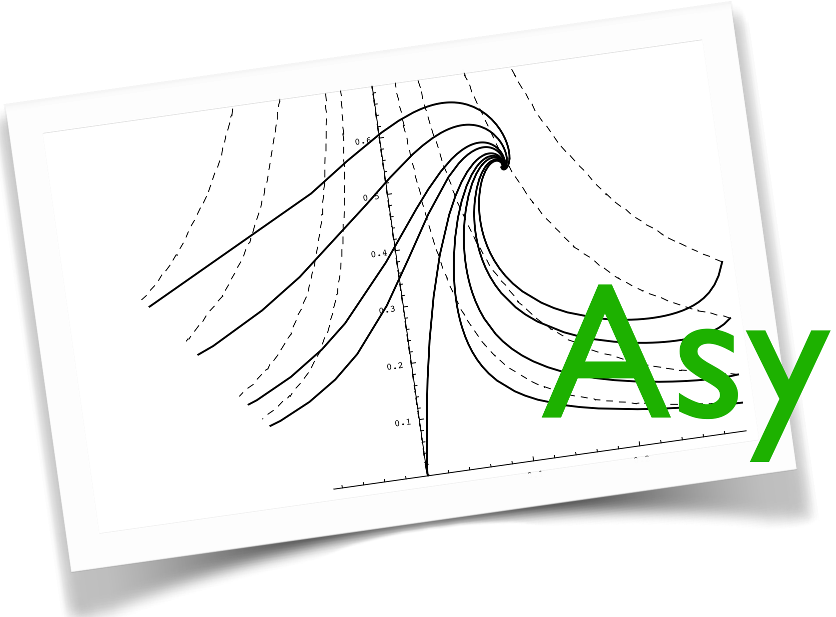


Asymptotic Safety
and/or
EFT of Quantum Gravity

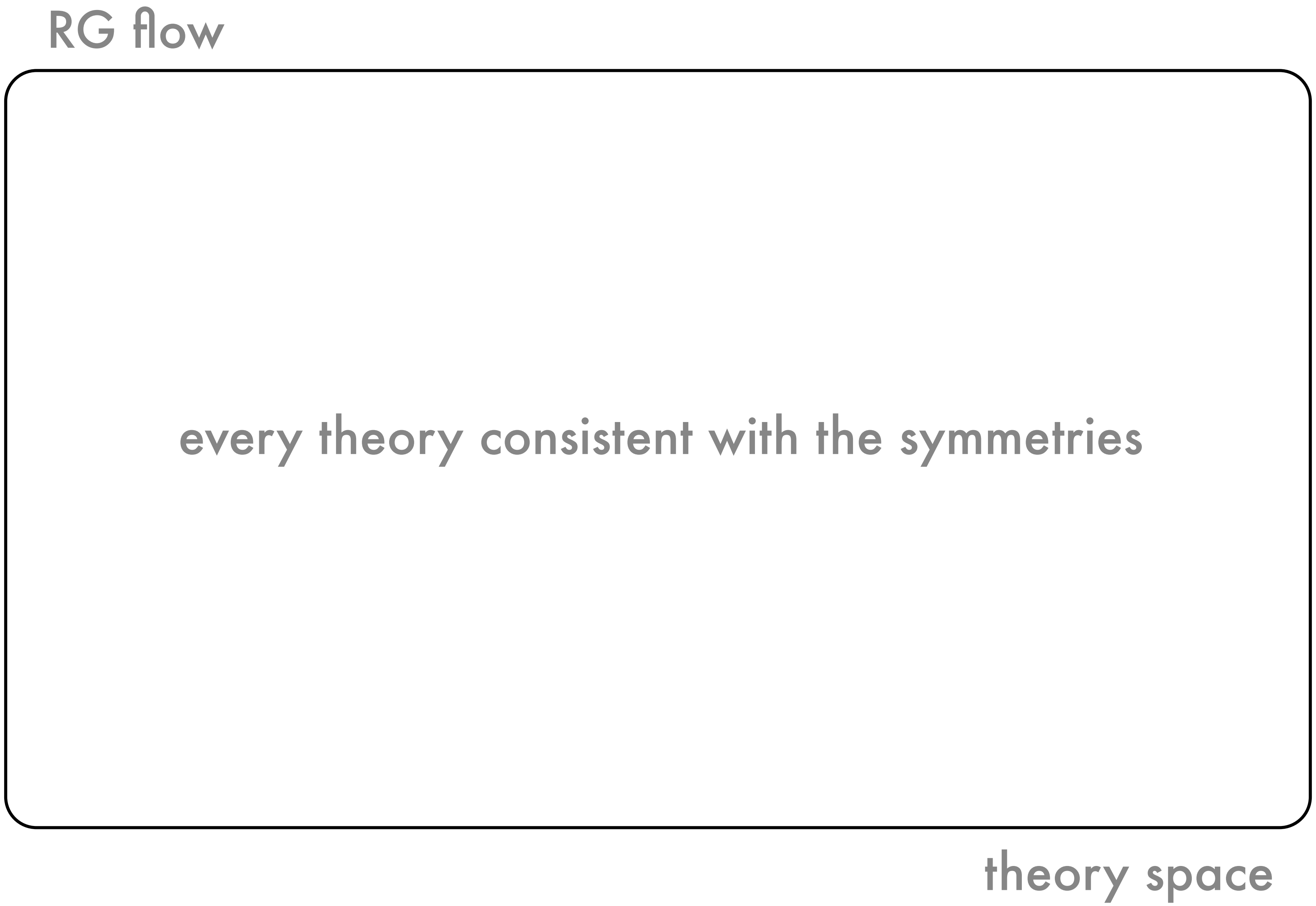
Alessandro Codello

Witnessing Quantum Aspects of Gravity in a Lab

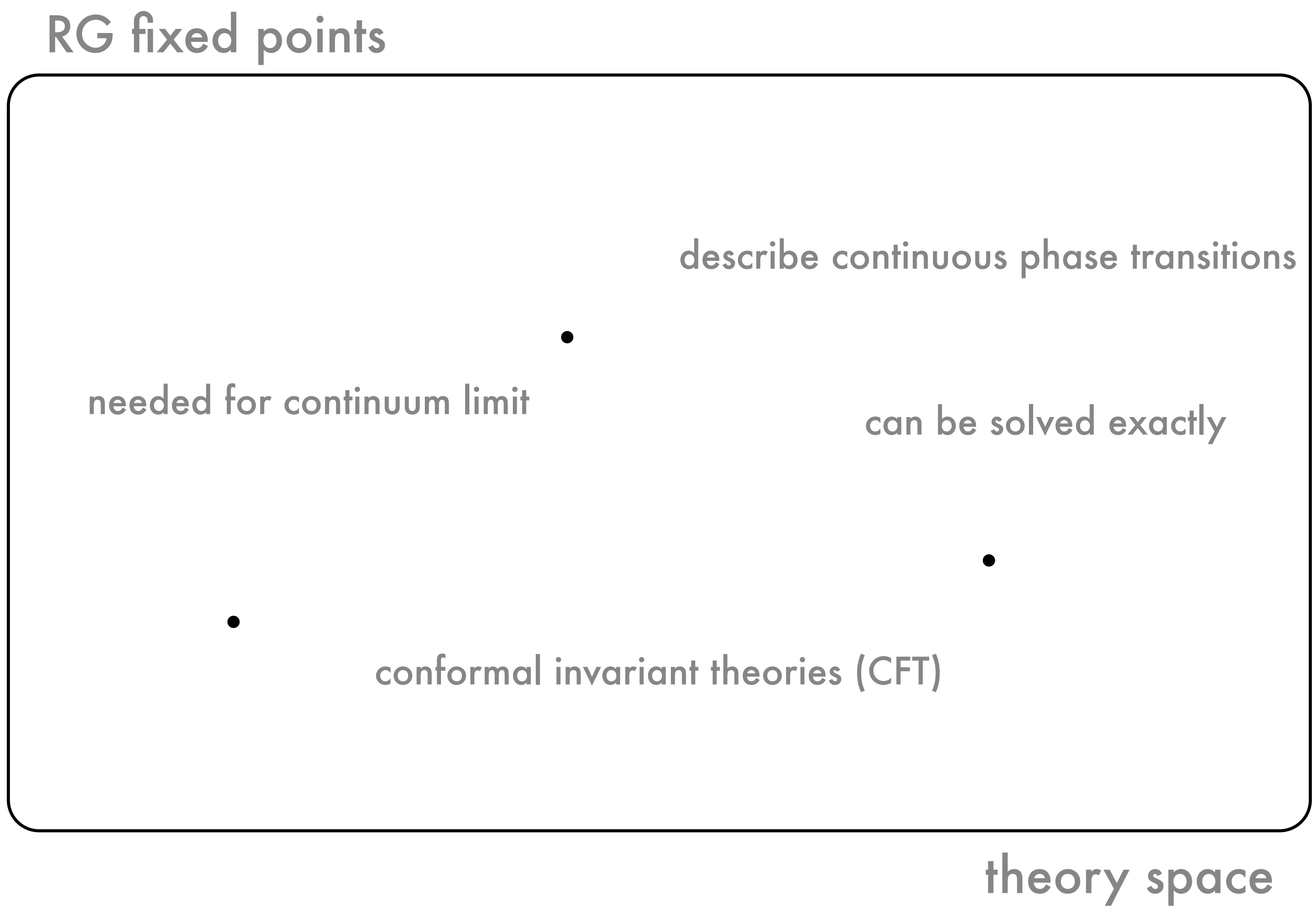
ICTP-SAIFR, Sao Paulo, Brasil

September 27, 2024

RG theory

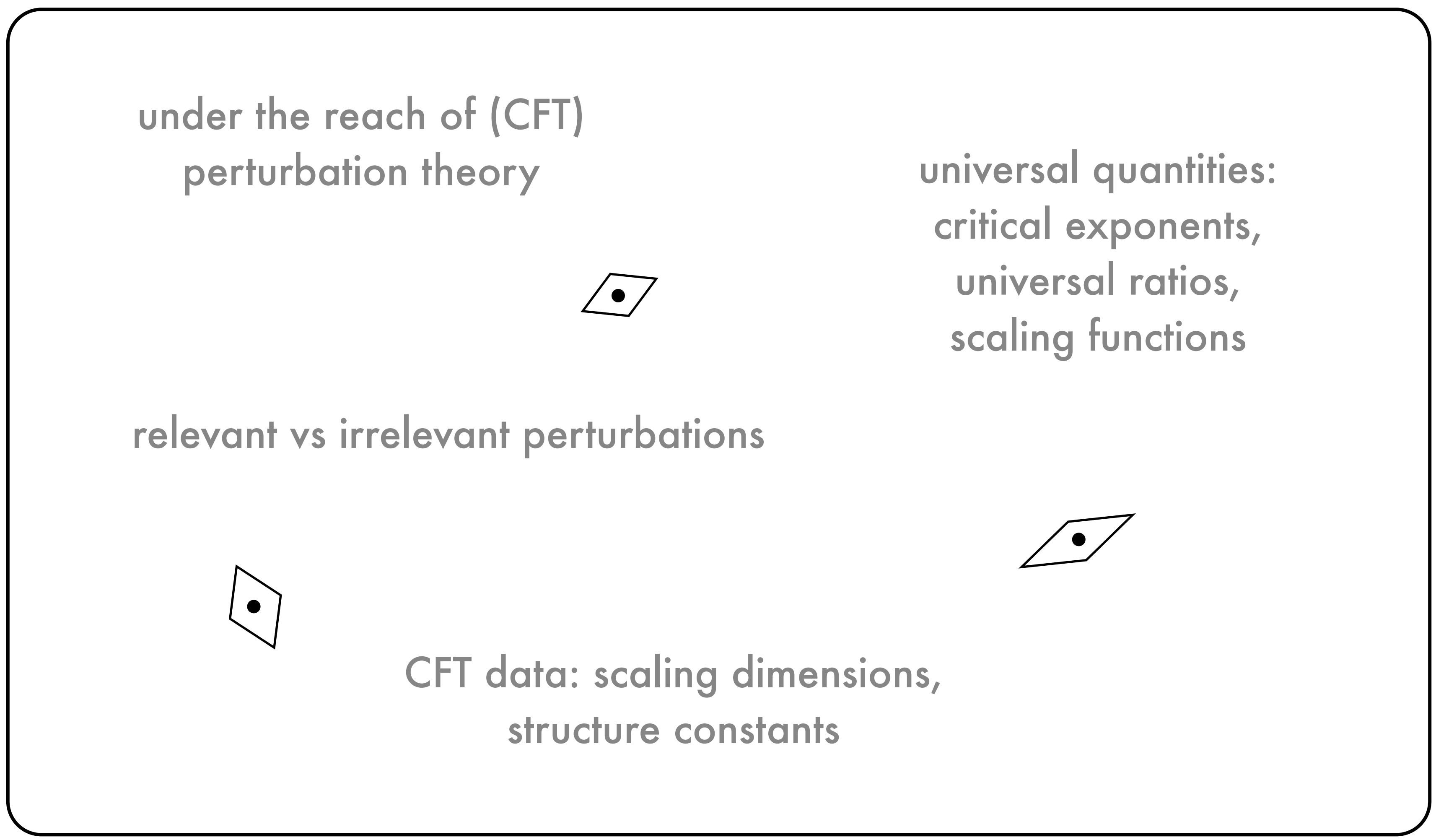

RG flow
every theory consistent with the symmetries
theory space

RG theory


RG fixed points
describe continuous phase transitions
needed for continuum limit
can be solved exactly
conformal invariant theories (CFT)
theory space

