Quantum-spacetime effects on nonrelativistic Schrödinger evolution

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Witnessing Quantum Aspects of Gravity in a Lab

ICTP-SAIFR/Principia Institute

São Paulo, 26.09.2024

Based on PRD 108, 066008 (2023), "Quantum-spacetime effects on nonrelativistic Schrödinger evolution" and based on arXiv:2405.13544 "Fractional quantum mechanics meets quantum gravity phenomenology".

Outlook

- Quantum Gravity Phenomenology in the UV and IR
- Two Approaches Considered for modified Schrödinger equation (Relation with Minimal-Length Models)
- Applications with rough bounds
- Relation with Fractional Quantum Mechanics
    - Equivalent Quantum System for In-Vacuo Dispersion
    - Fractional Dimension in the Deep Infrared

- In 1936, **Matvei Bronstein** ignited the area of quantum gravity, by imposing uncertainty relations to a perturbed metric and its momentum.

- Emergence of the idea of a **quantum spacetime**

A way to realize a fundamental minimal length

QUANTUM THEORY OF WEAK GRAVITATIONAL FIELDS[1]

By M. Bronstein.

(Received on 2. January 1936)

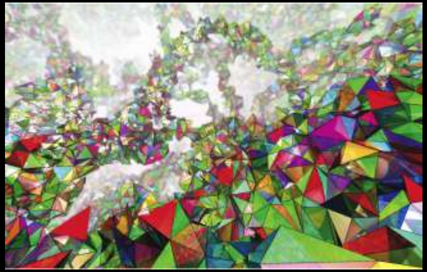
- In fact, analyzing the kinematics of particles in such spacetime, Bronstein showed that such effects would manifest in the form of an **extra uncertainty in momentum** that complements the Heisenberg one

$$\Delta p_x \approx \frac{h}{\Delta x} + G\rho^2 V \Delta x \Delta t.$$

where Δp_x is the uncertainty in momentum. Let the duration of the momentum measurement be Δt (of course, $\Delta t \ll T$); Δx be the uncertainty in the coordinate associated with measuring the momentum. The uncertainty Δp_x consists of two terms: the usual quantum-mechanical $h/\Delta x$ and one associated with the gravitational field produced by the test body itself because of its recoil due to the measurement. Because of E i n s t e i n 's equa-

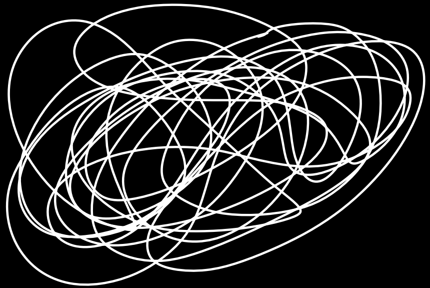
- We don't have a complete theory of quantum gravity, but along the decades we have discovered some common aspects that different proposals have.

- For example, the most fundamental prediction of Loop Quantum Gravity is the existence of a fundamental quantum of geometry of the order of the Planck scale.



http://bronsteinprize.org

- A limitation in the resolution of the location of string-like objects would also indicate the existence of a limitation in spacetime resolution, leading to a minimum physical length



Hossenfelder, Living Reviews in Relativity, 2013

- A phenomenological way to describe this behavior consists in assuming a **generalized uncertainty principle**

$$[\hat{x}, \hat{p}] = i + f(\hat{p})$$

- The physical properties of a system endowed with a generalized uncertainty principle can be described by a standard uncertainty principle, but with a modified Schrödinger equation

$$\hat{p} = g(\hat{k}) \qquad [\hat{x}, \hat{k}] = i \qquad \hat{H} = \frac{g^2(\hat{k})}{2M} + V(\hat{x})$$

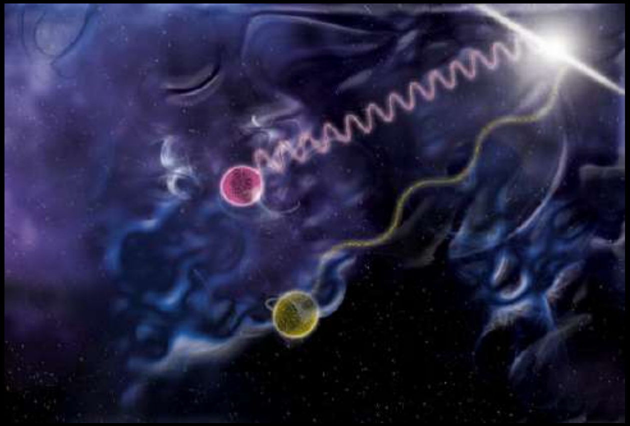
Bosso, EPJC (2024)

- This means that the foundations of quantum mechanics would be modified in order to have a quantum description of gravity

- But what about spacetime geometry? Does the smooth, 4D, Riemannian description still makes sense?

- How is the transition from the Riemannian description to a full quantum geometry of spacetime?

- One can expect that such departures should appear as small changes in the kinematics of particles that travel through spacetime



Credit: NASA/Sonoma State University/Aurore Simonnet

- It is possible to derive effective spacetime solutions considering Loop Quantum Cosmology and Loop Quantum Black Holes by relying on a procedure called Polymerization

- This introduces deformations in the algebra that describes the evolution of space hypersurfaces in GR.

