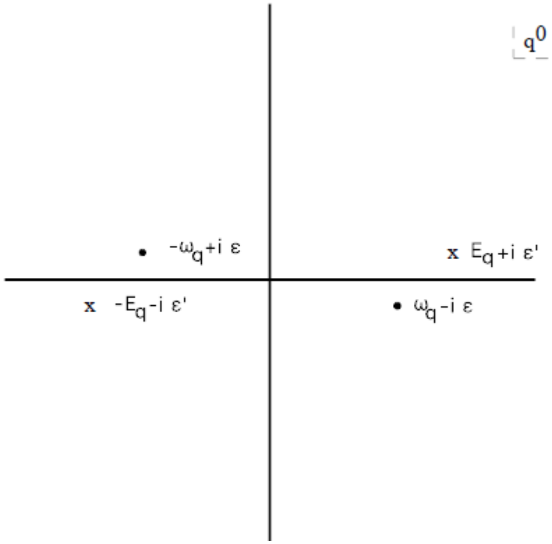
Has a positive imaginary component

$$\mathrm{Im}\Sigma \sim \frac{g^2}{32\pi} \equiv \gamma$$

Leads to exponential decay (not growth)

$$D^{\rm for}(t,\vec{x}) = \int \frac{d^3q}{(2\pi)^3}\left[\frac{e^{-i(\omega_q t - \vec{q}\cdot\vec{x})}}{2\omega_q} - \frac{e^{i(E_q t - \vec{q}\cdot\vec{x})}}{2(E_q + i\frac{\gamma}{2E_q})} e^{-\frac{\gamma t}{2E_q}}\right]$$

$$D^{\rm back}(t,\vec{x}) = \int \frac{d^3q}{(2\pi)^3}\left[\frac{e^{i(\omega_q t - \vec{q}\cdot\vec{x})}}{2\omega_q} - \frac{e^{-i(E_q t - \vec{q}\cdot\vec{x})}}{2(E_q + i\frac{\gamma}{2E_q})} e^{-\frac{\gamma |t|}{2E_q}}\right]$$



Merlin modes:

-Merlin (the wizard in the tales of King Arthur) ages backwards

"Now ordinary people are born forwards in Time, if you understand what I mean, and nearly everything in the world goes forward too. (...) But I unfortunately was born at the wrong end of time, and I have to live backwards from in front, while surrounded by a lot of people living forwards from behind."

T. H. White *Once and Future King*

Note, there is a key distinction with usual nomenclature "ghosts"

- ghost is anything with a minus sign in the numerator
- these Merlin modes refer to crucial sign $-i\gamma$ in denominator in addition

Higher Derivative Theories and Emergent Causality

Microcausality violation is generic in HD theories

Källen-Lehmann replaced by Coleman relation

- from days of Lee-Wick theories

$$D(q) = \frac{1}{q^2 + i\epsilon} - \frac{\beta}{q^2 - M_r^2} - \frac{\beta^*}{q^2 - M_r^{*2}} + \frac{1}{\pi}\int_0^\infty ds \frac{\rho(s)}{q^2 - s + i\epsilon}$$

with $M_r^2 = m^2 + im\Gamma$

If M is heavy, integrate it out

- causal EFT at low energy

<u>**This can be consistent will all experimental knowledge!**</u>

Are there Higher Derivative theories?

A) Theoretical – Natural in Wilsonian context

Wilsonian exact renormalization group

- changing scales brings in operators of all dimensions
- flow from HE to LE will induce **all operators**
- not natural to truncate to 2 derivatives only
- this is how Asymptotic Safety works

B) Phenomenological – Starobinsky inflation

Many inflation theories are now ruled out

Of remaining, Starobinsky inflation is natural from QFT perspective

- inflation slows because of dynamical R^2term
- consistency will require $(Weyl)^2$ term also
- **Quadratic Gravity**

<u>Quadratic gravity</u>:

$$S=\int d^4x\sqrt{-g}\left[-\Lambda_{vac}+\frac{1}{16\pi G}R+\frac{1}{6f_0^2}R^2-\frac{1}{2f_2^2}C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta}\right]$$

Renormalizable QFT for quantum gravity Stelle

- New but technical – can be tachyon free and asymptotically free

(Buccio, JFD, Menezes, Percacci PRL 2024)

This is a HD theory $R\sim\partial^2 g$ $R^2\sim\partial^2 g\partial^2 g$

The C^2 term leads to a spin 2 Merlin mode (partner with graviton)

$m_2^2 = f_2^2 M_P^2$

The R^2 term is spin 0 and ghost free (Ghost is gauge artifact)

$m_0^2 = f_0^2 M_P^2$

Mixed causal structure due to spin-2 Merlin

- near $m_2^2 = f_2^2 M_P^2$
- do we even expect usual causality in QG near Planck scale?

A Quadratic Gravity layer of reality?