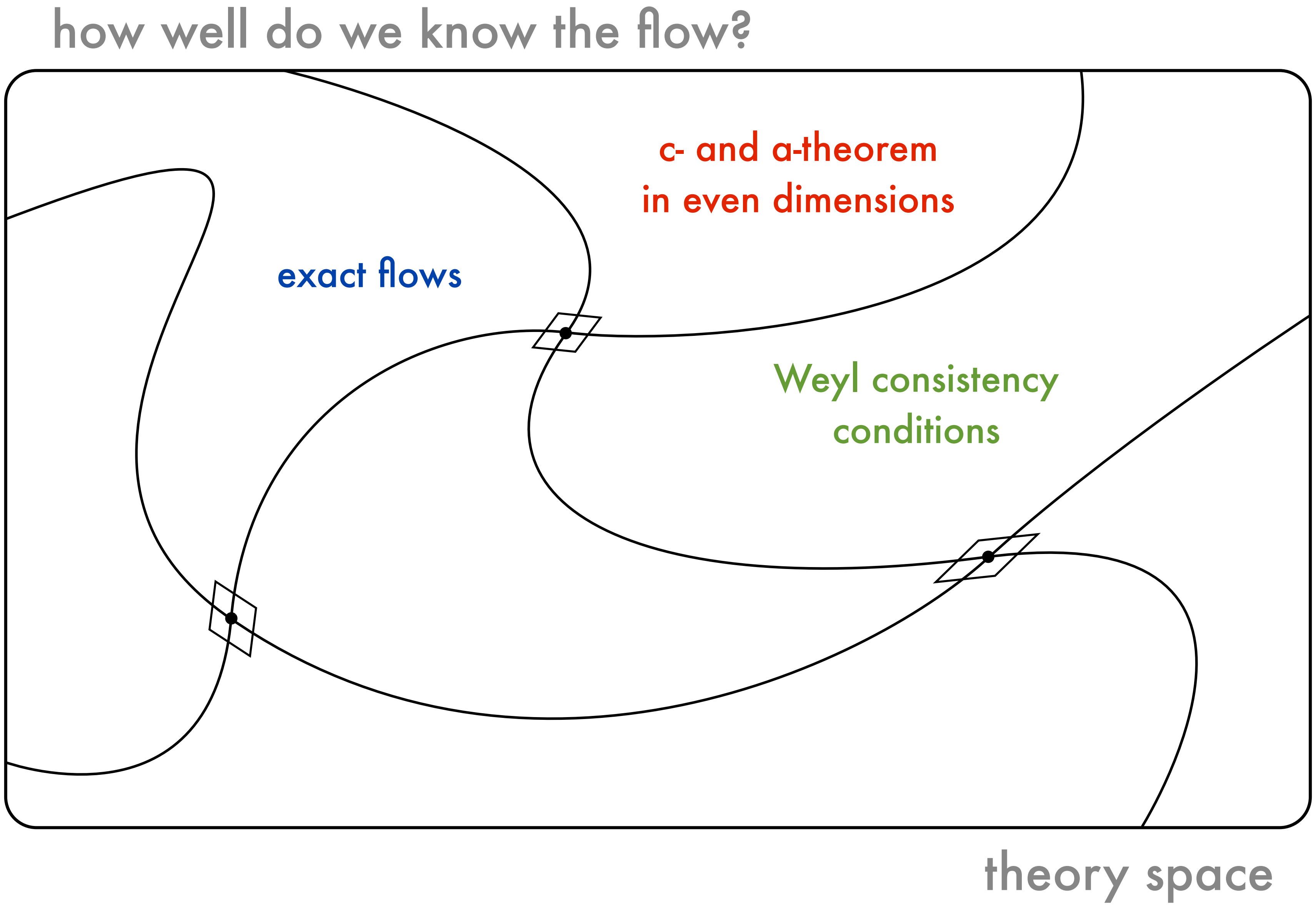
RG theory

scaling regions



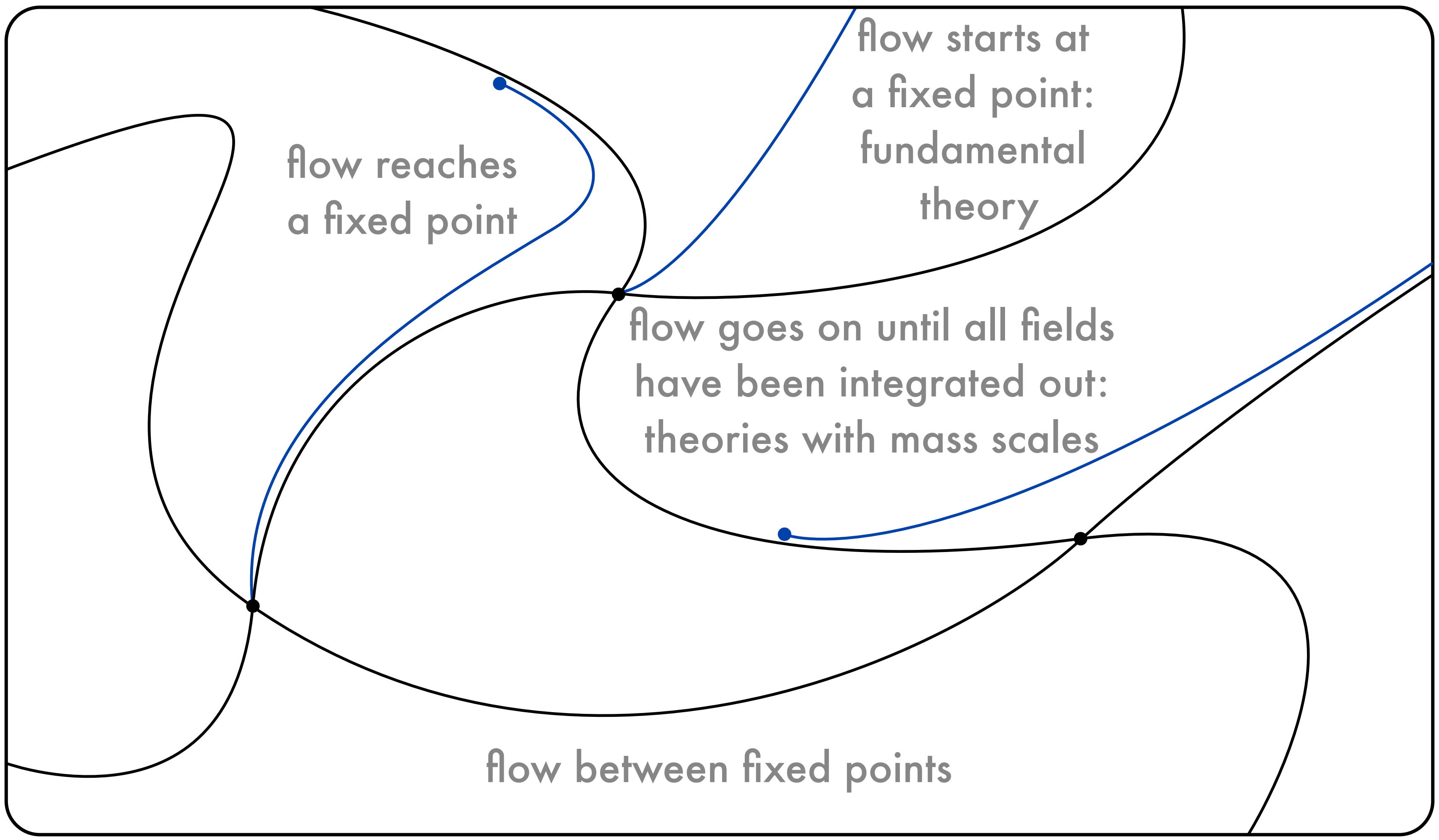
theory space

RG theory


how well do we know the flow?
c- and a-theorem
in even dimensions
exact flows
Weyl consistency
conditions
theory space

RG theory

types of flows



theory space

Asymptotic Safety

A Theory Space and an RG Flow for Quantum Gravity can be constructed!

Quantum Gravity may be Asymptotically Safe, i.e. non-perturbatively renormalizable at a non-trivial fixed point of the RG flow...

...there is evidence that a non-trivial fixed point with a finite dimensional UV critical surface exists!

A. Codello, R. Percacci and C. Rahmede, Annals Phys. 324 (2009) 414, arXiv:0805.2909 [hep-th]
A. Codello, R. Percacci and C. Rahmede, Int. J. Mod. Phys. A 23 (2008) 143, arXiv:0705.1769 [hep-th]

Asymptotic Safety

Effective Field Theory, Past and Future

Steven Weinberg*
Theory Group, Department of Physics, University of Texas
Austin, TX, 78712

Abstract

This is a written version of the opening talk at the 6th International Workshop on Chiral Dynamics, at the University of Bern, Switzerland, July 6, 2009, to be published in the proceedings of the Workshop. In it, I reminisce about the early development of effective field theories of the strong interactions, comment briefly on some other applications of effective field theories, and then take up the idea that the Standard Model and General Relativity are the leading terms in an effective field theory. Finally, I cite recent calculations that suggest that the effective field theory of gravitation and matter is asymptotically safe.

couplings that exploded at high energy, would lose all predictive value at high energy.

In just the last few years calculations have been done that allow more optimism. Codello, Percacci, and Rahmede[41] have considered a Lagrangian containing all terms $\sqrt{g}R^n$ with n running from zero to a maximum value $n_{\rm max}$, and find that the ultraviolet critical surface has dimensionality 3 even when $n_{\rm max}$ exceeds 2, up to the highest value $n_{\rm max} = 6$ that they considered, for which the space of coupling constants is 7-dimensional. Furthermore, the three eigenvalues they find with negative real part seem to converge as $n_{\rm max}$ increases, as shown in the following table of ultraviolet-attractive eigenvalues:

$n_{\rm max} = 2:$	$-1.38 \pm 2.32i$	-26.8
$n_{\rm max} = 3:$	$-2.71 \pm 2.27i$	-2.07
$n_{\rm max} = 4:$	$-2.86 \pm 2.45i$	-1.55
$n_{\rm max} = 5:$	$-2.53 \pm 2.69i$	-1.78
$n_{\rm max} = 6:$	$-2.41 \pm 2.42i$	-1.50

In a subsequent paper[42] they added matter fields, and again found just three ultraviolet-attractive eigenvalues. Further, this year Benedetti, Machado, and Saueressig[43] considered a truncation with a different four terms, terms proportional to $\sqrt{g}R^n$ with $n = 0, 1$ and 2 and also $\sqrt{g}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$ (where $C_{\mu\nu\rho\sigma}$ is the Weyl tensor) and they too find just three ultraviolet-attractive eigenvalues, also when matter is added. If this pattern of eigenvalues continues to hold in future calculations, it will begin to look as if there is a quantum field theory of gravitation that is well-defined at all energies, and that has just three free parameters.

The natural arena for application of these ideas is in the physics of gravitation at small distance scales and high energy — specifically, in the early universe. A start in this direction has been made by Reuter and his collaborators,[44] but much remains to be done.

41 A. Codello, R. Percacci, & C. Rahmede, Int. J. Mod. Phys. A23, 143 (2008)
42 A. Codello, R. Percacci, & C. Rahmede, Ann. Phys. 324, 414 (2009)
43 D. Benedetti, P. F. Machado, & F. Saueressig, 0901.2984, 0902.4630
44 A. Bonanno and M. Reuter, Phys. Rev. D 65, 043508 (2002); Phys. Lett. B527, 9

fRG and local symmetries

The coarse-graining procedure necessarily breaks gauge or space-time symmetries…

Preserve gauge symmetry while breaking space-time symmetry:

lattice gauge theory

dynamical triangulations

….

Preserve space-time symmetry while breaking gauge symmetry:

effective average action + modified ward identities

background effective average action

…

Background field method in Quantum Gravity

Quantize fluctuations around a general background:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

TECHNICAL!

Physical diffeomorphisms:

$$x_\alpha \to x_\alpha + \epsilon_\alpha$$

$$\delta g_{\mu\nu} = \nabla_\mu \epsilon_\nu + \nabla_\nu \epsilon_\mu$$

$$\delta h_{\mu\nu} = \nabla_\mu \epsilon_\nu + \nabla_\nu \epsilon_\mu \qquad \delta \bar{g}_{\mu\nu} = 0$$

Background diffeomorphisms:

$$\bar{\delta} h_{\mu\nu} = 0 \qquad \bar{\delta} \bar{g}_{\mu\nu} = \bar{\nabla}_\mu \epsilon_\nu + \bar{\nabla}_\nu \epsilon_\mu$$

Background effective average action (bEAA)

TECHNICAL!

Classical action:

$$S[h, \bar{C}, C; \bar{g}] = S[\bar{g} + h] + S_{gf}[h; \bar{g}] + S_{gh}[h, \bar{C}, C; \bar{g}]$$

$$(\delta + \bar{\delta}) S[\varphi; \bar{g}] = 0$$

Cutoff action:

$$\Delta S_k[\varphi; \bar{g}] = \frac{1}{2} \int d^d x \sqrt{\bar{g}} \varphi \, R_k[\bar{g}] \varphi$$

$$(\delta + \bar{\delta}) \Delta S_k[\varphi; \bar{g}] = 0$$

Background effective average action (bEAA)

$$e^{-\Gamma[\varphi;\bar{g}]} = \int D\chi \exp\left(-S[\varphi+\chi;\bar{g}] + \int d^dx\sqrt{\bar{g}}\,\Gamma^{(1,0)}[\varphi;\bar{g}]\chi\right)$$

$$\varphi = (h, \bar{C}, C)$$

Standard definition of the background effective action

Add the cutoff action!

$$e^{-\Gamma_k[\varphi;\bar{g}]} = \int D\chi \exp\left(-S[\varphi+\chi;\bar{g}] - \Delta S_k[\chi;\bar{g}] + \int d^dx\sqrt{\bar{g}}\,\Gamma_k^{(1;0)}[\varphi;\bar{g}]\chi\right)$$

The bEAA can be seen as a non-perturbative regularised version of the background effective action

We want to preserve the one-loop structure of the flow!

TECHNICAL!

Background effective average action (bEAA)

TECHNICAL!

The bEAA is invariant under physical + background diffeomorphisms:

$$(\delta + \bar{\delta})\Gamma_k[\varphi; \bar{g}] = 0$$

We can define a gauge invariant functional (gEAA):

$$\bar{\Gamma}_k[\bar{g}] = \Gamma_k[0; \bar{g}]$$

$$\Gamma_k[\varphi; \bar{g}] = \bar{\Gamma}_k[\bar{g} + h] + \hat{\Gamma}_k[\varphi; \bar{g}]$$

The gEAA is invariant under physical diffeomorphisms:

$$\delta\bar{\Gamma}_k[\bar{g}] = 0$$

Exact fRG equations for QG

The exact flow equation for the bEAA has a one-loop structure:

$$\partial_t \Gamma_k[\varphi;\bar{g}] = \frac{1}{2}\mathrm{Tr}\left(\Gamma_k^{(2;0)}[\varphi;\bar{g}] + R_k[\bar{g}]\right)^{-1} \partial_t R_k[\bar{g}]$$

Obtain the flow equation for the gEAA:

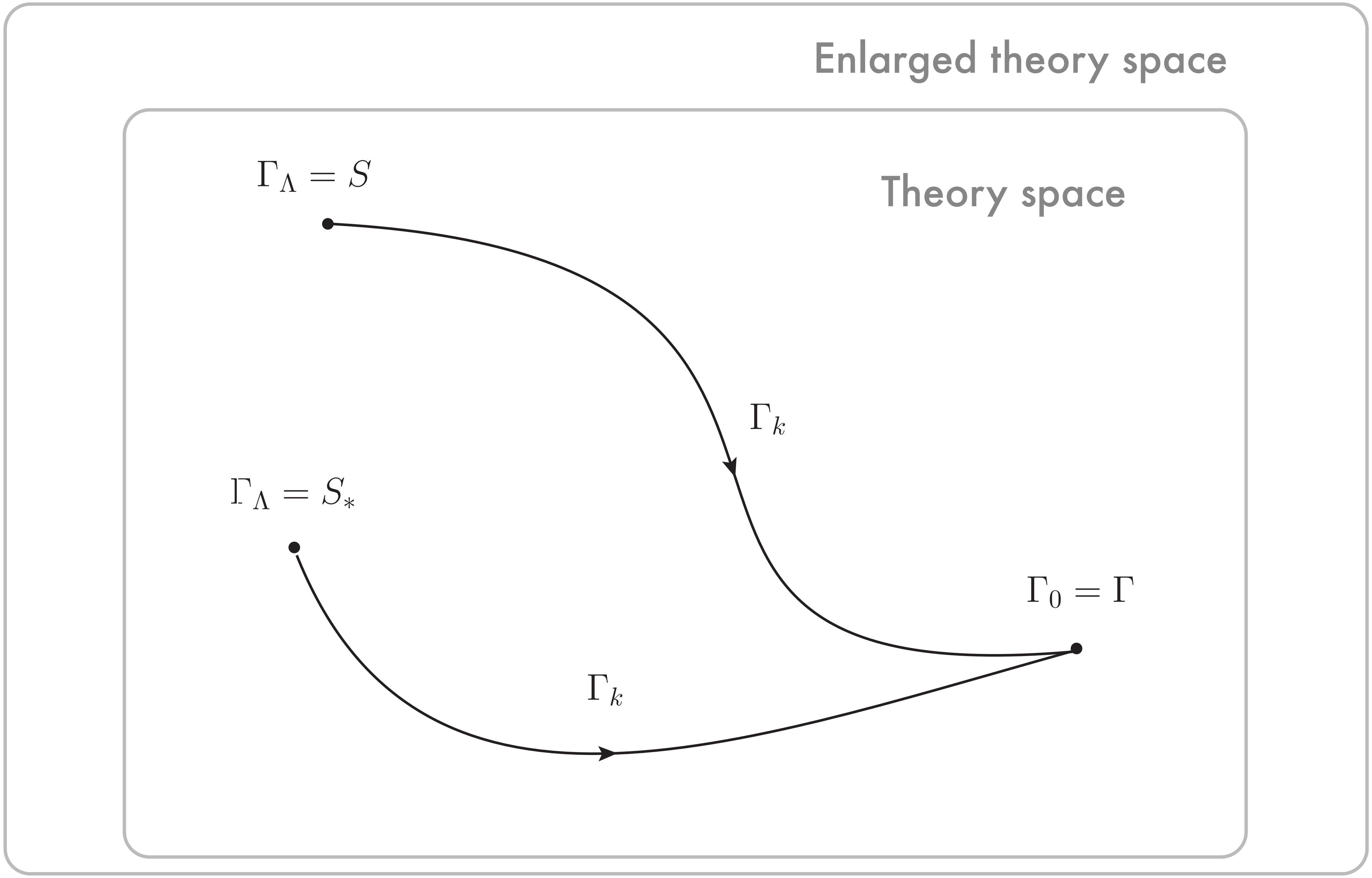
$$\partial_t \bar{\Gamma}_k[\bar{g}] = \partial_t \Gamma_k[0;\bar{g}]$$

$$\partial_t \bar{\Gamma}_k[g] = \frac{1}{2}\mathrm{Tr}\left(\Gamma_k^{(2;0)}[0;g] + R_k[g]\right)^{-1} \partial_t R_k[g]$$

The flow of the gEAA is not closed!