Deformed algebra of constrains that generate time/radial evolution and diffeomorphism

$$\{D[M^a], D[N^a]\} = D[\mathcal{L}_{\vec{M}} N^a],$$
$$\{D[N^a], H^Q[M]\} = H^Q[\mathcal{L}_{\vec{N}} M],$$
$$\{H^Q[M], H^Q[N]\} = D[\beta h^{ab}(M\partial_b N - N\partial_b M)],$$

$$\beta = \cos(2\delta K_\varphi)\,,$$

Taking the Minkowski limit of this algebra

$$N = \Delta t + v_k x^k$$
$$N^i = \Delta x^i + \phi^j \epsilon^{ijk} x^k$$

One finds a deformed commutator between the boost and the time translation

$$[B_r, P_0] = i P_r \cos(\lambda P_r)\,.$$

- This is non-trivial Minkowski limit governed by a length scale λ

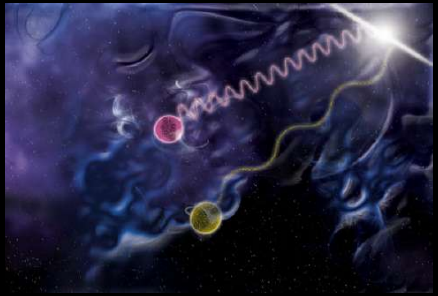
- The deformed general relativity, inspired by Loop Quantum Gravity, induces a deformation of the Poincaré algebra in the Minkowski limit.

The Casimir operator of this algebra is deformed

$$P_0^2 = \frac{2}{\lambda^2}\left(\lambda P_r \sin(\lambda P_r) + \cos(\lambda P_r) - 1\right)$$

Amelino-Camelia, da Silva, Ronco, Cesarini, Lecian, PRD (2017)

- The speed of particles would be modified since $v = \partial P_0 / \partial P_r$



- For a massless particle, its speed would be energy-dependent $c(P_0) = 1 + \mathcal{O}(\lambda P_0)$

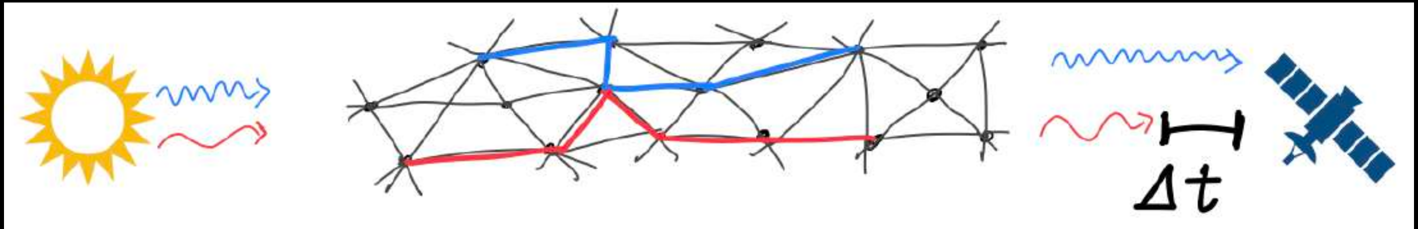
- This is just one example of how the symmetries of spacetime would be modified due to quantum gravity.

- There are several proposals that are capable of describing these kinds of symmetries in a geometric way, like **non-commutative geometry, Finsler geometry**, **Hamilton geometry, curved momentum spaces.** Majid, PLB (1994). Kowalski-Glikman, PLB (2002). Girelli et al. PRD (2007). **IPL**, Loret, Nettel, PRD (2017). Barcaroli et al. PRD (2015)…

All of them describe departures of the Riemannian nature of spacetime

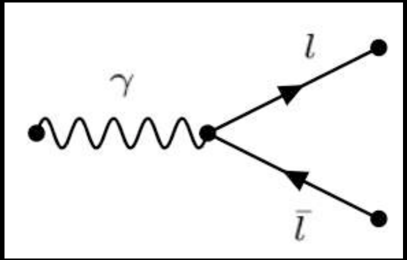
- Modified kinematics can have measurable effects in the propagation and production of high-energy particles, making **Astrophysics** and **Astroparticle Physics** the natural environment of research in this area also due to amplifiers that make Planckian effects measurable.

- There is a well stablished phenomenological community that explores these departures in the hight-energy (UV) regime



- Time delays between massless particles Vasileiou et al., PRD (2013)

- Changes in threshold energies for interactions and how this affect propagation and production of particles Albert et al. (HAWC Collaboration) PRL (2020)



- Bounds are placed with Planck scale sensitivity.

Drawback: uncertainties in astrophysics and hidden assumptions.

Which is alleviated in the case of **tabletop experiments**

Quantum gravity in the infrared

- Not only quantum mechanics would be modified, but also the geometry of spacetime.

- How these aspects of quantum gravity connect with each other?