If inflation occurs and is Starobinsky style:

- requires R^2 to be dynamically active
- not a small EFT perturbation
- then C^2 also expected to be dynamical

Both couplings are required with matter loops

- mixed under RG flow

$$\beta_{f_2^2} = -\frac{1}{(4\pi)^2}\frac{(539 f_2^2 + 20 f_0^2) f_2^2}{30} \,,$$

$$\beta_{f_0^2} = \frac{1}{(4\pi)^2}\frac{6 f_0^4 + 36 f_2^2 f_0^2 - 420 f_2^4}{36} \,,$$

This implies a layer of reality with active Quadratic Gravity

- need not be ultimate theory
- but at least is temporary a renormalizable theory

Fun fact: Planck mass becomes irrelevant

When $\boldsymbol{R^2}, \boldsymbol{C^2}$ dominant it is a scale invariant theory

- happens beyond $f_0^2 M_P^2$_
- $f_0^2 \sim 10^{-12}$ in Starobinsky

Nothing new happens at $\boldsymbol{M_P}$

- just a shadow of the low energy theory

Similar in Fermi theory vs SM

- Gauge boson scattering becomes strong at ~250 GeV in Fermi theory
- Higgs boson tames this at lower energy
- the strong scattering scale is a shadow of the Fermi theory

Briefly:

QFT of HD theories not fully understood

Tree-level stability seen in above calculations
- and lattice calculations (Jansen, Liu, Kuti)

Unitarity OK near Minkowski (JFD + GM)
- massive ghost not an asymptotic state

One loop results understood
- Lee-Wick contour
- D. Anselmi on higher orders

Singularity avoidance – Holdom, Stelle….

If questions are asked about Unitarity:

$$\langle f|T|i\rangle - \langle f|T^\dagger|i\rangle = i\sum_j \langle f|T^\dagger|j\rangle\langle j|T|i\rangle$$

Who counts in unitarity relation?

- Veltman 1963
- **only stable particles count**
- they form asymptotic Hilbert space
- **do not** make any cuts on unstable resonances

UNITARITY AND CAUSALITY IN A RENORMALIZABLE FIELD THEORY WITH UNSTABLE PARTICLES

M. VELTMAN *)

In HD theory, massive Merlin mode is not asymptotic state

- decay to light states

Veltman proof of unitarity goes through here also

- if only cuts are on the stable particles

Unitarity, stability, and loops of unstable ghosts

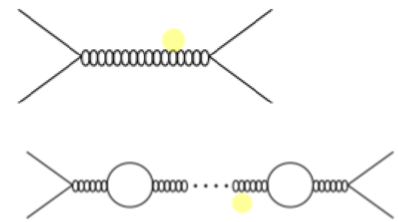
John F. Donoghue and Gabriel Menezes

Phys. Rev. D **100**, 105006 – Published 12 November 2019

Simple example: Unitarity in the spin two channel

Direct production of spin-2 ghost

First consider single scalar loop at low energy:



$$i\mathcal{M} = \left(\frac{1}{2}V_{\mu\nu}(q)\right)\left[iD^{\mu\nu\alpha\beta}(q^2)\right]\left(\frac{1}{2}V_{\alpha\beta}(-q)\right)$$

$$\mathcal{M} = 16\pi\sum_{J=0}^{\infty}(2J+1)T_J(s)P_J(\cos\theta)$$

Results in

$$T_2(s) = -\frac{N_{\rm eff}s}{640\pi}\bar{D}(s).$$

$N_{\rm eff} = 1/6$ for a single scalar field

$$\bar{D}^{-1}(s) \; = \; \frac{1}{\tilde{\kappa}^2}\left\{1 - \frac{\tilde{\kappa}^2 s}{2\xi^2(\mu)} - \frac{\tilde{\kappa}^2 s N_{\rm eff}}{640\pi^2}\ln\left(\frac{s}{\mu^2}\right) + \frac{i\tilde{\kappa}^2 s N_{\rm eff}}{640\pi}\right\}$$

$\uparrow$

$\xi = f_2$

Satisfies elastic unitarity:

$$\mathrm{Im} T_2 = |T_2|^2.$$

This implies the structure

$$T_2(s) = \frac{A(s)}{f(s) - iA(s)} = \frac{A(s)[f(s) + iA(s)]}{f^2(s) + A^2(s)}$$