Exact fRG equations for QG

The bEAA interpolates smoothly between the bare and the quantum action but theory space is enlarged to bi-field functionals!



Einstein-Hilbert truncation

Einstein-Hilbert truncation:

$$\bar{\Gamma}_k[g] = \frac{1}{16\pi G_k} \int d^d x \sqrt{g}\,(2\Lambda_k - R)$$

Running Newton's constant

Running cosmological constant

$$\tilde{\Lambda}_k = k^{-2}\Lambda_k \qquad \tilde{G}_k = k^{d-2} G_k$$

Einstein-Hilbert truncation

$$\bar{\Gamma}_k[\bar{g} + \kappa_k Z_{h,k}^{1/2} h] = \frac{1}{\kappa_k^2} \int d^d x \sqrt{\bar{g}} \left(2\Lambda_k - \bar{R}\right)$$
$$+ \frac{Z_{h,k}^{1/2}}{\kappa_k} \int d^d x \sqrt{\bar{g}} \left[-\bar{\Delta} h - \bar{\nabla}^\mu \bar{\nabla}^\nu h_{\mu\nu} + h_{\mu\nu} \bar{R}^{\mu\nu} + \frac{1}{2} h \left(2\Lambda_k - \bar{R}\right) \right]$$
$$+ \frac{1}{2} Z_{h,k} \int d^d x \left[\frac{1}{2} h^{\mu\nu} \bar{\Delta} h_{\mu\nu} - \frac{1}{2} h \bar{\Delta} h + h^{\mu\nu} \bar{\nabla}_\nu \bar{\nabla}_\alpha h^\alpha_\mu - h \bar{\nabla}^\mu \bar{\nabla}^\nu h_{\mu\nu} \right.$$
$$- h^{\mu\nu} h^\alpha_\mu \bar{R}_{\nu\alpha} - h^{\mu\nu} h^{\alpha\beta} \bar{R}_{\alpha\mu\beta\nu} - h \bar{R}^{\mu\nu} h_{\mu\nu}$$
$$\left. + \left(\frac{1}{4} h^2 - \frac{1}{2} h^{\alpha\beta} h_{\alpha\beta} \right) \left(2\Lambda_k - \bar{R}\right) \right]$$
$$+ O\left(\kappa_k^{3/2} h^3\right)$$

$$\kappa_k = \sqrt{16\pi G_k}$$

Details of the truncation:

Pauli-Fierz mass:

$$m_{h,k} = 0$$

$$\hat{\Gamma}_k[Z_{h,k}^{1/2} h, Z_{C,k}^{1/2} \bar{C}, Z_{C,k}^{1/2} C; \bar{g}] = \frac{1}{2} Z_{h,k} \int d^d x \sqrt{\bar{g}} \left(h_{\mu\nu} h^{\mu\nu} - h^2 \right) m_{h,k}^2$$
$$+ \frac{1}{2\alpha_k} Z_{h,k} \int d^d x \sqrt{\bar{g}} \bar{g}^{\mu\nu} \left(\bar{\nabla}^\alpha h_{\alpha\mu} - \frac{\beta_k^2}{2} \bar{\nabla}_\mu h \right)^2$$
$$- Z_{C,k} \int d^d x \sqrt{\bar{g}}\, \bar{C}^\mu \left[\bar{\nabla}^\alpha g_{\nu\alpha} \nabla_\mu + \bar{\nabla}^\alpha g_{\mu\nu} \nabla_\alpha \right.$$
$$\left. - \beta_k \bar{\nabla}_\mu g_{\nu\alpha} \nabla^\alpha \right] C^\nu$$

Gauge-fixing parameters:

$$\alpha_k = \beta_k = 1$$

TECHNICAL!

Einstein-Hilbert truncation

Insert in the flow equation:

$$\eta_{h,k} = -\partial_t \log Z_{h,k} \qquad \eta_{C,k} = -\partial_t \log Z_{C,k}$$

$$\partial_t \bar{\Gamma}_k[g] = \frac{1}{2} \mathrm{Tr} \frac{\partial_t R_k(\Delta) - \eta_{h,k} R_k(\Delta)}{\mathbf{1}\,(\Delta - 2\Lambda_k) + \mathbf{U}} - \mathrm{Tr} \frac{\partial_t R_k(\Delta) - \eta_{C,k} R_k(\Delta)}{\Delta \delta^{\mu\nu} - R^{\mu\nu}}$$

$$\begin{aligned} U^{\alpha\beta}_{\rho\sigma} = & \left(\delta^{\alpha\beta}_{\rho\sigma} - \frac{1}{2} g^{\alpha\beta} g_{\rho\sigma}\right) R + g^{\alpha\beta} R_{\rho\sigma} + R^{\alpha\beta} g_{\rho\sigma} + \\ & - \frac{1}{2} \left(\delta^{\alpha}_{\rho} R^{\beta}_{\sigma} + \delta^{\alpha}_{\sigma} R^{\beta}_{\rho} + R^{\alpha}_{\rho} \delta^{\beta}_{\sigma} + R^{\alpha}_{\sigma} \delta^{\beta}_{\rho}\right) - \left(R^{\beta}{}_{\rho}{}^{\alpha}{}_{\sigma} + R^{\beta}{}_{\sigma}{}^{\alpha}{}_{\rho}\right) + \\ & - \frac{d-4}{2(d-2)} \left(R\, g^{\alpha\beta} g_{\rho\sigma} + g^{\alpha\beta} R_{\rho\sigma} + R^{\alpha\beta} g_{\rho\sigma}\right) \end{aligned}$$

TECHNICAL!

Calculate the functional traces using the heat kernel techniques

Asymptotic Safety

$$\partial_t \tilde{\Lambda}_k = -2\tilde{\Lambda}_k + \frac{8\pi}{(4\pi)^{d/2}\Gamma\left(\frac{d}{2}+2\right)} \left\{ \frac{d(d+1)}{4} \frac{d+2-\eta_{h,k}}{1-2\tilde{\Lambda}_k} - d(d+2-\eta_{C,k}) \right.$$

$$- 2\tilde{\Lambda}_k \left[\frac{d(d+1)(d+2)}{48} \frac{d-\eta_{h,k}}{1-2\tilde{\Lambda}_k} - \frac{d(d+2)}{12}(d-\eta_{C,k}) \right.$$

$$\left.\left. - \frac{d(d-1)}{4} \frac{2+d-\eta_{h,k}}{(1-2\tilde{\Lambda}_k)^2} - (d+2-\eta_{C,k}) \right] \right\} \tilde{G}_k$$

$$\partial_t \tilde{G}_k = (d-2)\tilde{G}_k + \frac{16\pi}{(4\pi)^{d/2}\Gamma\left(\frac{d}{2}+2\right)} \left\{ \frac{d(d+1)(d+2)}{48} \frac{d-\eta_{h,k}}{1-2\tilde{\Lambda}_k} \right.$$

$$\left. - \frac{d(d+2)}{12}(d-\eta_{C,k}) - \frac{d(d-1)}{4} \frac{2+d-\eta_{h,k}}{(1-2\tilde{\Lambda}_k)^2} - (d+2-\eta_{C,k}) \right\} \tilde{G}_k^2$$

The beta function system is not closed!

Asymptotic Safety

How do we obtain a closed beta function system?

One-loop approximation:

$$\eta_{h,k} = 0 \qquad\qquad \eta_{C,k} = 0$$

Standard approximation:

$$\eta_{h,k} = \frac{\partial_t \kappa_k}{\kappa_k} \qquad\qquad \eta_{C,k} = 0$$

The NSVZ beta function
Can be derived this way!

Calculate the anomalous dimensions:

$$\eta_{h,k}(\tilde{\Lambda}_k, \tilde{G}_k) \qquad\qquad \eta_{C,k}(\tilde{\Lambda}_k, \tilde{G}_k)$$

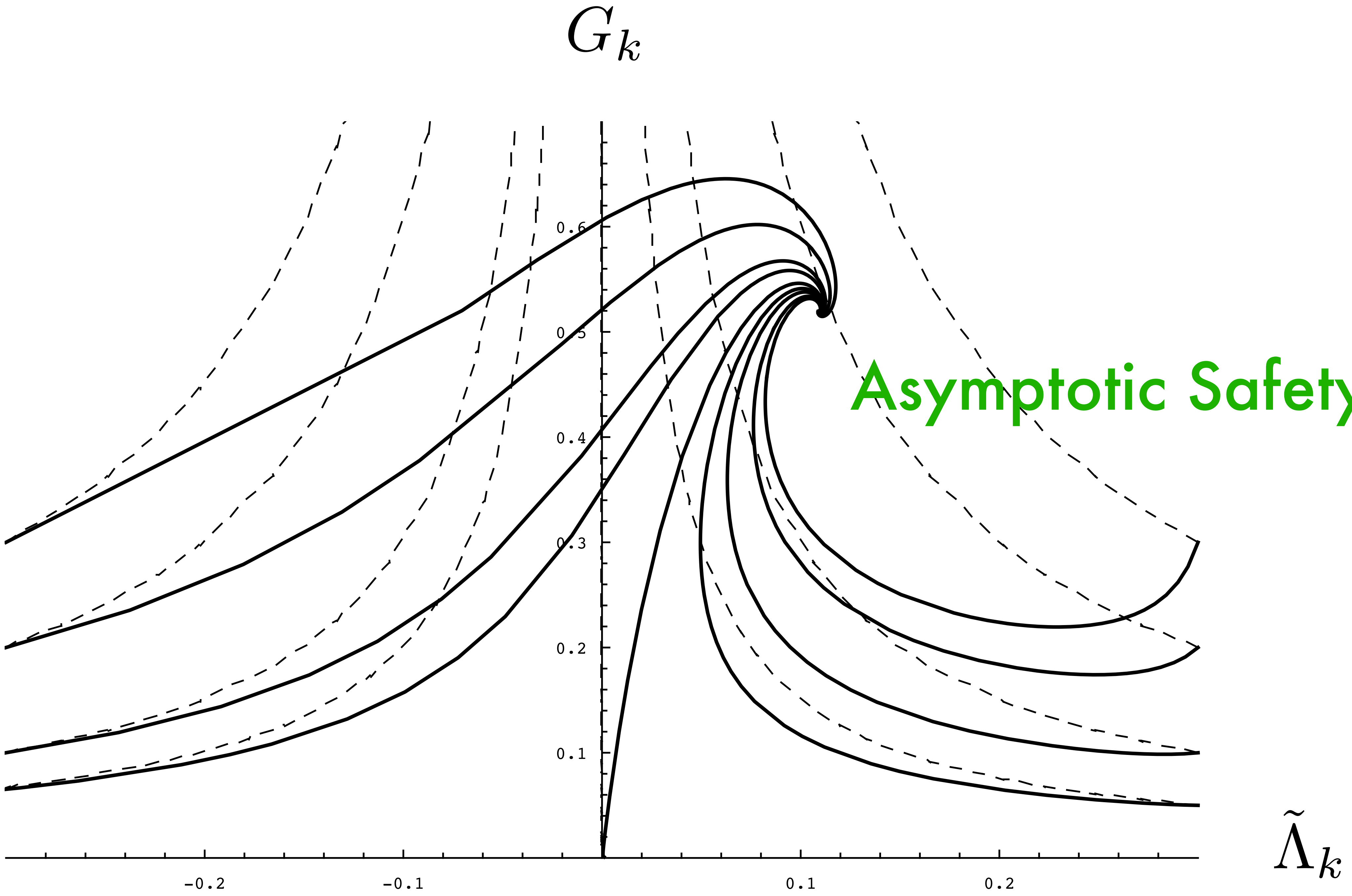
Asymptotic Safety

$$\eta_{h,k} = -\frac{1}{24\pi}\Big[(39 - 358\tilde{\Lambda}_k + 176\tilde{\Lambda}_k^2 + 1792\tilde{\Lambda}_k^3 - 2560\tilde{\Lambda}_k^4 + 1024\tilde{\Lambda}_k^5)\tilde{G}_k$$
$$-\frac{1}{48\pi}(381 - 2176\tilde{\Lambda}_k + 5040\tilde{\Lambda}_k^2 - 616\tilde{\Lambda}_k^3)\tilde{G}_k^2\Big]$$
$$\times\Big[(1-2\tilde{\Lambda}_k)^5 - \frac{1}{12\pi}(10 - 63\tilde{\Lambda}_k + 115\tilde{\Lambda}_k^2 - 56\tilde{\Lambda}_k^3 - 4\tilde{\Lambda}_k^4)\tilde{G}_k$$
$$+\frac{1}{576\pi^2}(18 - 125\tilde{\Lambda}_k + 307\tilde{\Lambda}_k^2 - 226\tilde{\Lambda}_k^3)\tilde{G}_k^2\Big]^{-1}$$

$$\eta_{C,k} = -\frac{1}{48\pi(1-2\tilde{\Lambda}_k)}\Big[(1-2\tilde{\Lambda}_k)^4(105 + 16\tilde{\Lambda}_k)\tilde{G}_k$$
$$-\frac{1}{192\pi}(12813 - 40496\tilde{\Lambda}_k + 85760\tilde{\Lambda}_k^2 - 107520\tilde{\Lambda}_k^3 + 57856\tilde{\Lambda}_k^4)\tilde{G}_k^2\Big]$$
$$\times\Big[(1-2\tilde{\Lambda}_k)^5 - \frac{1}{12\pi}(10 - 63\tilde{\Lambda}_k + 115\tilde{\Lambda}_k^2 - 56\tilde{\Lambda}_k^3 - 4\tilde{\Lambda}_k^4)\tilde{G}_k$$
$$+\frac{1}{576\pi^2}(18 - 125\tilde{\Lambda}_k + 307\tilde{\Lambda}_k^2 - 226\tilde{\Lambda}_k^3)\tilde{G}_k^2\Big]^{-1}$$