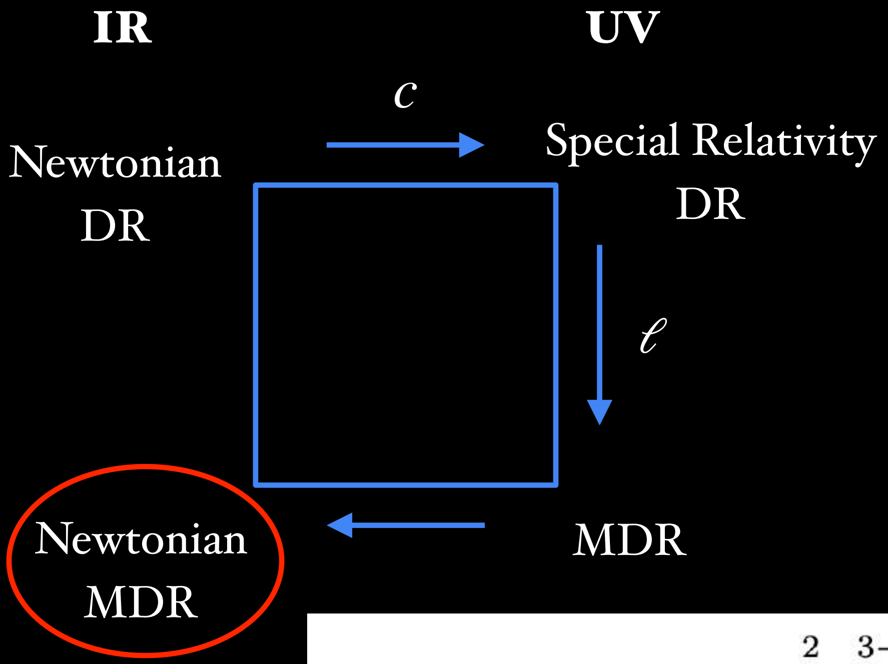
1. The first quantization is done over the Newtonian dispersion relation. Therefore a MDR should lead to a **modified Schrödinger equation**.

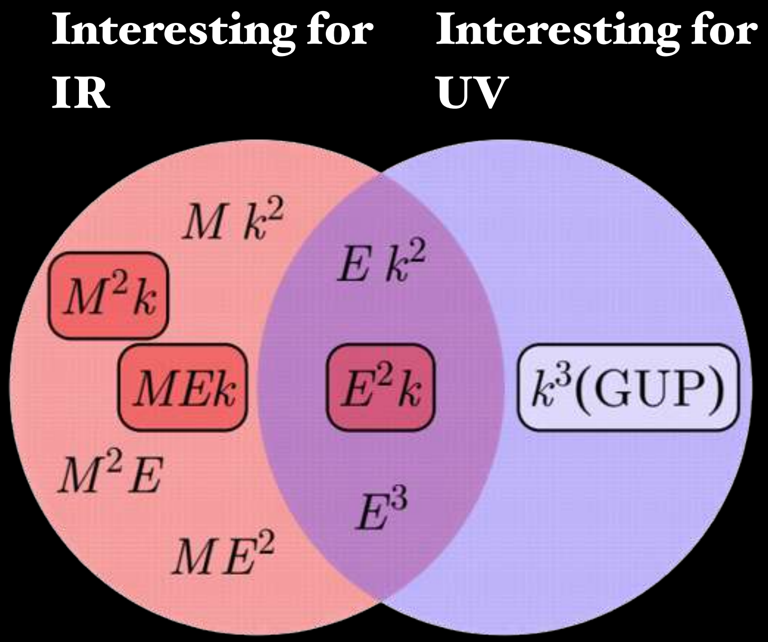
2. The dispersion relation also comes from a the geometric optical (eikonal) limit a field equation:

    3. From a field equation it is possible to derive the Schrödinger equation in the non relativistic limit.

We are going to explore these ideas and check what kind of Schrödinger equation we find.

Quantum gravity phenomenology in the UV and IR





$$M^2c^4 = E^2 - k^2c^2 + \ell \sum_{n=0}^{2} \sum_{m=0}^{3-n} a_{n,m,3-n-m} (Mc)^n k^m \left(\frac{E}{c}\right)^{3-n-m}$$

Main hypotheses

- We are going to perturb our equations for very large c's and pick up the leading terms