for any real *f(s)*

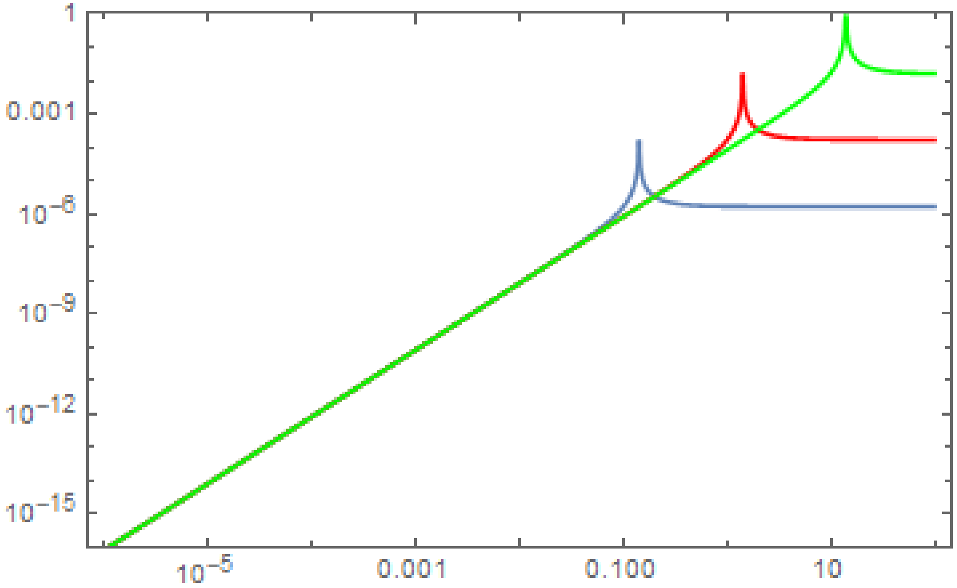
Signs and magnitudes work out for $A(s) = -\frac{N_{\mathrm{eff}}s}{640\pi}.$

Multi-particle problem**:**

- just diagonalize the J=2 channel
- same result but with general N

Lesson: Unitarity follows from the cuts on the stable particles

Tames the high energy behavior of the amplitude:



$\xi^2 = 0.1, 1, 10$

Can we utilize and/or observe lack of microcausality?

Possibly homogeneous initial conditions for Starobinsky inflation?
- recall acausal homogeneity arguments
- issues resurfaces in initial conditions for inflation

Backwards-in-time propagation can spread uniformity
- effective outside the light cone

Perhaps even a possible alternative to inflation?

Related:

Higher-Order Gravity, Finite Action, and a Safe Beginning for the Universe

Jean-Luc Lehners, K.S. Stelle (Dec 21, 2023)

e-Print: 2312.14048 [hep-th]

Emergent Causality as an alternative to inflation

JFD , GM
In progress

Homogeneity and Isotropy are motivations for inflation

- cannot be obtained from causal behavior in non-inflationary FLRW

But perhaps we do not have causal behavior in early universe!

- Merlin non-causality can be arbitrarily large at high energy
- perhaps observed homogeneity reflects lack of microcausality!

Other needed physics

- nearly scale invariant fluctuations
- slight tilt in perturbations

Alternate models exist for this.

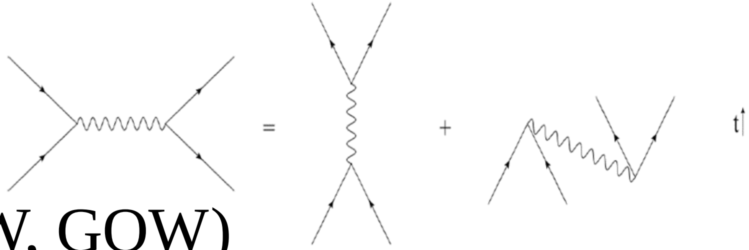
Dimension zero scalars (Boyle,Turok)

Scalar from R^2 would be a candidate for this

Testing causality:

Lee, Wick
Coleman
Grinstein, O'Connell, Wise
Alvarez, Da Roid, Schat, Szynkman

Vertex displacements: (ADSS)
- look for final state emergence
- before beam collision

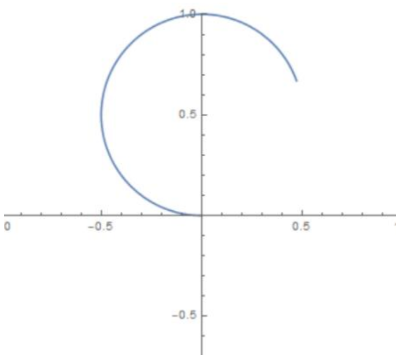


Form wavepackets – early arrival (LW, GOW)
- wavepacket description of scattering process
- some components arrive at detector early

Resonance Wigner time delay reversal
- normal resonaces counterclockwise on Argand diagram

$$\Delta t \sim \frac{\partial \delta}{\partial E} \sim > 0$$

- Merlin modes are clockwise resonance



Swampland EFT coefficients
- causality/analyticity constraints

Summary:

GR as EFT is a normal QFT in the modern sense

- reliable and predictive in its range of validity

Microcausality may be a HE frontier for QFT

Quadratic gravity is renormalizable QFT for gravity

- retains metric and general covariance as ingredients
- can be asymptotically free

Emergent causality is a feature/bug

- perhaps an opportunity
- Lab is CMB

Novel aspects of QFT get highlighted

- still not fully understood