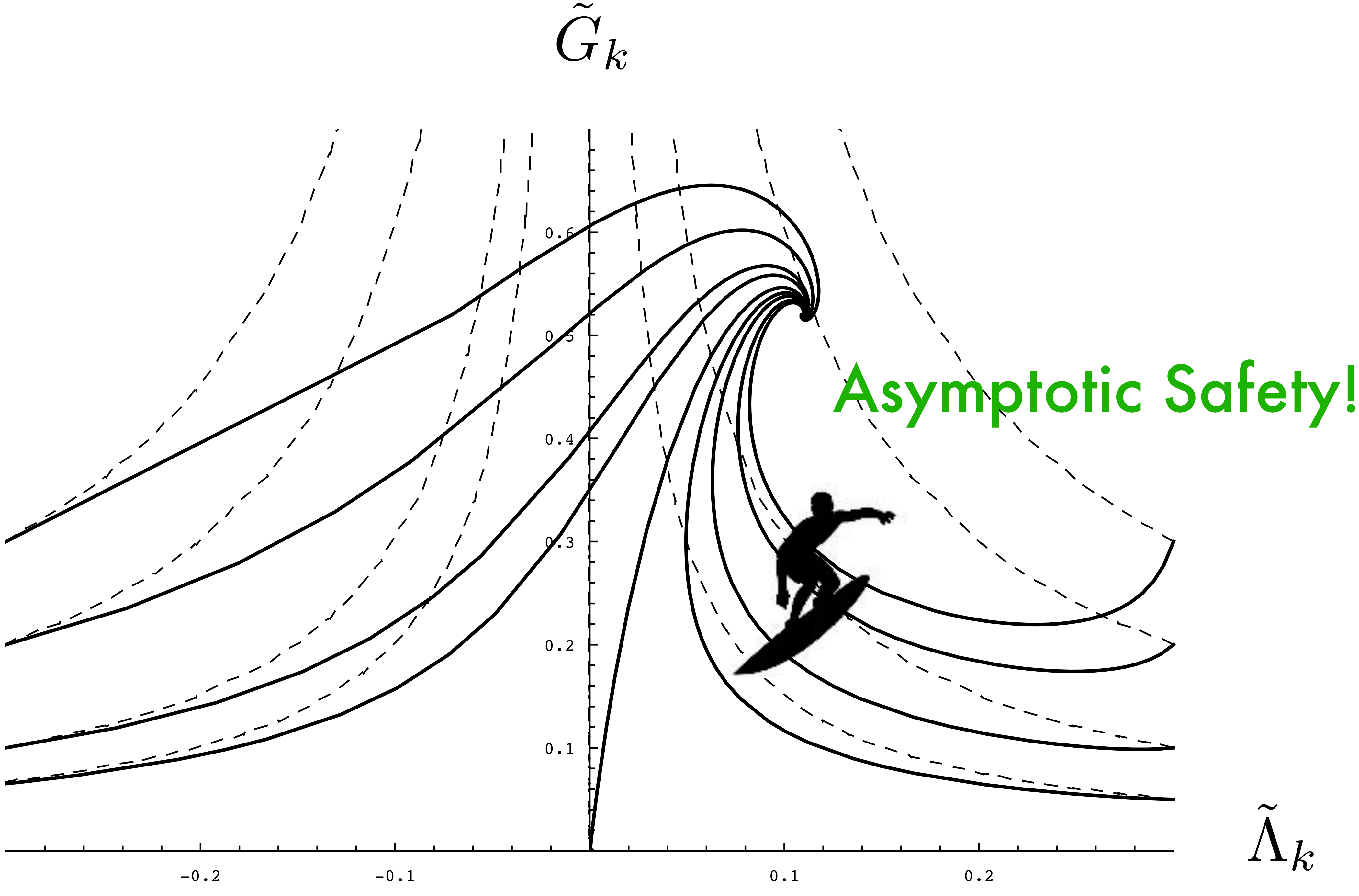
A. Codello, G. D'Odorico and C. Pagani Phys. Rev. D **89** (2014) 081701(R)

Asymptotic Safety



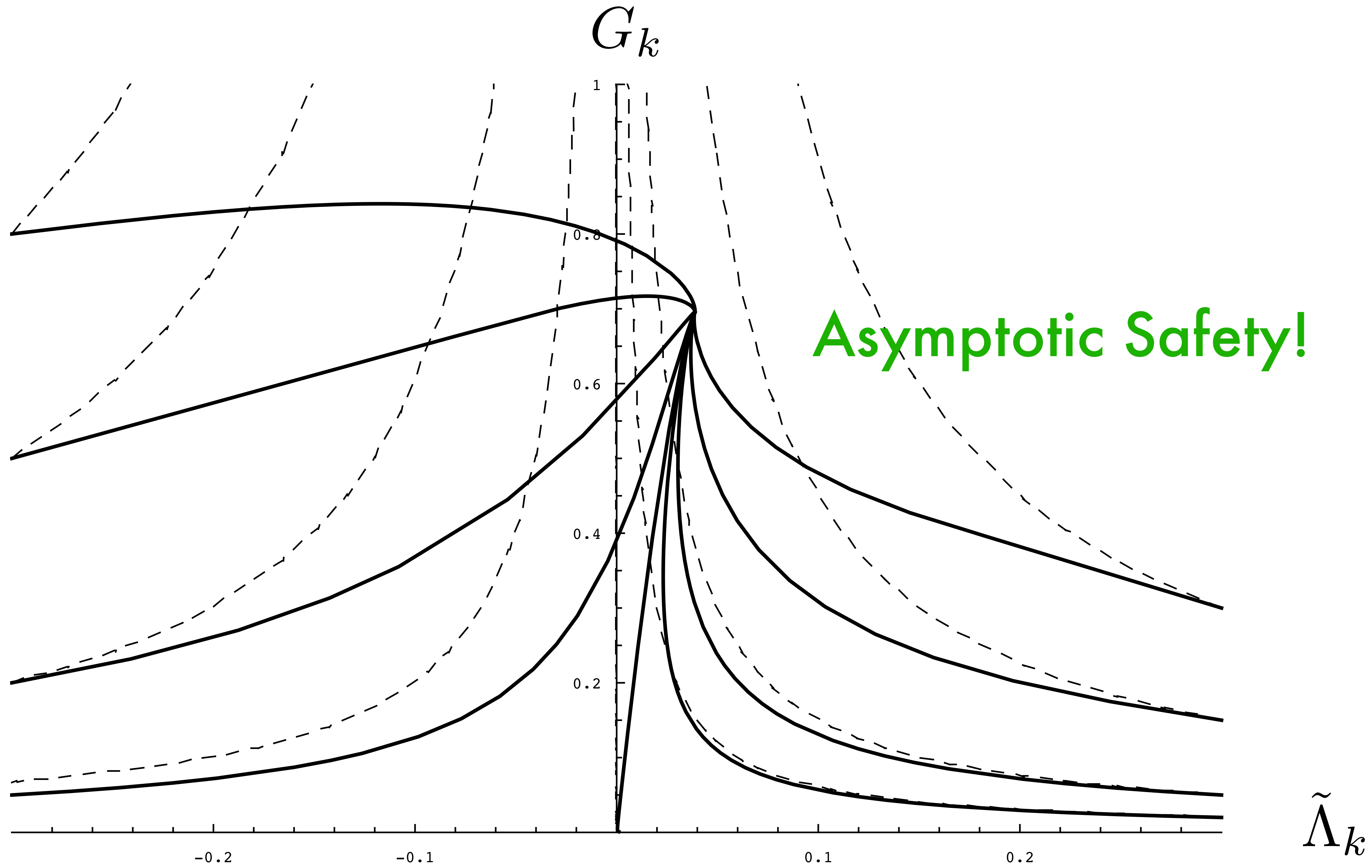
Flow in standard approximation

Asymptotic Safety



Flow in standard approximation

Asymptotic Safety



Flow after solving for the anomalous dimensions

Asymptotic Safety

More general approximation scheme:
curvature expansion

$$\bar{\Gamma}_k[g] = \int d^d x \sqrt{g} \left[\frac{1}{16\pi G_k} (2\Lambda_k - R) + R F_{1,k}(\Delta) R + R_{\mu\nu} F_{2,k}(\Delta) R^{\mu\nu} \right] + O(\mathcal{R}^3)$$

Running structure functions: full “functional” RG

$$\partial_t F_{i,k} = \mathcal{F}^d_{R_k} \left(\Lambda_k, G_k, F_{i,k}, F'_{i,k}, F''_{i,k} \right)$$

Integro-differential equations!

TECHNICAL!

Curvature expansion: an example

Minimally coupled scalar field on a two dimensional manifold:

$$\Delta\phi = -\frac{1}{\sqrt{g}}\partial_\mu\left(\sqrt{g}g^{\mu\nu}\partial_\nu\phi\right)$$

$$S[\phi,g] = \frac{1}{2}\int d^2x\sqrt{g}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = \frac{1}{2}\int d^2x\sqrt{g}\phi\Delta\phi$$

the bare action is Weyl invariant

The non trivial part of the EAA is purely gravitational:

$$\Gamma_k[g] = \int d^2x\sqrt{g}\left[a_k + b_k R + R\, c_k(\Delta) R\right] + O(R^3)$$

running structure function

The exact fRG equations a trace of a function of the covariant operator:

$$\partial_t\Gamma_k[g] = \frac{1}{2}\mathrm{Tr}\frac{\partial_t R_k(\Delta)}{\Delta + R_k(\Delta)}$$

TECHNICAL!

Curvature expansion: an example

TECHNICAL!

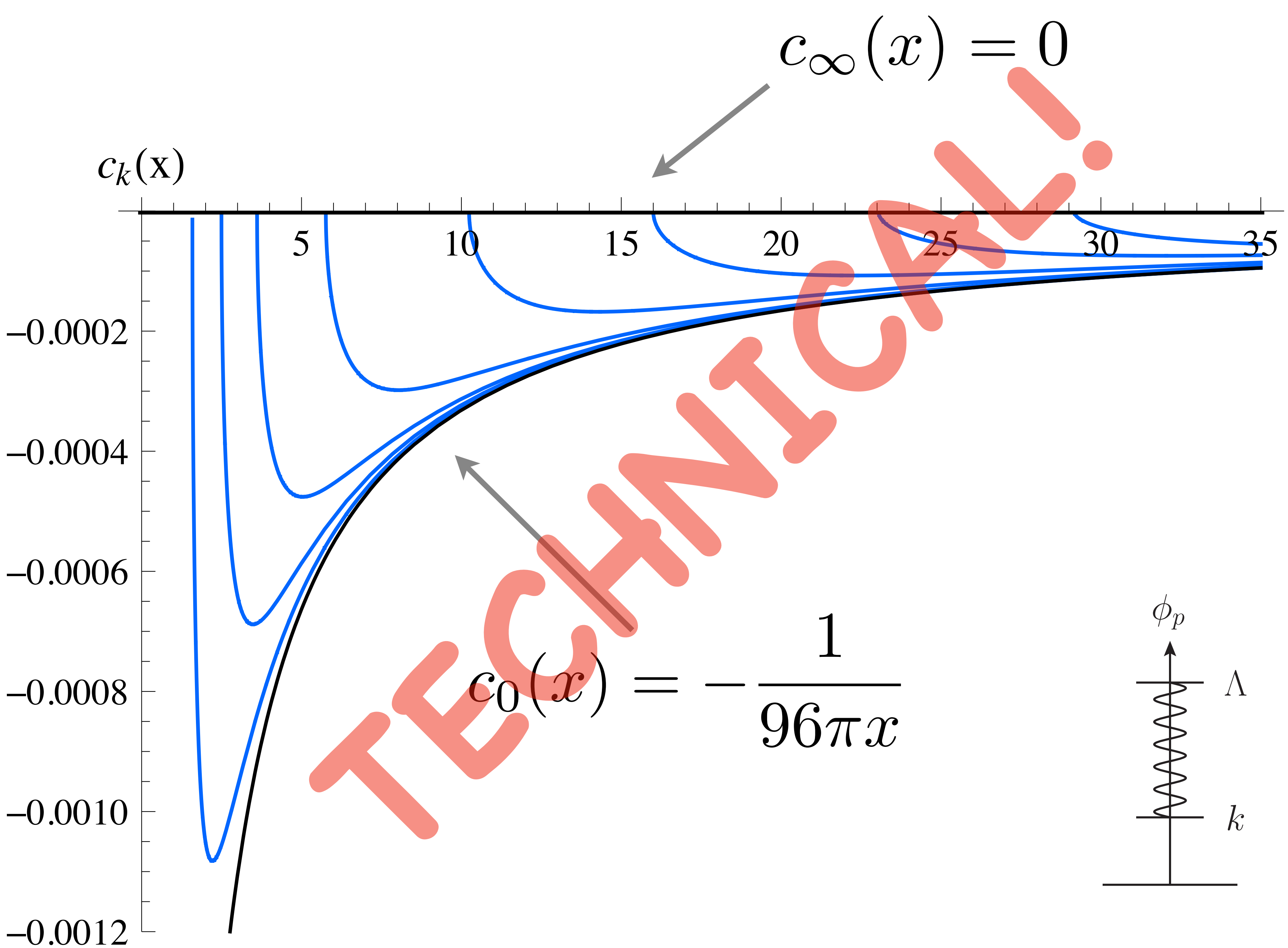
The EAA interpolates smoothly between the bare and the quantum action:

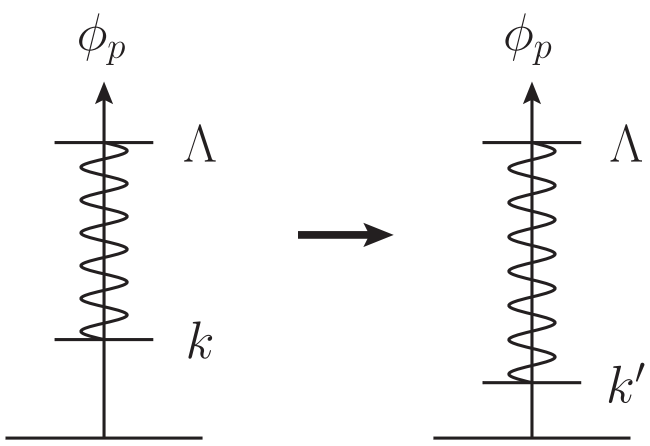
$$
\begin{aligned}
\Gamma_k[g] = {} & \frac{k^2}{4\pi}\int d^2x\sqrt{g} + \frac{\chi}{6}\log\frac{k}{k_0} \\
& - \frac{1}{96\pi}\int d^2x\sqrt{g}R\left[\frac{\sqrt{\Delta/k^2-4}\,(\Delta/k^2+2)}{\Delta\,(\Delta/k^2)^{3/2}}\theta(\Delta/k^2-4)\right]R \\
& + O(R^3)
\end{aligned}
$$

For $k=0$ we recover Polyakov's effective action:

$$
\Gamma_0[g] = -\frac{1}{96\pi}\int d^2x\sqrt{g}R\frac{1}{\Delta}R
$$

Curvature expansion: an example





A. Codello, Annals Phys. 325 (2010) 1727-1738

Curvature expansion: corrections to Newton's law

4d flow induced by the Einstein-Hilbert truncation

$$\bar{\Gamma}_0[g]\big|_{\mathcal{R}^2} = \frac{1}{32\pi^2}\int d^4x\sqrt{g}\left[\frac{1}{60}R\,\log\left(\frac{\Delta}{k_0^2}\right)R + \frac{7}{10}R_{\mu\nu}\,\log\left(\frac{\Delta}{k_0^2}\right)R^{\mu\nu}\right]$$

leads to Quantum Gravitational corrections to Newton's law:

$$V(r) = -\frac{MG_0}{r}\left[1 + \frac{43G_0}{30\pi r^2}\right]$$

A. Satz, A. Codello and F. D. Mazzitelli, Low energy Quantum Gravity from the Effective Average Action, Phys. Rev. D 82 (2010) 084011, arXiv:1006.3808 [hep-th]

Curvature expansion: corrections to Newto... law

4d flow induced by the Einst...

$$\bar{\Gamma}_0[g]\big|_{\mathcal{R}^2} = \frac{1}{32\pi^2}\int d^4x\sqrt{g}\,\Big[\ \ldots\ \Big]$$

UV shows up starting from $\mathcal{R}^3$-terms!