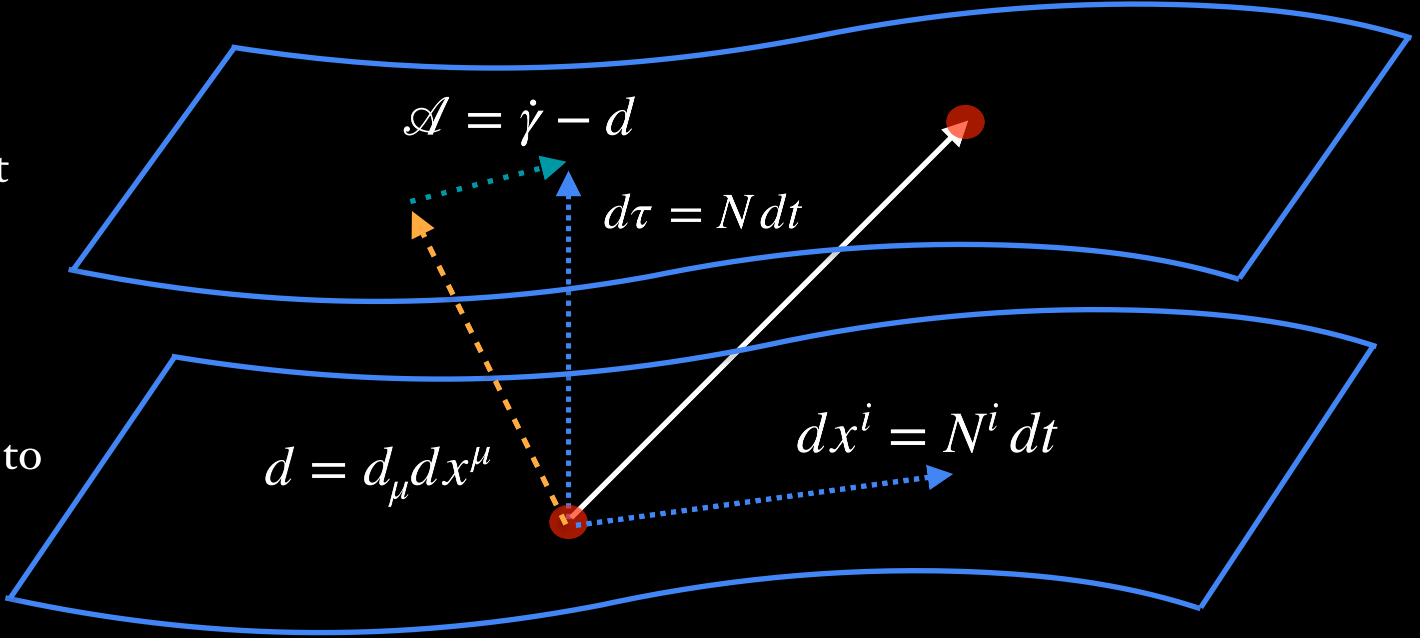
- We are going to keep the Planckian quantities fixed. This can be done since the Planck length is exceedingly small, independently of the appearance of c in our equations.
  So, we have two scales: c and ℓ.

- We assume that the spatial part of the metric is time-independent. This is to avoid issues related to time-dependent measures in our Hilbert space and particle creation.

$$\langle \psi | \phi \rangle = \int_{\Sigma_t} \mathrm{d}^d x \sqrt{h} \psi^* \phi,$$

d is a Lorentz-violating vector that defines the MDR

$$g_\mu^\nu = \delta_\mu^\nu + d_\mu d^\nu$$



$\dot{\gamma}$ defines the foliation to the Newtonian world

$$g_\mu^\nu = \delta_\mu^\nu + d_\mu d^\nu$$

$$\mathscr{A} = \dot{\gamma} - d$$

$\mathscr{A}$ measures the velocity difference between lab and the Lorentz Violating frame $\sim 10^3 c$

$$\mathcal{C}(E,k,M) = M^2c^2,$$

$$M^2c^4 = E^2 - k^2c^2 + \ell \sum_{n=0}^{2} \sum_{m=0}^{3-n} a_{n,m,3-n-m} (Mc)^n k^m \left(\frac{E}{c}\right)^{3-n-m},$$

- Modified Dispersion relation

$$\left[\hat{\mathcal{C}}(-iD_0, -iD_i, M) - M^2\right] \varphi = 0.$$

$$D_\mu = \nabla_\mu - ieA_\mu.$$

- Modified Klein-Gordon Equation

Schwartz, Giulini CQG (2019)

$$\varphi = e^{-iMc\lambda}\psi = e^{-iMc\lambda} \sum_{n=0}^{\infty} \frac{\psi_n}{c^n},$$

- WKB-like expansion

$$A_i^{\mathrm{g}} = -N_i^{(1)}$$

Undeformed Hamiltonian

$$H_{0,\mathrm{NR}} = \frac{h_{(0)}^{ij}}{2M} \left(k_i - MA_i^{\mathrm{g}}\right)\left(k_j - MA_j^{\mathrm{g}}\right) + \phi_{\mathrm{g}} M,$$

$$\phi_{\mathrm{g}} = (N_{(2)} - N_{(1)}^i N_j^{(1)}/2)$$

Two Approaches Considered

$$\begin{array}{ccc} \text{dispersion relation} & \xrightarrow{\text{inspires}} & \text{wave equation} \\ \Big\downarrow{\scriptstyle c\to\infty} & & \Big\downarrow{\scriptstyle c\to\infty} \\ \text{nonrel. Hamiltonian} & \xrightarrow[\text{quantise}]{} & \text{Schrödinger Eq.} \end{array}$$

$$\hat{H}_{\ell,NR,EM} = \hat{H}_{0,NR,EM} - \frac{\ell}{2}\left(\xi_1 Mc^2 \hat{k}_{\mathscr{A}} + \xi_2 c\hat{k}^2_{\mathscr{A}} + \xi_3 \frac{\hat{k}^3_{\mathscr{A}}}{M}\right)$$

Alfaro, Morales-Tecotl, Urrutia PRL (2000)

$$\boxed{\begin{aligned}\hat{H}_{\ell,\mathrm{NR,EM}}\psi = \Bigg[& -\frac{h^{ij}}{2M}\left(\nabla_i - ieA^e_i - iMA^g_i\right)\left(\nabla_j - ieA^e_j - iMA^g_j\right) + e\phi_e + M\phi_{\mathrm{g}} \\ & -\frac{\ell}{2M}\sum_{n=1}^{3}\xi_n (Mc)^{3-n}\left[-h^{ij}\left(\nabla_i - ieA^e_i - iM\mathcal{A}_i\right)\left(\nabla_j - ieA^e_j - iM\mathcal{A}_j\right)\right]^{\frac{n}{2}}\Bigg]\psi.\end{aligned}}$$

Fabian Wagner, Gislaine Varão, **IPL**, Valdir. B. Bezerra, PRD (2023)

Relations with minimal length models

$$[\hat{x}^i, \hat{k}_j] = i\delta^i_j, \qquad \hat{H} = \frac{\hat{k}^2}{2M} + V(\hat{x}) + \delta H(\hat{k}).$$

$$[\hat{x}^i, \hat{p}_j] = i f^i_j(\hat{p}), \qquad \hat{H} = \frac{\hat{p}^2}{2M} + V(\hat{x}),$$

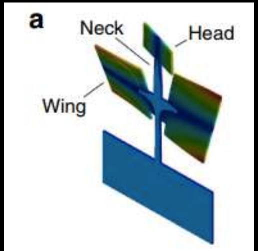
$$\hat{p}_i|_{\mathcal{A}_i=0} = \hat{k}_i\left[1 - \frac{\ell}{2}\left(\frac{\xi_1 (Mc)^2}{\hat{k}} + \xi_2 Mc + \xi_3 \hat{k}\right)\right].$$

Equivalent Isotropic Generalized Uncertainty Principle $\mathscr{A} = 0$

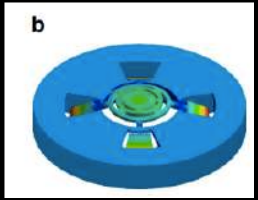
$$[\hat{x}^i, \hat{p}_j] = i\left[1 - \frac{\ell}{2}\left(\frac{\xi_1 (Mc)^2}{\hat{p}} + \xi_2 Mc + \xi_3 \hat{p}\right)\right]\delta^i_j - \frac{i\ell}{2}\left(\frac{\xi_1 (Mc)^2}{\hat{p}^3} + \frac{\xi_3}{\hat{p}}\right)\hat{p}^i \hat{p}_j.$$

Application 1: Harmonic Oscillator (shift in energy levels)

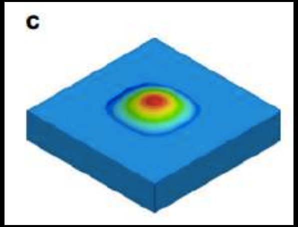
300-μm thick silicon plates



70-μm wheel



30 nm thick, square membrane of silicon nitride



Bawaj et al. "Probing deformed commutators with macroscopic harmonic oscillators", Nature Communications, 2015, arXiv:1411.6410

We use the map between GUP and MDRs for a rescaled ℓ/N to find a rough estimate that **improves** previous (quadratic) bounds in 17 **orders of magnitude**

$$\underline{\xi_1 \leq \mathcal{O}(10^{-1}),} \qquad \xi_2 \leq \mathcal{O}(10^{10}), \qquad \xi_3 = \mathcal{O}(10^{21}).$$

Improves the linear bound of Amelino-Camelia, Laemmerzahl, Mercati, Tino, PRL (2009) in 1 order of magnitude

Nonrelativistic experiments are competitive in quantum gravity phenomenology

Application 2: Violation of the Weak Equivalence Principle

$$\hat{H}_{\ell,\mathrm{NR,EM}} = \frac{\hat{k}^2}{2M} + Mgz - \frac{\ell}{2M}\sum_{n=1}^{3}\xi_n (Mc)^{3-n}\hat{k}_A^n. = \frac{\hat{k}^2}{2M_I} + M_g gz$$

$$M_I \approx M_g\left[1 + \frac{\ell M_g c}{2}\left(10^3\xi_1 + \xi_2 + 10^{-3}\xi_3\right)\right]$$

Inertial mass $\neq$ Gravitational mass

The **violation of the weak equivalence** principle can be summarized in the **Eötvös parameter** (we used ℓ/N to avoid the soccer ball problem)

$$\eta(A,B) = 2\left\langle \frac{\frac{M_{g,A}}{M_{I,A}} - \frac{M_{g,B}}{M_{I,B}}}{\frac{M_{g,A}}{M_{I,A}} + \frac{M_{g,B}}{M_{I,B}}} \right\rangle$$

The MICROSCOPE Collaboration gives $|\eta| < 10^{-14}$

$|\xi_1| \leq \mathcal{O}(10^1)$

$|\xi_2| \leq \mathcal{O}(10^4)$

$|\xi_3| \leq \mathcal{O}(10^7)$

Featured in Physics   Editors' Suggestion

MICROSCOPE Mission: Final Results of the Test of the Equivalence Principle

Pierre Touboul *et al.* (MICROSCOPE Collaboration)
Phys. Rev. Lett. **129**, 121102 – Published 14 September 2022