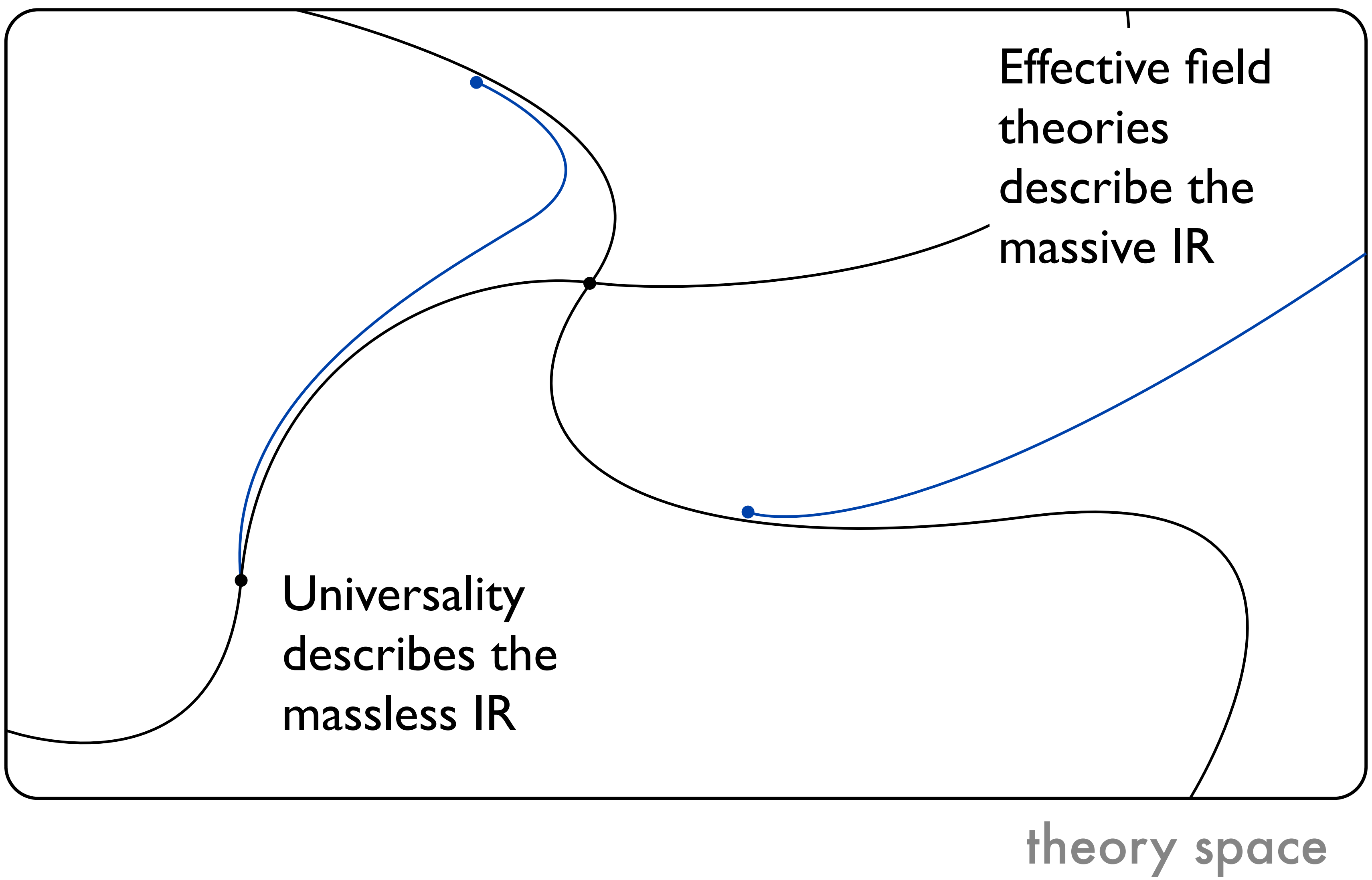
...al

...s law:

$$\ldots\frac{G_0}{r}\left[1+\frac{43G_0}{30\pi r^2}\right]$$

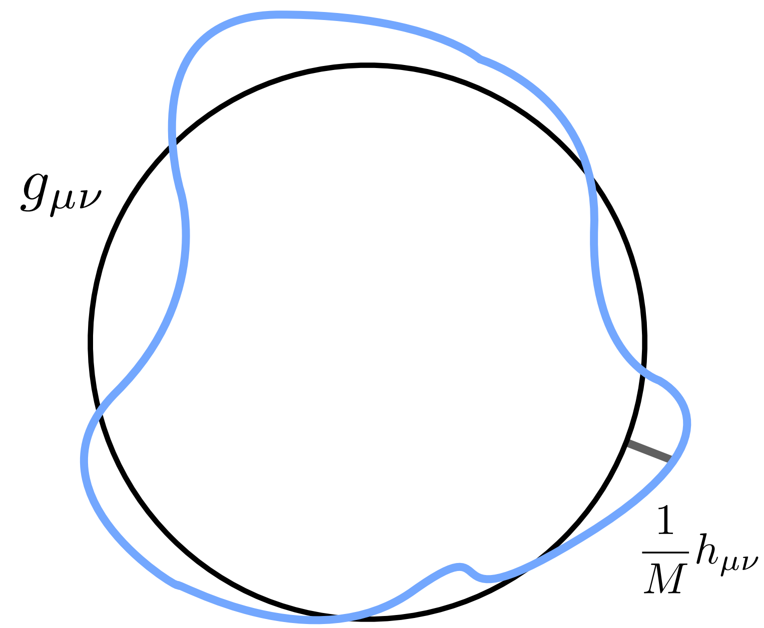
...ello and F. D. Mazzitelli, Low energy Quantum Gravity from the Effective Average Action, ... D 82 (2010) 084011, arXiv:1006.3808 [hep-th]

Effectivity vs Universality



Two main reasons why mathematical modeling of nature actually works

EFT of Gravity



- The theory of small fluctuations of the metric

$$g_{\mu\nu} \to g_{\mu\nu} + \sqrt{16\pi G} h_{\mu\nu} = g_{\mu\nu} + \frac{1}{M} h_{\mu\nu}$$

- Planck's scale is the characteristic scale of gravity

$$M \equiv \frac{1}{\sqrt{16\pi G}} = \frac{M_{Planck}}{\sqrt{16\pi}}$$

$$M_{Planck} = \frac{1}{\sqrt{G}} = 1.2 \times 10^{19}\,\mathrm{GeV}$$

- Classical theory (CT) is successful over many orders of magnitude

EFT of Gravity

$$S_{eff}[g] = M^2 \left[I_1[g] + \frac{1}{M^2} I_2[g] + \frac{1}{M^4} I_3[g] + ... \right]$$

UV action contains all couplings expressed in terms of the scale M

$$I_1[g] = \int d^4x \sqrt{g} \left[M^2 c_0 - c_1 R \right]$$

$$I_2[g] = \int d^4x \sqrt{g} \left[c_{2,1} R^2 + c_{2,2} \mathrm{Ric}^2 + c_{2,3} \mathrm{Riem}^2 \right]$$

$$I_3[g] = \int d^4x \sqrt{g} \left[c_{3,1} R \Box R + c_{3,2} R_{\mu\nu} \Box R^{\mu\nu} + c_{3,3} R^3 + ... \right]$$

Derivative expansion of the UV action

EFT of Gravity

$$S_{eff}[g] = M^2 \left[I_1[g] + \frac{1}{M^2} I_2[g] + \frac{1}{M^4} I_3[g] + ... \right]$$

UV action contains all couplings expressed in terms of the scale M

$$I_1[g] = \int d^4x \sqrt{g} \left[M^2 c_0 - c_1 R \right]$$

$$I_2[g] = \int d^4x \sqrt{g} \left[c_{2,1} R^2 + c_{2,2} \mathrm{Ric}^2 + c_{2,3} \mathrm{Riem}^2 \right]$$

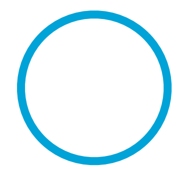
$$I_3[g] = \int d^4x \sqrt{g} \left[c_{3,1} R \Box R + c_{3,2} R_{\mu\nu} \Box R^{\mu\nu} + c_{3,3} R^3 + ... \right]$$

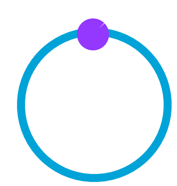
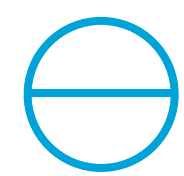
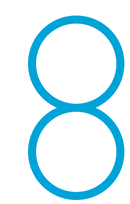
UV predicts the $\mathscr{R}^3$ coefficients only!

Covariant EFT of Gravity

The EFT recipe in three lines

$\Gamma = \underset{I_1}{\bullet}$ CT

$+ \frac{1}{M^2}\Big[\underset{I_2}{\bullet} + \frac{1}{2}$  $\Big]$ LO

$+ \frac{1}{M^4}\Big[\underset{I_3}{\bullet} + \frac{1}{2}$  $- \frac{1}{12}$  $+ \frac{1}{8}$  $\Big]$ NLO

$+ \cdots$ NNLO

1) the general lagrangian of order E^2 is to be used both at tree level and in loop diagrams

2) the general lagrangian of order $E^{n\geq 4}$ is to be used at tree level and as an insertion in loop diagrams

3) the renormalization program is carried out order by order

Curvature expansion

- $$+\frac{1}{2}\bigcirc = -\frac{1}{2(4\pi)^{d/2}}\int d^dx\sqrt{g}\,\mathrm{tr}\,\mathcal{R}\,\gamma_i\left(\frac{-\Box}{m^2}\right)\mathcal{R}+\ldots$$

TECHNICAL!

The finite physical part of the effective action is covariantly encoded in the structure functions which can be computed using the non-local heat kernel expansion

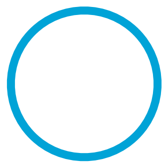
$$\gamma_i\left(\frac{X}{m^2}\right) \equiv \lim_{\Lambda_{UV}\to\infty}\int_{1/\Lambda_{UV}^2}^{\infty}\frac{ds}{s}\,s^{-d/2+2}\left[f_i(sX)-f_i(0)\right]e^{-sm^2}$$

Non-local heat kernel structure functions

Non-local heat kernel

A. O. Barvinsky and G. A. Vilkovisky, Nucl. Phys. B 282 (1987) 163
I. G. Avramidi, Lect. Notes Phys. M 64 (2000) 1
A. Codello and O. Zanusso, J. Math. Phys. 54 (2013) 013513

Curvature expansion

- $+\dfrac{1}{2}$  $= -\dfrac{1}{2(4\pi)^2}\displaystyle\int d^4x\sqrt{g}\,\mathrm{tr}\Big[\mathbf{1}R_{\mu\nu}\gamma_{Ric}\Big(\frac{-\Box}{m^2}\Big)R^{\mu\nu} + \frac{\mathbf{1}}{120}R\gamma_R\Big(\frac{-\Box}{m^2}\Big)R$

$$-\frac{1}{6}R\gamma_{RU}\Big(\frac{-\Box}{m^2}\Big)\mathbf{U} + \frac{1}{2}\mathbf{U}\gamma_U\Big(\frac{-\Box}{m^2}\Big)\mathbf{U} + \frac{1}{12}\mathbf{\Omega}_{\mu\nu}\gamma_\Omega\Big(\frac{-\Box}{m^2}\Big)\mathbf{\Omega}^{\mu\nu}\Big]$$

TECHNICAL!

Explicit form for the structure functions

$$\gamma_{Ric}(u) = \frac{1}{40} + \frac{1}{12u} - \frac{1}{2}\int_0^1 d\xi\left[\frac{1}{u} + \xi(1-\xi)\right]^2 \log\left[1 + u\,\xi(1-\xi)\right]$$

$$\gamma_R(u) = -\frac{23}{960} - \frac{1}{96u} + \frac{1}{32}\int_0^1 d\xi\left\{\frac{2}{u^2} + \frac{4}{u}\left[1 + \xi(1-\xi)\right]\right.$$
$$\left. - 1 + 2\xi(2-\xi)(1-\xi^2)\right\}\log\left[1 + u\,\xi(1-\xi)\right]$$

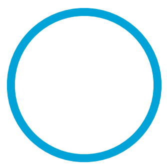
$$\gamma_{RU}(u) = \frac{1}{12} - \frac{1}{2}\int_0^1 d\xi\left[\frac{1}{u} - \frac{1}{2} + \xi(1-\xi)\right]\log\left[1 + u\,\xi(1-\xi)\right]$$

$$\gamma_U(u) = -\frac{1}{2}\int_0^1 d\xi\,\log\left[1 + u\,\xi(1-\xi)\right]$$

$$\gamma_\Omega(u) = \frac{1}{12} - \frac{1}{2}\int_0^1 d\xi\left[\frac{1}{u} + \xi(1-\xi)\right]\log\left[1 + u\,\xi(1-\xi)\right]$$

$$u \equiv \frac{-\Box}{m^2}$$

Curvature expansion

$$+\frac{1}{2}$$  $$= -\frac{1}{2(4\pi)^2}\int d^4x\sqrt{g}\,\mathrm{tr}\Big[\mathbf{1}R_{\mu\nu}\gamma_{Ric}\Big(\frac{-\Box}{m^2}\Big)R^{\mu\nu} + \frac{\mathbf{1}}{120}R\gamma_R\Big(\frac{-\Box}{m^2}\Big)R$$
$$-\frac{1}{6}R\gamma_{RU}\Big(\frac{-\Box}{m^2}\Big)\mathbf{U} + \frac{1}{2}\mathbf{U}\gamma_U\Big(\frac{-\Box}{m^2}\Big)\mathbf{U} + \frac{1}{12}\mathbf{\Omega}_{\mu\nu}\gamma_\Omega\Big(\frac{-\Box}{m^2}\Big)\mathbf{\Omega}^{\mu\nu}\Big]$$

TECHNICAL!

$$u \gg 1$$

$$\gamma_{Ric}(u) = \frac{23}{450} - \frac{1}{60}\log u + \frac{5}{18u} - \frac{\log u}{6u} + \frac{1}{4u^2} - \frac{\log u}{2u^2} + O\Big(\frac{1}{u^3}\Big)$$
$$\gamma_R(u) = \frac{1}{1800} - \frac{1}{120}\log u - \frac{2}{9u} + \frac{\log u}{12u} + \frac{1}{8u^2} + \frac{\log u}{4u^2} + O\Big(\frac{1}{u^3}\Big)$$
$$\gamma_{RU}(u) = -\frac{5}{18} + \frac{1}{6}\log u + \frac{1}{u} - \frac{1}{2u^2} - \frac{\log u}{u^2} + O\Big(\frac{1}{u^3}\Big)$$
$$\gamma_U(u) = 1 - \frac{1}{2}\log u - \frac{1}{u} - \frac{\log u}{u} - \frac{1}{2u^2} + \frac{\log u}{u^2} + O\Big(\frac{1}{u^3}\Big)$$
$$\gamma_\Omega(u) = \frac{2}{9} - \frac{1}{12}\log u + \frac{1}{2u} - \frac{\log u}{2u} - \frac{3}{4u^2} - \frac{\log u}{2u^2} + O\Big(\frac{1}{u^3}\Big)$$

$$u \equiv \frac{-\Box}{m^2}$$

LO effective action to $\mathcal{R}^2$

$$\Gamma[g] = \frac{1}{16\pi G}\int d^4x\sqrt{g}\,(2\Lambda - R) + \frac{1}{2\lambda}\int d^4x\sqrt{g}\,C^2 + \frac{1}{\xi}\int d^4x\sqrt{g}\,R^2$$
$$+ \int d^4x\sqrt{g}\,C_{\mu\nu\alpha\beta}\,\mathcal{G}\left(\frac{-\Box}{m^2}\right)C^{\mu\nu\alpha\beta} + \int d^4x\sqrt{g}\,R\,\mathcal{F}\left(\frac{-\Box}{m^2}\right)R + O(\mathcal{R}^3)$$

Graviton contributions:

$$\mathcal{G}_2(u) = -\frac{1}{2(4\pi)^2}\Big(5\gamma_{Ric}(u) + 3\gamma_U(u) - 12\gamma_\Omega(u) - 4\gamma_{Ric}(u) - \gamma_U(u) + 4\gamma_\Omega(u)\Big)$$

CEFT of Gravity I

A. C. and K. J. Jain

arXiv:1507.06308

$$\mathcal{F}_2(u) = -\frac{1}{2(4\pi)^2}\left(\frac{10}{3}\gamma_{Ric}(u) + 10\gamma_R(u) + 6\gamma_{RU}(u) + 4\gamma_U(u) - 2\gamma_\Omega(u) - \frac{8}{3}\gamma_{Ric}(u) - 8\gamma_R(u) + 2\gamma_{RU}(u) - \frac{2}{3}\gamma_U(u) + \frac{2}{3}\gamma_\Omega(u)\right)$$

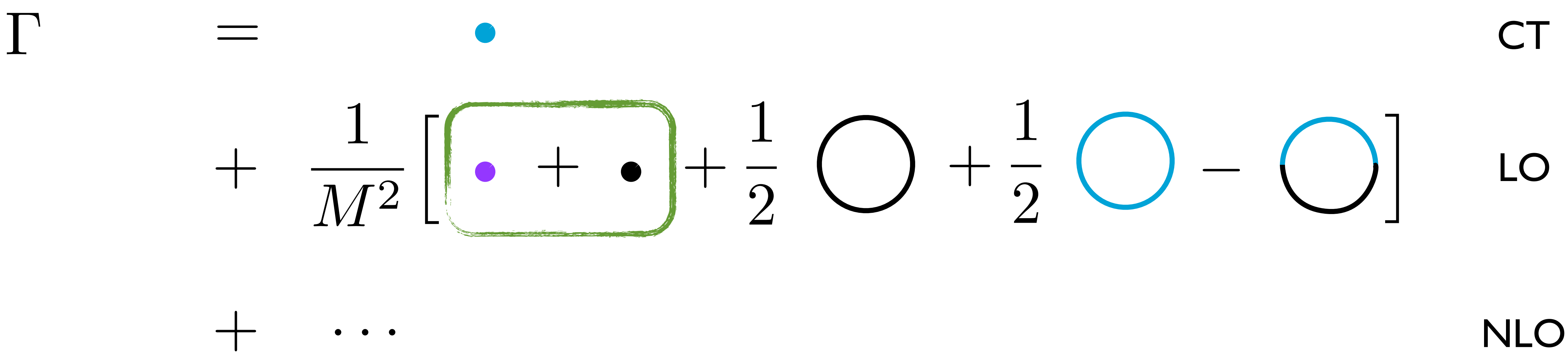
Matter contributions:

$$\mathcal{G}_0(u), \mathcal{F}_0(u), \mathcal{G}_{1/2}(u), \mathcal{F}_{1/2}(u), \mathcal{G}_1(u), \mathcal{F}_1(u)$$

Adding matter

$$
\begin{aligned}
\Gamma \quad &= \quad \bullet && \text{CT} \\
&+ \quad \frac{1}{M^2}\left[\, \bullet + \bullet + \frac{1}{2} \bigcirc + \frac{1}{2} \bigcirc - \bigcirc \,\right] && \text{LO} \\
&+ \quad \cdots && \text{NLO}
\end{aligned}
$$

Adding matter



UV divergencies and renormalization with matter

Scalars

A. O. Barvinsky, A. Y. .Kamenshchik and I. P. Karmazin, Phys. Rev. D 48 (1993) 3677

...