Physics See Viewpoint: Satellite Confirms the Principle of Falling

Fractional Quantum Mechanics

Theoretical motivation

- Barrow, PLB (2022) has proposed that the Bekenstein formula would be modified due to quantum gravity. Where the surface of the black hole would be a fractal

$$S_{\text{frac}} = (A/4\ell_{\mathrm{P}})^{d/2}$$

- We can derive the same kind of law by assuming a fractional MDR (*d* is the fractal dimension)
  Varão, Bezerra, **IPL**, arXiv:2405.13544

$$E = \frac{2^{d-1}}{d}\left(\frac{G}{\pi}\right)^{\frac{d}{2}-1} p^{d-1} \equiv \xi_d \ell_{\mathrm{P}}^{d-2} p^{d-1},$$

- These kinds of MDRs have also been analyzed in Multifractional Spacetime Theory
  Calcagni, PRL (2010)

- MDRs with fractional powers are usually considered for gravitational waves phenomenology
  Yunes, Yagi, Pretorius PRD (2016).

$$E^2 = c^2p^2 + M^2c^4 + \xi_\alpha \ell_{\mathrm{P}}^{-\beta} c^2 (Mc)^{2+\beta-\alpha} p^\alpha .$$

α and β are real numbers

In a nutshell, this is the form of the MDR in the nonrelativistic limit ($\mathscr{A} = 0$)

$$E = \frac{p^2}{2M} + \xi_\alpha \ell_{\mathrm{P}}^{-\beta} \frac{(Mc)^{2+\beta-\alpha}}{2M} p^\alpha \equiv \frac{p^2}{2M} + D_\alpha^{(\beta)} p^\alpha .$$

If α and β runs freely from $0 \leq \alpha \leq 2$, the quantization of this MDR is related to the area of **FRACTIONAL QUANTUM MECHANICS**

Nikolai Laskin, PLA (2000)

$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{x}, t) = \left(-\frac{\hbar^2}{2M} \Delta + V(\mathbf{x}, t) + \xi_\alpha p_{\mathrm{P}}^{-\beta} \frac{(Mc)^{2-\alpha+\beta}}{2M} \left(-\hbar^2 \Delta\right)^{\alpha/2} \right) \Psi(\mathbf{x}, t)$$

Riez derivative

The deep infrared regime

- UV-IR Mixing

If the dispersion relation is exact $E = \dfrac{p^2}{2M} + \xi_\alpha \dfrac{p_P^{-\beta}}{2M}(Mc)^{2+\beta-\alpha} p^\alpha$, then if $\alpha \leq 2$, there is a regime in which the new contribution dominates.

We call this the **Deep Infrared Regime**

$$p \ll \left[2MD_\alpha^{(\beta)}\right]^{1/(2-\alpha)}$$

$$E = D_\alpha^{(\beta)} p^\alpha = \xi_\alpha p_{\mathrm{P}}^{-\beta} \frac{(Mc)^{2+\beta-\alpha}}{2M} p^\alpha$$

Its quantization leads to pure fractional quantum mechanics

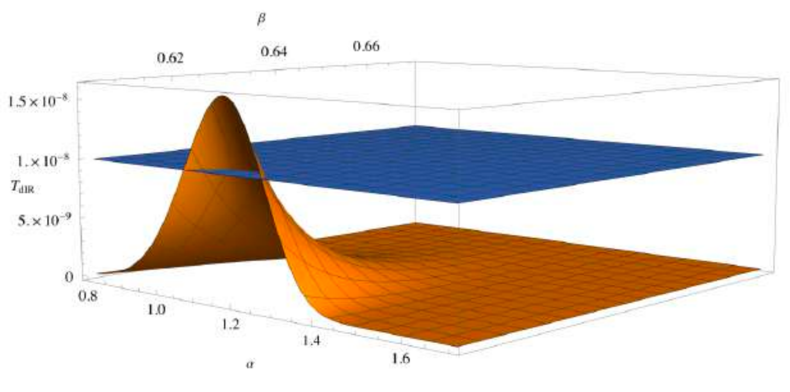
- Considering a fractional Bose-Einstein condensate

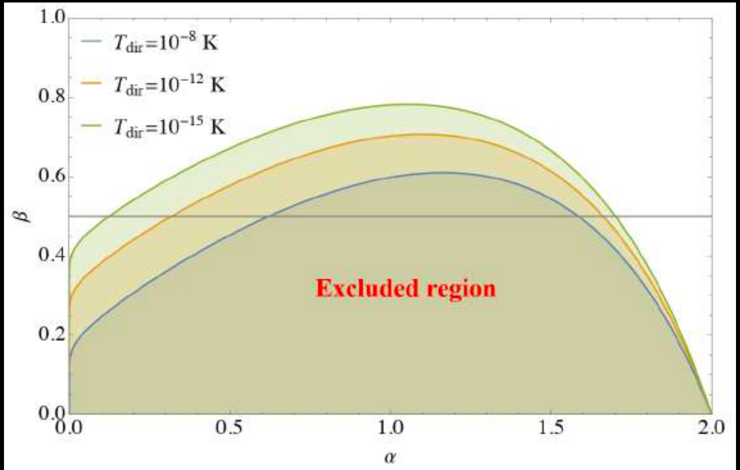
$$E \sim N \frac{k_B T}{2\alpha}.$$

- We translate momentum to temperature conditions

$$T \ll T_{\rm dir} = \frac{\alpha}{M k_B}\left[\xi_\alpha (Mc)^{2+\beta-\alpha} p_P^{-\beta}\right]^{\frac{3-\alpha}{2-\alpha}},$$

If one prepares a system at very low temperatures and observes no deviations, then T_{dir} must be at least on the same order of the achieved temperature.





- Record low temperature with BEC (10^{-12} K) Deppner et al. PRL (2021)

Fractional harmonic dimension

The energy levels of the pure fractional 1 -dimensional harmonic oscillator have been calculated by Laskin (see for instance [Laskin, arXiv:1009.5533])

$$E_{n,1}^{(\alpha)} = \left(\frac{\pi \hbar D_\alpha^{(\beta)^{1/\alpha}} q}{B\left(\frac{1}{2}, \frac{1}{\alpha} + 1\right)} \right)^{\frac{2\alpha}{2+\alpha}} \left(n + \frac{1}{2} \right)^{\frac{2\alpha}{2+\alpha}}$$

Comparing with the energy levels of the usual harmonic oscillator of dimension d_α, the fractional system works as an usual oscillator, but with fractional dimension

$$d_\alpha = \left[\frac{\pi}{2B\left(\frac{1}{2}, \frac{1}{\alpha} + 1\right)} \right]^{2\alpha/(2+\alpha)}$$

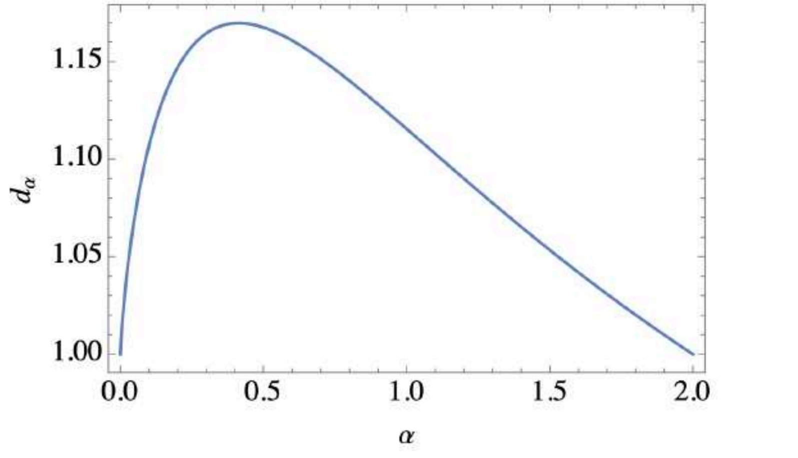
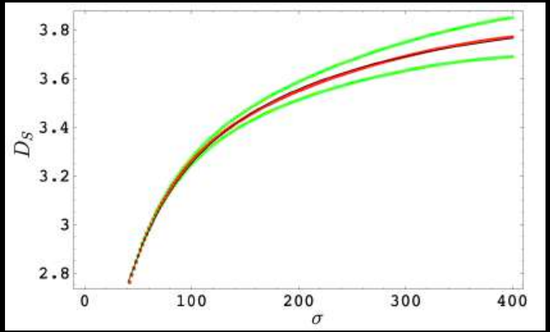


FIG. 2: Harmonic fractional dimension d_α as a function of the Lévy index α.

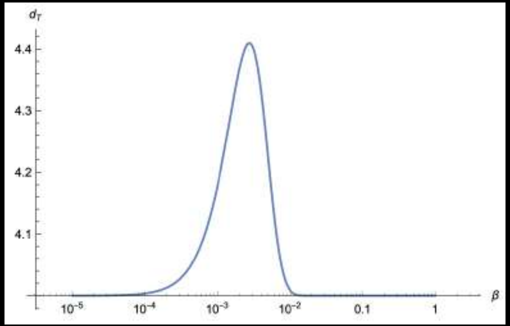
- Fractal dimensions are not a novelty in quantum gravity and they appear in the context of test particles with energies beyond the Planck scale.

- In this case, dimensional reduction appears in Causal Dynamical Triangulations, Horava-Lifschitz gravity, Causal Sets, Asymptotic Safe Gravity, Noncommutative Geometry, LQG, etc. Sotitiou et al. PRD (2011). Horava, PRL (2009). Ambjorn et al. PRL (2009). Modesto CQG (2009). Litim PRL (2009)...

- Its relation with dispersion relations is well known in the UV. Sotitiou et al. PRD (2011).



Ambjorn et al. PRL (2009),



Amelino-Camelia et al. PLB (2016)

- Now, we are suggesting that if there are new terms in the dispersion relation that have momentum powers **smaller than 2**, a similar issue can happen, but in the Infrared regime of ultra low temperatures.