Gauge

S. P. Robinson and F. Wilczek, Phys. Rev. Lett. 96, 231601 (2006)

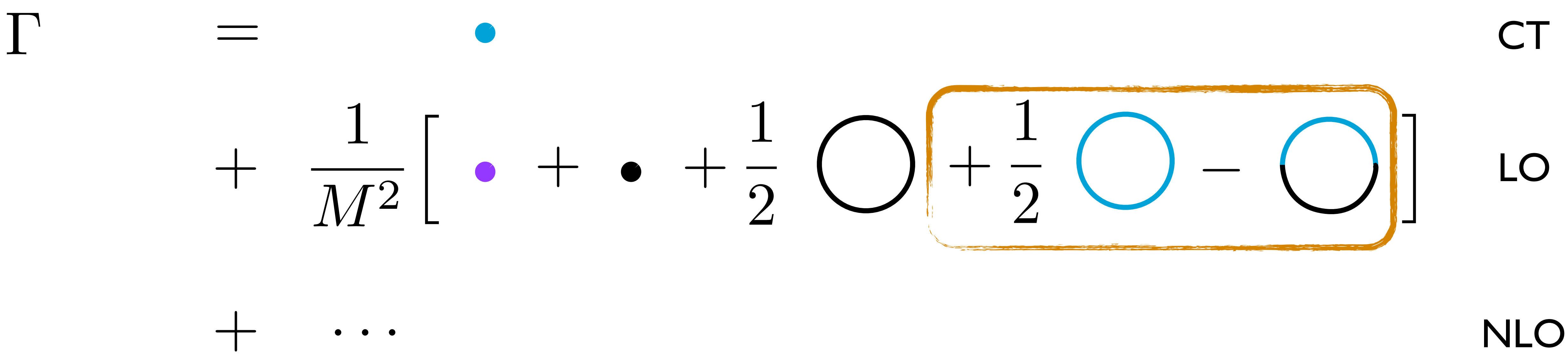
...

Yukawa

A. Rodigast and T. Schuster, Phys. Rev. Lett. 104, 081301 (2010)

...

Adding matter



Finite LO terms with matter

Flat space corrections to Newton's potential

J.F. Donoghue, Phys. Rev. Lett. 72, 2996 (1994)

N.E.J. Bjerrum-Bohr, J.F. Donoghue and B.R. Holstein (2003b) Phys. Rev. D 67

I.B. Khriplovich, G.G. Kirilin (2004) J. Exp. Theor. Phys. 98, 1063-1072

Covariant leading logs

A C, R Percacci, L Rachwal and A Tonero Eur. Phys. J. C **76** (2016) 4, 226

Adding matter

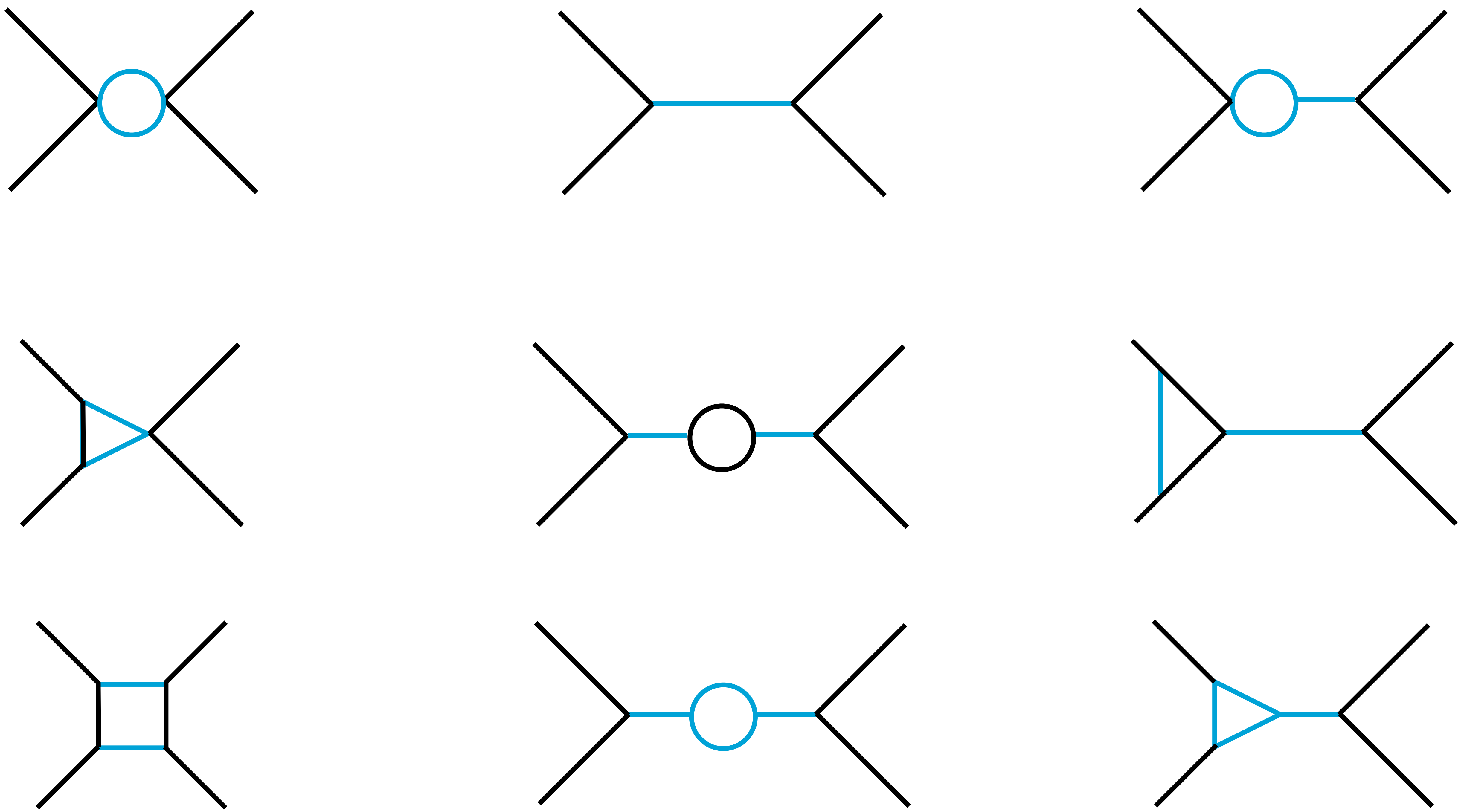
$$\Gamma = \bullet \qquad \text{CT}$$

$$+ \frac{1}{M^2}\left[\bullet + \bullet \boxed{+ \frac{1}{2} \bigcirc} + \frac{1}{2} \bigcirc - \bigcirc \right] \qquad \text{LO}$$

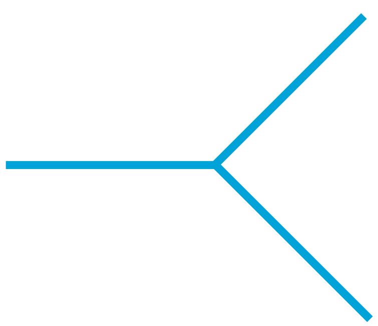
$$+ \cdots \qquad \text{NLO}$$

Matter induced effective action

Corrections to Newton's potential



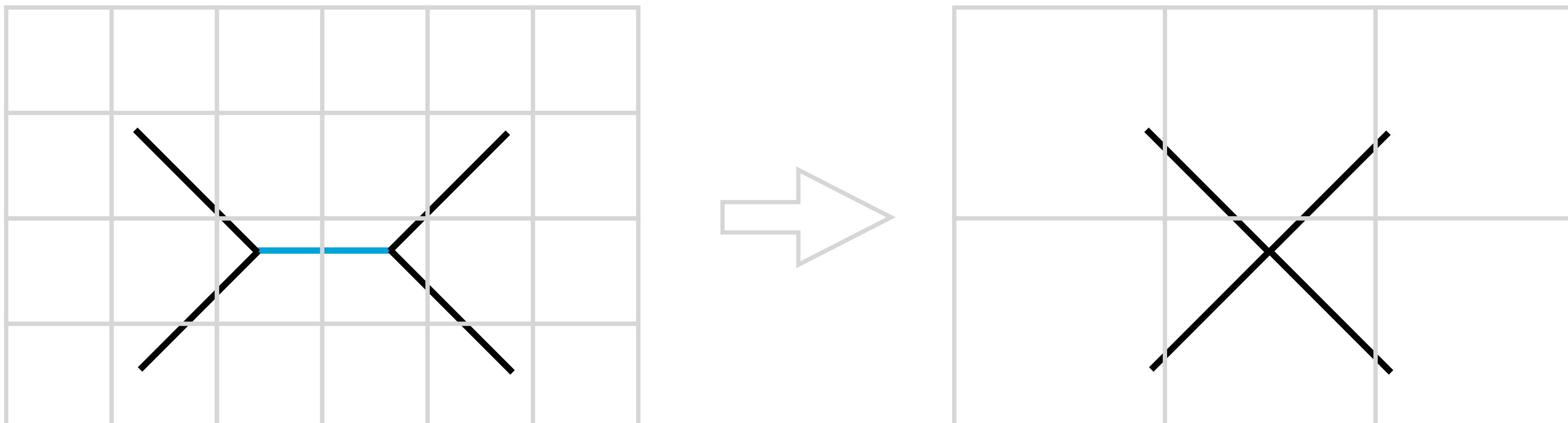
Corrections to Newton's interaction



$$
\begin{aligned}
= \; & \frac{i\kappa}{2}\Bigg(P_{\alpha\beta,\gamma\delta}\Big[k^\mu k^\nu + (k-q)^\mu(k-q)^\nu + q^\mu q^\nu - \frac{3}{2}\eta^{\mu\nu}q^2\Big] \\
& +2q_\lambda q_\sigma \Big[I^{\lambda\sigma,}{}_{\alpha\beta} I^{\mu\nu,}{}_{\gamma\delta} + I^{\lambda\sigma,}{}_{\gamma\delta} I^{\mu\nu,}{}_{\alpha\beta} - I^{\lambda\mu,}{}_{\alpha\beta} I^{\sigma\nu,}{}_{\gamma\delta} - I^{\sigma\nu,}{}_{\alpha\beta} I^{\lambda\mu,}{}_{\gamma\delta}\Big] \\
& + \Big[q_\lambda q^\mu \Big(\eta_{\alpha\beta} I^{\lambda\nu,}{}_{\gamma\delta} + \eta_{\gamma\delta} I^{\lambda\nu,}{}_{\alpha\beta}\Big) + q_\lambda q^\nu \Big(\eta_{\alpha\beta} I^{\lambda\mu,}{}_{\gamma\delta} + \eta_{\gamma\delta} I^{\lambda\mu,}{}_{\alpha\beta}\Big) \\
& -q^2\Big(\eta_{\alpha\beta}I^{\mu\nu,}{}_{\gamma\delta} + \eta_{\gamma\delta}I^{\mu\nu,}{}_{\alpha\beta}\Big) - \eta^{\mu\nu}q^\lambda q^\sigma\left(\eta_{\alpha\beta}I_{\gamma\delta,\lambda\sigma} + \eta_{\gamma\delta}I_{\alpha\beta,\lambda\sigma}\right)\Big] \\
& + \Big[2q^\lambda \Big(I^{\sigma\nu,}{}_{\alpha\beta} I_{\gamma\delta,\lambda\sigma}(k-q)^\mu + I^{\sigma\mu,}{}_{\alpha\beta} I_{\gamma\delta,\lambda\sigma}(k-q)^\nu \\
& - I^{\sigma\nu,}{}_{\gamma\delta} I_{\alpha\beta,\lambda\sigma} k^\mu - I^{\sigma\mu,}{}_{\gamma\delta} I_{\alpha\beta,\lambda\sigma} k^\nu\Big) \\
& +q^2\Big(I^{\sigma\mu,}{}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^{\nu} + I_{\alpha\beta,\sigma}{}^{\nu} I^{\sigma\mu,}{}_{\alpha\delta}\Big) + \eta^{\mu\nu}q^\lambda q_\sigma \Big(I_{\alpha\beta,\lambda\rho}I^{\rho\sigma,}{}_{\gamma\delta} + I_{\gamma\delta,\lambda\rho}I^{\rho\sigma,}{}_{\alpha\beta}\Big)\Big] \\
& + \Big\{\left(k^2+(k-q)^2\right)\Big(I^{\sigma\mu,}{}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^{\nu} + I^{\sigma\nu,}{}_{\alpha\beta}I_{\gamma\delta,\sigma}{}^{\mu} - \frac{1}{2}\eta^{\mu\nu}P_{\alpha\beta,\gamma\delta}\Big) \\
& - \Big(k^2\eta_{\gamma\delta}I^{\mu\nu,}{}_{\alpha\beta} + (k-q)^2\eta_{\alpha\beta}I^{\mu\nu,}{}_{\gamma\delta}\Big)\Big\}\Bigg)
\end{aligned}
$$

The truth behind Feynman diagrams...

Corrections to Newton's interaction

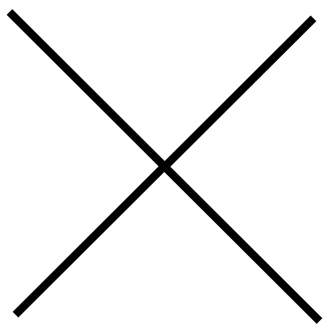


$$V = -\frac{GMm}{r}\left[1 + 3\frac{G(M+m)}{c^2 r} + \frac{41}{10\pi}\frac{G\hbar}{c^3 r^2} + \cdots\right]$$

Leading quantum corrections to Newton's potential

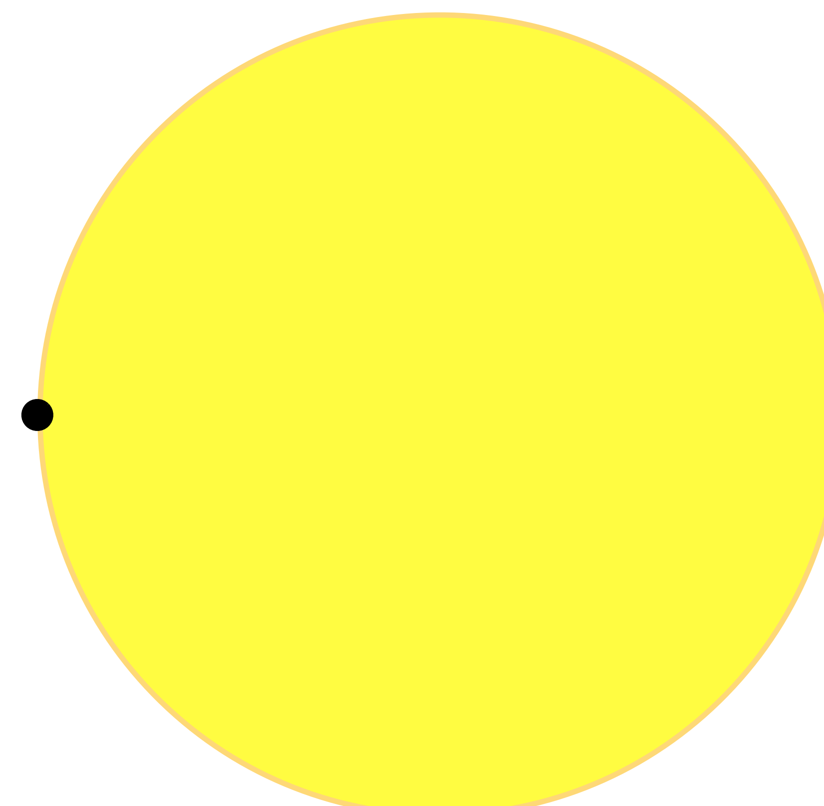
J.F. Donoghue, Phys. Rev. Lett. 72, 2996 (1994)

Corrections to Newton's interaction



$$\frac{GM_\odot}{c^2 r_\odot} \sim 10^{-6}$$

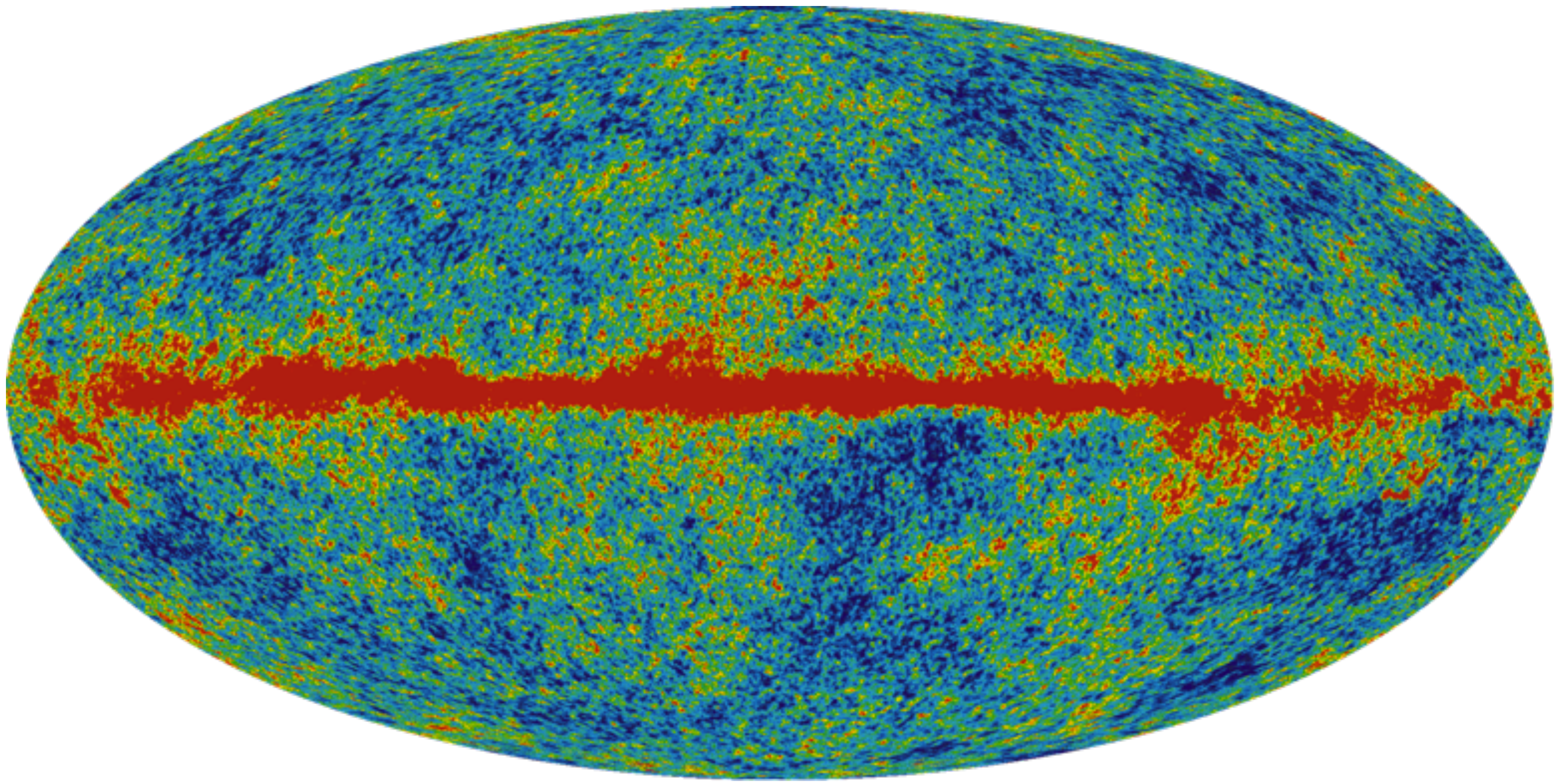
$$\frac{G\hbar}{c^3 r_\odot^2} \sim 10^{-88}$$



sun

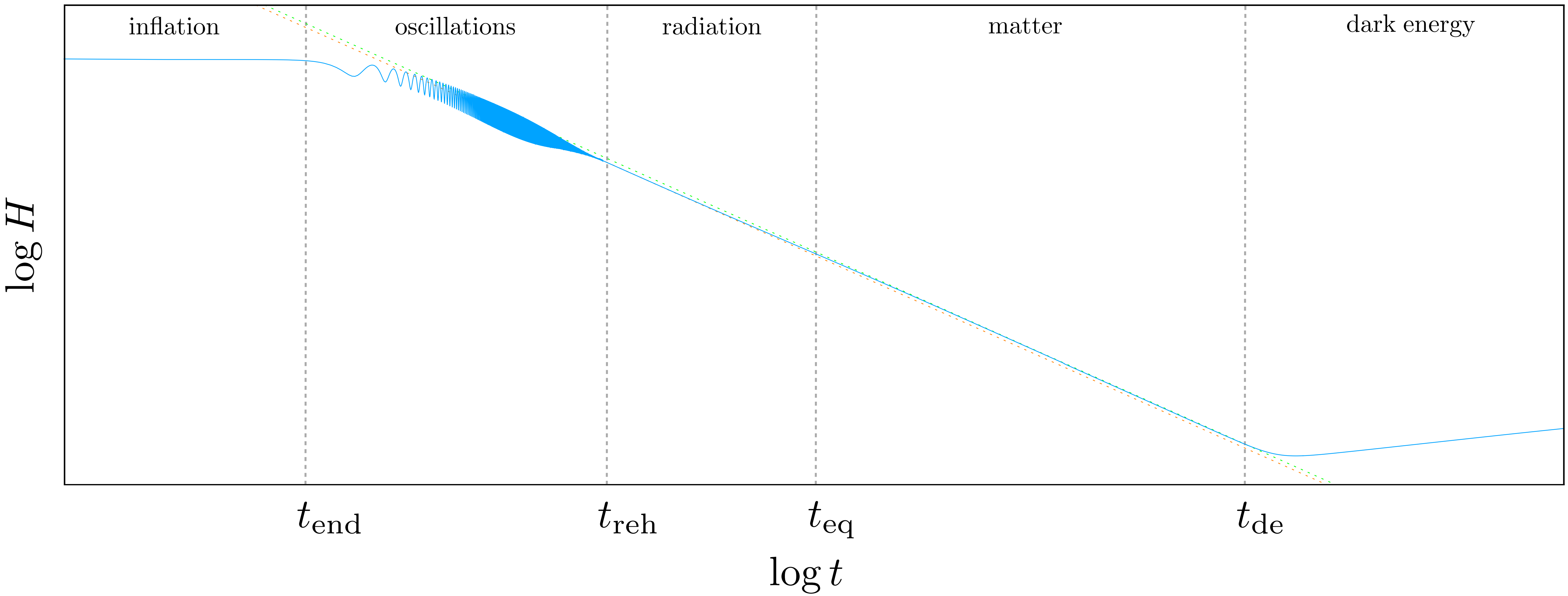
Leading quantum corrections to Newton's law are incredibly small!

Can we ever observe quantum gravity effects?



Look for physical situations where LO corrections are enhanced

Unified evolution of the universe

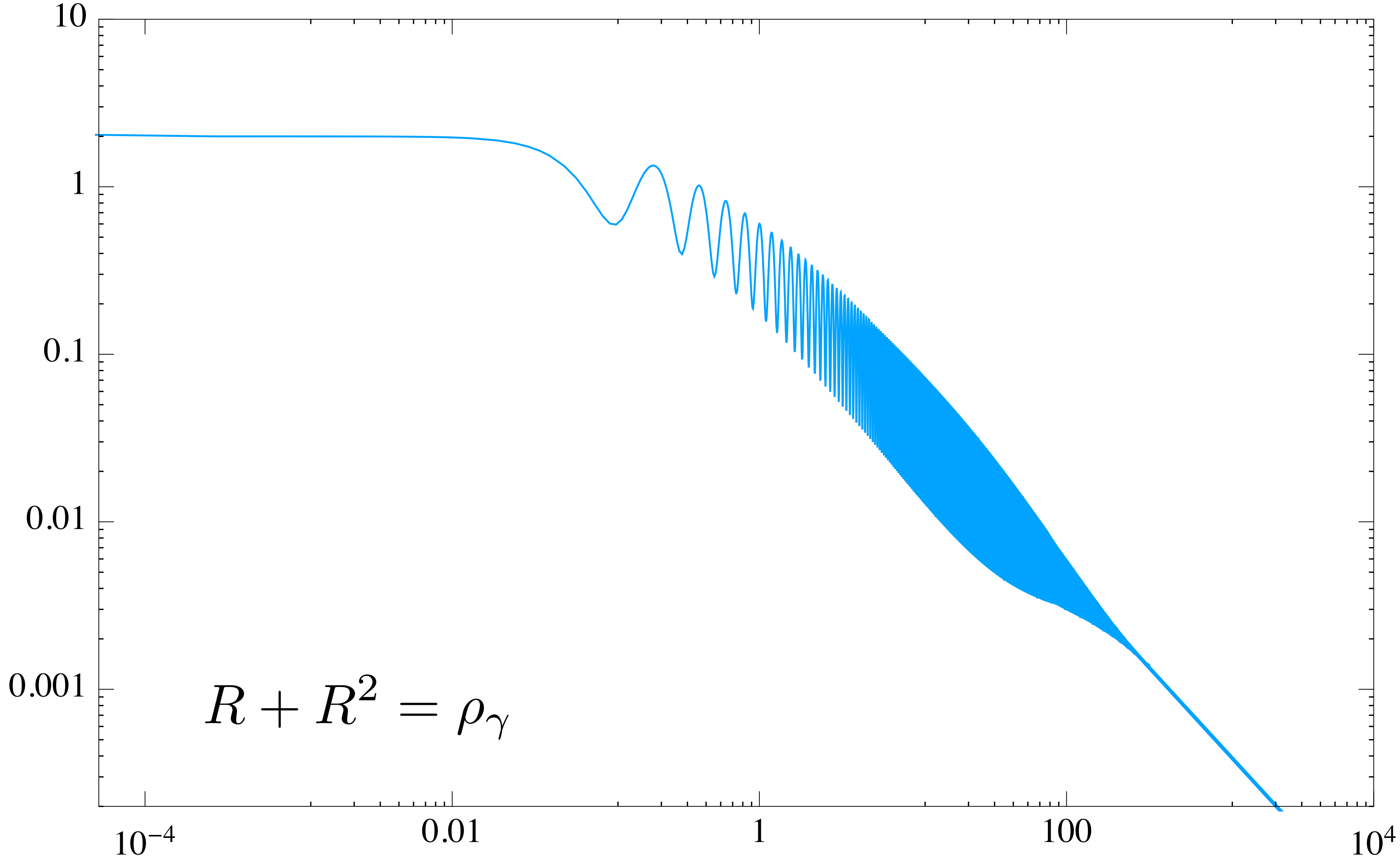


$$\Gamma = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{\xi}R^2 + M^4 R\frac{1}{\Box^2}R\right] + S_{\rm m}$$