Experimental analogue in optics

[Shilong Liu, Yingwen Zhang, Boris A. Malomed, Ebrahim Karimi, Nature Commun. 14 (2023) 1, 222, arXiv:2208.01128]

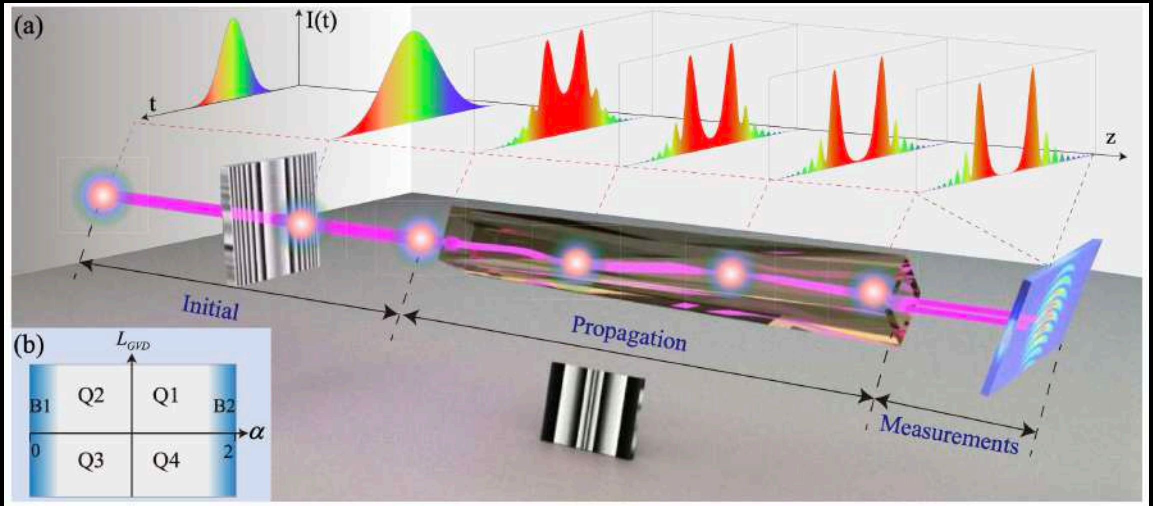
It has been measured amplitudes related to a fractional-like Schrödinger equation

$$i\frac{\partial}{\partial x}\Psi = \left[-\frac{|\beta_2|}{2}\frac{\partial^2}{\partial t^2} + i\frac{\beta_3}{6}\frac{\partial^3}{\partial t^3} + \frac{D}{2}\left(-\frac{\partial^2}{\partial t^2}\right)^{\alpha/2}\right]\Psi,$$

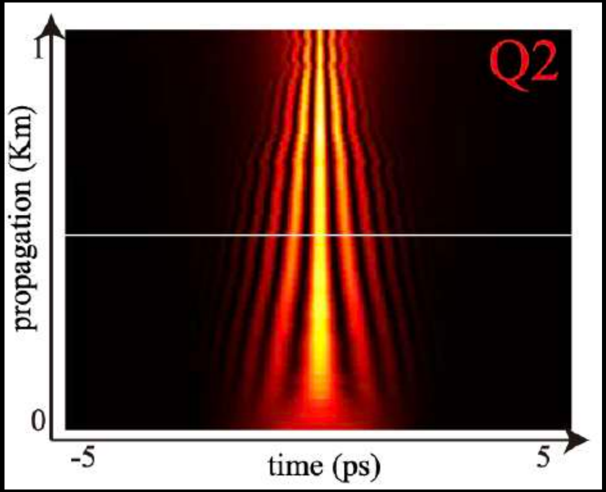
We relate $x = ct$,

$$i\hbar\frac{\partial}{\partial t}\Psi = \left[-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial x^2} - i\frac{\xi_3\hbar^3}{2Mp_{\rm P}}\frac{\partial^3}{\partial x^3} \right.$$
$$\left. + \xi_\alpha p_{\rm P}^{-\beta}\frac{(Mc)^{3-\alpha+\beta}\hbar^\alpha}{2M}\left(-\frac{\partial^2}{\partial x^2}\right)^{\alpha/2}\right]\Psi,$$

Experiments for Fractional Quantum Mechanics could be serving as analog systems to the quantization of in-vacuo dispersion in the IR



The setup used for the realisation of the fractional Schrödinger equation. From [Liu, Zhang, Malomed, Karimi, Nature Commun. 2023]



Detected soliton rain for $\alpha = 0.25$ or $p^{1/8}$. From [Liu, Zhang, Malomed, Karimi, Nature Commun. 2023]

Delocalization in time means delocalization in space from our analogy.

This suggests that a wave packet would exhibit spatial delocalization in a quantum spacetime.

- The relation between optics experiments and quantum gravity is not a novelty [Ford, De Lorenci, Menezes, Svaiter AoP (2013)], but here we also take advantage of models that study fractional Schrödinger-like equations.

- Modified kinematics due to Planck scale effects lead to a Modified Schrödinger Equation
- This can be studied via tabletop experiments with Planck scale sensitivity (oscillators, equivalence principle…)
- If the powers involved are fractional, then one enters into the realm of Fractional Quantum Mechanics
- Fractal structures may appear at very low temperatures if the corrections are exact
- Experiments that test terms with powers of the Laplacian are analog models of *in-vacuo dispersion*

Work in progress

- Two-particle system
- Composition of energies is usually modified in scenarios that preserve the Relativity Principle at the Planck scale leading to modified Hamiltonians for composed systems
- On the other hand, quantum gravity effects usually contribute with Lindblad operators in this scenario
- How do these effects get along?

Research group and some collaborators

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Thank you!

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