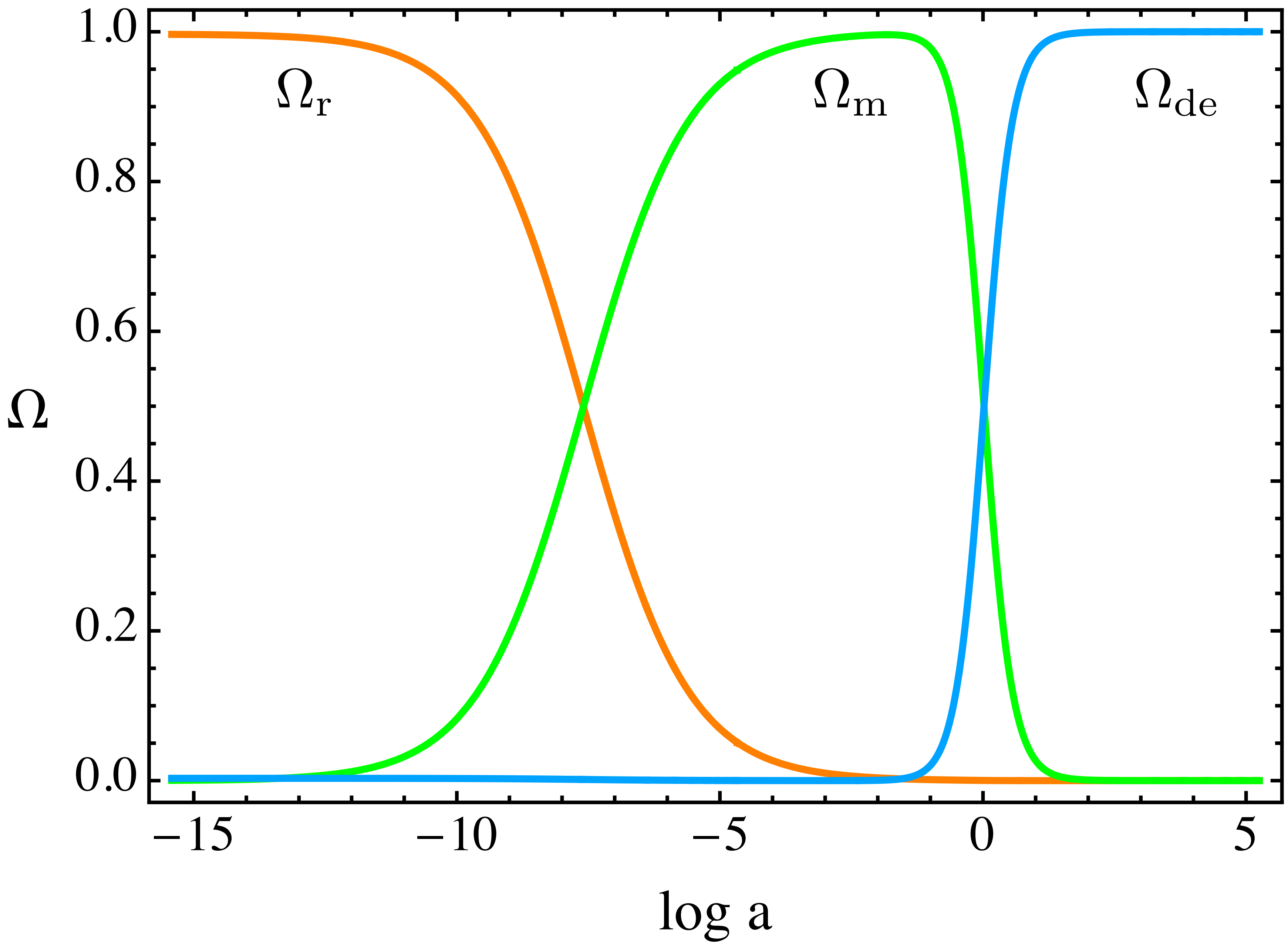
A unified evolution of the universe

A. C. and K. J. Jain, arXiv:1603.00028

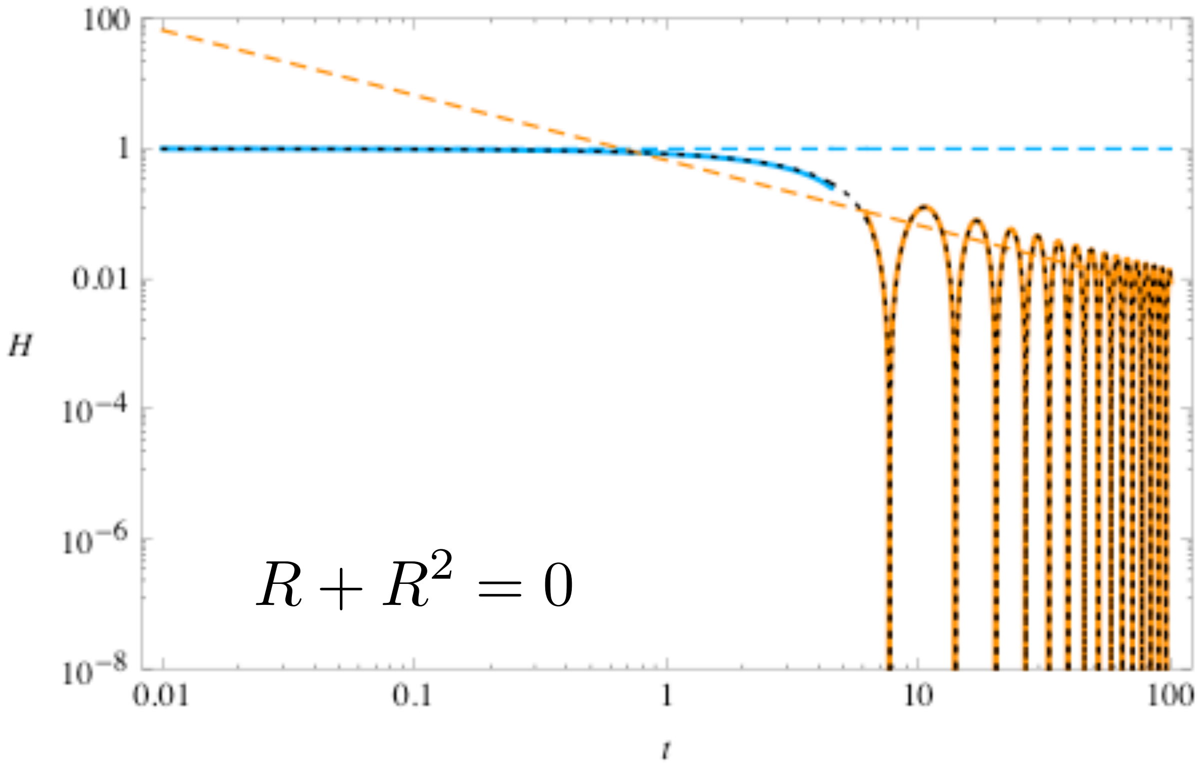
Early times: Inflation and reheating

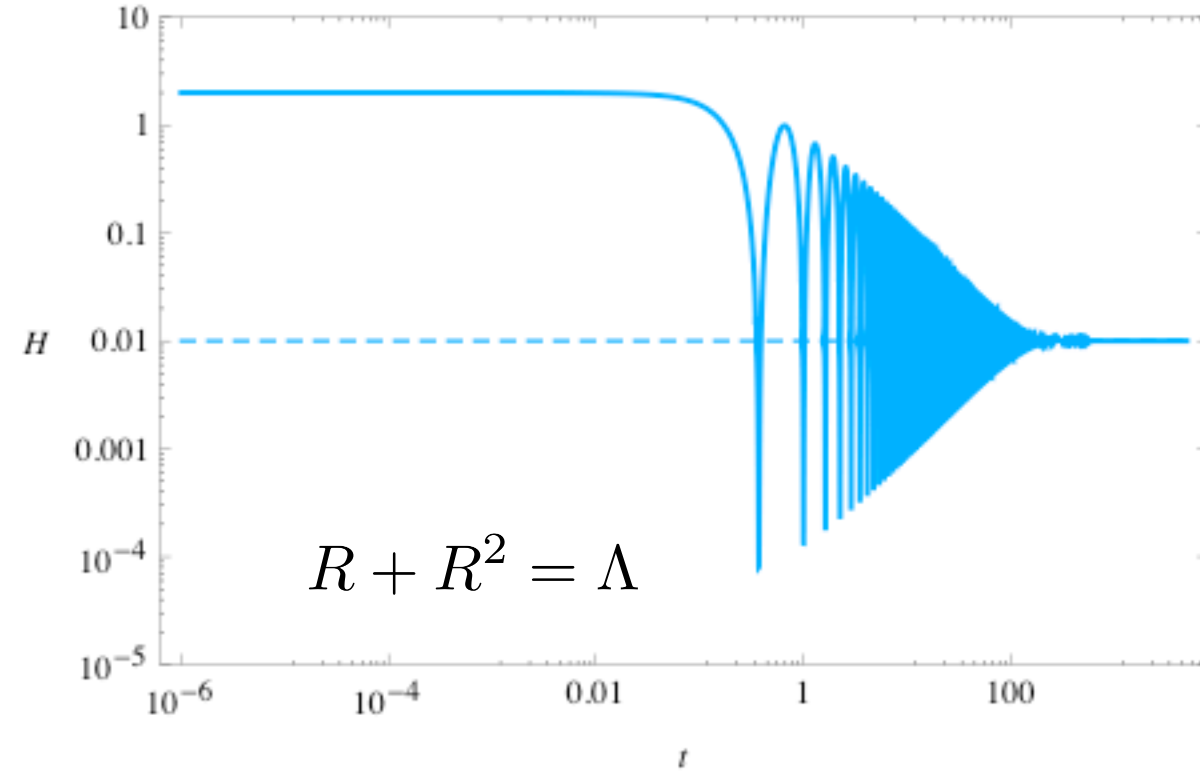


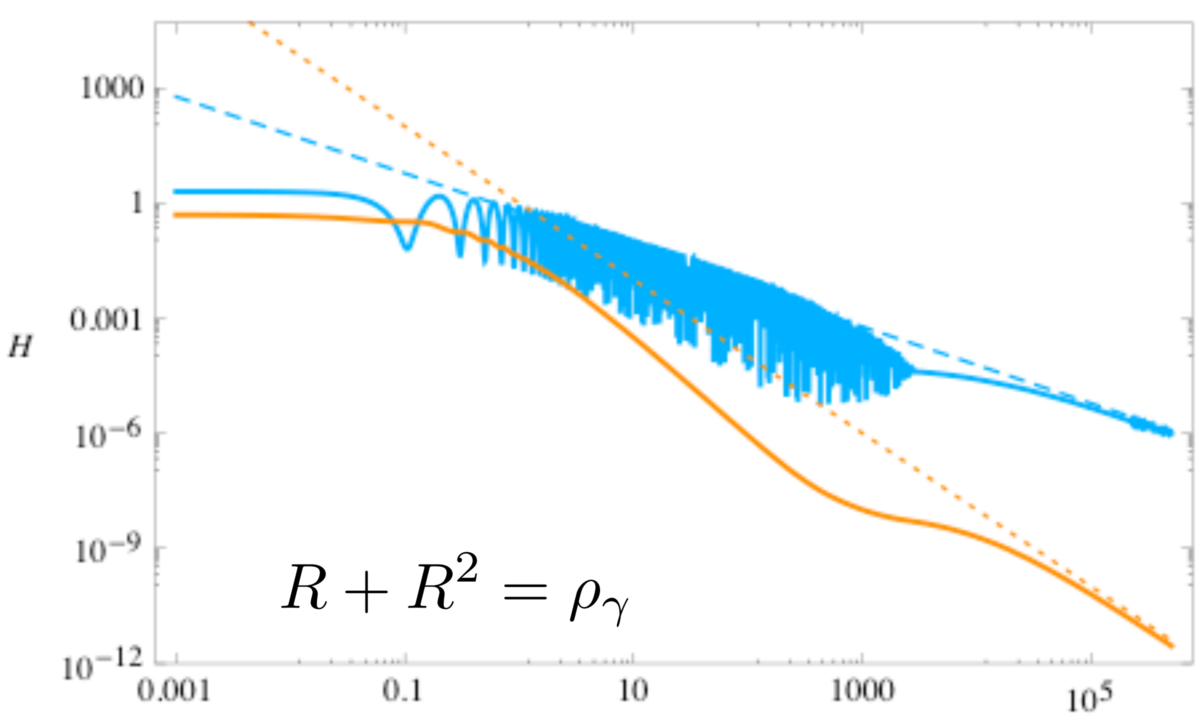
Late times: Dark energy

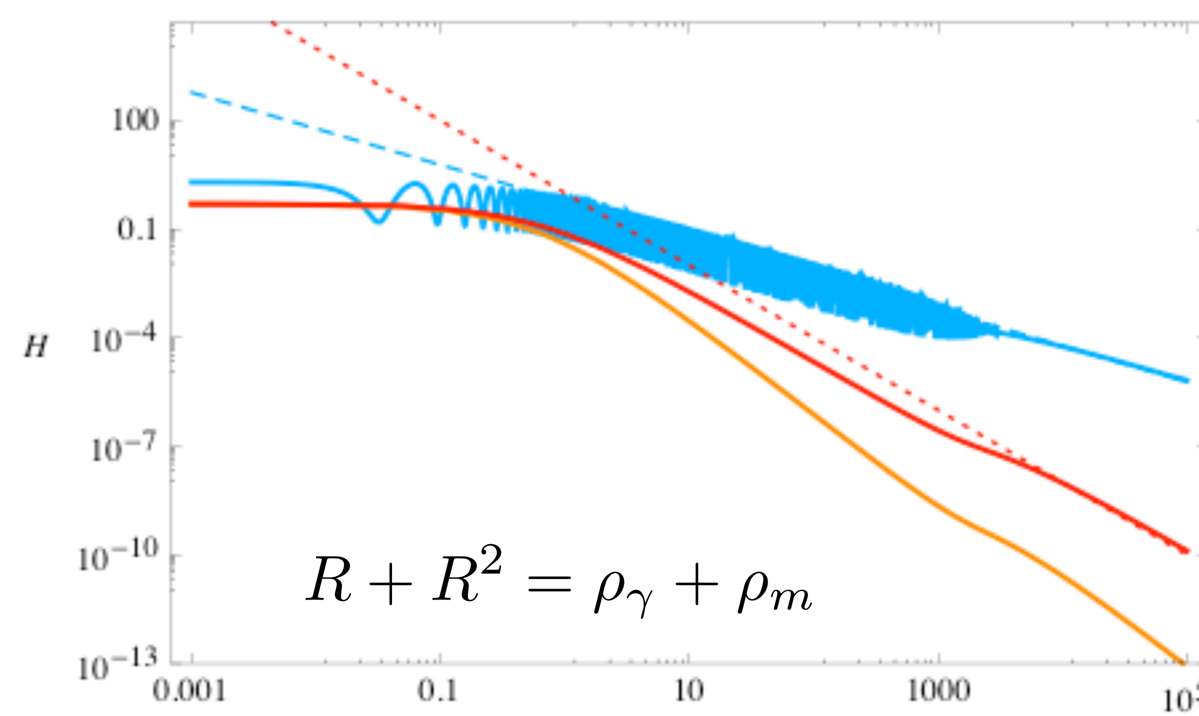


Effective Friedmann equations

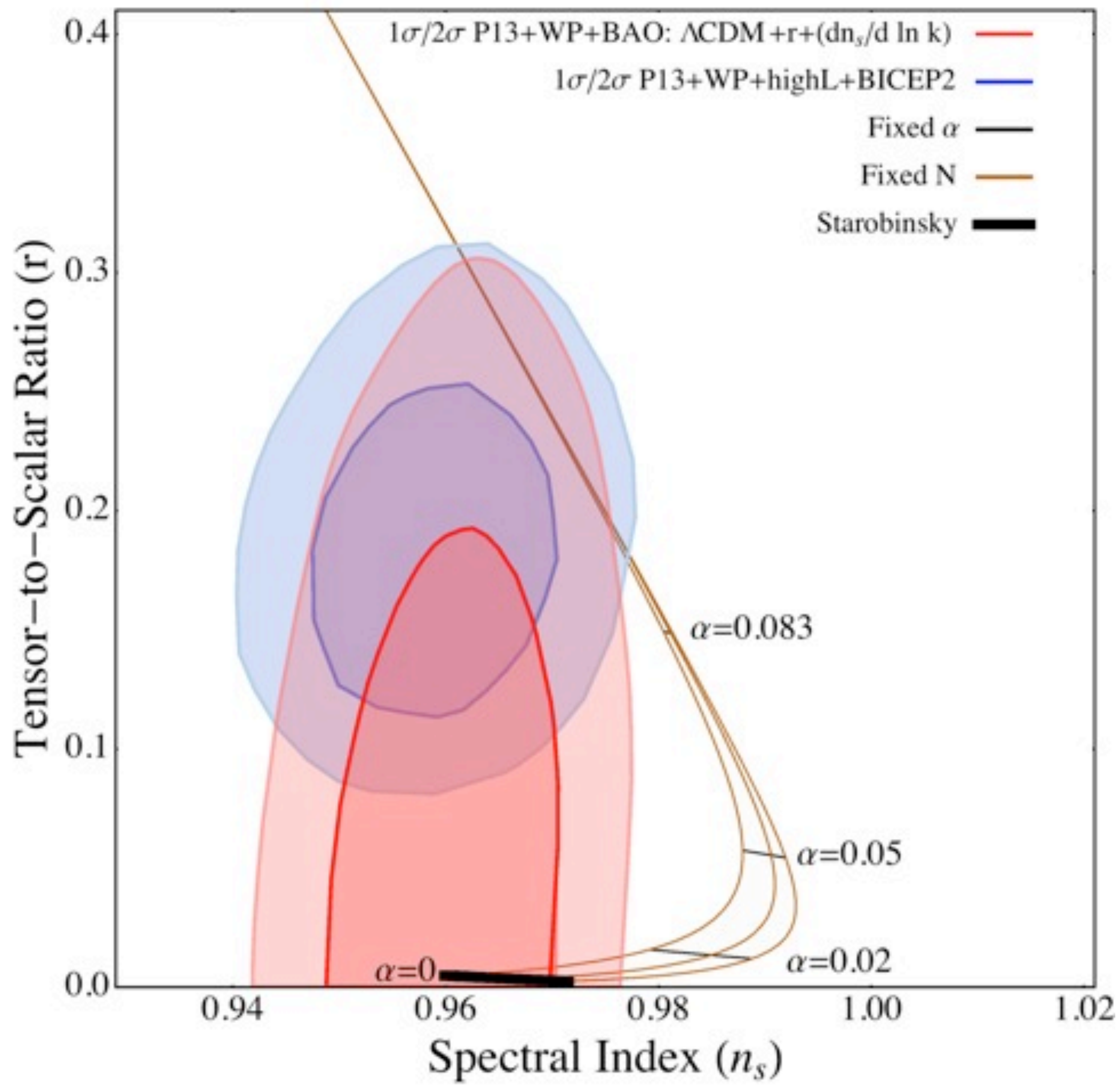








Marginally deformed Starobinsky



$$F(R) = \log \frac{R}{m^2}$$

Leading quantum corrections to tensor-to-scalar ratio

A. C, J. Joergensen, F. Sannino and O. Svendsen, JHEP 1502, 050 (2015)